## Financial Instruments Toolbox ${ }^{\text {TM }}$

 User's GuideR2014a

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## Financial Instruments Toolbox ${ }^{\mathrm{TM}}$ User's Guide

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## Getting Started

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- "Interest-Rate-Based Derivatives" on page 1-4
- "Equity-Based Derivatives" on page 1-5
- "Expected Users" on page 1-6
- "Portfolio Creation" on page 1-7
- "Pricing a Portfolio Using the Black-Derman-Toy Model" on page 1-11
- "Portfolio Management" on page 1-17


## Financial Instruments Toolbox Product Description

## Design, price, and hedge complex financial instruments

Financial Instruments Toolbox ${ }^{\mathrm{TM}}$ provides functions for pricing, modeling, and analyzing fixed-income, credit, and equity instrument portfolios. You can use the toolbox to perform cash flow modeling and yield curve fitting analysis, compute prices and sensitivities, view price evolutions, and perform hedging analyses using common equity and fixed-income modeling methods. The toolbox lets you create new financial instrument types and fit yield curves to market data using parametric fitting models and bootstrapping.

Financial Instruments Toolbox includes functions for pricing and analyzing fixed-income and equity instruments. Fixed-income modeling tools let you calculate price, yield, spread, and sensitivity values for several types of securities and derivatives, including mortgage-backed securities, treasury bills, bonds, bonds with embedded options, swaps, caps, floors, and floating-rate notes. For equities, the toolbox lets you compute price, implied volatility, and greek values of vanilla equity options and of several exotic equity derivatives such as Bermuda, basket, barrier, digital, and rainbow options.

## Key Features

- Yield curve fitting with bootstrapping and parametric fitting models, and term-structure analysis
- Black Scholes, Black, Garman-Kohlhagen, Roll-Geske-Whaley, Bjerksund-Stensland, Nengjiu Ju, Stulz, and Longstaff-Schwartz models
- Fixed-income and equity derivative calculations for price, yield, discount rate, cash-flow schedule, spread, implied volatility, option adjusted spread (OAS), and greeks
- Tree models: CRR, EQP, LR, ITT, HJM, BDT, BK, and HW
- Interest-rate instruments: bonds, stepped-coupon bonds, futures, vanilla options, Bermuda options, bonds with embedded options, vanilla swaps, forward swaps, amortizing swaps, swaptions, caps, floors, range notes, floating-rate notes, and collared floating-rate notes
- Equity instruments: stocks, vanilla options, Bermuda options, Asian options, lookback options, barrier options, digital options, rainbow options, basket options, compound options, and chooser options
- Credit instruments: mortgage pools, balloon mortgages, and credit default swaps


## Interest-Rate-Based Derivatives

The toolbox provides functionality that supports the creation and management of these interest-rate-based instruments:

- Bonds
- Bond options (puts and calls)
- Bond with embedded options
- Caps
- Fixed-rate notes
- Floating-rate notes
- Floors
- Swaps
- Swaption

Additionally, the toolbox provides functions to create arbitrary cash flow instruments. The toolbox provides pricing and sensitivity routines for these instruments. For more information, see "Pricing Using Interest-Rate Term Structure" on page 2-53 "Pricing Using Interest-Rate Tree Models" on page 2-81, and"Interest-Rate Derivatives Using Closed-Form Solutions" on page 2-104.

## Equity-Based Derivatives

The toolbox also provides functions to create and manage various equity-based derivatives, including the following:

- Asian options
- Barrier options
- Basket options
- Compound options
- Digital options
- Lookback options
- Rainbow options
- Vanilla stock options (put and call options)

The toolbox also provides pricing and sensitivity routines for these instruments. (See "Pricing Equity Derivatives Using Trees" on page 3-76, "Equity Derivatives Using Closed-Form Solutions" on page 3-95, and "Basket Option" on page 3-25.)

## Expected Users

In general, this guide assumes experience working with financial derivatives and some familiarity with the underlying models.

In designing Financial Instruments Toolbox documentation, we assume your title is similar to one of these:

- Analyst, quantitative analyst
- Risk manager
- Portfolio manager
- Fund manager, asset manager
- Financial engineer
- Trader
- Student, professor, or other academic

We also assume your background, education, training, and responsibilities match some aspects of this profile:

- Finance, economics, perhaps accounting
- Engineering, mathematics, physics, other quantitative sciences
- Bachelor's degree minimum; MS or MBA likely; Ph.D. perhaps; CFA
- Comfortable with probability theory, statistics, and algebra
- Understand linear or matrix algebra, calculus, and differential equations
- Previously done traditional programming (C, Fortran, etc.)
- Responsible for instruments or analyses involving large sums of money
- Perhaps new to MATLAB ${ }^{\circledR}$


## Portfolio Creation

In this section...<br>"Introduction" on page 1-7<br>"Interest-Rate-Based Derivatives" on page 1-7<br>"Equity Derivatives" on page 1-8<br>"Adding Instruments to an Existing Portfolio" on page 1-9

## Introduction

The instadd function creates a set of instruments (portfolio) or adds instruments to an existing instrument collection. The TypeString argument specifies the type of the investment instrument. For interest-rate-based derivatives, the types are: Bond, OptBond, CashFlow, Fixed, Float, Cap, Floor, and Swap. For equity derivatives, the types are Asian, Barrier, Compound, Lookback, and OptStock.

The input arguments following TypeString are specific to the type of investment instrument. Thus, the TypeString argument determines how the remainder of the input arguments is interpreted. For example, instadd with the type string Bond creates a portfolio of bond instruments.

```
InstSet = instadd('Bond', CouponRate, Settle, Maturity, Period,
Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate,
StartDate, Face)
```


## Interest-Rate-Based Derivatives

In addition to the bond instrument already described, the toolbox can create portfolios containing the following set of interest-rate-based derivatives:

- Bond option

InstSet $=$ instadd('OptBond', BondIndex, OptSpec, Strike, ExerciseDates, AmericanOpt)

- Arbitrary cash flow instrument

```
InstSet = instadd('CashFlow', CFlowAmounts, CFlowDates, Settle, Basis)
```

- Fixed-rate note instrument

InstSet $=$ instadd('Fixed', CouponRate, Settle, Maturity, FixedReset, Basis, Principal)

- Floating-rate note instrument

InstSet = instadd('Float', Spread, Settle, Maturity, FloatReset, Basis, Principal)

- Cap instrument

```
InstSet = instadd('Cap', Strike, Settle, Maturity, CapReset, Basis, Principal)
```

- Floor instrument

InstSet = instadd('Floor', Strike, Settle, Maturity, FloorReset, Basis, Principal)

- Swap instrument InstSet = instadd('Swap', LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType)
- Swaption instrument

```
InstSet = instadd('Swaption', OptSpec, Strike, ExerciseDates, Spread, ...
Settle, Maturity, AmericanOpt, SwapReset, Basis, Principal)
```

- Bond with embedded option instrument

```
InstSet = instadd('OptEmBond', CouponRate, Settle, Maturity, OptSpec, Strike, ...
ExerciseDates, 'AmericanOpt', AmericanOpt, 'Period', Period,'Basis', Basis, ...
'EndMonthRule', EndMonthRule,'Face',Face,'IssueDate', IssueDate, 'FirstCouponDate', ...
FirstCouponDate, 'LastCouponDate', LastCouponDate,'StartDate', StartDate)
```


## Equity Derivatives

The toolbox can create portfolios containing the following set of equity derivatives:

- Asian instrument

InstSet = instadd('Asian', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt, ... AvgType, AvgPrice, AvgDate)

- Barrier instrument

```
InstSet = instadd('Barrier', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt, ...
BarrierType, Barrier, Rebate)
```

- Compound instrument

```
InstSet = instadd('Compound', UOptSpec, UStrike, USettle, UExerciseDates, UAmericanOpt, ...
COptSpec, CStrike, CSettle, CExerciseDates, CAmericanOpt)
```

- Lookback instrument

InstSet = instadd('Lookback', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt)

- Stock option instrument

```
InstSet = instadd('OptStock', OptSpec, Strike, Settle, Maturity, AmericanOpt)
```


## Adding Instruments to an Existing Portfolio

To use the instadd function to add additional instruments to an existing instrument portfolio, provide the name of an existing portfolio as the first argument to the instadd function.

Consider, for example, a portfolio containing two cap instruments only:

```
Strike = [0.06; 0.07];
Settle = '08-Feb-2000';
Maturity = '15-Jan-2003';
Port_1 = instadd('Cap', Strike, Settle, Maturity);
```

These commands create a portfolio containing two cap instruments with the same settlement and maturity dates, but with different strikes. In general, the input arguments describing an instrument can be either a scalar, or a number of instruments (NumInst)-by-1 vector in which each element corresponds to an instrument. Using a scalar assigns the same value to all instruments passed in the call to instadd.

Use the instdisp command to display the contents of the instrument set:

```
instdisp(Port_1)
Index Type Strike Settle Maturity CapReset Basis Principal
```

| 1 | Cap | 0.06 | $08-$ Feb-2000 | 15-Jan-2003 | 1 | 0 | 100 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | Cap | 0.07 | $08-$ Feb-2000 | 15-Jan-2003 | 1 | 0 | 100 |

Now add a single bond instrument to Port_1. The bond has a $4.0 \%$ coupon and the same settlement and maturity dates as the cap instruments.

```
CouponRate = 0.04;
Port_1 = instadd(Port_1, 'Bond', CouponRate, Settle, Maturity);
```

Use instdisp again to see the resulting instrument set:

```
instdisp(Port_1)
\begin{tabular}{lllllll} 
Index & Type Strike Settle & Maturity & \multicolumn{2}{l}{ CapReset } & Basis Principal \\
1 & Cap & 0.06 & 08-Feb-2000 & 15-Jan-2003 & 1 & 0 \\
100 \\
2 & Cap & 0.07 & \(08-\) Feb-2000 & 15-Jan-2003 & 1 & 0
\end{tabular}
Index Type CouponRate Settle Maturity Period Basis EndMonthRule IssueDate ... Face
```


## Pricing a Portfolio Using the Black-Derman-Toy Model

This example illustrates how the Financial Instruments Toolbox ${ }^{\text {TM }}$ can be used to create a Black-Derman-Toy (BDT) tree and price a portfolio of instruments using the BDT model.

## Create the Interest Rate Term Structure

The structure RateSpec is an interest rate term structure that defines the initial forward-rate specification from which the tree rates are derived. Use the information of annualized zero coupon rates in the table below to populate the RateSpec structure.

```
\begin{tabular}{cll} 
From & \multicolumn{2}{c}{ To } \\
01 Jan 2005 & 01 Jan 2006 & 0.0275 \\
01 Jan 2005 & 01 Jan 2007 & 0.0312 \\
01 Jan 2005 & 01 Jan 2008 & 0.0363 \\
01 Jan 2005 & 01 Jan 2009 & 0.0415 \\
01 Jan 2005 & 01 Jan 2010 & 0.0458
\end{tabular}
StartDates = ['01 Jan 2005'];
EndDates = ['01 Jan 2006';
    '01 Jan 2007';
    '01 Jan 2008';
    '01 Jan 2009';
    '01 Jan 2010'];
ValuationDate = ['01 Jan 2005'];
Rates = [0.0275; 0.0312; 0.0363; 0.0415; 0.0458];
Compounding = 1;
RateSpec = intenvset('Compounding',Compounding,'StartDates', StartDates,...
                            'EndDates', EndDates, 'Rates', Rates,'ValuationDate',
```

RateSpec =
FinObj: 'RateSpec'
Compounding: 1

```
                    Disc: [5x1 double]
                    Rates: [5x1 double]
        EndTimes: [5x1 double]
        StartTimes: [5x1 double]
        EndDates: [5x1 double]
        StartDates: 732313
ValuationDate: 732313
        Basis: 0
    EndMonthRule: 1
```


## Specify the Volatility Model

Create the structure VolSpec that specifies the volatility process with the following data.

```
Volatility = [0.005; 0.0055; 0.006; 0.0065; 0.007];
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Volatility)
```


## BDTVolSpec =

```
            FinObj: 'BDTVolSpec'
    ValuationDate: 732313
        VolDates: [5x1 double]
        VolCurve: [5x1 double]
    VolInterpMethod: 'linear'
```


## Specify the Time Structure of the Tree

The structure TimeSpec specifies the time structure for an interest rate tree. This structure defines the mapping between the observation times at each level of the tree and the corresponding dates.

```
Maturity = EndDates;
BDTTimeSpec = bdttimespec(ValuationDate, Maturity, Compounding)
BDTTimeSpec =
```

```
            FinObj: 'BDTTimeSpec'
ValuationDate: 732313
    Maturity: [5x1 double]
    Compounding: 1
            Basis: 0
EndMonthRule: 1
```


## Create the BDT Tree

Use the previously computed values for RateSpec, VolSpec and TimeSpec to create the BDT tree.

BDTTree = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec)

## BDTTree =

```
    FinObj: 'BDTFwdTree'
VolSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
        tObs: [0 1 1 2 3 4]
        dObs: [732313 732678 733043 733408 733774]
        TFwd: {[5x1 double] [4x1 double] [3x1 double] [2x1 double] [4]}
    CFlowT: {[5x1 double] [4x1 double] [3x1 double] [2x1 double] [5]}
FwdTree: {1x5 cell}
```


## Observe the Interest Rate Tree

Visualize the interest rate evolution along the tree by looking at the output structure BDTTree. BDTTree returns an inverse discount tree, which you can convert into an interest rate tree with the cvtree function.

BDTTreeR = cvtree(BDTTree);
Look at the upper branch and lower branch paths of the tree:

```
%Rate at root node:
RateRoot = treepath(BDTTreeR.RateTree, [0])
%Rates along upper branch:
RatePathUp = treepath(BDTTreeR.RateTree, [[\begin{array}{llll}{1}&{1}&{1}&{1}\end{array}])
%Rates along lower branch:
RatePathDown = treepath(BDTTreeR.RateTree, [2 2 2 2])
RateRoot =
    0.0275
RatePathUp =
    0.0275
    0.0347
    0.0460
    0.0560
    0.0612
RatePathDown =
    0.0275
    0.0351
    0.0472
    0.0585
    0.0653
```

You can also display a graphical representation of the tree to examine interactively the rates on the nodes of the tree until maturity. The function treeviewer displays the structure of the rate tree in the left pane. The tree visualization in the right pane is blank, but by selecting Diagram and clicking on the nodes you can examine the rates along the paths.
treeviewer (BDTTreeR)

## Create an Instrument Portfolio

Create a portfolio consisting of two bond instruments and a option on the 5\% Bond.

```
% Bonds
CouponRate = [0.04;0.05];
Settle = '01 Jan 2005';
Maturity = ['01 Jan 2009';'01 Jan 2010'];
Period = 1;
```

\% Option
OptSpec = \{'call'\};
Strike = 98;
ExerciseDates = ['01 Jan 2010'];
AmericanOpt = 1;
InstSet = instadd('Bond', CouponRate, Settle, Maturity, Period);
InstSet = instadd(InstSet,'OptBond', 2, OptSpec, Strike, ExerciseDates, Ame

Examine the set of instruments contained in the variable InstSet.
instdisp(InstSet)

| Index | Type CouponRate | Settle | Maturity | Period Basis EndMonthRu |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | Bond 0.04 | $01-$ Jan-2005 | $01-\operatorname{Jan}-2009$ | 1 | 0 | 1 |
| 2 | Bond 0.05 | $01-\operatorname{Jan}-2005$ | $01-\operatorname{Jan}-2010$ | 1 | 0 | 1 |


| Index | Type UnderInd | OptSpec | Strike | ExerciseDates | AmericanOpt |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 3 | OptBond 2 | call | 98 | $01-$ Jan-2010 | 1 |

## Price the Portfolio Using a BDT Tree

Calculate the price of each instrument in the instrument set.
Price = bdtprice(BDTTree, InstSet)

Price =
99.6374
102.2460
4.2460

The prices in the output vector Price correspond to the prices at observation time zero ( $\mathrm{tObs}=0$ ), which is defined as the Valuation Date of the interest-rate tree.

In the Price vector, the first element, 99.6374, represents the price of the first instrument ( $4 \%$ Bond); the second element, 102.2460, represents the price of the second instrument ( $5 \%$ Bond), and 4.2460 represents the price of the Option.

## Portfolio Management

```
In this section...
"Instrument Constructors" on page 1-17
"Creating Instruments or Properties" on page 1-18
"Searching or Subsetting a Portfolio" on page 1-20
```


## Instrument Constructors

The toolbox provides constructors for the most common financial instruments. A constructor is a function that builds a structure dedicated to a certain type of object; in this toolbox, an object is a type of market instrument.

The instruments and their constructors in this toolbox are listed below.

| Instrument | Constructor |
| :--- | :--- |
| Asian option | instasian |
| Barrier option | instbarrier |
| Bond | instbond |
| Bond option | instoptbnd |
| Arbitrary cash flow | instcf |
| Compound option | instcompound |
| Fixed-rate note | instfixed |
| Floating-rate note | instfloat |
| Cap | instcap |
| Floor | instfloor |
| Lookback option | instlookback |
| Stock option | instoptstock |
| Swap | instswap |
| Swaption | instswaption |

Each instrument has parameters (fields) that describe the instrument. The toolbox functions let you do the following:

- Create an instrument or portfolio of instruments.
- Enumerate stored instrument types and information fields.
- Enumerate instrument field data.
- Search and select instruments.

The instrument structure consists of various fields according to instrument type. A field is an element of data associated with the instrument. For example, a bond instrument contains the fields CouponRate, Settle, Maturity, and so on. Additionally, each instrument has a field that identifies the investment type (bond, cap, floor, and so on).

In reality, the set of parameters for each instrument is not fixed. You have the ability to add additional parameters. These additional fields are ignored by the toolbox functions. They may be used to attach additional information to each instrument, such as an internal code describing the bond.

Parameters not specified when creating an instrument default to NaN, which, in general, means that the functions using the instrument set (such as intenvprice or hjmprice) will use default values. At the time of pricing, an error occurs if any of the required fields is missing, such as Strike in a cap or CouponRate in a bond.

## Creating Instruments or Properties

Use the instaddfield function to create a kind of instrument or to add new properties to the instruments in an existing instrument collection.

To create a kind of instrument with instaddfield, you must specify three arguments:

- Type
- FieldName
- Data

Type defines the type of the new instrument, for example, Future. FieldName names the fields uniquely associated with the new type of instrument. Data contains the data for the fields of the new instrument.

An optional fourth argument is ClassList. ClassList specifies the data types of the contents of each unique field for the new instrument.

Use either syntax to create a kind of instrument using instaddfield:

```
InstSet = instaddfield('FieldName', FieldList, 'Data', DataList,...
'Type', TypeString)
InstSet = instaddfield('FieldName', FieldList, 'FieldClass',...
ClassList, 'Data' , DataList, 'Type', TypeString)
```

To add new instruments to an existing set, use:

```
InstSetNew = instaddfield(InstSetOld, 'FieldName', FieldList,...
'Data', DataList, 'Type', TypeString)
```

As an example, consider a futures contract with a delivery date of July 15, 2000, and a quoted price of $\$ 104.40$. Since Financial Instruments Toolbox software does not directly support this instrument, you must create it using the function instaddfield. Use these parameters to create instruments:

- Type: Future
- Field names: Delivery and Price
- Data: Delivery is July 15,2000 , and price is $\$ 104.40$.

Enter the data into MATLAB software:

```
Type = 'Future';
FieldName = {'Delivery', 'Price'};
Data = {'Jul-15-2000', 104.4};
```

Finally, create the portfolio with a single instrument:

```
Port = instaddfield('Type', Type, 'FieldName', FieldName,...
'Data', Data);
```

Now use the function instdisp to examine the resulting single-instrument portfolio:

```
instdisp(Port)
Index Type Delivery Price
1 Future Jul-15-2000 104.4
```

Because your portfolio Port has the same structure as those created using the function instadd, you can combine portfolios created using instadd with portfolios created using instaddfield. For example, you can now add two cap instruments to Port with instadd.

```
Strike = [0.06; 0.07];
Settle = '08-Feb-2000';
Maturity = '15-Jan-2003';
Port = instadd(Port, 'Cap', Strike, Settle, Maturity);
```

View the resulting portfolio using instdisp.

```
instdisp(Port)
Index }\begin{array}{lll}{\mathrm{ Type Delivery }}&{\mathrm{ Price }}\\{1}&{\mathrm{ Future 15-Jul-2000 }}&{104.4}
Index Type Strike Settle Maturity CapReset Basis Principal
2 Cap 0.06 08-Feb-2000 15-Jan-2003 1 0 100
3 Cap 0.07 08-Feb-2000 15-Jan-2003 1 00
```


## Searching or Subsetting a Portfolio

Financial Instruments Toolbox software provides functions that enable you to:

- Find specific instruments within a portfolio.
- Create a subset portfolio consisting of instruments selected from a larger portfolio.

The instfind function finds instruments with a specific parameter value; it returns an instrument index (position) in a large instrument set. The instselect function, on the other hand, subsets a large instrument set into
a portfolio of instruments with designated parameter values; it returns an instrument set (portfolio) rather than an index.

## instfind

The general syntax for instfind is

IndexMatch = instfind(InstSet, 'FieldName', FieldList, 'Data',...
DataList, 'Index', IndexSet, 'Type', TypeList)
InstSet is the instrument set to search. Within InstSet instruments categorized by type, each type can have different data fields. The stored data field is a row vector or string for each instrument.

The FieldList, DataList, and TypeList arguments indicate values to search for in the FieldName, Data, and Type data fields of the instrument set. FieldList is a cell array of field name(s) specific to the instruments. DataList is a cell array or matrix of acceptable values for the parameter(s) specified in FieldList. FieldName and Data (consequently, FieldList and DataList) parameters must appear together or not at all.

IndexSet is a vector of integer index(es) designating positions of instruments in the instrument set to check for matches; the default is all indices available in the instrument set. TypeList is a string or cell array of strings restricting instruments to match one of the TypeList types; the default is all types in the instrument set.

IndexMatch is a vector of positions of instruments matching the input criteria. Instruments are returned in IndexMatch if all the FieldName, Data, Index, and Type conditions are met. An instrument meets an individual field condition if the stored FieldName data matches any of the rows listed in the DataList for that FieldName.
instfind Examples. The examples use the provided MAT-file deriv.mat.
The MAT-file contains an instrument set, HJMInstSet, that contains eight instruments of seven types.


Find all instruments with a maturity date of January 01, 2003.

```
Mat2003 = ...
instfind(HJMInstSet,'FieldName','Maturity','Data','01-Jan-2003')
Mat2003 =
1
4
5
8
```

Find all cap and floor instruments with a maturity date of January 01, 2004.

```
CapFloor = instfind(HJMInstSet,...
'FieldName','Maturity','Data','01-Jan-2004', 'Type',...
{'Cap';'Floor'})
CapFloor =
    6
    7
```

Find all instruments where the portfolio is long or short a quantity of 50 .
Pos50 = instfind(HJMInstSet,'FieldName',...
'Quantity', 'Data', \{'50';'-50'\})
Pos50 =
2
3

## instselect

The syntax for instselect is the same syntax as for instfind. instselect returns a full portfolio instead of indexes into the original portfolio. Compare the values returned by both functions by calling them equivalently.

Previously you used instfind to find all instruments in HJMInstSet with a maturity date of January 01, 2003.

Mat2003 = ...
instfind(HJMInstSet,'FieldName','Maturity','Data','01-Jan-2003')
Mat2003 =
1
4
5
8

Now use the same instrument set as a starting point, but execute the instselect function instead, to produce a new instrument set matching the identical search criteria.

```
Select2003 = ...
instselect(HJMInstSet,'FieldName','Maturity','Data',...
'01-Jan-2003')
instdisp(Select2003)
```


instselect Examples. These examples use the portfolio ExampleInst provided with the MAT-file InstSetExamples.mat.
load InstSetExamples.mat
instdisp(ExampleInst)

| Index | Type | Strike | Price | Opt | Contracts |
| :--- | :--- | :--- | :--- | :--- | :---: |
| 1 | Option 95 | 12.2 | Call | 0 |  |
| 2 | Option 100 | 9.2 | Call | 0 |  |
| 3 | Option 105 | 6.8 | Call | 1000 |  |


| Index | Type | Delivery | F | Contracts |
| :--- | :--- | :--- | :--- | :--- |
| 4 | Futures | 01 -Jul-1999 | 104.4 | -1000 |


| Index | Type | Strike | Price | Opt | Contracts |
| :--- | :--- | :---: | :---: | :---: | :---: |
| 5 | Option | 105 | 7.4 | Put | -1000 |
| 6 | Option | 95 | 2.9 | Put | 0 |

```
Index Type Price Maturity Contracts
```

The instrument set contains 3 instrument types: Option, Futures, and TBill. Use instselect to make a new instrument set containing only options struck at 95 . In other words, select all instruments containing the field Strike and with the data value for that field equal to 95 .

```
InstSet = instselect(ExampleInst,'FieldName','Strike','Data',95);
instdisp(InstSet)
\begin{tabular}{llcccc} 
Index & Type & Strike & Price & Opt & Contracts \\
1 & Option & 95 & 12.2 & Call & 0 \\
2 & Option & 95 & 2.9 & Put & 0
\end{tabular}
```

You can use all the various forms of instselect and instfind to locate specific instruments within this instrument set.

## Interest-Rate Derivatives

- "Supported Interest-Rate Instruments" on page 2-2
- "Calibrate the SABR Model" on page 2-20
- "Price a Swaption Using the SABR Model" on page 2-26
- "Overview of Interest-Rate Tree Models" on page 2-34
- "Understanding the Interest-Rate Term Structure" on page 2-38
- "Pricing Using Interest-Rate Term Structure" on page 2-53
- "Understanding Interest-Rate Tree Models" on page 2-60
- "Pricing Using Interest-Rate Tree Models" on page 2-81
- "Interest-Rate Derivatives Using Closed-Form Solutions" on page 2-104
- "Price Swaptions With Interest-Rate Models Using Simulation" on page 2-105
- "Pricing Bermudan Swaptions with Monte Carlo Simulation" on page 2-127
- "Graphical Representation of Trees" on page 2-139


## Supported Interest-Rate Instruments

In this section...
"Bond" on page 2-3
"Stepped Coupon Bonds" on page 2-4
"Sinking Fund Bonds" on page 2-4
"Bonds with an Amortization Schedule" on page 2-5
"Bond Options" on page 2-5
"Bond with Embedded Options" on page 2-6
"Stepped Coupon Bonds with Calls and Puts" on page 2-7
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"Floating-Rate Note" on page 2-9
"Floating-Rate Note with an Amortization Schedule" on page 2-10
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"Floating-Rate Note with Embedded Options" on page 2-12
"Cap" on page 2-13
"Floor" on page 2-13
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"Swap with an Amortization Schedule" on page 2-15
"Forward Swap" on page 2-16
"Swaption" on page 2-16
"Bond Futures" on page 2-17

## Bond

A bond is a long-term debt security with a preset interest-rate and maturity. At maturity you must pay the principal and interest.

The price or value of a bond is determined by discounting the expected cash flows of the bond to the present, using the appropriate discount rate. The following equation represents the relationship of the expected cash flows and discount rate:
$B_{0}=\frac{C}{2}\left[\frac{1-\left(1+\frac{r}{2}\right)^{-2 t}}{\frac{r}{2}}\right]+\frac{F}{\left(1+\frac{r}{2}\right)^{2 t}}$
where:
$B_{0}$ is the bond value.
$C$ is the annual coupon payment.
$F$ is the face value of the bond.
$r$ is the required return on the bond.
$t$ is the number of years remaining until maturity.
Financial Instruments Toolbox supports the following for pricing and specifying a bond.

| Function | Purpose |
| :--- | :--- |
| bondbybdt | Price a bond using a BDT interest-rate tree. |
| bondbyhw | Price a bond using an HW interest-rate tree. |
| bondbybk | Price a bond using a BK interest-rate tree. |
| bondbyhjm | Price a bond using an HJM interest-rate tree. |
| bondbyzero | Price a bond using a set of zero curves. |
| instbond | Construct a bond instrument. |

## Stepped Coupon Bonds

A step-up and step-down bond is a debt security with a predetermined coupon structure over time. With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond. For more information on options features (call and puts), see "Stepped Coupon Bonds with Calls and Puts" on page 2-7. The following functions have a modified CouponRate argument to support a new variable coupon schedule allowing pricing of stepped coupon bonds.

| Function | Purpose |
| :--- | :--- |
| bondbyzero | Price bonds using a term structure model. |
| bondbybdt | Price bonds using a BDT tree model. |
| bondbyhjm | Price bonds using an HJM tree model. |
| bondbyhw | Price bonds using an HW tree model. |
| bondbybk | Price bonds using a BK tree model. |
| instbond | Construct a bond instrument. |
| instoptbnd | Construct a bond option instrument. |
| instdisp | Display instruments stored in a variable. |

## Sinking Fund Bonds

A sinking fund bond is a coupon bond with a sinking fund provision. This provision obligates the issuer to amortize portions of the principal before maturity, affecting bond prices since the time of the principal repayment changes. This means that investors receive the coupon and a portion of the principal paid back over time. These types of bonds reduce credit risk, since it lowers the probability of investors not receiving their principal payment at maturity. For more information on options support for sinking fund bonds, see "Sinking Fund Bonds with an Embedded Option" on page 2-8. The following functions have a modified Face argument to support a variable face schedule for pricing bonds with a sinking provisions.

| Function | Purpose |
| :--- | :--- |
| bondbyzero | Price bonds using a term structure model. |
| bondbybdt | Price bonds using a BDT tree model. |


| Function | Purpose |
| :--- | :--- |
| bondbyhjm | Price bonds using an HJM tree model. |
| bondbyhw | Price bonds using an HW tree model. |
| bondbybk | Price bonds using a BK tree model. |
| instbond | Construct a bond instrument. |
| instoptbnd | Construct a bond option instrument. |
| instdisp | Display instruments stored in a variable. |

## Bonds with an Amortization Schedule

A bond with an amortization schedule repays part of the principal (face value) along with the coupon payments. An amortizing bond is a special case of a sinking fund bond when there is no market purchase option and no call provision.. The following functions have a modified Face argument to support an amortization schedule.

| Function | Purpose |
| :--- | :--- |
| bondbyzero | Price bonds using a term structure model. |
| bondbybdt | Price bonds using a BDT tree model. |
| bondbyhjm | Price bonds using an HJM tree model. |
| bondbyhw | Price bonds using an HW tree model. |
| bondbybk | Price bonds using a BK tree model. |

## Bond Options

Financial Instruments Toolbox software supports three types of put and call options on bonds:

- American option: An option that you exercise any time until its expiration date.
- European option: An option that you exercise only on its expiration date.
- Bermuda option: A Bermuda option resembles a hybrid of American and European options. You can exercise it on predetermined dates only, usually monthly.

Financial Instruments Toolbox supports the following for pricing and specifying a bond option.

| Function | Purpose |
| :--- | :--- |
| optbndbybdt | Price a bond option price using a BDT <br> interest-rate tree. |
| optbndbyhw | Price a bond option price using an HW <br> interest-rate tree. |
| optbndbybk | Price a bond option price using a BK <br> interest-rate tree. |
| optbndbyhjm | Price a bond option price using an HJM <br> interest-rate tree. |
| instoptbnd | Construct a bond option instrument. |

## Bond with Embedded Options

A bond with embedded options allows the issuer to buy back or redeem the bond at a predetermined price at specified future dates. Financial Instruments Toolbox software supports American, European, and Bermuda callable and puttable bonds.

The pricing for a bond with embedded options is as follows:

- For a callable bond: PriceCallableBond $=$ BondPrice - BondCallOption
- For a puttable bond: PricePuttableBond = PriceBond + PricePutOption

Financial Instruments Toolbox supports the following for pricing and specifying a bond with embedded options.

| Function | Purpose |
| :--- | :--- |
| optembndbybdt | Price a bond with embedded options using a <br> BDT interest-rate tree. |
| optembndbyhw | Price a bond with embedded options using an <br> HW interest rate tree. |
| optembndbybk | Price a bond with embedded options using a <br> BK interest-rate tree. |
| optembndbyhjm | Price a bond with embedded options using an <br> HJM interest-rate tree. |
| instoptembnd | Construct a bond-with-embedded-options <br> instrument. |

## Stepped Coupon Bonds with Calls and Puts

A step-up and step-down bond is a debt security with a predetermined coupon structure over time. For more information on stepped coupon bonds, see "Stepped Coupon Bonds" on page 2-4. Stepped coupon bonds can have options features (call and puts). The following functions have a modified CouponRate argument to support a new variable coupon schedule allowing pricing stepped coupon bonds with callable and puttable features:

| Function | Purpose |
| :--- | :--- |
| optembndbybdt | Price bonds with embedded options using a BDT <br> model tree. |
| optembndbyhjm | Price bonds with embedded options using an HJM <br> model tree. |
| optembndbybk | Price bonds with embedded options using a BK model <br> tree. |
| optembndbyhw | Price bonds with embedded options using an HW <br> model tree. |
| instbond | Construct a bond instrument. |
| instoptbnd | Construct a bond option instrument. |


| Function | Purpose |
| :--- | :--- |
| instoptembnd | Construct a bond with an embedded option <br> instrument. |
| instdisp | Display instruments stored in a variable. |

## Sinking Fund Bonds with an Embedded Option

A sinking fund bond is a coupon bond with a sinking fund provision. For more information on sinking fund bonds, see "Sinking Fund Bonds" on page 2-4. The sinking fund bond can have a sinking fund option provision allowing the issuer to retire the sinking fund obligation either by purchasing the bonds to be redeemed from the market or by calling the bond via a sinking fund call, whichever is cheaper.

If interest rates are high, then the issuer buys back the required amount of bonds from the market since bonds will be cheap. But if interest rates are low (bond prices are high), then most likely the issuer buys the bonds at the call price. Unlike a call feature, however, if a bond has a sinking fund option provision, it is an obligation, not an option, for the issuer to buy back the increments of the issue as stated. Because of this, a sinking fund bond trades at a lower price than a nonsinking fund bond. The following functions have a modified Face argument to support a variable face schedule for pricing bonds with a sinking fund option provision.

| Function | Purpose |
| :--- | :--- |
| optembndbybdt | Price bonds with embedded options using a BDT <br> model tree. |
| optembndbyhjm | Price bonds with embedded options using an HJM <br> model tree. |
| optembndbybk | Price bonds with embedded options using a BK model <br> tree. |
| optembndbyhw | Price bonds with embedded options using an HW <br> model tree. |
| instbond | Construct a bond instrument. |


| Function | Purpose |
| :--- | :--- |
| instoptbnd | Construct a bond option instrument. |
| instdisp | Display instruments stored in a variable. |

## Fixed-Rate Note

A fixed-rate note is a long-term debt security with a preset interest rate and maturity, by which the interest must be paid. The principal may or may not be paid at maturity. In Financial Instruments Toolbox software, the principal is always paid at maturity.

| Function | Purpose |
| :--- | :--- |
| fixedbybdt | Price a fixed-rate note using a BDT <br> interest-rate tree. |
| fixedbyhw | Price a fixed-rate note using an HW <br> interest-rate tree. |
| fixedbybk | Price a fixed-rate note using a BK interest-rate <br> tree. |
| fixedbyhjm | Price a fixed-rate note using an HJM <br> interest-rate tree. |
| fixedbyzero | Price a fixed-rate note using a set of zero <br> curves. |
| instfixed | Construct a fixed-rate instrument. |

## Floating-Rate Note

A floating-rate note is a security like a bond, but the interest rate of the note is reset periodically, relative to a reference index rate, to reflect fluctuations in market interest rates.

| Function | Purpose |
| :--- | :--- |
| floatbybdt | Price a floating-rate note using a BDT <br> interest-rate tree. |
| floatbyhw | Price a floating-rate note using an HW <br> interest-rate tree. |
| floatbybk | Price a floating-rate note using a BK <br> interest-rate tree. |
| floatbyhjm | Price a floating-rate note using an HJM <br> interest-rate tree. |
| floatbyzero | Price a floating-rate note using a set of zero <br> curves. |
| instfloat | Construct a floating-rate note instrument. |

## Floating-Rate Note with an Amortization Schedule

A floating-rate note with an amortization schedule repays part of the principal (face value) along with the coupon payments. The following functions have a Principal argument to support an amortization schedule.

| Function | Purpose |
| :--- | :--- |
| floatbyzero | Price floating-rate note from set of zero curves. |
| floatbybdt | Price floating-rate note from Black-Derman-Toy <br> interest-rate tree. |
| floatbyhjm | Price floating-rate note from Heath-Jarrow-Morton <br> interest-rate tree. |
| floatbyhw | Price floating-rate note from Hull-White interest-rate <br> tree. |
| floatbybk | Price floating-rate note from Black-Karasinski <br> interest-rate tree. |

## Floating-Rate Note with Caps, Collars, and Floors

A floating-rate note with caps, collars, and floors This type of instrument can carry restrictions on the maximum (cap) or minimum (floor) coupon rate are paid. A cap is an unattractive feature for an investor, since they constrain the coupon rates from increasing. A floor, on the other hand, is an attractive feature, since it allows investors to get a minimum coupon rate when market rates decrease below a certain level. Also, a floating-rate note can have a collar which is a combination of a cap and a floor together. The following functions have a CapRate and FloorRate argument to support a capped, collared, or floored floating-rate note.

| Function | Purpose |
| :--- | :--- |
| floatbybdt | Price a capped floating-rate note from a <br> Black-Derman-Toy interest-rate tree. |
| floatbyhjm | Price a capped floating-rate note from a <br> Heath-Jarrow-Morton interest-rate tree. |
| floatbyhw | Price a capped floating-rate note from a Hull-White <br> interest-rate tree. |
| floatbybk | Price a capped floating-rate note from a <br> Black-Karasinskiinterest-rate tree. |
| instfloat | Create a capped floating-rate note instrument. |
| instadd | Add a capped floating-rate note instrument to a <br> portfolio. |

## Floating-Rate Note with Options

Financial Instruments Toolbox supports three types of put and call options on floating rate-notes:

- American option - An option that you exercise any time until its expiration date.
- European option - An option that you exercise only on its expiration date.
- Bermuda option - A Bermuda option resembles a hybrid of American and European options; you can only exercise it on predetermined dates, usually monthly.

Financial Instruments Toolbox supports the following for pricing and specifying a floating rate-note option:

| Function | Purpose |
| :--- | :--- |
| optfloatbybdt | Price an option for floating-rate note using a <br> Black-Derman-Toy interest-rate tree. |
| optfloatbyhjm | Price an option for floating-rate note using a <br> Heath-Jarrow-Morton interest-rate tree. |
| optfloatbyhw | Price an option for floating-rate note using a <br> Hull-White interest-rate tree. |
| optfloatbybk | Price an option for floating-rate note using a <br> Black-Karasinski interest-rate tree. |
| instoptfloat | Define the option instrument for floating-rate note. |

## Floating-Rate Note with Embedded Options

A floating-rate note with an embedded option enables floating-rate notes to have early redemption features. A FRN with an embedded option gives investors or issuers the option to retire the outstanding principal prior to maturity. An embedded call option gives the right to retire the note prior to the maturity date (callable floater), and an embedded put option gives the right to sell the note back at a specific price (puttable floater).

Financial Instruments Toolbox supports the following for pricing and specifying a floating rate-note with an embedded option:

| Function | Purpose |
| :--- | :--- |
| optemfloatbybdt | Price an embedded option for floating-rate note using <br> a Black-Derman-Toy interest-rate tree. |
| optemfloatbybk | Price an embedded option for floating-rate note using <br> a Black-Karasinski interest-rate tree. |
| optemfloatbyhjm | Price an embedded option for floating-rate note using <br> a Heath-Jarrow-Morton interest-rate tree. |


| Function | Purpose |
| :--- | :--- |
| optemfloatbyhw | Price an embedded option for floating-rate note using <br> a Hull-White interest-rate tree. |
| instoptemfloat | Define the floating-rate note with embedded option <br> instrument. |

## Cap

A cap is a contract that includes a guarantee that sets the maximum interest rate to be paid by the holder, based on an otherwise floating interest rate. The payoff for a cap is:
$\max ($ CurrentRate - CapRate, 0 )

| Function | Purpose |
| :--- | :--- |
| capbybdt | Price a cap instrument using a BDT <br> interest-rate tree. |
| capbyhw | Price a cap instrument using an HW <br> interest-rate tree. |
| capbybk | Price a cap instrument using a BK interest-rate <br> tree. |
| capbyhjm | Price a cap instrument using an HJM <br> interest-rate tree. |
| capbyblk | Price a cap instrument using the Black option <br> pricing model. |
| instcap | Construct a cap instrument. |

## Floor

A floor is a contract that includes a guarantee setting the minimum interest rate to be received by the holder, based on an otherwise floating interest rate. The payoff for a floor is:
$\max ($ FloorRate - CurrentRate,0)

| Function | Purpose |
| :--- | :--- |
| floorbybdt | Price a floor instrument using a BDT <br> interest-rate tree. |
| floorbyhw | Price a floor instrument using an HW <br> interest-rate tree. |
| floorbybk | Price a floor instrument using a BK <br> interest-rate tree. |
| floorbyhjm | Price a floor instrument using an HJM <br> interest-rate tree. |
| instfloor | Construct a floor instrument. |

## Range Note

A range note is a structured (market-linked) security whose coupon-rate is equal to the reference rate as long as the reference rate is within a certain range. If the reference rate is outside of the range, the coupon-rate is 0 for that period. This type of instrument entitles the holder to cash flows that depend on the level of some reference interest-rate that are floored to be positive and gives the holder of the note direct exposure to the reference rate. This type of instrument is useful for cases where you believe that interest rates will stay within a certain range. In return for the drawback that no interest will be paid for the time the range is left, a range note offers higher coupon rates than comparable standard products, like vanilla floating notes.

| Function | Purpose |
| :--- | :--- |
| instrangefloat | Create a range note instrument. |
| rangefloatbybdt | Price range floating note using a BDT tree. |
| rangefloatbybk | Price range floating note using a BK tree. |
| rangefloatbyhjm | Price range floating note using an HJM tree. |
| rangefloatbyhw | Price range floating note using an HW tree. |

## Swap

A swap is contract between two parties obligating the parties to exchange future cash flows. This toolbox version handles only the vanilla swap, which is composed of a floating-rate leg and a fixed-rate leg.

| Function | Purpose |
| :--- | :--- |
| swapbybdt | Price a swap instrument using a BDT <br> interest-rate tree. |
| swapbyhw | Price a swap instrument using an HW <br> interest-rate tree. |
| swapbybk | Price a swap instrument using a BK <br> interest-rate tree. |
| swapbyhjm | Price a swap instrument using an HJM <br> interest-rate tree. |
| swapbyzero | Price a swap instrument using a set of zero <br> curves. |
| instswap | Construct a swap instrument. |

## Swap with an Amortization Schedule

A swap with an amortization schedule repays part of the principal (face value) along with the coupon payments. A swap with an amortization schedule is used to manage interest rate risk and serve as a cash flow management tool. For this particular type of swap, the notional amount decreases over time. This means that interest payments will decrease not only on the floating leg but also on the fixed leg. The following swap functions have a Principal argument to support an amortization schedule.

| Function | Purpose |
| :--- | :--- |
| swapbyzero | Price swap instrument from set of zero curves. |
| swapbybdt | Price swap instrument from Black-Derman-Toy <br> interest-rate tree. |
| swapbyhjm | Price swap instrument from Heath-Jarrow-Morton <br> interest-rate tree. |


| Function | Purpose |
| :--- | :--- |
| swapbyhw | Price swap instrument from Hull-White interest-rate <br> tree. |
| swapbybk | Price swap instrument from Black-Karasinski <br> interest-rate tree. |
| instswap | Construct swap instrument. |

## Forward Swap

In a forward interest-rate swap, a fixed interest rate loan is exchanged for a floating interest rate loan at a future specified date The following functions have a StartDate argument to support the future date for the forward swap.

| Function | Purpose |
| :--- | :--- |
| swapbyzero | Price a forward swap from a zero curve. |
| swapbybdt | Price a forward swap from a Black-Derman-Toy <br> interest-rate tree. |
| swapbyhjm | Price a forward swap from a Heath-Jarrow-Morton <br> interest-rate tree. |
| swapbyhw | Price a forward swap from a Hull-White interest-rate <br> tree. |
| swapbybk | Price a forward swap from a <br> Black-Karasinskiinterest-rate tree. |
| instswap | Create a forward swap instrument. |
| instadd | Add a capped floating-rate note instrument to a <br> portfolio. |

## Swaption

A swaption is an option to enter into an interest-rate swap contract. A call swaption allows the option buyer to enter into an interest-rate swap where the buyer of the option pays the fixed-rate and receives the floating-rate. A put
swaption allows the option buyer to enter into an interest-rate swap where the buyer of the option receives the fixed-rate and pays the floating-rate.

| Function | Purpose |
| :--- | :--- |
| swaptionbybdt | Price a swaption instrument using a BDT <br> interest-rate tree. |
| swaptionbyhw | Price a swaption instrument using an HW <br> interest-rate tree. |
| swaptionbybk | Price a swaption instrument using a BK <br> interest-rate tree. |
| swaptionbyhjm | Price a swaption instrument using an HJM <br> interest-rate tree. |
| swaptionbyblk | Price swaptions using the Black model with <br> a forward on a swap. |
| instswaption | Construct a swaption instrument. |

Use swaptionbyblk to price a swaption using the Black model. The Black model is standard model used in the swaption market when pricing European swaptions. This type of model is widely used by when speed is important to quickly obtain a price at settlement date, even if the price is less accurate than other swaption pricing models based on interest-rate tree models.

## Bond Futures

Bond futures are futures contracts where the commodity for delivery is a government bond. There are established global markets for government bond futures. Bond futures provide a liquid alternative for managing interest-rate risk.

In the U.S. market, the Chicago Mercantile Exchange (CME) offers futures on Treasury bonds and notes with maturities of $2,5,10$, and 30 years. Typically, the following bond future contracts from the CME have maturities of $3,6,9$, and 12 months:

- 30-year U.S. Treasury bond
- 10-year U.S. Treasury bond
- 5-year U.S. Treasury bond
- 2-year U.S. Treasury bond

The short position in a Treasury bond or note future contract must deliver to the long position in one of many possible existing Treasury bonds. For example, in a 30 -year Treasury bond future, the short position must deliver a Treasury bond with at least 15 years to maturity. Because these bonds have different values, the bond future contract is standardized by computing a conversion factor. The conversion factor normalizes the price of a bond to a theoretical bond with a coupon of $6 \%$. The price of a bond future contract is represented as:

InvoicePrice $=$ FutPrice $\times C F+A I$
where:
FutPrice is the price of the bond future.
$C F$ is the conversion factor for a bond to deliver in a futures contract.
$A I$ is the accrued interest.
You can reference these conversion factors at U.S. Treasury Bond Futures Contract. The short position in a futures contract has the option of which bond to deliver and, in the U.S. bond market, when in the delivery month to deliver the bond. The short position typically chooses to deliver the bond known as the Cheapest to Deliver (CTD). The CTD bond most often delivers on the last delivery day of the month.

Financial Instruments Toolbox software supports the following bond futures:

- U.S. Treasury bonds and notes
- German Bobl, Bund, Buxl, and Schatz
- UK gilts
- Japanese government bonds (JGBs)

The functions supporting all bond futures are:

| Function | Purpose |
| :--- | :--- |
| convfactor | Calculates bond conversion factors for U.S. Treasury <br> bonds, German Bobl, Bund, Buxl, and Schatz, U.K. <br> gilts, and JGBs. |
| bndfutprice | Prices bond future given repo rates. |
| bndfutimprepo | Calculates implied repo rates for a bond future given <br> price. |

The functions supporting U.S. Treasury bond futures are:

| Function | Purpose |
| :--- | :--- |
| tfutbyprice | Calculates future prices of Treasury bonds given the <br> spot price. |
| tfutbyyield | Calculates future prices of Treasury bonds given <br> current yield. |
| tfutimprepo | Calculates implied repo rates for the Treasury bond <br> future given price. |
| tfutpricebyrepo | Calculates Treasury bond futures price given the <br> implied repo rates. |
| tfutyieldbyrepo | Calculates Treasury bond futures yield given the <br> implied repo rates. |

For more information on bond futures, see "Supported Bond Futures" on page 7-12.

## Calibrate the SABR Model

This example shows how to use two different methods to calibrate the SABR stochastic volatility model from market implied Black volatilities. Both approaches use blackvolbysabr.

```
In this section...
"Load Market Implied Black Volatility Data" on page 2-20
"Method 1: Calibrate Alpha, Rho and Nu Directly" on page 2-21
"Method 2: Calibrate Rho and Nu by Implying Alpha From At-The-Money
Volatility" on page 2-22
"Use the Calibrated Models" on page 2-24
"References" on page 2-25
```


## Load Market Implied Black Volatility Data

This example shows how to set up hypothetical market implied Black volatilities for European swaptions over a range of strikes before calibration. The swaptions expire in 3 years from the Settle date and have 10 year swaps as the underlying instrument. Note that the rates are expressed in decimals. (Changing the units will affect the numerical value and interpretation of the Alpha input parameter to the function blackvolbysabr.)

Load the market implied Black volatility data for swaptions expiring in 3 years.

```
Settle = '12-Jun-2013';
ExerciseDate = '12-Jun-2016';
MarketStrikes = [2.0 2.5 3.0 3.5 4.0 4.5 5.0]'/100;
MarketVolatilities = [45.6 41.6 37.9 36.6 37.8 39.2 40.0]'/100;
```

At the time of Settle, define the underlying forward rate and the at-the-money volatility.

CurrentForwardValue = MarketStrikes(4)
ATMVolatility = MarketVolatilities(4)
0.0350

ATMVolatility =
0.3660

## Method 1: Calibrate Alpha, Rho and Nu Directly

This example shows how to calibrate the Alpha, Rho, and Nu input parameters directly. The value of Beta is predetermined either by fitting historical market volatility data or by choosing a value deemed appropriate for that market [1].

Define the predetermined Beta.
Beta1 = 0.5;

After fixing the value of $\beta$ (Beta), the parameters $\alpha$ (Aplha), $\rho$ (Rho) and $v(\mathrm{Nu})$ are all fitted directly. The Optimization Toolbox ${ }^{\mathrm{TM}}$ function lsqnonlin generates the parameter values that minimize the squared error between the market volatilities and the volatilities computed by blackvolbysabr.

```
% Calibrate Alpha, Rho, and Nu
objFun = @(X) MarketVolatilities - ...
    blackvolbysabr(X(1), Beta1, X(2), X(3), Settle, ...
    ExerciseDate, CurrentForwardValue, MarketStrikes);
X = lsqnonlin(objFun, [0.5 0 0.5], [0 -1 0], [Inf 1 Inf]);
Alpha1 = X(1);
Rho1 = X(2);
Nu1 = X(3);
Local minimum possible.
lsqnonlin stopped because the final change in the sum of squares relative
its initial value is less than the default value of the function tolerance.
```


## Method 2: Calibrate Rho and Nu by Implying Alpha From At-The-Money Volatility

This example shows how to use an alternative calibration method where the value of $\beta$ (Beta) is again predetermined as in Method 1.

Define the predetermined Beta.
Beta2 $=0.5$;
However, after fixing the value of $\beta$ (Beta), the parameters $\rho$ (Rho) and $v(\mathrm{Nu})$ are fitted directly while $\alpha$ (Alpha) is implied from the market at-the-money volatility. Models calibrated using this method produce at-the-money volatilities that are equal to market quotes. This approach is widely used in swaptions, where at-the-money volatilities are quoted most frequently and are important to match. In order to imply $\alpha$ (Alpha) from market at-the-money volatility ( $\sigma_{A T M}$ ), the following cubic polynomial is solved for $\alpha$ (Alpha), and the smallest positive real root is selected [2].

$$
\frac{(1-\beta)^{2} T}{24 F^{(2-2 \beta)}} \alpha^{3}+\frac{\rho \beta v T}{4 F^{(1-\beta)}} \alpha^{2}+\left(1+\frac{2-3 \rho^{2}}{24} v^{2} T\right) \alpha-\sigma_{A T M} F^{(1-\beta)}=0
$$

where:

- $F$ is the current forward value.
- $T$ is the year fraction to maturity.

To accomplish this, define an anonymous function as:

```
% Year fraction from Settle to option maturity
T = yearfrac(Settle, ExerciseDate, 1);
% This function solves the SABR at-the-money volatility equation as a
% polynomial of Alpha
alpharoots = @(Rho,Nu) roots([...
    (1 - Beta2)^2*T/24/CurrentForwardValue^(2 - 2*Beta2) ...
    Rho*Beta2*Nu*T/4/CurrentForwardValue^(1 - Beta2) ...
    (1 + (2 - 3*Rho^2)*Nu^2*T/24) ...
    -ATMVolatility*CurrentForwardValue^(1 - Beta2)]);
```

```
% This function converts at-the-money volatility into Alpha by picking the
% smallest positive real root
atmVol2SabrAlpha = @(Rho,Nu) min(real(arrayfun(@(x) ...
    x*(x>0) + realmax*(x<0 || abs(imag(x))>1e-6), alpharoots(Rho,Nu))));
```

The function atmVol2SabrAlpha converts at-the-money volatility into $\alpha$ (Alpha) for a given set of $\rho$ (Rho) and $v$ (Nu). This function is then used in the objective function to fit parameters $\rho$ (Rho) and $v(\mathrm{Nu})$.

```
% Calibrate Rho and Nu (while converting at-the-money volatility into Alpha
% using atmVol2SabrAlpha)
objFun = @(X) MarketVolatilities - ...
    blackvolbysabr(atmVol2SabrAlpha(X(1), X(2)), ...
    Beta2, X(1), X(2), Settle, ExerciseDate, CurrentForwardValue, ...
    MarketStrikes);
X = lsqnonlin(objFun, [0 0.5], [-1 0], [1 Inf]);
Rho2 = X(1);
Nu2 = X(2);
Local minimum found.
Optimization completed because the size of the gradient is less than
the default value of the function tolerance.
```

The calibrated parameter $\alpha$ (Alpha) is computed using the calibrated parameters $\rho$ (Rho) and $v(\mathrm{Nu})$.
\% Obtain final Alpha from at-the-money volatility using calibrated paramete Alpha2 = atmVol2SabrAlpha(Rho2, Nu2);
\% Display calibrated parameters
C = \{Alpha1 Beta1 Rho1 Nu1;Alpha2 Beta2 Rho2 Nu2\};
CalibratedPrameters = cell2table(C,...
'VariableNames', \{'Alpha' 'Beta' 'Rho' 'Nu'\},...
'RowNames', \{'Method 1';'Method 2'\})
CalibratedPrameters =

| Alpha |  | Beta | Rho |  |
| :--- | :--- | :--- | :--- | :--- | | Nu |
| :---: |
|  |

## Use the Calibrated Models

This example shows how to use the calibrated models to compute new volatilities at any strike value.

Compute volatilities for models calibrated using Method 1 and Method 2 and plot the results.

PlottingStrikes $=(1.75: 0.1: 5.50) ' / 100 ;$
\% Compute volatilities for model calibrated by Method 1
ComputedVols1 = blackvolbysabr(Alpha1, Beta1, Rho1, Nu1, Settle, ...
ExerciseDate, CurrentForwardValue, PlottingStrikes);
\% Compute volatilities for model calibrated by Method 2
ComputedVols2 = blackvolbysabr(Alpha2, Beta2, Rho2, Nu2, Settle, ... ExerciseDate, CurrentForwardValue, PlottingStrikes);
figure;
plot (MarketStrikes,MarketVolatilities,'xk',...
PlottingStrikes,ComputedVols1,'b', ...
PlottingStrikes,ComputedVols2,'r', ...
CurrentForwardValue,ATMVolatility, 'ok',...
'MarkerSize',10);
xlim([0.01 0.06]);
ylim([0.35 0.5]);
xlabel('Strike', 'FontWeight', 'bold');
ylabel('Implied Black Volatility', 'FontWeight', 'bold');
legend('Market Volatilities', 'SABR Model (Method 1)',...
'SABR Model (Method 2)', 'At-the-money volatility');


Note that the model calibrated using Method 2 reproduces the market at-the-money volatility (marked with a circle) exactly.

## References

[1] Hagan, P. S., Kumar, D., Lesniewski, A. S. and Woodward, D. E., Managing smile risk, Wilmott Magazine, 2002.
[2] West, G., "Calibration of the SABR Model in Illiquid Markets," Applied Mathematical Finance, 12(4), pp. 371-385, 2004.

See Also
blackvolbysabr | swaptionbyblk

## Related

- "Price a Swaption Using the SABR Model" on page 2-26


## Price a Swaption Using the SABR Model

This example shows how to price a swaption using the SABR model. First, a swaption volatility surface is constructed from market volatilities. This is done by calibrating the SABR model parameters separately for each swaption maturity. The swaption price is then computed by using the implied Black volatility on the surface as an input to the swaptionbyblk function.

```
In this section...
"Load Market Swaption Volatility Data " on page 2-26
"Calibrate the SABR Model Parameters for Each Swaption Maturity" on
page 2-27
"Construct a Volatility Surface" on page 2-29
"Use swaptionbyblk to Price a Swaption" on page 2-31
"References" on page 2-33
```


## Load Market Swaption Volatility Data

This example shows how to set up hypothetical market implied Black volatilities for European swaptions over a range of strikes (rows) and exercise dates (columns). All swaptions have 10 year swaps as the underlying instruments and the rates are expressed in decimals.

Load the market implied Black volatility data for swaptions.

```
Settle = '12-Jun-2013';
ExerciseDates = {'12-Sep-2013';'12-Jun-2014';'12-Jun-2015';...
    '12-Jun-2016';'12-Jun-2017';'12-Jun-2018';'12-Jun-2020'; ...
    '12-Jun-2023'};
YearsToExercise = yearfrac(Settle, ExerciseDates, 1);
NumMaturities = length(YearsToExercise);
MarketVolatilities = [ ...
    57.6 53.7 49.4 45.6 44.1 41.1 35.2 32.0
    46.6 46.9 44.8 41.6 39.8 37.4 33.4 31.0
    35.9 39.3 39.6 37.9 37.2 34.7 30.5 28.9
```

$\left.\begin{array}{rlllllllll}34.1 & 36.5 & 37.8 & 36.6 & 35.0 & 31.9 & 28.1 & 26.6 \\ 41.0 & 41.3 & 39.5 & 37.8 & 36.0 & 32.6 & 29.0 & 26.0 \\ 45.8 & 43.4 & 41.9 & 39.2 & 36.9 & 33.2 & 29.6 & 26.3 \\ 50.3 & 46.9 & 44.0 & 40.0 & 37.5 & 33.8 & 30.2 & 27.3\end{array}\right] / 100 ;$

The current underlying forward rates and the corresponding at-the-money volatilities across the eight swaption maturities are represented in the fourth rows of the two matrices.

```
CurrentForwardValues = MarketStrikes(4,:)
ATMVolatilities = MarketVolatilities(4,:)
```

CurrentForwardValues =

$$
\begin{array}{lllllll}
0.0250 & 0.0275 & 0.0318 & 0.0350 & 0.0376 & 0.0391 & 0.0408
\end{array}
$$

## ATMVolatilities =

| 0.3410 | 0.3650 | 0.3780 | 0.3660 | 0.3500 | 0.3190 | 0.2810 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Calibrate the SABR Model Parameters for Each Swaption Maturity

This example shows how to use the model implemented in the function blackvolbysabr, a static SABR model, where the model parameters are assumed to be constant with respect to time. The parameters are calibrated separately for each swaption maturity (years to exercise) in a for loop. To better represent market at-the-money volatilities, the $\alpha$ (Alpha) parameter values are implied by the market at-the-money volatilities (see "Method 2" for "Calibrate the SABR Model" on page 2-20).

Define the predetermined Beta.

```
Beta = 0.5;
Betas = repmat(Beta, NumMaturities, 1);
Alphas = zeros(NumMaturities, 1);
Rhos = zeros(NumMaturities, 1);
Nus = zeros(NumMaturities, 1);
```

Calibrate SABR model parameters for each swaption maturity.

```
options = optimoptions('lsqnonlin','Display','none');
for k = 1:NumMaturities
    % This function solves the SABR at-the-money volatility equation as a
    % polynomial of Alpha
    alpharoots = @(Rho,Nu) roots([...
            (1 - Beta)^2*YearsToExercise(k)/24/CurrentForwardValues(k)^(2 - 2*B
            Rho*Beta*Nu*YearsToExercise(k)/4/CurrentForwardValues(k)^(1 - Beta)
            (1 + (2 - 3*Rho^2)*Nu^2*YearsToExercise(k)/24) ...
            -ATMVolatilities(k)*CurrentForwardValues(k)^(1 - Beta)]);
```

    \% This function converts at-the-money volatility into Alpha by picking
    \% smallest positive real root
    atmVol2SabrAlpha \(=@(R h o, N u)\) min(real(arrayfun( \(@(x) . .\).
            \(x^{*}(x>0)+r e a l m a x *(x<0\) || abs(imag(x))>1e-6), alpharoots(Rho,Nu))))
    \% Fit Rho and Nu (while converting at-the-money volatility into Alpha)
    objFun = @(X) MarketVolatilities(:,k) - ...
            blackvolbysabr(atmVol2SabrAlpha(X(1), X(2)), ...
            Beta, X(1), X(2), Settle, ExerciseDates(k), CurrentForwardValues(k)
            MarketStrikes(:,k));
    X = lsqnonlin(objFun, [0 0.5], [-1 0], [1 Inf], options);
    Rho \(=X(1)\);
    \(\mathrm{Nu}=\mathrm{X}(2)\);
    \% Get final Alpha from the calibrated parameters
    Alpha = atmVol2SabrAlpha(Rho, Nu);
    Alphas(k) = Alpha;
    ```
    Rhos(k) = Rho;
    Nus(k) = Nu;
end
```

Display calibrated parameters in a table.
CalibratedPrameters = array2table([Alphas Betas Rhos Nus],...
'VariableNames',\{'Alpha' 'Beta' 'Rho' 'Nu'\},...
'RowNames',\{'3M into 10Y';'1Y into 10Y';...
'2Y into 10Y';'3Y into 10Y';'4Y into $10 Y^{\prime} ; .$. '5Y into 10Y';'7Y into 10Y';'10Y into 10Y'\})

CalibratedPrameters =

| Alpha | Beta |  | Rho |  |
| ---: | ---: | ---: | ---: | ---: |

## Construct a Volatility Surface

This example shows how to use the calibrated model to compute new volatilities at any strike value to produce a smooth smile for a given maturity. This can be repeated for each maturity to form a volatility surface.

Compute volatilities using the calibrated models for each maturity.

```
PlottingStrikes = (0.95:0.1:5.8)'/100;
ComputedVols = zeros(length(PlottingStrikes), NumMaturities);
```

\% Compute volatilities using the calibrated models for each maturity
for $k=1:$ NumMaturities
ComputedVols(:,k) = blackvolbysabr(Alphas(k), Betas(k), Rhos(k), Nus(k), Se
ExerciseDates(k), CurrentForwardValues(k), PlottingStrikes);

## end

Plot the volatility surface.
figure;
surf(YearsToExercise, PlottingStrikes, ComputedVols);
xlim([0 10]); ylim([0.0095 0.06]); zlim([0.2 0.8]);
view $(113,32)$;
set(gca, 'Position', [0.13 0.110 .7750 .815$], .$.
'PlotBoxAspectRatioMode', 'manual');
xlabel('Years to exercise', 'Fontweight', 'bold');
ylabel('Strike', 'Fontweight', 'bold');
zlabel('Implied Black volatility', 'Fontweight', 'bold');


Note, in this volatility surface, the smiles tend to get flatter for longer swaption maturities (years to exercise). This is consistent with the $v(\mathrm{Nu})$ parameter values tending to decrease with swaption maturity, as shown previously in the table for CalibratedPrameters.

## Use swaptionbyblk to Price a Swaption

This example shows how to use the volatility surface to price a swaption that matures in 5 years.

Define a swaption (for a 10 year swap) that matures in 5 years.

```
SwaptionSettle = '12-Jun-2013';
SwaptionExerciseDate = '12-Jun-2018';
SwapMaturity = '12-Jun-2028';
Reset = 1;
OptSpec = 'call';
Strike = 0.0263;
```

Assume that the interest-rate term structure at the time of the swaption Settle date is:

ValuationDate = '12-Jun-2013';
EndDates = \{'12-Jul-2013';'12-Sep-2013';'12-Dec-2013';'12-Jun-2014'; ...
'12-Jun-2015';'12-Jun-2016';'12-Jun-2017';'12-Jun-2018'; ...
'12-Jun-2019';'12-Jun-2020';'12-Jun-2021';'12-Jun-2022'; ...
'12-Jun-2023';'12-Jun-2025';'12-Jun-2028';'12-Jun-2033'\};
Rates $=\left[\begin{array}{ll}0.2 & 0.3 \\ 0.4 & 0.7 \\ 0.5 & 0.7 \\ 1.0 & 1.41 .71 .9\end{array}\right.$...
$2.12 .32 .52 .83 .13 .3] / 100 ;$
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', Valuatio
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
RateSpec =

```
            FinObj: 'RateSpec'
    Compounding: 1
            Disc: [16x1 double]
            Rates: [16x1 double]
            EndTimes: [16x1 double]
        StartTimes: [16x1 double]
            EndDates: [16x1 double]
        StartDates: 735397
        ValuationDate: 735397
            Basis: 0
```


## EndMonthRule: 1

Using this RateSpec, compute the current forward swap rate using the function swapbyzero.

LegRate = [NaN 0]; \% To compute the forward swap rate, set the coupon rate [~, CurrentForwardSwapRate] = swapbyzero(RateSpec, LegRate, SwaptionSettle, 'StartDate', SwaptionExerciseDate);

Compute the SABR implied Black volatility for this swaption using the function blackvolbysabr (and it is marked with a red arrow in the figure that follows).

SABRBlackVolatility = blackvolbysabr(Alphas(6), Betas(6), Rhos(6), Nus(6), SwaptionExerciseDate, CurrentForwardSwapRate, Strike)
text (YearsToExercise(6), Strike, SABRBlackVolatility, '\leftarrow',... 'Color', 'r', 'FontWeight', 'bold', 'FontSize', 22);


Price the swaption using the SABR implied Black volatility as an input to the function swaptionbyblk.

```
Price = swaptionbyblk(RateSpec, OptSpec, Strike, SwaptionSettle, SwaptionEx
SwapMaturity, SABRBlackVolatility, 'Reset', Reset)
SABRBlackVolatility =
```

    0.3932
    Price =
14.2403

## References

[1] Hagan, P. S., Kumar, D., Lesniewski, A. S. and Woodward, D. E., "Managing Smile Risk," Wilmott Magazine, 2002.
[2] West, G., "Calibration of the SABR Model in Illiquid Markets," Applied Mathematical Finance, 12(4), pp. 371-385, 2004.

See Also blackvolbysabr \| swaptionbyblk | swapbyzero

## Related

- "Calibrate the SABR Model " on page 2-20


## Overview of Interest-Rate Tree Models

In this section...<br>"Interest-Rate Modeling" on page 2-34<br>"Rate and Price Trees" on page 2-35<br>"Viewing Rate or Price Movement" on page 2-36

## Interest-Rate Modeling

Financial Instruments Toolbox software computes prices and sensitivities of interest-rate contingent claims based on several methods of modeling changes in interest rates over time:

- The interest-rate term structure

This model uses sets of zero-coupon bonds to predict changes in interest rates.

- Heath-Jarrow-Morton (HJM) model

The HJM model considers a given initial term structure of interest rates and a specification of the volatility of forward rates to build a tree representing the evolution of the interest rates, based on a statistical process.

- Black-Derman-Toy (BDT) model

In the BDT model, all security prices and rates depend on the short rate (annualized 1-period interest rate). The model uses long rates and their volatilities to construct a tree of possible future short rates. The resulting tree can then be used to determine the value of interest-rate sensitive securities from this tree.

- Hull-White (HW) model

The Hull-White model incorporates the initial term structure of interest rates and the volatility term structure to build a trinomial recombining tree of short rates. The resulting tree is used to value interest-rate dependent securities. The implementation of the HW model in Financial Instruments Toolbox software is limited to one factor.

- Black-Karasinski (BK) model

The BK model is a single-factor, log-normal version of the HW model.
For detailed information about interest-rate models, see:

- "Pricing Using Interest-Rate Term Structure" on page 2-53 for a discussion of price and sensitivity based on portfolios of zero-coupon bonds
- "Pricing Using Interest-Rate Tree Models" on page 2-81 for a discussion of price and sensitivity based on the HJM and BDT interest-rate models

> Note Historically, the initial version of Financial Instruments Toolbox software provided only the HJM interest-rate model. A later version added the BDT model. The current version adds both the HW and BK models. This section provides extensive examples of using the HJM and BDT models to compute prices and sensitivities of interest-rate based financial derivatives.

> The HW and BK tree structures are similar to the BDT tree structure. To avoid needless repetition throughout this section, documentation is provided only where significant deviations from the BDT structure exist. Specifically, "HW and BK Tree Structures" on page 2-77 explains the few noteworthy differences among the various formats.

## Rate and Price Trees

The interest-rate or price trees supported in this toolbox can be either binomial (two branches per node) or trinomial (three branches per node). Typically, binomial trees assume that underlying interest rates or prices can only either increase or decrease at each node. Trinomial trees allow for a more complex movement of rates or prices. With trinomial trees the movement of rates or prices at each node is unrestricted (for example, up-up-up or unchanged-down-down).

## Types of Trees

Financial Instruments Toolbox trees can be classified as bushy or recombining. A bushy tree is a tree in which the number of branches increases exponentially relative to observation times; branches never recombine. In this context, a recombining tree is the opposite of a bushy tree. A recombining tree has
branches that recombine over time. From any given node, the node reached by taking the path up-down is the same node reached by taking the path down-up. A bushy tree and a recombining binomial tree are illustrated next.


Bushy Tree


Recombining Binomial Tree

In this toolbox the Heath-Jarrow-Morton model works with bushy trees. The Black-Derman-Toy model, on the other hand, works with recombining binomial trees.

The other two interest rate models supported in this toolbox, Hull-White and Black-Karasinski, work with recombining trinomial trees.

## Viewing Rate or Price Movement

This toolbox provides the data file deriv.mat that contains four interest-rate based trees:

- HJMTree - A bushy binomial tree
- BDTTree - A recombining binomial tree
- HWTree and BKTree - Recombining trinomial trees

The toolbox also provides the treeviewer function, which graphically displays the shape and data of price, interest rate, and cash flow trees. Viewed with treeviewer, the bushy shape of an HJM tree and the recombining shape of a BDT tree are apparent.


With treeviewer, you can also see the recombining shape of HW and BK trinomial trees.


HWTree and BKTree (recombining)

# Understanding the Interest-Rate Term Structure 

In this section...<br>"Introduction" on page 2-38<br>"Interest Rates Versus Discount Factors" on page 2-38<br>"Interest-Rate Term Conversions" on page 2-43<br>"Modeling the Interest-Rate Term Structure" on page 2-47

## Introduction

The interest-rate term structure represents the evolution of interest rates through time. In MATLAB software, the interest-rate environment is encapsulated in a structure called RateSpec (rate specification). This structure holds all information required to completely identify the evolution of interest rates. Several functions included in Financial Instruments Toolbox software are dedicated to the creating and managing of the RateSpec structure. Many others take this structure as an input argument representing the evolution of interest rates.

Before looking further at the RateSpec structure, examine three functions that provide key functionality for working with interest rates: disc2rate, its opposite, rate2disc, and ratetimes. The first two functions map between discount factors and interest rates. The third function, ratetimes, calculates the effect of term changes on the interest rates.

## Interest Rates Versus Discount Factors

Discount factors are coefficients commonly used to find the current value of future cash flows. As such, there is a direct mapping between the rate applicable to a period of time, and the corresponding discount factor. The function disc2rate converts discount factors for a given term (period) into interest rates. The function rate2disc does the opposite; it converts interest rates applicable to a given term (period) into the corresponding discount factors.

## Calculating Discount Factors from Rates

As an example, consider these annualized zero-coupon bond rates.

| From | To | Rate |
| :--- | :--- | :--- |
| 15 Feb 2000 | 15 Aug 2000 | 0.05 |
| 15 Feb 2000 | 15 Feb 2001 | 0.056 |
| 15 Feb 2000 | 15 Aug 2001 | 0.06 |
| 15 Feb 2000 | 15 Feb 2002 | 0.065 |
| 15 Feb 2000 | 15 Aug 2002 | 0.075 |

To calculate the discount factors corresponding to these interest rates, call rate2disc using the syntax

```
Disc = rate2disc(Compounding, Rates, EndDates, StartDates,
ValuationDate)
```

where:

- Compounding represents the frequency at which the zero rates are compounded when annualized. For this example, assume this value to be 2 .
- Rates is a vector of annualized percentage rates representing the interest rate applicable to each time interval.
- EndDates is a vector of dates representing the end of each interest-rate term (period).
- StartDates is a vector of dates representing the beginning of each interest-rate term.
- ValuationDate is the date of observation for which the discount factors are calculated. In this particular example, use February 15, 2000 as the beginning date for all interest-rate terms.

Next, set the variables in MATLAB.

```
StartDates = ['15-Feb-2000'];
EndDates = ['15-Aug-2000'; '15-Feb-2001'; '15-Aug-2001';...
'15-Feb-2002'; '15-Aug-2002'];
Compounding = 2;
ValuationDate = ['15-Feb-2000'];
```

```
Rates = [0.05; 0.056; 0.06; 0.065; 0.075];
```

Finally, compute the discount factors.
Disc = rate2disc(Compounding, Rates, EndDates, StartDates,... ValuationDate)

Disc =
0.9756
0.9463
0.9151
0.8799
0.8319

By adding a fourth column to the rates table (see "Calculating Discount Factors from Rates" on page 2-38) to include the corresponding discounts, you can see the evolution of the discount factors.

| From | To | Rate | Discount |
| :--- | :--- | :--- | :--- |
| 15 Feb 2000 | 15 Aug 2000 | 0.05 | 0.9756 |
| 15 Feb 2000 | 15 Feb 2001 | 0.056 | 0.9463 |
| 15 Feb 2000 | 15 Aug 2001 | 0.06 | 0.9151 |
| 15 Feb 2000 | 15 Feb 2002 | 0.065 | 0.8799 |
| 15 Feb 2000 | 15 Aug 2002 | 0.075 | 0.8319 |

## Optional Time Factor Outputs

The function rate2disc optionally returns two additional output arguments: EndTimes and StartTimes. These vectors of time factors represent the start dates and end dates in discount periodic units. The scale of these units is determined by the value of the input variable Compounding.

To examine the time factor outputs, find the corresponding values in the previous example.
[Disc, EndTimes, StartTimes] = rate2disc(Compounding, Rates,...

```
EndDates, StartDates, ValuationDate);
```

Arrange the two vectors into a single array for easier visualization.

```
Times = [StartTimes, EndTimes]
```

Times =

| 0 | 1 |
| :--- | :--- |
| 0 | 2 |
| 0 | 3 |
| 0 | 4 |
| 0 | 5 |

Because the valuation date is equal to the start date for all periods, the StartTimes vector is composed of 0 s . Also, since the value of Compounding is 2 , the rates are compounded semiannually, which sets the units of periodic discount to 6 months. The vector EndDates is composed of dates separated by intervals of 6 months from the valuation date. This explains why the EndTimes vector is a progression of integers from 1 to 5.

## Alternative Syntax (rate2disc)

The function rate2disc also accommodates an alternative syntax that uses periodic discount units instead of dates. Since the relationship between discount factors and interest rates is based on time periods and not on absolute dates, this form of rate2disc allows you to work directly with time periods. In this mode, the valuation date corresponds to 0 , and the vectors StartTimes and EndTimes are used as input arguments instead of their date equivalents, StartDates and EndDates. This syntax for rate2disc is:

Disc = rate2disc(Compounding, Rates, EndTimes, StartTimes)
Using as input the StartTimes and EndTimes vectors computed previously, you should obtain the previous results for the discount factors.

```
Disc = rate2disc(Compounding, Rates, EndTimes, StartTimes)
```

Disc =
0.9463
0.9151
0.8799
0.8319

## Calculating Rates from Discounts

The function disc2rate is the complement to rate2disc. It finds the rates applicable to a set of compounding periods, given the discount factor in those periods. The syntax for calling this function is:

Rates = disc2rate(Compounding, Disc, EndDates, StartDates, ValuationDate)

Each argument to this function has the same meaning as in rate2disc. Use the results found in the previous example to return the rate values you started with.

```
Rates = disc2rate(Compounding, Disc, EndDates, StartDates,...
ValuationDate)
```

Rates =
0.0500
0.0560
0.0600
0.0650
0.0750

## Alternative Syntax (disc2rate)

As in the case of rate2disc, disc2rate optionally returns StartTimes and EndTimes vectors representing the start and end times measured in discount periodic units. Again, working with the same values as before, you should obtain the same numbers.
[Rates, EndTimes, StartTimes] = disc2rate(Compounding, Disc,... EndDates, StartDates, ValuationDate);

Arrange the results in a matrix convenient to display.

```
Result = [StartTimes, EndTimes, Rates]
Result =
\begin{tabular}{lll}
0 & 1.0000 & 0.0500 \\
0 & 2.0000 & 0.0560 \\
0 & 3.0000 & 0.0600 \\
0 & 4.0000 & 0.0650 \\
0 & 5.0000 & 0.0750
\end{tabular}
```

As with rate2disc, the relationship between rates and discount factors is determined by time periods and not by absolute dates. Consequently, the alternate syntax for disc2rate uses time vectors instead of dates, and it assumes that the valuation date corresponds to time $=0$. The time-based calling syntax is:

Rates $=$ disc2rate(Compounding, Disc, EndTimes, StartTimes);
Using this syntax, you again obtain the original values for the interest rates.
Rates $=$ disc2rate(Compounding, Disc, EndTimes, StartTimes)

Rates =
0.0500
0.0560
0.0600
0.0650
0.0750

## Interest-Rate Term Conversions

Interest rate evolution is typically represented by a set of interest rates, including the beginning and end of the periods the rates apply to. For zero rates, the start dates are typically at the valuation date, with the rates extending from that valuation date until their respective maturity dates.

## Spot Curve to Forward Curve Conversion

Frequently, given a set of rates including their start and end dates, you may be interested in finding the rates applicable to different terms (periods). This
problem is addressed by the function ratetimes. This function interpolates the interest rates given a change in the original terms. The syntax for calling ratetimes is
[Rates, EndTimes, StartTimes] = ratetimes(Compounding, RefRates, RefEndDates, RefStartDates, EndDates, StartDates, ValuationDate);
where:

- Compounding represents the frequency at which the zero rates are compounded when annualized.
- RefRates is a vector of initial interest rates representing the interest rates applicable to the initial time intervals.
- RefEndDates is a vector of dates representing the end of the interest rate terms (period) applicable to RefRates.
- RefStartDates is a vector of dates representing the beginning of the interest rate terms applicable to RefRates.
- EndDates represent the maturity dates for which the interest rates are interpolated.
- StartDates represent the starting dates for which the interest rates are interpolated.
- ValuationDate is the date of observation, from which the StartTimes and EndTimes are calculated. This date represents time $=0$.

The input arguments to this function can be separated into two groups:

- The initial or reference interest rates, including the terms for which they are valid
- Terms for which the new interest rates are calculated

As an example, consider the rate table specified in "Calculating Discount Factors from Rates" on page 2-38.

| From | To | Rate |
| :--- | :--- | :--- |
| 15 Feb 2000 | 15 Aug 2000 | 0.05 |
| 15 Feb 2000 | 15 Feb 2001 | 0.056 |


| From | To | Rate |
| :--- | :--- | :--- |
| 15 Feb 2000 | 15 Aug 2001 | 0.06 |
| 15 Feb 2000 | 15 Feb 2002 | 0.065 |
| 15 Feb 2000 | 15 Aug 2002 | 0.075 |

Assuming that the valuation date is February 15, 2000, these rates represent zero-coupon bond rates with maturities specified in the second column. Use the function ratetimes to calculate the forward rates at the beginning of all periods implied in the table. Assume a compounding value of 2.

```
% Reference Rates.
RefStartDates = ['15-Feb-2000'];
RefEndDates = ['15-Aug-2000'; '15-Feb-2001'; '15-Aug-2001';...
'15-Feb-2002'; '15-Aug-2002'];
Compounding = 2;
ValuationDate = ['15-Feb-2000'];
RefRates = [0.05; 0.056; 0.06; 0.065; 0.075];
% New Terms.
StartDates = ['15-Feb-2000'; '15-Aug-2000'; '15-Feb-2001';...
'15-Aug-2001'; '15-Feb-2002'];
EndDates = ['15-Aug-2000'; '15-Feb-2001'; '15-Aug-2001';...
'15-Feb-2002'; '15-Aug-2002'];
% Find the new rates.
Rates = ratetimes(Compounding, RefRates, RefEndDates,...
RefStartDates, EndDates, StartDates, ValuationDate)
```

Rates $=$
0.0500
0.0620
0.0680
0.0801
0.1155

Place these values in a table like the previous one. Observe the evolution of the forward rates based on the initial zero-coupon rates.

From
15 Feb 2000
15 Aug 2000
15 Feb 2001
15 Aug 2001
15 Feb 2002

## To

15 Aug 2000
15 Feb 2001
15 Aug 2001
15 Feb 2002
15 Aug 2002

## Rate

0.0500
0.0620
0.0680
0.0801
0.1155

## Alternative Syntax (ratetimes)

The ratetimes function can provide the additional output arguments StartTimes and EndTimes, which represent the time factor equivalents to the StartDates and EndDates vectors. The ratetimes function uses time factors for interpolating the rates. These time factors are calculated from the start and end dates, and the valuation date, which are passed as input arguments. ratetimes can also use time factors directly, assuming time $=$ 0 as the valuation date. This alternate syntax is:
[Rates, EndTimes, StartTimes] = ratetimes(Compounding, RefRates, RefEndTimes, RefStartTimes, EndTimes, StartTimes);

Use this alternate version of ratetimes to find the forward rates again. In this case, you must first find the time factors of the reference curve. Use date2time for this.

RefEndTimes = date2time(ValuationDate, RefEndDates, Compounding)

RefEndTimes =

1
2
3
4
5
RefStartTimes $=$ date2time(ValuationDate, RefStartDates,... Compounding)

RefStartTimes =

0

These are the expected values, given semiannual discounts (as denoted by a value of 2 in the variable Compounding), end dates separated by 6 -month periods, and the valuation date equal to the date marking beginning of the first period (time factor $=0$ ).

Now call ratetimes with the alternate syntax.

```
[Rates, EndTimes, StartTimes] = ratetimes(Compounding,...
RefRates, RefEndTimes, RefStartTimes, EndTimes, StartTimes);
Rates =
    0.0500
    0.0620
    0.0680
    0.0801
    0.1155
```

EndTimes and StartTimes have, as expected, the same values they had as input arguments.

Times = [StartTimes, EndTimes]
Times =

| 0 | 1 |
| :--- | :--- |
| 1 | 2 |
| 2 | 3 |
| 3 | 4 |
| 4 | 5 |

## Modeling the Interest-Rate Term Structure

Financial Instruments Toolbox software includes a set of functions to encapsulate interest-rate term information into a single structure. These functions present a convenient way to package all information related to
interest-rate terms into a common format, and to resolve interdependencies when one or more of the parameters is modified. For information, see:

- "Creating or Modifying (intenvset)" on page 2-48 for a discussion of how to create or modify an interest-rate term structure (RateSpec) using the intenvset function
- "Obtaining Specific Properties (intenvget)" on page 2-50 for a discussion of how to extract specific properties from a RateSpec


## Creating or Modifying (intenvset)

The main function to create or modify an interest-rate term structure RateSpec (rates specification) is intenvset. If the first argument to this function is a previously created RateSpec, the function modifies the existing rate specification and returns a new one. Otherwise, it creates a RateSpec.

When using RateSpec to specify the rate term structure to price instruments based on yields (zero coupon rates) or forward rates, specify zero rates or forward rates as the input argument. However, the RateSpec structure is not limited or specific to this problem domain. RateSpec is an encapsulation of rates-times relationships; intenvset acts as either a constructor or a modifier, and intenvget as an accessor. The interest rate models supported by the Financial Instruments Toolbox software work either with zero coupon rates or forward rates.

The other intenvset arguments are property-value pairs, indicating the new value for these properties. The properties that can be specified or modified are:

- Basis
- Compounding
- Disc
- EndDates
- EndMonthRule
- Rates
- StartDates
- ValuationDate

To learn about the properties EndMonthRule and Basis, type help ftbEndMonthRule and help ftbBasis or see the Financial Toolbox ${ }^{\text {TM }}$ documentation.

Consider again the original table of interest rates (see "Calculating Discount Factors from Rates" on page 2-38).

| From | To | Rate |
| :--- | :--- | :--- |
| 15 Feb 2000 | 15 Aug 2000 | 0.05 |
| 15 Feb 2000 | 15 Feb 2001 | 0.056 |
| 15 Feb 2000 | 15 Aug 2001 | 0.06 |
| 15 Feb 2000 | 15 Feb 2002 | 0.065 |
| 15 Feb 2000 | 15 Aug 2002 | 0.075 |

Use the information in this table to populate the RateSpec structure.
StartDates = ['15-Feb-2000'];
EndDates $=$ ['15-Aug-2000';
'15-Feb-2001';
'15-Aug-2001';
'15-Feb-2002';
'15-Aug-2002'];
Compounding = 2;
ValuationDate = ['15-Feb-2000'];
Rates $=$ [0.05; 0.056; 0.06; 0.065; 0.075];
rs = intenvset('Compounding', Compounding,'StartDates',...
StartDates, 'EndDates', EndDates, 'Rates', Rates,...
'ValuationDate', ValuationDate)
rs =
FinObj: 'RateSpec'
Compounding: 2
Disc: [5x1 double]
Rates: [5x1 double]
EndTimes: [5x1 double]

```
    StartTimes: [5x1 double]
    EndDates: [5x1 double]
    StartDates: 730531
ValuationDate: 730531
            Basis: 0
EndMonthRule: 1
```

Some of the properties filled in the structure were not passed explicitly in the call to RateSpec. The values of the automatically completed properties depend on the properties that are explicitly passed. Consider for example the StartTimes and EndTimes vectors. Since the StartDates and EndDates vectors are passed in, and the ValuationDate, intenvset has all the information required to calculate StartTimes and EndTimes. Hence, these two properties are read-only.

## Obtaining Specific Properties (intenvget)

The complementary function to intenvset is intenvget, which gets function specific properties from the interest-rate term structure. Its syntax is:

```
ParameterValue = intenvget(RateSpec, 'ParameterName')
```

To obtain the vector EndTimes from the RateSpec structure, enter:

```
EndTimes = intenvget(rs, 'EndTimes')
EndTimes =
```

    1
    2
    3
    4
    5
    To obtain Disc, the values for the discount factors that were calculated automatically by intenvset, type:

```
Disc = intenvget(rs, 'Disc')
```

Disc =
0.9756
0.9463
0.9151
0.8799
0.8319

These discount factors correspond to the periods starting from StartDates and ending in EndDates.

Caution Although you can directly access these fields within the structure instead of using intenvget, it is advised not to do so. The format of the interest-rate term structure could change in future versions of the toolbox. Should that happen, any code accessing the RateSpec fields directly would stop working.

Now use the RateSpec structure with its functions to examine how changes in specific properties of the interest-rate term structure affect those depending on it. As an exercise, change the value of Compounding from 2 (semiannual) to 1 (annual).

```
rs = intenvset(rs, 'Compounding', 1);
```

Since StartTimes and EndTimes are measured in units of periodic discount, a change in Compounding from 2 to 1 redefines the basic unit from semiannual to annual. This means that a period of 6 months is represented with a value of 0.5 , and a period of 1 year is represented by 1 . To obtain the vectors StartTimes and EndTimes, enter:

```
StartTimes = intenvget(rs, 'StartTimes');
```

EndTimes = intenvget(rs, 'EndTimes');
Times = [StartTimes, EndTimes]

Times =

```
0 0.5000
0 1.0000
0 1.5000
0.0000
```

Since all the values in StartDates are the same as the valuation date, all StartTimes values are 0 . On the other hand, the values in the EndDates vector are dates separated by 6 -month periods. Since the redefined value of compounding is 1, EndTimes becomes a sequence of numbers separated by increments of 0.5.

## Pricing Using Interest-Rate Term Structure

## In this section...

"Introduction" on page 2-53
"Computing Instrument Prices" on page 2-54
"Computing Instrument Sensitivities" on page 2-56
"OAS for Callable and Puttable Bonds" on page 2-57
"Agency OAS" on page 2-58

## Introduction

The instruments can be presented to the functions as a portfolio of different types of instruments or as groups of instruments of the same type. The current version of the toolbox can compute price and sensitivities for four instrument types using interest-rate curves:

- Bonds
- Fixed-rate notes
- Floating-rate notes
- Swaps
- OAS for callable and puttable bonds
- Agency OAS

In addition to these instruments, the toolbox also supports the calculation of price and sensitivities of arbitrary sets of cash flows.

Note that options and interest-rate floors and caps are absent from the above list of supported instruments. These instruments are not supported because their pricing and sensitivity function require a stochastic model for the evolution of interest rates. The interest-rate term structure used for pricing is treated as deterministic, and as such is not adequate for pricing these instruments.

Financial Instruments Toolbox software also contains functions that use the Heath-Jarrow-Morton (HJM) and Black-Derman-Toy (BDT) models to compute prices and sensitivities for financial instruments. These models support computations involving options and interest-rate floors and caps. See "Pricing Using Interest-Rate Tree Models" on page 2-81 for information on computing price and sensitivities of financial instruments using the HJM and BDT models.

## Computing Instrument Prices

The main function used for pricing portfolios of instruments is intenvprice. This function works with the family of functions that calculate the prices of individual types of instruments. When called, intenvprice classifies the portfolio contained in InstSet by instrument type, and calls the appropriate pricing functions. The map between instrument types and the pricing function intenvprice calls is

| bondbyzero: | Price a bond by a set of zero curves |
| :--- | :--- |
| fixedbyzero: | Price a fixed-rate note by a set of zero curves |
| floatbyzero: | Price a floating-rate note by a set of zero curves |
| swapbyzero: | Price a swap by a set of zero curves |

You can use each of these functions individually to price an instrument. Consult the reference pages for specific information on using these functions.
intenvprice takes as input an interest-rate term structure created with intenvset, and a portfolio of interest-rate contingent derivatives instruments created with instadd.

The syntax for using intenvprice to price an entire portfolio is

```
Price = intenvprice(RateSpec, InstSet)
```

where:

- RateSpec is the interest-rate term structure.
- InstSet is the name of the portfolio.


## Example: Pricing a Portfolio of Instruments

Consider this example of using the intenvprice function to price a portfolio of instruments supplied with Financial Instruments Toolbox software.

The provided MAT-file deriv.mat stores a portfolio as an instrument set variable ZeroInstSet. The MAT-file also contains the interest-rate term structure ZeroRateSpec. You can display the instruments with the function instdisp.

```
load deriv.mat;
instdisp(ZeroInstSet)
```

| Index | Type | CouponRate S | Settle | Maturity |  | Period | Basis. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Bond | 0.04 | 01 -Jan-2000 | 01-Jan-2003 |  | 1 | NaN. |
| 2 | Bond | 0.04 | 01 -Jan-2000 | 01-Jan-2004 |  | 2 | NaN. . . |
| Index | Type | CouponRate | Settle | Maturity F |  | dReset B | Basis. |
| 3 | Fixed | 0.04 | 01 -Jan-2000 | 01 -Jan-2003 | 1 |  | NaN. |
| Index | Type | Spread Sett | tle | Maturity |  | oatReset | t Basis. |
| 4 | Float | 20 01-J | Jan-2000 | $01-J a n-2003$ | 1 |  | NaN... |
| Index | Type | LegRate | Settle | Maturity |  | LegRese | et Basis. |
| 5 | Swap | [0.06 20] | 01-Jan-2000 | 01-Jan-2003 |  | $\left[\begin{array}{ll}1 & 1\end{array}\right]$ | NaN |

Use intenvprice to calculate the prices for the instruments contained in the portfolio ZeroInstSet.
format bank
Prices = intenvprice(ZeroRateSpec, ZeroInstSet)

Prices =
98.72
97.53
98.72
100.55
3.69

The output Prices is a vector containing the prices of all the instruments in the portfolio in the order indicated by the Index column displayed by instdisp. Consequently, the first two elements in Prices correspond to the first two bonds; the third element corresponds to the fixed-rate note; the fourth to the floating-rate note; and the fifth element corresponds to the price of the swap.

## Computing Instrument Sensitivities

In general, you can compute sensitivities either as dollar price changes or as percentage price changes. The toolbox reports all sensitivities as dollar sensitivities.

Using the interest-rate term structure, you can calculate two types of derivative price sensitivities, delta and gamma. Delta represents the dollar sensitivity of prices to shifts in the observed forward yield curve. Gamma represents the dollar sensitivity of delta to shifts in the observed forward yield curve.

The intenvsens function computes instrument sensitivities and instrument prices. If you need both the prices and sensitivity measures, use intenvsens. A separate call to intenvprice is not required.

Here is the syntax
[Delta, Gamma, Price] = intenvsens(RateSpec, InstSet)
where, as before:

- RateSpec is the interest-rate term structure.
- InstSet is the name of the portfolio.


## Example: Sensitivities and Prices

Here is an example that uses intenvsens to calculate both sensitivities and prices.
format bank
load deriv.mat;
[Delta, Gamma, Price] = intenvsens(ZeroRateSpec, ZeroInstSet);

Display the results in a single matrix in bank format.

```
All = [Delta Gamma Price]
```

All =

| -272.64 | 1029.84 | 98.72 |
| ---: | ---: | ---: |
| -347.44 | 1622.65 | 97.53 |
| -272.64 | 1029.84 | 98.72 |
| -1.04 | 3.31 | 100.55 |
| -282.04 | 1059.62 | 3.69 |

To view the per-dollar sensitivity, divide the first two columns by the last one.
[Delta./Price, Gamma./Price, Price]
ans =

| -2.76 | 10.43 | 98.72 |
| ---: | ---: | ---: |
| -3.56 | 16.64 | 97.53 |
| -2.76 | 10.43 | 98.72 |
| -0.01 | 0.03 | 100.55 |
| -76.39 | 286.98 | 3.69 |

## OAS for Callable and Puttable Bonds

Option Adjusted Spread (OAS) is a useful way to value and compare securities with embedded options, like callable or puttable bonds. Basically, when the constant or flat spread is added to the interest-rate curve/rates in the tree, the pricing model value equals the market price. Financial Instruments Toolbox supports pricing American, European and Bermuda callable and puttable bonds using different interest rate models. The pricing for a bond with embedded options is:

- For a callable bond, where the holder has bought a bond and sold a call option to the issuer:

```
Price callable bond = Price Option free bond Price call
option
```

- For a puttable bond, where the holder has bought a bond and a put option:

```
Price puttable bond=Price Option free bond + Price put option
```

There are two additional sensitivities related to OAS for bonds with embedded options: Option Adjusted Duration and Option Adjusted Convexity. These are similar to the concepts of modified duration and convexity for option-free bonds. The measure Duration is a general term that describes how sensitive a bond's price is to a parallel shift in the yield curve. Modified Duration and Modified Convexity assume that the bond's cash flows do not change when the yield curve shifts. This is not true for OA Duration or OA Convexity because the cash flows may change due to the option risk component of the bond.

| Function | Purpose |
| :--- | :--- |
| oasbybdt | Compute OAS using a BDT model. |
| oasbybk | Compute OAS using a BK model. |
| oasbyhjm | Compute OAS using an HJM model. |
| oasbyhw | Compute OAS using an HW model. |

## Agency OAS

Often bonds are issued with embedded options, which then makes standard price/yield or spread measures irrelevant. For example, a municipality concerned about the chance that interest rates may fall in the future might issue bonds with a provision that allows the bond to be repaid before the bond's maturity. This is a call option on the bond and must be incorporated into the valuation of the bond. Option-adjusted spread (OAS), which adjusts a bond spread for the value of the option, is the standard measure for valuing bonds with embedded options. Financial Instruments Toolbox software supports computing option-adjusted spreads for bonds with single embedded options using the agency model.

The Securities Industry and Financial Markets Association (SIFMA) has a simplified approach to compute OAS for agency issues (Government Sponsored Entities like Fannie Mae and Freddie Mac) termed "Agency OAS". In this approach, the bond has only one call date (European call) and uses Black's model (see The BMA European Callable Securities Formula at http://www.sifma.org) to value the bond option. The price of the bond is computed as follows:

Price $_{\text {Callable }}=$ Price $_{\text {NonCallable }}-$ Price $_{\text {Option }}$
where
Price $_{\text {Callable }}$ is the price of the callable bond.
Price $_{\text {NonCallable }}$ is the price of the noncallable bond, i.e., price of the bond using bndspread.

Price $_{\text {Option }}$ is the price of the option, i.e., price of the option using Black's model.

The Agency OAS is the spread, when used in the previous formula, yields the market price. Financial Instruments Toolbox software supports these functions:

| Agency OAS <br> Functions | Purpose |
| :--- | :--- |
| agencyoas | Compute the OAS of the callable bond using the <br> Agency OAS model. |
| agencyprice | Price the callable bond OAS using Agency using <br> the OAS model. |

For more information on agency OAS, see "Agency Option-Adjusted Spreads" on page 6-2.

## Understanding Interest-Rate Tree Models

In this section...<br>"Introduction" on page 2-60<br>"Building a Tree of Forward Rates" on page 2-61<br>"Specifying the Volatility Model (VolSpec)" on page 2-63<br>"Specifying the Interest-Rate Term Structure (RateSpec)" on page 2-66<br>"Specifying the Time Structure (TimeSpec)" on page 2-67<br>"Creating Trees" on page 2-69<br>"Examining Trees" on page 2-70

## Introduction

Financial Instruments Toolbox software supports the Black-Derman-Toy (BDT), Black-Karasinski (BK), Heath-Jarrow-Morton (HJM), and Hull-White (HW) interest-rate models. The Heath-Jarrow-Morton model is one of the most widely used models for pricing interest-rate derivatives. The model considers a given initial term structure of interest rates and a specification of the volatility of forward rates to build a tree representing the evolution of the interest rates, based on a statistical process. For further explanation, see the book Modelling Fixed Income Securities and Interest Rate Options by Robert A. Jarrow.

The Black-Derman-Toy model is another analytical model commonly used for pricing interest-rate derivatives. The model considers a given initial zero rate term structure of interest rates and a specification of the yield volatilities of long rates to build a tree representing the evolution of the interest rates. For further explanation, see the paper "A One Factor Model of Interest Rates and its Application to Treasury Bond Options" by Fischer Black, Emanuel Derman, and William Toy.

The Hull-White model incorporates the initial term structure of interest rates and the volatility term structure to build a trinomial recombining tree of short rates. The resulting tree is used to value interest rate dependent securities. The implementation of the Hull-White model in Financial Instruments Toolbox software is limited to one factor.

The Black-Karasinski model is a single factor, log-normal version of the Hull-White model.

For further information on the Hull-White and Black-Karasinski models, see the book Options, Futures, and Other Derivatives by John C. Hull.

## Building a Tree of Forward Rates

The tree of forward rates is the fundamental unit representing the evolution of interest rates in a given period of time. This section explains how to create a forward-rate tree using Financial Instruments Toolbox software.

Note To avoid needless repetition, this document uses the HJM and BDT models to illustrate the creation and use of interest-rate trees. The HW and BK models are similar to the BDT model. Where specific differences exist, they are documented in "HW and BK Tree Structures" on page 2-77.

The MATLAB functions that create rate trees are hjmtree and bdttree. The hjmtree function creates the structure, HJMTree, containing time and forward-rate information for a bushy tree. The bdttree function creates a similar structure, BDTTree, for a recombining tree.

This structure is a self-contained unit that includes the tree of rates (found in the FwdTree field of the structure) and the volatility, rate, and time specifications used in building this tree.

These functions take three structures as input arguments:

- The volatility model VolSpec. (See "Specifying the Volatility Model (VolSpec)" on page 2-63.)
- The interest-rate term structure RateSpec. (See "Specifying the Interest-Rate Term Structure (RateSpec)" on page 2-66.)
- The tree time layout TimeSpec. (See "Specifying the Time Structure (TimeSpec)" on page 2-67.)

An easy way to visualize any trees you create is with the treeviewer function, which displays trees in a graphical manner. See "Graphical Representation of Trees" on page 2-139 for information about treeviewer.

## Calling Sequence

The calling syntax for hjmtree is
HJMTree = hjmtree(VolSpec, RateSpec, TimeSpec)
Similarly, the calling syntax for bdttree is
BDTTree = bdttree(VolSpec, RateSpec, TimeSpec)
Each of these functions requires VolSpec, RateSpec, and TimeSpec input arguments:

- VolSpec is a structure that specifies the forward-rate volatility process. You create VolSpec using either of the functions hjmvolspec or bdtvolspec.
The hjmvolspec function supports the specification of up to three factors.
It handles these models for the volatility of the interest-rate term structure:
- Constant
- Stationary
- Exponential
- Vasicek
- Proportional

A one-factor model assumes that the interest term structure is affected by a single source of uncertainty. Incorporating multiple factors allows you to specify different types of shifts in the shape and location of the interest-rate structure. See hjmvolspec for details.

The bdtvolspec function supports only a single volatility factor. The volatility remains constant between pairs of nodes on the tree. You supply the input volatility values in a vector of decimal values. See bdtvolspec for details.

- RateSpec is the interest-rate specification of the initial rate curve. You create this structure with the function intenvset. (See "Modeling the Interest-Rate Term Structure" on page 2-47.)
- TimeSpec is the tree time layout specification. You create this variable with the functions hjmtimespec or bdttimespec. It represents the mapping between level times and level dates for rate quoting. This structure indirectly determines the number of levels in the tree.


## Specifying the Volatility Model (VolSpec)

Because HJM supports multifactor (up to 3) volatility models while BDT (also, BK and HW) supports only a single volatility factor, the hjmvolspec and bdtvolspec functions require different inputs and generate slightly different outputs. For examples, see "Creating an HJM Volatility Model" on page 2-63. For BDT examples see "Creating a BDT Volatility Model" on page 2-65.

## Creating an HJM Volatility Model

The function hjmvolspec generates the structure VolSpec, which specifies the volatility process $\sigma(t, T)$ used in the creation of the forward-rate trees. In this context capital $T$ represents the starting time of the forward rate, and $t$ represents the observation time. The volatility process can be constructed from a combination of factors specified sequentially in the call to function that creates it. Each factor specification starts with a string specifying the name of the factor, followed by the pertinent parameters.

HJM Volatility Specification Example. Consider an example that uses a single factor, specifically, a constant-sigma factor. The constant factor specification requires only one parameter, the value of $\sigma$. In this case, the value corresponds to 0.10 .

```
HJMVolSpec = hjmvolspec('Constant', 0.10)
HJMVolSpec =
            FinObj: 'HJMVolSpec'
FactorModels: {'Constant'}
    FactorArgs: {{1x1 cell}}
    SigmaShift: 0
    NumFactors: 1
```

```
    NumBranch: 2
    PBranch: [0.5000 0.5000]
Fact2Branch: [-1 1]
```

The NumFactors field of the VolSpec structure, VolSpec. NumFactors = 1, reveals that the number of factors used to generate VolSpec was one. The FactorModels field indicates that it is a Constant factor, and the NumBranches field indicates the number of branches. As a consequence, each node of the resulting tree has two branches, one going up, and the other going down.

Consider now a two-factor volatility process made from a proportional factor and an exponential factor.

```
% Exponential factor
Sigma_0 = 0.1;
Lambda = 1;
% Proportional factor
CurveProp = [0.11765; 0.08825; 0.06865];
CurveTerm = [ 1 ; 2 ; 3 ];
% Build VolSpec
HJMVolSpec = hjmvolspec('Proportional', CurveProp, CurveTerm,...
1e6,'Exponential', Sigma_0, Lambda)
HJMVolSpec =
            FinObj: 'HJMVolSpec'
FactorModels: {'Proportional' 'Exponential'}
    FactorArgs: {{1x3 cell} {1x2 cell}}
    SigmaShift: 0
    NumFactors: 2
        NumBranch: 3
            PBranch: [0.2500 0.2500 0.5000]
    Fact2Branch: [2x3 double]
```

The output shows that the volatility specification was generated using two factors. The tree has 3 branches per node. Each branch has probabilities of $0.25,0.25$, and 0.5 , going from top to bottom.

## Creating a BDT Volatility Model

The function bdtvolspec generates the structure VolSpec, which specifies the volatility process. The function requires three input arguments:

- The valuation date ValuationDate
- The yield volatility end dates VolDates
- The yield volatility values VolCurve

An optional fourth argument InterpMethod, specifying the interpolation method, can be included.

The syntax used for calling bdtvolspec is:
BDTVolSpec = bdtvolspec(ValuationDate, VolDates, VolCurve,... InterpMethod)
where:

- ValuationDate is the first observation date in the tree.
- VolDates is a vector of dates representing yield volatility end dates.
- VolCurve is a vector of yield volatility values.
- InterpMethod is the method of interpolation to use. The default is linear.

BDT Volatility Specification Example. Consider the following example:

```
ValuationDate = datenum('01-01-2000');
EndDates = datenum(['01-01-2001'; '01-01-2002'; '01-01-2003';
'01-01-2004'; '01-01-2005']);
Volatility = [.2; .19; .18; .17; .16];
```

Use bdtvolspec to create a volatility specification. Because no interpolation method is explicitly specified, the function uses the linear default.

```
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Volatility)
BDTVolSpec =
    FinObj: 'BDTVolSpec'
    ValuationDate: 730486
```


## VolDates: [5x1 double] <br> VolCurve: [5x1 double] <br> VolInterpMethod: 'linear'

## Specifying the Interest-Rate Term Structure (RateSpec)

The structure RateSpec is an interest term structure that defines the initial forward-rate specification from which the tree rates are derived. "Modeling the Interest-Rate Term Structure" on page 2-47 explains how to create these structures using the function intenvset, given the interest rates, the starting and ending dates for each rate, and the compounding value.

## Rate Specification Creation Example

Consider the following example:
Compounding = 1;
Rates = [0.02; 0.02; 0.02; 0.02];
StartDates = ['01-Jan-2000';
'01-Jan-2001';
'01-Jan-2002';
'01-Jan-2003'];
EndDates = ['01-Jan-2001';
'01-Jan-2002';
'01-Jan-2003';
'01-Jan-2004'];
ValuationDate = '01-Jan-2000';

RateSpec = intenvset('Compounding',1,'Rates', Rates,...
'StartDates', StartDates, 'EndDates', EndDates,...
'ValuationDate', ValuationDate)
RateSpec =

| FinObj: | 'RateSpec' |
| ---: | :--- |
| Compounding: | 1 |
| Disc: | $[4 \times 1$ |
| Rates: | double $]$ |
| EndTimes: | $[4 \times 1$ |
| double | double $]$ |
| StartTimes: | $[4 \times 1$ |
| double $]$ |  |
| EndDates: | $[4 \times 1$ |
| double |  |

## StartDates: [4x1 double] ValuationDate: 730486 <br> Basis: 0 <br> EndMonthRule: 1

Use the function datedisp to examine the dates defined in the variable RateSpec. For example:
datedisp(RateSpec.ValuationDate)
01-Jan-2000

## Specifying the Time Structure (TimeSpec)

The structure TimeSpec specifies the time structure for an interest-rate tree. This structure defines the mapping between the observation times at each level of the tree and the corresponding dates.

TimeSpec is built using either the hjmtimespec or bdttimespec function. These functions require three input arguments:

- The valuation date ValuationDate
- The maturity date Maturity
- The compounding rate Compounding

For example, the syntax used for calling hjmtimespec is
TimeSpec = hjmtimespec(ValuationDate, Maturity, Compounding)
where:

- ValuationDate is the first observation date in the tree.
- Maturity is a vector of dates representing the cash flow dates of the tree. Any instrument cash flows with these maturities fall on tree nodes.
- Compounding is the frequency at which the rates are compounded when annualized.


## Creating a Time Specification

Calling the time specification creation functions with the same data used to create the interest-rate term structure, RateSpec builds the structure that specifies the time layout for the tree.

HJM Time Specification Example. Consider the following example:

```
Maturity = EndDates;
HJMTimeSpec = hjmtimespec(ValuationDate, Maturity, Compounding)
HJMTimeSpec =
    FinObj: 'HJMTimeSpec'
ValuationDate: 730486
    Maturity: [4x1 double]
    Compounding: 1
        Basis: 0
    EndMonthRule: 1
```

Note that maturities specified when building TimeSpec need not coincide with the EndDates of the rate intervals in RateSpec. Since TimeSpec defines the time-date mapping of the tree, the rates in RateSpec are interpolated to obtain the initial rates with maturities equal to those in TimeSpec.

Creating a BDT Time Specification. Consider the following example:
Maturity = EndDates;
BDTTimeSpec $=$ bdttimespec(ValuationDate, Maturity, Compounding)

BDTTimeSpec =
FinObj: 'BDTTimeSpec'
ValuationDate: 730486
Maturity: [4x1 double]
Compounding: 1
Basis: 0
EndMonthRule: 1

## Creating Trees

Use the VolSpec, RateSpec, and TimeSpec you have previously created as inputs to the functions used to create HJM and BDT trees.

## Creating an HJM Tree

```
% Reset the volatility factor to the Constant case
HJMVolSpec = hjmvolspec('Constant', 0.10);
HJMTree = hjmtree(HJMVolSpec, RateSpec, HJMTimeSpec)
HJMTree =
    FinObj: 'HJMFwdTree'
VolSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
    tObs: [0 1 1 2 3]
        TFwd: {[4x1 double] [3x1 double] [2x1 double] [3]}
    CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4]}
FwdTree:{[4x1 double][3x1x2 double][2x2x2 double][1x4x2 double]}
```


## Creating a BDT Tree

Now use the previously computed values for VolSpec, RateSpec, and TimeSpec as input to the function bdttree to create a BDT tree.

```
BDTTree = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec)
BDTTree =
    FinObj: 'BDTFwdTree'
VolSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
    tObs: [lllll}
    TFwd: {[4\times1 double] [3\times1 double] [2x1 double] [3.00]}
    CFlowT: {[4\times1 double] [3x1 double] [2x1 double] [4.00]}
FwdTree: {[1.02] [1.02 1.02] [1.01 1.02 1.03] [1.01 1.02 1.02 1.03]}
```


## Examining Trees

When working with the models, Financial Instruments Toolbox software uses trees to represent forward rates, prices, and so on. At the highest level, these trees have structures wrapped around them. The structures encapsulate information required to interpret completely the information contained in a tree.

Consider this example, which uses the interest rate and portfolio data in the MAT-file deriv.mat included in the toolbox.

Load the data into the MATLAB workspace.
load deriv.mat
Display the list of the variables loaded from the MAT-file.
whos

| Name | Size | Bytes | Class | Attributes |
| :--- | :--- | ---: | :--- | :--- |
|  |  |  |  |  |
| BDTInstSet | $1 \times 1$ | 15956 | struct |  |
| BDTTree | $1 \times 1$ | 5138 | struct |  |
| BKInstSet | $1 \times 1$ | 15946 | struct |  |
| BKTree | $1 \times 1$ | 5904 | struct |  |
| CRRInstSet | $1 \times 1$ | 12434 | struct |  |
| CRRTree | $1 \times 1$ | 5058 | struct |  |
| EQPInstSet | $1 \times 1$ | 12434 | struct |  |
| EQPTree | $1 \times 1$ | 5058 | struct |  |
| HJMInstSet | $1 \times 1$ | 15948 | struct |  |
| HJMTree | $1 \times 1$ | 5838 | struct |  |
| HWInstSet | $1 \times 1$ | 15946 | struct |  |
| HWTree | $1 \times 1$ | 5904 | struct |  |
| ITTInstSet | $1 \times 1$ | 12438 | struct |  |
| ITTTree | $1 \times 1$ | 8862 | struct |  |
| ZeroInstSet | $1 \times 1$ | 10282 | struct |  |
| ZeroRateSpec | $1 \times 1$ | 1580 | struct |  |

## HJM Tree Structure

You can now examine in some detail the contents of the HJMTree structure contained in this file.

```
HJMTree
```

```
HJMTree =
```

HJMTree =
FinObj: 'HJMFwdTree'
VolSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
tObs: [0 0 1 2 3]
TFwd: {[4\times1 double] [3\times1 double] [2\times1 double] [3]}
CFlowT: {[4\times1 double] [3x1 double] [2x1 double] [4]}
FwdTree:{[4\times1 double][3\times1\times2 double][2\times2\times2 double][1\times4\times2 double]}

```

FwdTree contains the actual forward-rate tree. MATLAB software represents it as a cell array with each cell array element containing a tree level.

The other fields contain other information relevant to interpreting the values in FwdTree. The most important are VolSpec, TimeSpec, and RateSpec, which contain the volatility, time structure, and rate structure information respectively.

First Node. Observe the forward rates in FwdTree. The first node represents the valuation date, \(\mathrm{t} 0 \mathrm{bs}=0\).

HJMTree.FwdTree\{1\}
```

ans =
1.0356
1.0468
1.0523
1.0563

```

Note Financial Instruments Toolbox software uses inverse discount notation for forward rates in the tree. An inverse discount represents a factor by which the current value of an asset is multiplied to find its future value. In general, these forward factors are reciprocals of the discount factors.

Look closely at the RateSpec structure used in generating this tree to see where these values originate. Arrange the values in a single array.
[HJMTree.RateSpec.StartTimes HJMTree.RateSpec.EndTimes... HJMTree.RateSpec.Rates]
ans =
\begin{tabular}{rrr}
0 & 1.0000 & 0.0356 \\
1.0000 & 2.0000 & 0.0468 \\
2.0000 & 3.0000 & 0.0523 \\
3.0000 & 4.0000 & 0.0563
\end{tabular}

If you find the corresponding inverse discounts of the interest rates in the third column, you have the values at the first node of the tree. You can turn interest rates into inverse discounts using the function rate2disc.

Disc = rate2disc(HJMTree.TimeSpec.Compounding, ...
HJMTree.RateSpec.Rates, HJMTree.RateSpec.EndTimes, ...
HJMTree.RateSpec.StartTimes);
FRates = 1./Disc

FRates =
1.0356
1.0468
1.0523
1.0563

Second Node. The second node represents the first-rate observation time, tObs \(=1\). This node displays two states: one representing the branch going up and the other representing the branch going down.

Note that HJMTree.VolSpec.NumBranch \(=2\).
```

HJMTree.VolSpec
ans =
FinObj: 'HJMVolSpec'
FactorModels: {'Constant'}
FactorArgs: {{1x1 cell}}
SigmaShift: 0
NumFactors: 1
NumBranch: 2
PBranch: [0.5000 0.5000]
Fact2Branch: [-1 1]
Examine the rates of the node corresponding to the up branch.

```
```

HJMTree.FwdTree{2}(:,:,1)

```
HJMTree.FwdTree{2}(:,:,1)
ans =
    1.0364
    1.0420
    1.0461
Now examine the corresponding down branch.
HJMTree.FwdTree\{2\}(:,:,2)
ans =
1.0574
1.0631
1.0672
```

Third Node. The third node represents the second observation time, tObs $=2$. This node contains a total of four states, two representing the branches going up and the other two representing the branches going down. Examine the rates of the node corresponding to the up states.

HJMTree.FwdTree\{3\}(: , : , 1)
ans =

```
1.0317 1.0526
1.0358 1.0568
```

Next examine the corresponding down states.
HJMTree.FwdTree\{3\}(: , : , 2)
ans =
1.05261 .0738
1.05681 .0781

Isolating a Specific Node. Starting at the third level, indexing within the tree cell array becomes complex, and isolating a specific node can be difficult. The function bushpath isolates a specific node by specifying the path to the node as a vector of branches taken to reach that node. As an example, consider the node reached by starting from the root node, taking the branch up, then the branch down, and then another branch down. Given that the tree has only two branches per node, branches going up correspond to a 1 , and branches going down correspond to a 2 . The path up-down-down becomes the vector [1 2 2].

FRates = bushpath(HJMTree.FwdTree, [lll)
FRates =
1.0356
1.0364
1.0526
1.0674
bushpath returns the spot rates for all the nodes touched by the path specified in the input argument, the first one corresponding to the root node, and the last one corresponding to the target node.

Isolating the same node using direct indexing obtains
HJMTree.FwdTree\{4\}(:, 3, 2)
ans =

As expected, this single value corresponds to the last element of the rates returned by bushpath.

You can use these techniques with any type of tree generated with Financial Instruments Toolbox software, such as forward-rate trees or price trees.

## BDT Tree Structure

You can now examine in some detail the contents of the BDTTree structure.

```
BDTTree
```

```
BDTTree =
    FinObj: 'BDTFwdTree
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        t0bs: [\begin{array}{lllll}{0}&{1.00}&{2.00}&{3.00}\end{array}]
    TFwd: {[4x1 double] [3\times1 double] [2x1 double] [3.00]}
    CFlowT: {[4\times1 double] [3\times1 double] [2\times1 double] [4.00]}
    FwdTree: {[1.10] [1.10 1.14] [1.10 1.14 1.19] [ll.09 1.12 1.16 1.22]}
```

FwdTree contains the actual rate tree. MATLAB software represents it as a cell array with each cell array element containing a tree level.

The other fields contain other information relevant to interpreting the values in FwdTree. The most important are VolSpec, TimeSpec, and RateSpec, which contain the volatility, time structure, and rate structure information respectively.

Look at the RateSpec structure used in generating this tree to see where these values originate. Arrange the values in a single array.
[BDTTree.RateSpec.StartTimes BDTTree.RateSpec.EndTimes... BDTTree.RateSpec.Rates]

| 0 | 1.0000 | 0.1000 |
| :--- | :--- | :--- |
| 0 | 2.0000 | 0.1100 |
| 0 | 3.0000 | 0.1200 |
| 0 | 4.0000 | 0.1250 |

Look at the rates in FwdTree. The first node represents the valuation date, tObs $=0$. The second node represents tObs $=1$. Examine the rates at the second, third, and fourth nodes.

BDTTree.FwdTree\{2\}
ans =

$$
1.0979 \quad 1.1432
$$

The second node represents the first observation time, tObs $=1$. This node contains a total of two states, one representing the branch going up (1.0979) and the other representing the branch going down (1.1432).

Note The convention in this document is to display prices going up on the upper branch. Consequently, when displaying rates, rates are falling on the upper branch and increasing on the lower branch.

BDTTree.FwdTree\{3\}
ans =
$1.0976 \quad 1.1377 \quad 1.1942$
The third node represents the second observation time, tObs $=2$. This node contains a total of three states, one representing the branch going up (1.0976), one representing the branch in the middle (1.1377) and the other representing the branch going down (1.1942).

BDTTree.FwdTree\{4\}
ans =

### 1.08721 .11831 .16061 .2179

The fourth node represents the third observation time, tObs $=3$. This node contains a total of four states, one representing the branch going up (1.0872), two representing the branches in the middle (1.1183 and 1.1606), and the other representing the branch going down (1.2179).

Isolating a Specific Node. The function treepath isolates a specific node by specifying the path to the node as a vector of branches taken to reach that node. As an example, consider the node reached by starting from the root node, taking the branch up, then the branch down, and finally another branch down. Given that the tree has only two branches per node, branches going up correspond to a 1 , and branches going down correspond to a 2 . The path up-down-down becomes the vector [ $\left.\begin{array}{lll}1 & 2 & 2\end{array}\right]$.


FRates =
1.1000
1.0979
1.1377
1.1606
treepath returns the short rates for all the nodes touched by the path specified in the input argument, the first one corresponding to the root node, and the last one corresponding to the target node.

## HW and BK Tree Structures

The HW and BK tree structures are similar to the BDT tree structure. You can see this if you examine the sample HW tree contained in the file deriv.mat.
load deriv.mat;
HWTree

```
HWTree =
    FinObj: 'HWFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
```

```
RateSpec: [1x1 struct]
    t0bs: [0 1.00 2.00 3.00]
    dObs: [731947.00 732313.00 732678.00 733043.00]
    CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4.00]}
    Probs: {[3\times1 double] [3x3 double] [3\times5 double]}
Connect: {[2.00] [2.00 3.00 4.00] [2.00 2.00 3.00 4.00 4.00]}
FwdTree: {[1.03] [1.05 1.04 1.02] [1.08 1.07 1.05 1.03 1.01] [1.09 1.08 1.06 1.04 1.02]
```

All fields of this structure are similar to their BDT counterparts. There are two additional fields not present in BDT: Probs and Connect. The Probs field represents the occurrence probabilities at each branch of each node in the tree. The Connect field describes the connectivity of the nodes of a given tree level to nodes to the next tree level.

Probs Field. While BDT and one-factor HJM models have equal probabilities for each branch at a node, HW and BK do not. For HW and BK trees, the Probs field indicates the likelihood that a particular branch will be taken in moving from one node to another node on the next level.

The Probs field consists of a cell array with 1 cell per tree level. Each cell is a 3 -by-NUMNODES array with the top row representing the probability of an up movement, the middle row representing the probability of a middle movement, and the last row the probability of a down movement.

As an illustration, consider the first two elements of the Probs field of the structure, corresponding to the first (root) and second levels of the tree.

HWTree. Probs \{1\}

> 0.16666666666667
> 0.6666666666667
> 0.16666666666667

```
HWTree.Probs{2}
```

| 0.12361333418768 | 0.1666666666667 | 0.21877591615172 |
| :--- | :--- | :--- |
| 0.65761074966060 | 0.66666666666667 | 0.65761074966060 |
| 0.21877591615172 | 0.16666666666667 | 0.12361333418768 |

Reading from top to bottom, the values in HWTree. Probs $\{1\}$ correspond to the up, middle, and down probabilities at the root node.

HWTree. Probs \{2\} is a 3-by-3 matrix of values. The first column represents the top node, the second column represents the middle node, and the last column represents the bottom node. As with the root node, the first, second, and third rows hold the values for up, middle, and down branching off each node.

As expected, the sum of all the probabilities at any node equals 1.

```
sum(HWTree.Probs{2})
```

$1.0000 \quad 1.0000 \quad 1.0000$
Connect Field. The other field that distinguishes HW and BK tree structures from the BDT tree structure is Connect. This field describes how each node in a given level connects to the nodes of the next level. The need for this field arises from the possibility of nonstandard branching in a tree.

The Connect field of the HW tree structure consists of a cell array with 1 cell per tree level.

HWTree. Connect
ans =
[2] [1x3 double] [1x5 double]
Each cell contains a 1-by-NUMNODES vector. Each value in the vector relates to a node in the corresponding tree level and represents the index of the node in the next tree level that the middle branch of the node connects to.

If you subtract 1 from the values contained in Connect, you reveal the index of the nodes in the next level that the up branch connects to. If you add 1 to the values, you reveal the index of the corresponding down branch.

As an illustration, consider HWTree.Connect \{1\}:

```
HWTree.Connect{1}
```

ans =

2

This indicates that the middle branch of the root node connects to the second (from the top) node of the next level, as expected. If you subtract 1 from this value, you obtain 1 , which tells you that the up branch goes to the top node. If you add 1 , you obtain 3 , which points to the last node of the second level of the tree.

Now consider level 3 in this example:
HWTree.Connect $\{3\}$
$\begin{array}{lllll}2 & 2 & 3 & 4 & 4\end{array}$
On this level, there is nonstandard branching. This can be easily recognized because the middle branch of two nodes is connected to the same node on the next level.

To visualize this, consider the following illustration of the tree.


Here it becomes apparent that there is nonstandard branching at the third level of the tree, on the top and bottom nodes. The first and second nodes connect to the same trio of nodes on the next level. Similar branching occurs at the bottom and next-to-bottom nodes of the tree.

## Pricing Using Interest-Rate Tree Models

In this section...<br>"Introduction" on page 2-81<br>"Computing Instrument Prices" on page 2-81<br>"Computing Instrument Sensitivities" on page 2-90<br>"Calibrating Hull-White Model Using Market Data" on page 2-93

## Introduction

For purposes of illustration, this section relies on the HJM and BDT models. The HW and BK functions that perform price and sensitivity computations are not explicitly shown here. Functions that use the HW and BK models operate similarly to the BDT model.

## Computing Instrument Prices

The portfolio pricing functions hjmprice and bdtprice calculate the price of any set of supported instruments, based on an interest-rate tree. The functions are capable of pricing these instrument types:

- Bonds
- Bond options
- Bond with embedded options
- Arbitrary cash flows
- Fixed-rate notes
- Floating-rate notes
- Floating-rate notes with options or embedded options
- Caps
- Floors
- Range Notes
- Swaps
- Swaptions

For example, the syntax for calling hjmprice is:
[Price, PriceTree] = hjmprice(HJMTree, InstSet, Options)
Similarly, the calling syntax for bdtprice is:

```
[Price, PriceTree] = bdtprice(BDTTree, InstSet, Options)
```

Each function requires two input arguments: the interest-rate tree and the set of instruments, InstSet. An optional argument Options further controls the pricing and the output displayed. (See Appendix B, "Derivatives Pricing Options" for information about the Options argument.)

HJMTree is the Heath-Jarrow-Morton tree sampling of a forward-rate process, created using hjmtree. BDTTree is the Black-Derman-Toy tree sampling of an interest-rate process, created using bdttree. See "Building a Tree of Forward Rates" on page 2-61 to learn how to create these structures.

InstSet is the set of instruments to be priced. This structure represents the set of instruments to be priced independently using the model.

Options is an options structure created with the function derivset. This structure defines how the tree is used to find the price of instruments in the portfolio, and how much additional information is displayed in the command window when calling the pricing function. If this input argument is not specified in the call to the pricing function, a default Options structure is used. The pricing options structure is described in "Pricing Options Structure" on page $B-2$.

The portfolio pricing functions classify the instruments and call the appropriate instrument-specific pricing function for each of the instrument types. The HJM instrument-specific pricing functions are bondbyhjm, cfbyhjm, fixedbyhjm, floatbyhjm, optbndbyhjm, rangefloatbyhjm, swapbyhjm, and swaptionbyhjm. A similarly named set of functions exists for BDT models. You can also use these functions directly to calculate the price of sets of instruments of the same type.

## HJM Pricing Example

Consider the following example, which uses the portfolio and interest-rate data in the MAT-file deriv.mat included in the toolbox. Load the data into the MATLAB workspace.
load deriv.mat

Use the MATLAB whos command to display a list of the variables loaded from the MAT-file.
whos

| Name | Size | Bytes Class Attributes |  |
| :--- | :--- | ---: | :--- |
|  |  |  |  |
| BDTInstSet | $1 \times 1$ | 15956 | struct |
| BDTTree | $1 \times 1$ | 5138 | struct |
| BKInstSet | $1 \times 1$ | 15946 | struct |
| BKTree | $1 \times 1$ | 5904 | struct |
| CRRInstSet | $1 \times 1$ | 12434 | struct |
| CRRTree | $1 \times 1$ | 5058 | struct |
| EQPInstSet | $1 \times 1$ | 12434 | struct |
| EQPTree | $1 \times 1$ | 5058 | struct |
| HJMInstSet | $1 \times 1$ | 5948 | struct |
| HJMTree | $1 \times 1$ | 15946 | struct |
| HWInstSet | $1 \times 1$ | 5904 | struct |
| HWTree | $1 \times 1$ | 12438 | struct |
| ITTInstSet | $1 \times 1$ | 8862 | struct |
| ITTTree | $1 \times 1$ | 10282 | struct |
| ZeroInstSet | $1 \times 1$ | 1580 | struct |

HJMTree and HJMInstSet are the input arguments required to call the function hjmprice.

Use the function instdisp to examine the set of instruments contained in the variable HJMInstSet.

## instdisp(HJMInstSet)



| Index | Type CouponRate Settle | Maturity | Period Basis | $\ldots$ | Name | Quantity |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | Bond 0.04 | 01-Jan-2000 | 01-Jan-2003 | 1 | NaN | $\ldots$ | $4 \%$ | bond |
| 2 | Bond 0.04 | 01-Jan-2000 | 01-Jan-2004 | 2 | NaN | $\ldots$ | $4 \%$ bond | 50 |

Note that there are eight instruments in this portfolio set: two bonds, one bond option, one fixed-rate note, one floating-rate note, one cap, one floor, and one swap. Each instrument has a corresponding index that identifies the instrument prices in the price vector returned by hjmprice.

Now use hjmprice to calculate the price of each instrument in the instrument set.

Price = hjmprice(HJMTree, HJMInstSet)

```
Warning: Not all cash flows are aligned with the tree. Result will
be approximated
Price =
    98.7159
    97.5280
        0.0486
        98.7159
    100.5529
        6.2831
        0.0486
        3.6923
```

Note The warning shown above appears because some of the cash flows for the second bond do not fall exactly on a tree node.

## BDT Pricing Example

Load the MAT-file deriv.mat into the MATLAB workspace.
load deriv.mat

Use the MATLAB whos command to display a list of the variables loaded from the MAT-file.
whos

| Name | Size | Bytes | Class | Attributes |
| :--- | :--- | ---: | :--- | :--- |
|  |  |  |  |  |
| BDTInstSet | $1 \times 1$ | 15956 | struct |  |
| BDTTree | $1 \times 1$ | 5138 | struct |  |
| BKInstSet | $1 \times 1$ | 15946 | struct |  |
| BKTree | $1 \times 1$ | 5904 | struct |  |
| CRRInstSet | $1 \times 1$ | 12434 | struct |  |
| CRRTree | $1 \times 1$ | 5058 | struct |  |
| EQPInstSet | $1 \times 1$ | 12434 | struct |  |
| EQPTree | $1 \times 1$ | 5058 | struct |  |


| HJMInstSet | $1 \times 1$ | 15948 | struct |
| :--- | :--- | ---: | :--- |
| HJMTree | $1 \times 1$ | 5838 | struct |
| HWInstSet | $1 \times 1$ | 15946 | struct |
| HWTree | $1 \times 1$ | 5904 | struct |
| ITTInstSet | $1 \times 1$ | 12438 | struct |
| ITTTree | $1 \times 1$ | 8862 | struct |
| ZeroInstSet | $1 \times 1$ | 10282 | struct |
| ZeroRateSpec | $1 \times 1$ | 1580 | struct |

BDTTree and BDTInstSet are the input arguments required to call the function bdtprice.

Use the function instdisp to examine the set of instruments contained in the variable BDTInstSet.

## instdisp(BDTInstSet)



Note that there are eight instruments in this portfolio set: two bonds, one bond option, one fixed-rate note, one floating-rate note, one cap, one floor, and one swap. Each instrument has a corresponding index that identifies the instrument prices in the price vector returned by bdtprice.

Now use bdtprice to calculate the price of each instrument in the instrument set.

```
Price = bdtprice(BDTTree, BDTInstSet)
```

Warning: Not all cash flows are aligned with the tree. Result will
be approximated.
Price =
95.5030
93.9079
1.7657
95.5030
100.4865
1.4863
0.0245
7.4222

## Price Vector Output

The prices in the output vector Price correspond to the prices at observation time zero (tObs $=0$ ), which is defined as the valuation date of the interest-rate tree. The instrument indexing within Price is the same as the indexing within InstSet.

In the HJM example, the prices in the Price vector correspond to the instruments in this order.

```
InstNames = instget(HJMInstSet, 'FieldName','Name')
InstNames =
4% bond
4% bond
Option 101
```

```
4% Fixed
20BP Float
3% Cap
3% Floor
6%/20BP Swap
```

Consequently, in the Price vector, the fourth element, 98.7159, represents the price of the fourth instrument ( $4 \%$ fixed-rate note); the sixth element, 6.2831 , represents the price of the sixth instrument ( $3 \%$ cap).

In the BDT example, the prices in the Price vector correspond to the instruments in this order.

InstNames = instget(BDTInstSet, 'FieldName','Name')

InstNames =

10\% Bond
10\% Bond
Option 95
10\% Fixed
20BP Float
15\% Cap
9\% Floor
15\%/10BP Swap
Consequently, in the Price vector, the fourth element, 95.5030, represents the price of the fourth instrument ( $10 \%$ fixed-rate note); the sixth element, 1.4863 , represents the price of the sixth instrument ( $15 \%$ cap).

## Price Tree Structure Output

If you call a pricing function with two output arguments, for example,
[Price, PriceTree] = hjmprice(HJMTree, HJMInstSet)
you generate a price tree along with the price information.
The optional output price tree structure PriceTree holds all the pricing information.

HJM Price Tree. In the HJM example, the first field of this structure, FinObj, indicates that this structure represents a price tree. The second field, PBush, is the tree holding the price of the instruments in each node of the tree. The third field, AIBush, is the tree holding the accrued interest of the instruments in each node of the tree. Finally, the fourth field, tObs, represents the observation time of each level of PBush and AIBush, with units in terms of compounding periods.

In this example, the price tree looks like

```
PriceTree =
FinObj: 'HJMPriceTree'
    PBush: {[8x1 double] [8x1x2 double] ...[8x8 double]}
AIBush: {[8x1 double] [8x1x2 double] ... [8x8 double]}
    tObs: [[0 1 1 2 3 4]
```

Both PBush and AIBush are 1-by-5 cell arrays, consistent with the five observation times of tObs. The data display has been shortened here to fit on a single line.

Using the command line interface, you can directly examine PriceTree. PBush, the field within the PriceTree structure that contains the price tree with the price vectors at every state. The first node represents tObs $=0$, corresponding to the valuation date.

PriceTree. PBush\{1\}
ans =
98.7159
97.5280
0.0486
98.7159
100.5529
6.2831
0.0486
3.6923

With this interface, you can observe the prices for all instruments in the portfolio at a specific time.

BDT Price Tree. The BDT output price tree structure PriceTree holds all the pricing information. The first field of this structure, FinObj, indicates that this structure represents a price tree. The second field, PTree, is the tree holding the price of the instruments in each node of the tree. The third field, AITree, is the tree holding the accrued interest of the instruments in each node of the tree. The fourth field, tObs, represents the observation time of each level of PTree and AITree, with units in terms of compounding periods.

You can directly examine the field within the PriceTree structure, which contains the price tree with the price vectors at every state. The first node represents tObs $=0$, corresponding to the valuation date.

```
[Price, PriceTree] = bdtprice(BDTTree, BDTInstSet)
PriceTree.PTree{1}
ans =
    95.5030
    93.9079
        1.7657
        95.5030
        100.4865
        1.4863
        0.0245
        7.4222
```


## Computing Instrument Sensitivities

Sensitivities can be reported either as dollar price changes or percentage price changes. The delta, gamma, and vega sensitivities that the toolbox computes are dollar sensitivities.

The functions hjmsens and bdtsens compute the delta, gamma, and vega sensitivities of instruments using an interest-rate tree. They also optionally return the calculated price for each instrument. The sensitivity functions require the same two input arguments used by the pricing functions (HJMTree and HJMInstSet for HJM; BDTTree and BDTInstSet for BDT).

Sensitivity functions calculate the dollar value of delta and gamma by shifting the observed forward yield curve by 100 basis points in each direction, and
the dollar value of vega by shifting the volatility process by $1 \%$. To obtain the per-dollar value of the sensitivities, divide the dollar sensitivity by the price of the corresponding instrument.

## HJM Sensitivities Example

The calling syntax for the function is:

```
[Delta, Gamma, Vega, Price] = hjmsens(HJMTree, HJMInstSet)
```

Use the previous example data to calculate the price of instruments.

```
load deriv.mat
[Delta, Gamma, Vega, Price] = hjmsens(HJMTree, HJMInstSet);
Warning: Not all cash flows are aligned with the tree. Result will
be approximated.
```

Note The warning appears because some of the cash flows for the second bond do not fall exactly on a tree node.

You can conveniently examine the sensitivities and the prices by arranging them into a single matrix.

All = [Delta, Gamma, Vega, Price]

All =

| -272.65 | 1029.90 | 0.00 | 98.72 |
| ---: | ---: | ---: | ---: |
| -347.43 | 1622.69 | -0.04 | 97.53 |
| -8.08 | 643.40 | 34.07 | 0.05 |
| -272.65 | 1029.90 | 0.00 | 98.72 |
| -1.04 | 3.31 | 0 | 100.55 |
| 294.97 | 6852.56 | 93.69 | 6.28 |
| -47.16 | 8459.99 | 93.69 | 0.05 |
| -282.05 | 1059.68 | 0.00 | 3.69 |

As with the prices, each row of the sensitivity vectors corresponds to the similarly indexed instrument in HJMInstSet. To view the per-dollar
sensitivities, divide each dollar sensitivity by the corresponding instrument price.

## BDT Sensitivities Example

The calling syntax for the function is:
[Delta, Gamma, Vega, Price] = bdtsens(BDTTree, BDTInstSet);
Arrange the sensitivities and prices into a single matrix.
All = [Delta, Gamma, Vega, Price]

All $=$

| -232.67 | 803.71 | -0.00 | 95.50 |
| ---: | ---: | ---: | ---: |
| -281.05 | 1181.93 | -0.01 | 93.91 |
| -50.54 | 246.02 | 5.31 | 1.77 |
| -232.67 | 803.71 | 0 | 95.50 |
| 0.84 | 2.45 | 0 | 100.49 |
| 78.38 | 748.98 | 13.54 | 1.49 |
| -4.36 | 382.06 | 2.50 | 0.02 |
| -253.23 | 863.81 | 0 | 7.42 |

To view the per-dollar sensitivities, divide each dollar sensitivity by the corresponding instrument price.

All = [Delta ./ Price, Gamma ./ Price, Vega ./ Price, Price]

All =

| -2.44 | 8.42 | -0.00 | 95.50 |
| ---: | ---: | ---: | ---: |
| -2.99 | 12.59 | -0.00 | 93.91 |
| -28.63 | 139.34 | 3.01 | 1.77 |
| -2.44 | 8.42 | 0 | 95.50 |
| 0.01 | 0.02 | 0 | 100.49 |
| 52.73 | 503.92 | 9.11 | 1.49 |
| -177.89 | 15577.42 | 101.87 | 0.02 |
| -34.12 | 116.38 | 0 | 7.42 |

## Calibrating Hull-White Model Using Market Data

The pricing of interest rate derivative securities relies on models that describe the underlying process. These interest rate models depend on one or more parameters that you must determine by matching the model predictions to the existing data available in the market. In the Hull-White model, there are two parameters related to the short rate process: mean reversion and volatility. Calibration is used to determine these parameters, such that the model can reproduce, as close as possible, the prices of caps or floors observed in the market. The calibration routines find the parameters that minimize the difference between the model price predictions and the market prices for caps and floors.

For a Hull-White model, the minimization is two dimensional, with respect to mean reversion ( $\alpha$ ) and volatility ( $\sigma$ ). That is, calibrating the Hull-White model minimizes the difference between the model's predicted prices and the observed market prices of the corresponding caplets or floorlets.

## Hull-White Model Calibration Example

Use market data to identify the implied volatility ( $\sigma$ ) and mean reversion (a) coefficients needed to build a Hull-White tree to price an instrument. The ideal case is to use the volatilities of the caps or floors used to calculate Alpha (a) and Sigma ( $\sigma$ ). This will most likely not be the case, so market data must be interpolated to obtain the required values.

Consider a cap with these parameters:

```
Settle = ' Jan-21-2008';
Maturity = 'Mar-21-2011';
Strike = 0.0690;
Reset = 4;
Principal = 1000;
Basis = 0;
```

The caplets for this example would fall in:

```
capletDates = cfdates(Settle, Maturity, Reset, Basis);
datestr(capletDates')
ans =
```

```
21-Mar-2008
21-Jun-2008
21-Sep-2008
21-Dec-2008
21-Mar-2009
21-Jun-2009
21-Sep-2009
21-Dec-2009
21-Mar-2010
21-Jun-2010
21-Sep-2010
21-Dec-2010
21-Mar-2011
```

In the best case, look up the market volatilities for caplets with a Strike = 0.0690 , and maturities in each reset date listed, but the likelihood of finding these exact instruments is low. As a consequence, use data that is available in the market and interpolate to find appropriate values for the caplets.

Based on the market data, you have the cap information for different dates and strikes. Assume that instead of having the data for Strike $=0.0690$, you have the data for Strike1 $=0.0590$ and Strike2 $=0.0790$

| Maturity | Strike $\mathbf{1}=\mathbf{0 . 0 5 9 0}$ | Strike2 = 0.0790 |
| :--- | :--- | :--- |
| 21-Mar-2008 | 0.1533 | 0.1526 |
| 21-Jun-2008 | 0.1731 | 0.1730 |
| 21-Sep-2008 | 0.1727 | 0.1726 |
| 21-Dec-2008 | 0.1752 | 0.1747 |
| 21-Mar-2009 | 0.1809 | 0.1808 |
| 21-Jun-2009 | 0.1809 | 0.1792 |
| 21-Sep-2009 | 0.1805 | 0.1797 |
| 21-Dec-2009 | 0.1802 | 0.1794 |
| 21-Mar-2010 | 0.1802 | 0.1733 |
| 21-Jun-2010 | 0.1757 | 0.1751 |
| 21-Sep-2010 | 0.1755 | 0.1750 |


| Maturity | Strike $\mathbf{1}=\mathbf{0 . 0 5 9 0}$ | Strike2 $=\mathbf{0 . 0 7 9 0}$ |
| :--- | :--- | :--- |
| 21-Dec-2010 | 0.1755 | 0.1745 |
| 21-Mar-2011 | 0.1726 | 0.1719 |

The nature of this data lends itself to matrix nomenclature, which is perfect for MATLAB. hwcalbycap requires that the dates, the strikes, and the actual volatility be separated into three variables: MarketStrike, MarketMat, and MarketVol.

```
MarketStrike = [0.0590; 0.0790];
MarketMat = {'21-Mar-2008';
'21-Jun-2008';
'21-Sep-2008';
'21-Dec-2008';
'21-Mar-2009';
'21-Jun-2009';
'21-Sep-2009';
'21-Dec-2009';
'21-Mar-2010';
'21-Jun-2010';
'21-Sep-2010';
'21-Dec-2010';
'21-Mar-2011'};
```


0.17550 .17550 .1726 ; First column in table corresponding to Strike1
$0.15260 .17300 .17260 .1747 \quad 0.1808 \quad 0.17920 .17970 .1794 \quad 0.1733 \quad 0.1751 \ldots$
$0.17500 .17450 .1719]$ \% Second column in table corresponding to Strike2

Complete the input arguments using this data for RateSpec:

```
Rates= [0.0627;
```

0.0657 ;
0.0691 ;
0.0717 ;
0.0739 ;
0.0755 ;
0.0765 ;

```
0.0772;
0.0779;
0.0783;
0.0786;
0.0789;
0.0792;
0.0793];
ValuationDate = '21-Jan-2008';
EndDates = {'21-Mar-2008';'21-Jun-2008';'21-Sep-2008';'21-Dec-2008'; ...
    '21-Mar-2009';'21-Jun-2009';'21-Sep-2009';'21-Dec-2009'; ....
    '21-Mar-2010';'21-Jun-2010';'21-Sep-2010';'21-Dec-2010'; ....
    '21-Mar-2011';'21-Jun-2011'};
Compounding = 4;
Basis = 0;
RateSpec = intenvset('ValuationDate', ValuationDate, ...
'StartDates', ValuationDate, 'EndDates', EndDates, ...
'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis);
RateSpec =
                    Fin0bj: 'RateSpec'
        Compounding: 4
            Disc: [14x1 double]
                    Rates: [14\times1 double]
                EndTimes: [14\times1 double]
            StartTimes: [14\times1 double]
            EndDates: [14\times1 double]
        StartDates: 733428
        ValuationDate: 733428
            Basis: 0
        EndMonthRule: 1
```

```
Call the calibration routine to find values for volatility parameters Alpha and Sigma. Use hwcalbycap to calculate the values of Alpha and Sigma based on market data. Internally, hwcalbycap calls the Optimization Toolbox function lsqnonlin. You can customize lsqnonlin by passing an optimization options structure created by optimoptions and then this can be passed to hwcalbycap using the name-value pair argument for OptimOptions. For example, optimoptions defines the target objective function tolerance as 100*eps and then calls hwcalbycap:
```

```
o=optimoptions('lsqnonlin','TolFun',100*eps);
```

o=optimoptions('lsqnonlin','TolFun',100*eps);
[Alpha, Sigma] = hwcalbycap(RateSpec, MarketStrike, MarketMat, MarketVol,...
[Alpha, Sigma] = hwcalbycap(RateSpec, MarketStrike, MarketMat, MarketVol,...
Strike, Settle, Maturity, 'Reset', Reset, 'Principal', Principal, 'Basis',...
Strike, Settle, Maturity, 'Reset', Reset, 'Principal', Principal, 'Basis',...
Basis, 'OptimOptions', o)
Basis, 'OptimOptions', o)
Local minimum possible.
Local minimum possible.
lsqnonlin stopped because the size of the current step is less than
the default value of the step size tolerance.
Warning: LSQNONLIN did not converge to an optimal solution. It exited with exitflag = 2.
> In hwcalbycapfloor at 93
In hwcalbycap at }7
Alpha =
1.0000e-06
Sigma =
0.0127

```

The previous warning indicates that the conversion was not optimal. The search algorithm used by the Optimization Toolbox \({ }^{\mathrm{TM}}\) function lsqnonlin did not find a solution that conforms to all the constraints. To discern whether the solution is acceptable, look at the results of the optimization by specifying a third output (OptimOut) for hwcalbycap:
```

[Alpha, Sigma, OptimOut] = hwcalbycap(RateSpec, MarketStrike, MarketMat,...

```
```

MarketVol, Strike, Settle, Maturity, 'Reset', Reset, 'Principal', Principal,...
'Basis', Basis, 'OptimOptions', o);

```

The OptimOut.residual field of the OptimOut structure is the optimization residual. This value contains the difference between the Black caplets and those calculated during the optimization. You can use the OptimOut.residual value to calculate the percentual difference (error) compared to Black caplet prices and then decide whether the residual is acceptable. There is almost always some residual, so decide if parametrizing the market with a single value of Alpha and Sigma is acceptable.

Price caplets using market data and Black's formula to obtain reference caplet values. To determine the effectiveness of the optimization, calculate reference caplet values using Black's formula and the market data. Note, you must first interpolate the market data to obtain the caplets for calculation:
```

MarketMatNum = datenum(MarketMat);
[Mats, Strikes] = meshgrid(MarketMatNum, MarketStrike);
FlatVol = interp2(Mats, Strikes, MarketVol, datenum(Maturity), Strike, 'spline');

```

Compute the price of the cap using the Black model:
```

[CapPrice, Caplets] = capbyblk(RateSpec, Strike, Settle, Maturity, FlatVol,...
'Reset', Reset, 'Basis', Basis, 'Principal', Principal);
Caplets = Caplets(2:end)';
Caplets =
0.3210
1.6355
2.4863
3.1903
3.4110
3.2685
3.2385
3.4803
3.2419
3.1949
3.2991
3.3750

```

\section*{Compare optimized values and Black values and display graphically.}

After calculating the reference values for the caplets, compare the values, analytically and graphically, to determine whether the calculated single values of Alpha and Sigma provide an adequate approximation:
```

OptimCaplets = Caplets+OptimOut.residual;
disp(' ');
disp(' Black76 Calibrated Caplets');
disp([Caplets OptimCaplets])
plot(MarketMatNum(2:end), Caplets, 'or', MarketMatNum(2:end), OptimCaplets, '*b');
datetick('x', 2)
xlabel('Caplet Maturity');
ylabel('Caplet Price');
title('Black and Calibrated Caplets');
h = legend('Black Caplets', 'Calibrated Caplets');
set(h, 'color', [0.9 0.9 0.9]);
set(h, 'Location', 'SouthEast');
set(gcf, 'NumberTitle', 'off')
grid on
Black76 Calibrated Caplets
0.3210 0.3636
1.6355 1.6603
2.4863 2.4974
3.1903 3.1874
3.4110 3.4040
3.2685 3.2639
3.2385 3.2364
3.4803 3.4683
3.2419 3.2408
3.1949 3.1957
3.2991 3.2960
3.3750 3.3663

```


Compare cap prices using the Black, HW analytical, and HW tree models. Using the calculated caplet values, compare the prices of the corresponding cap using the Black model, Hull-White analytical, and Hull-White tree models. To calculate a Hull-White tree based on Alpha and Sigma, use these calibration routines:
- Black model:
```

CapPriceBLK = CapPrice;

```
- HW analytical model:

CapPriceHWAnalytical = sum(OptimCaplets);
- HW tree model to price the cap derived from the calibration process:

1 Create VolSpec from the calibration parameters Alpha and Sigma:
\begin{tabular}{rl} 
VolDates & \(=\) EndDates; \\
VolCurve & \(=\) Sigma*ones \((14,1) ;\) \\
AlphaDates & \(=\) EndDates; \\
AlphaCurve & \(=\) Alpha*ones \((14,1) ;\)
\end{tabular}
```

HWVolSpec = hwvolspec(ValuationDate, VolDates, VolCurve,AlphaDates, AlphaCurve);

```

2 Create the TimeSpec:
```

HWTimeSpec = hwtimespec(ValuationDate, EndDates, Compounding);

```

3 Build the HW tree using the HW2000 method:
```

HWTree = hwtree(HWVolSpec, RateSpec, HWTimeSpec, 'Method', 'HW2000');

```

4 Price the cap:
```

Price = capbyhw(HWTree, Strike, Settle, Maturity, Reset, Basis, Principal);
disp(' ');
disp([' CapPrice Black76 ..................: ', num2str(CapPriceBLK,'%15.5f')]);
disp([' CapPrice HW analytical..........: ', num2str(CapPriceHWAnalytical,'%15.5f')]);
disp([' CapPrice HW from capbyhw ..: ', num2str(Price,'%15.5f')]);
disp(' ');
CapPrice Black76 ..........: 34.14220
CapPrice HW analytical.....: 34.18008
CapPrice HW from capbyhw :.: 34.14192

```

Price a portfolio of instruments using the calibrated HW tree. After building a Hull-White tree, based on parameters calibrated from market data, use HWTree to price a portfolio of these instruments:
- Two bonds

CouponRate = [0.07; 0.09];
Settle= ' Jan-21-2008';
Maturity = \{'Mar-21-2010';'Mar-21-2011'\};
Period = 1;
Face = 1000;
Basis = 0;
- Bond with an embedded American call option

CouponRateOEB = 0.08;
SettleOEB = ' Jan-21-2008';
MaturityOEB = 'Mar-21-2011';
```

OptSpec = 'call';
StrikeOEB = 950;
ExerciseDates0EB = 'Mar-21-2011';
AmericanOpt= 1;
Period =1;
Face = 1000;
Basis =0;

```

To price this portfolio of instruments using the calibrated HWTree:
1 Use instadd to create the portfolio InstSet:
```

InstSet = instadd('Bond', CouponRate, Settle, Maturity, Period, Basis, [], [], [], [], [], Face);
InstSet = instadd(InstSet,'OptEmBond', CouponRateOEB, SettleOEB, MaturityOEB, OptSpec,...
StrikeOEB, ExerciseDatesOEB, 'AmericanOpt', AmericanOpt, 'Period', Period,...
'Face',Face, 'Basis', Basis);

```

2 Add the cap instrument used in the calibration:
```

SettleCap = ' Jan-21-2008';
MaturityCap = 'Mar-21-2011';
StrikeCap = 0.0690;
Reset = 4;
Principal = 1000;
InstSet = instadd(InstSet,'Cap', StrikeCap, SettleCap, MaturityCap, Reset, Basis, Principal);

```

3 Assign names to the portfolio instruments:
```

Names = {'7% Bond'; '8% Bond'; 'BondEmbCall'; '6.9% Cap'};
InstSet = instsetfield(InstSet, 'Index',1:4, 'FieldName', {'Name'}, 'Data', Names );

```

4 Examine the set of instruments contained in InstSet:
```

instdisp(InstSet)

```
IdxType CoupRate Settle Mature Period Basis EOMRule IssueDate 1stCoupDate LastCoupDate StartDate Face Nam
\begin{tabular}{llllllllllllll}
1 & Bond 0.07 & 21-Jan-2008 & \(21-M a r-2010\) & 1 & 0 & \(N a N\) & \(N a N\) & \(N a N\) & NaN & NaN & 1000 & \(7 \%\) & Bond \\
2 & Bond 0.09 & \(21-J a n-2008\) & \(21-M a r-2011\) & 1 & 0 & NaN & NaN & NaN & NaN & NaN & 1000 & \(8 \%\) & Bond
\end{tabular}
```

IdxType CoupRate Settle Mature OptSpec Stke ExDate Per Basis EOMRule IssDate 1stCoupDate LstCoupDate Strt
3 OptEmBond 0.08 21-Jan-2008 21-Mar-2011 call 950 21-Jan-2008 21-Mar-2011 1 0 1 NaN NaN NaN NaN
Index Type Strike Settle Maturity CapReset Basis Principal Name
4 Cap 0.069 21-Jan-2008 21-Mar-2011 4 0 1000 6.9% Cap

```

5 Use hwprice to price the portfolio using the calibrated HWTree:
```

format bank
PricePortfolio =
980.45
1023.05
945.73
34.14

```
PricePortfolio = hwprice(HWTree, InstSet)

\section*{Interest-Rate Derivatives Using Closed-Form Solutions}

\section*{Pricing Caps and Floors Using the Black Option Model}

Caps and floors are contracts that allow the holder to be protected if interest rates rise or decrease. The Black model uses a forward price as an underlier in place of a spot price. The assumption is that the forward price at maturity of the option is log-normally distributed.

Closed-form solutions for pricing caps and floors using the Black model support the following tasks:
\begin{tabular}{l|l}
\hline Task & Function \\
\hline \begin{tabular}{l} 
Price the interest rate caps using the Black \\
option pricing model.
\end{tabular} & capbyblk \\
\hline \begin{tabular}{l} 
Price the interest rate floors using the Black \\
option pricing model.
\end{tabular} & floorbyblk \\
\hline
\end{tabular}

\section*{Price Swaptions With Interest-Rate Models Using Simulation}

\section*{In this section...}
"Introduction" on page 2-105
"Construct Zero Curve" on page 2-106
"Define Swaption Parameters" on page 2-108
"Compute the Black Model and the Swaption Volatility Matrix" on page 2-108
"Select Calibration Instruments" on page 2-109
"Compute Swaption Prices Using Black's Model" on page 2-109
"Define Simulation Parameters" on page 2-110
"Simulate Interest-Rate Paths Using the Hull-White One-Factor Model" on page 2-111
"Simulate Interest-Rate Paths Using the Linear Gaussian Two-Factor Model" on page 2-114
"Simulate Interest-Rate Paths Using the LIBOR Market Model" on page 2-118
"Compare Interest-Rate Modeling Results" on page 2-125
"References" on page 2-125

\section*{Introduction}

This example shows how to price European swaptions using interest-rate models in Financial Instruments Toolbox. Specifically, a Hull-White one factor model, a Linear Gaussian two-factor model, and a LIBOR Market Model are calibrated to market data and then used to generate interest-rate paths using Monte Carlo simulation.

The following sections set up the data that is then used with examples for "Simulate Interest-Rate Paths Using the Hull-White One-Factor Model" on page 2-111, "Simulate Interest-Rate Paths Using the Linear Gaussian

Two-Factor Model" on page 2-114, and "Simulate Interest-Rate Paths Using the LIBOR Market Model" on page 2-118:
- "Construct Zero Curve" on page 2-106
- "Define Swaption Parameters" on page 2-108
- "Compute the Black Model and the Swaption Volatility Matrix" on page 2-108
- "Select Calibration Instruments" on page 2-109
- "Compute Swaption Prices Using Black's Model" on page 2-109
- "Define Simulation Parameters" on page 2-110

\section*{Construct Zero Curve}

This example shows how to use ZeroRates for a zero curve that is hard-coded. You can also create a zero curve by bootstrapping the zero curve from market data (for example, deposits, futures/forwards, and swaps)

The hard-coded data for the zero curve is defined as:
```

Settle = datenum('21-Jul-2008');
% Zero Curve
CurveDates = daysadd(Settle,360*([1 3 5 7 10 20]),1);
ZeroRates = [1.9 2.6 3.1 3.5 4 4.3]'/100;
plot(CurveDates,ZeroRates)
datetick
title(['Zero Curve for ' datestr(Settle)]);

```


Construct an IRCurve object.
irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);
Construct the RateSpec.
RateSpec = intenvset('Rates',ZeroRates,'EndDates',CurveDates,'StartDate',Se

\section*{RateSpec =}
```

    FinObj: 'RateSpec'
    Compounding: 2
            Disc: [6x1 double]
            Rates: [6x1 double]
            EndTimes: [6x1 double]
    StartTimes: [6x1 double]
    EndDates: [6x1 double]
    StartDates: 733610
    ValuationDate: 733610
Basis: 0
EndMonthRule: 1

```

\section*{Define Swaption Parameters}

While Monte Carlo simulation is typically used to value more sophisticated derivatives (for example, Bermudan swaptions), in this example, the price of a European swaption is computed with an exercise date of 5 years and an underlying swap of 5 years.
```

InstrumentExerciseDate = datenum('21-Jul-2013');
InstrumentMaturity = datenum('21-Jul-2018');
InstrumentStrike = .045;

```

\section*{Compute the Black Model and the Swaption Volatility Matrix}

Black's model is often used to price and quote European exercise interest-rate options, that is, caps, floors and swaptions. In the case of swaptions, Black's model is used to imply a volatility given the current observed market price. The following matrix shows the Black implied volatility for a range of swaption exercise dates (columns) and underlying swap maturities (rows).
```

SwaptionBlackVol = [22 21 19 17 15 13 12
21 19 17 16 15 13 11
20}18181615151412 11
19 17 15 14 13 12 10
18 16 14 13 12 11 10
15 14 13 12 12 11 10
13 13 12 11 11 10 9]/100;
ExerciseDates = [1:5 7 10];

```
```

Tenors = [1:5 7 10];
EurExDatesFull = repmat(daysadd(Settle,ExerciseDates*360,1)',...
length(Tenors),1);
EurMatFull = reshape(daysadd(EurExDatesFull,...
repmat(360*Tenors,1,length(ExerciseDates)),1),size(EurExDatesFull));

```

\section*{Select Calibration Instruments}

Selecting the instruments to calibrate the model to is one of the tasks in calibration. For Bermudan swaptions, it is typical to calibrate to European swaptions that are co-terminal with the Bermudan swaption to be priced. In this case, all swaptions having an underlying tenor that matures before the maturity of the swaption to be priced (21-Jul-2018) are used in the calibration.
```

% Find the swaptions that expire on or before the maturity date of the
% sample swaption
relidx = find(EurMatFull <= InstrumentMaturity);

```

\section*{Compute Swaption Prices Using Black's Model}

This example shows how to compute swaption prices using Black's Model. The swaption prices are then used to compare the model's predicted values that are obtained from the calibration process.

To compute the swaption prices using Black's model:
```

SwaptionBlackPrices = zeros(size(SwaptionBlackVol));
SwaptionStrike = zeros(size(SwaptionBlackVol));
for iSwaption=1:length(ExerciseDates)
for iTenor=1:length(Tenors)
[~,SwaptionStrike(iTenor,iSwaption)] = swapbyzero(RateSpec,[NaN 0], Settle, EurMatFull(iTenor,iSwapt
StartDate',EurExDatesFull(iTenor,iSwaption),'LegReset',[1 1]);
SwaptionBlackPrices(iTenor,iSwaption) = swaptionbyblk(RateSpec, 'call', SwaptionStrike(iTenor,iSwapt
EurExDatesFull(iTenor,iSwaption), EurMatFull(iTenor,iSwaption), SwaptionBlackVol(iTenor,iSwaptio
end
end

```

\section*{Define Simulation Parameters}

This example shows how to use the simTermStructs method with HullWhite1F, LinearGaussian2F, and LiborMarketModel objects.

To demonstrate using the simTermStructs method with HullWhite1F, LinearGaussian2F, and LiborMarketModel objects, use the following simulation parameters:
```

nPeriods = 5;
DeltaTime = 1;
nTrials = 1000;
Tenor = (1:10)';
SimDates = daysadd(Settle,360*DeltaTime*(0:nPeriods),1)
SimTimes = diff(yearfrac(SimDates(1),SimDates))

```
\% For 1 year periods and an evenly spaced tenor, the exercise row will be
\% the sixth row and the swaption maturity will be the 5 th column
exRow = 6;
endCol = 5;
SimDates =
    733610
    733975
    734340
    734705
    735071
    735436
SimTimes =
    1.0000
    1.0000
    1.0000
    1.0027
    1.0000

\section*{Simulate Interest-Rate Paths Using the Hull-White One-Factor Model}

This example shows how to simulate interest-rate paths using the Hull-White one-factor model. Before beginning this example that uses a HullWhite1F model, make sure you have set up the data as described in:
- "Construct Zero Curve" on page 2-106
- "Define Swaption Parameters" on page 2-108
- "Compute the Black Model and the Swaption Volatility Matrix" on page 2-108
- "Select Calibration Instruments" on page 2-109
- "Compute Swaption Prices Using Black's Model" on page 2-109
- "Define Simulation Parameters" on page 2-110

The Hull-White one-factor model describes the evolution of the short rate and is specified using the zero curve, alpha, and sigma parameters for the equation
\[
d r=[\theta(t)-a(t) r] d t+\sigma(t) d W
\]
where:
\(d r\) is the change in the short-term interest rate over a small interval, \(d t\).
\(r\) is the short-term interest rate.
\(\Theta(t)\) is a function of time determining the average direction in which \(r\) moves, chosen such that movements in \(r\) are consistent with today's zero coupon yield curve.
\(\alpha\) is the mean reversion rate.
\(d t\) is a small change in time.
\(\sigma\) is the annual standard deviation of the short rate.
\(W\) is the Brownian motion.

The Hull-White model is calibrated using the function swaptionbyhw, which constructs a trinomial tree to price the swaptions. Calibration consists of minimizing the difference between the observed market prices (computed above using the Black's implied swaption volatility matrix, see "Compute the Black Model and the Swaption Volatility Matrix" on page 2-108) and the model's predicted prices.

In this example, the Optimization Toolbox function lsqnonlin is used to find the parameter set that minimizes the difference between the observed and predicted values. However, other approaches (for example, simulated annealing) may be appropriate. Starting parameters and constraints for \(\alpha\) and \(\sigma\) are set in the variables \(\mathrm{x} 0, \mathrm{lb}\) and ub; these could also be varied depending upon the particular calibration approach.

Calibrate the set of parameters that minimize the difference between the observed and predicted values using swaptionbyhw and lsqnonlin.
```

TimeSpec = hwtimespec(Settle,daysadd(Settle,360*(1:11),1), 2);
HW1Fobjfun = @(x) SwaptionBlackPrices(relidx) - ...
swaptionbyhw(hwtree(hwvolspec(Settle,'11-Aug-2015',x(2),'11-Aug-2015',x(1)), RateSpec, TimeSpec), 'call'
EurExDatesFull(relidx), 0, Settle, EurMatFull(relidx));
options = optimset('disp','iter','MaxFunEvals',1000,'TolFun',1e-5);
% Find the parameters that minimize the difference between the observed and
% predicted prices
x0 = [.1 .01];
lb = [0 0];
ub = [lll}10
HW1Fparams = lsqnonlin(HW1Fobjfun, x0,lb,ub,options);
HW_alpha = HW1Fparams(1)
HW_sigma = HW1Fparams(2)

```
\begin{tabular}{|c|c|c|c|c|c|}
\hline Iteration & Func-count & \(f(x)\) & Norm of step & First-order optimality & CG-iterations \\
\hline 0 & 3 & 0.481427 & & 8.91 & \\
\hline 1 & 6 & 0.321311 & 0.00637941 & 0.188 & 0 \\
\hline 2 & 9 & 0.307638 & 0.0102855 & 17.1 & 0 \\
\hline 3 & 12 & 0.306546 & 0.0128066 & 0.0889 & 0 \\
\hline 4 & 15 & 0.306518 & 0.00342934 & 0.0245 & 0 \\
\hline
\end{tabular}
```

5 18 0.306518 5.66218e-05 0.0.0

```
Local minimum possible.
lsqnonlin stopped because the final change in the sum of squares relative to
its initial value is less than the selected value of the function tolerance.
HW_alpha =
    0.0909
HW sigma =
    0.0092

Construct the HullWhite1F model using the HullWhite1F constructor.
HW1F = HullWhite1F(RateSpec,HW_alpha,HW_sigma)
HW1F =
    HullWhite1F with properties:
    ZeroCurve: [1x1 IRDataCurve]
        Alpha: @(t,V)inAlpha
    Sigma: @(t,V)inSigma

Use Monte Carlo simulation to generate the interest-rate paths with HullWhite1F.simTermStructs.
```

HW1FSimPaths = HW1F.simTermStructs(nPeriods,'NTRIALS',nTrials,...
'DeltaTime',DeltaTime,'Tenor',Tenor,'antithetic',true);
trialIdx = 1;
figure
surf(Tenor,SimDates,HW1FSimPaths(:,:,trialIdx))
datetick y keepticks keeplimits
title(['Evolution of the Zero Curve for Trial:' num2str(trialIdx) ' of Hull
xlabel('Tenor (Years)')

```


Price the European swaption.
```

DF $=\exp ($ bsxfun(@times, - HW1FSimPaths, repmat(Tenor',[nPeriods+1 1])));
SwapRate = (1-DF(exRow,endCol,:))./sum(bsxfun(@times,1,DF(exRow, 1:endCol,:)));
PayoffValue $=100 * \max ($ SwapRate-InstrumentStrike, 0$) . * \operatorname{sum}($ bsxfun(@times, $1, \mathrm{DF}($ exRow, $1:$ endCol, : ) ) ) ;
RealizedDF $=\operatorname{prod}(\exp (b s x f u n(@ t i m e s,-H W 1 F S i m P a t h s(2: \operatorname{exRow}, 1,:), \operatorname{SimTimes}(1: \operatorname{exRow}-1))), 1)$;
HW1F_SwaptionPrice $=$ mean(RealizedDF.*PayoffValue);

```

HW1F_SwaptionPrice =
2.7649

\section*{Simulate Interest-Rate Paths Using the Linear Gaussian Two-Factor Model}

This example shows how to simulate interest-rate paths using the Linear Gaussian two-factor model. Before beginning this example that uses a LinearGaussian2F model, make sure you have set up the data as described in:
- "Construct Zero Curve" on page 2-106
- "Define Swaption Parameters" on page 2-108
- "Compute the Black Model and the Swaption Volatility Matrix" on page 2-108
- "Select Calibration Instruments" on page 2-109
- "Compute Swaption Prices Using Black's Model" on page 2-109
- "Define Simulation Parameters" on page 2-110

The Linear Gaussian two-factor model (called the G2++ by Brigo and Mercurio, see "Interest-Rate Modeling Using Monte Carlo Simulation" on page C-12) is also a short rate model, but involves two factors. Specifically:
\[
\begin{aligned}
& r(t)=x(t)+y(t)+\phi(t) \\
& d x(t)=-a(t) x(t) d t+\sigma(t) d W_{1}(t), x(0)=0 \\
& d y(t)=-b(t) y(t) d t+\eta(t) d W_{2}(t), y(0)=0
\end{aligned}
\]
where \(d W_{1}(t) d W_{2}(t)=\rho d t\) is a two-dimensional Brownian motion with correlation \(\rho\), and \(\varphi\) is a function chosen to match the initial zero curve.

The function swaptionbylg2f is used to compute analytic values of the swaption price for model parameters and consequently can be used to calibrate the model. Calibration consists of minimizing the difference between the observed market prices (computed above using the Black's implied swaption volatility matrix, see "Compute the Black Model and the Swaption Volatility Matrix" on page 2-108) and the model's predicted prices.

In this example, the approach is similar to "Simulate Interest-Rate Paths Using the Hull-White One-Factor Model" on page 2-111 and the Optimization Toolbox function lsqnonlin is used to minimize the difference between the observed swaption prices and the predicted swaption prices. However, other approaches (for example, simulated annealing) may also be appropriate. Starting parameters and constraints for \(a, b, \eta, \rho\), and \(\sigma\) are set in the variables \(\times 0,1 b\), and ub; these could also be varied depending upon the particular calibration approach.

Calibrate the set of parameters that minimize the difference between the observed and predicted values using swaptionbylg2f and lsqnonlin.

G2PPobjfun \(=@(x)\) SwaptionBlackPrices(relidx) - swaptionbylg2f(irdc, \(x(1), x(2), x(3), x(4), x(5)\), SwaptionStrike( EurExDatesFull(relidx), EurMatFull(relidx), 'Reset', 1);
\(x_{0}=\left[\begin{array}{llll}\text {. } 2 & .1 & .02 & .01 \\ -.5\end{array}\right]\);
\(l b=\left[\begin{array}{lllll}0 & 0 & 0 & 0 & -1\end{array}\right] ;\)
ub = [ \(\left.\begin{array}{lllll}1 & 1 & 1 & 1 & 1\end{array}\right]\);
LG2Fparams = lsqnonlin(G2PPobjfun, \(x 0, l b, u b, o p t i o n s) ;\)
\begin{tabular}{cccccc} 
& & & \multicolumn{1}{c}{ Norm of } & First-order \\
Iteration & Func-count & \(f(x)\) & \multicolumn{1}{c}{ step } & optimality & CG-iterations \\
0 & 6 & 4.63874 & & 27.7 & \\
1 & 12 & 3.12695 & 0.116051 & 58.3 & 0 \\
2 & 18 & 0.466886 & 0.110096 & 12.5 & 0 \\
3 & 24 & 0.17667 & 0.0440836 & 3.2 & 0 \\
4 & 30 & 0.17667 & 0.0688327 & 3.2 & 0 \\
5 & 36 & 0.154169 & 0.0172082 & 0.246 & 0 \\
6 & 42 & 0.150196 & 0.0344163 & 0.0356 & 0 \\
7 & 48 & 0.143753 & 0.0688327 & 5.35 & 0 \\
8 & 54 & 0.143753 & 0.0669431 & 5.35 & 0 \\
9 & 60 & 0.140589 & 0.0167358 & 0.522 & 0 \\
10 & 66 & 0.13942 & 0.0334716 & 0.42 & 0 \\
11 & 72 & 0.13768 & 0.0669431 & 0.147 & 0 \\
12 & 78 & 0.136894 & 0.133886 & 28.6 & 0 \\
13 & 84 & 0.135024 & 0.0250289 & 3.56 & 0 \\
14 & 90 & 0.13492 & 0.0139061 & 4.24 & 0 \\
15 & 96 & 0.134879 & 0.00348578 & 0.232 & 0 \\
16 & 102 & 0.134879 & 0.038352 & 0.232 & 0 \\
17 & 108 & 0.134869 & 0.00836789 & 0.294 & 0
\end{tabular}

Local minimum possible.
lsqnonlin stopped because the final change in the sum of squares relative to its initial value is less than the selected value of the function tolerance.

LG2Fparams =
0.1694
0.0355
0.0073
0.0106
\(-0.7291\)

Create the G2PP object and use Monte Carlo simulation to generate the interest-rate paths with LinearGaussian2F.simTermStructs.
```

LG2f_a = LG2Fparams(1);
LG2f_b = LG2Fparams(2);
LG2f_sigma = LG2Fparams(3);
LG2f_eta = LG2Fparams(4);
LG2f_rho = LG2Fparams(5);
G2PP = LinearGaussian2F(RateSpec,LG2f_a,LG2f_b,LG2f_sigma,LG2f_eta,LG2f_rho
G2PPSimPaths = G2PP.simTermStructs(nPeriods,'NTRIALS',nTrials,...
'DeltaTime',DeltaTime,'Tenor',Tenor,'antithetic',true);
trialIdx = 1;
figure
surf(Tenor,SimDates,G2PPSimPaths(:,:,trialIdx))
datetick y keepticks keeplimits
title(['Evolution of the Zero Curve for Trial:' num2str(trialIdx) ' of G2++
xlabel('Tenor (Years)')

```


Price the European swaption.

DF \(=\exp (\) bsxfun(@times, -G2PPSimPaths, repmat(Tenor', [nPeriods+1 1])));
SwapRate \(=(1-\operatorname{DF}(\) exRow, endCol, : ) ) ./sum(bsxfun(@times, \(1, \operatorname{DF}(\) exRow, 1 :endCol, : ) ));
PayoffValue \(=100 * \max (\) SwapRate-InstrumentStrike, 0\() . *\) sum(bsxfun(@times, \(1, \mathrm{DF}(\) exRow, 1:endCol, : ) )) ;
RealizedDF \(=\operatorname{prod}(\exp (b s x f u n(@ t i m e s,-G 2 P P S i m P a t h s(2: \operatorname{exRow}, 1,:), \operatorname{SimTimes}(1: \operatorname{exRow}-1))), 1)\);
G2PP_SwaptionPrice \(=\) mean(RealizedDF.*PayoffValue)
G2PP_SwaptionPrice =

\subsection*{2.6186}

\section*{Simulate Interest-Rate Paths Using the LIBOR Market Model}

This example shows how to simulate interest-rate paths using the LIBOR market model. Before beginning this example that uses a LiborMarketModel, make sure you have set up the data as described in:
- "Construct Zero Curve" on page 2-106
- "Define Swaption Parameters" on page 2-108
- "Compute the Black Model and the Swaption Volatility Matrix" on page 2-108
- "Select Calibration Instruments" on page 2-109
- "Compute Swaption Prices Using Black's Model" on page 2-109
- "Define Simulation Parameters" on page 2-110

The LIBOR Market Model (LMM) differs from short rate models in that it evolves a set of discrete forward rates. Specifically, the lognormal LMM specifies the following diffusion equation for each forward rate
\[
\frac{d F_{i}(t)}{F_{i}}=-\mu_{i} d t+\sigma_{i}(t) d W_{i}
\]
where:
\(W\) is an N -dimensional geometric Brownian motion with
\[
d W_{i}(t) d W_{j}(t)=\rho_{i j}
\]

The LMM relates the drifts of the forward rates based on no-arbitrage arguments. Specifically, under the Spot LIBOR measure, the drifts are expressed as
\[
\mu_{i}(t)=-\sigma_{i}(t) \sum_{j=q(t)}^{i} \frac{\tau_{j} \rho_{i, j} \sigma_{j}(t) F_{j}(t)}{1+\tau_{j} F_{j}(t)}
\]
where:
\(\tau_{i}\) is the time fraction associated with the \(i\) th forward rate
\(q(t)\) is an index defined by the relation
\[
T_{q(t)-1}<t<T_{q(t)}
\]
and the Spot LIBOR numeraire is defined as
\[
B(t)=P\left(t, T_{q(t)}\right) \prod_{n=0}^{q(t)-1}\left(1+\tau_{n} F_{n}\left(T_{n}\right)\right)
\]

The choice with the LMM is how to model volatility and correlation and how to estimate the parameters of these models for volatility and correlation. In practice, you may use a combination of historical data (for example, observed correlation between forward rates) and current market data. For this example, only swaption data is used. Further, many different parameterizations of the volatility and correlation exist. For this example, two relatively straightforward parameterizations are used.

One of the most popular functional forms in the literature for volatility is:
\[
\sigma_{i}(t)=\varphi_{i}\left(a\left(T_{i}-t\right)+b\right) e^{c\left(T_{i}-t\right)}+d
\]
where \(\varphi\) adjusts the curve to match the volatility for the \(i\) th forward rate. For this example, all of the \(\varphi\) 's will be taken to be 1. For the correlation, the following functional form is used:
\[
\rho_{i, j}=e^{-\beta|i-j|}
\]

Once the functional forms have been specified, these parameters need to be estimated using market data. One useful approximation, initially developed by Rebonato, is the following, which relates the Black volatility for a European swaption, given a set of volatility functions and a correlation matrix
\[
\left(v_{\alpha, \beta}^{L F M}\right)^{2}=\sum_{i, j=\alpha+1}^{\beta} \frac{w_{i}(0) w_{j}(0) F_{i}(0) F_{j}(0) \rho_{i, j}}{S_{\alpha, \beta}(0)^{2}} \int_{0}^{T_{\alpha}} \sigma_{i}(t) \sigma_{j}(t) d t
\]
where:
\[
w_{i}(t)=\frac{\tau_{i} P\left(t, T_{i}\right)}{\sum_{k=\alpha+1}^{\beta} \tau_{\kappa} P\left(t, t_{\kappa}\right)}
\]

This calculation is done using the function blackvolbyrebonato to compute analytic values of the swaption price for model parameters, and consequently, is then used to calibrate the model. Calibration consists of minimizing the difference between the observed implied swaption Black volatilities and the predicted Black volatilities.

In this example, the approach is similar to "Simulate Interest-Rate Paths Using the Hull-White One-Factor Model" on page 2-111 and "Simulate Interest-Rate Paths Using the Linear Gaussian Two-Factor Model" on page 2-114 where the Optimization Toolbox function lsqnonlin is used to minimize the difference between the observed swaption prices and the predicted swaption prices. However, other approaches (for example, simulated annealing) may also be appropriate. Starting parameters and constraints for \(a, b, d\), and \(\beta\) are set in the variables \(\mathrm{x} 0,1 \mathrm{~b}\), and ub; these could also be varied depending upon the particular calibration approach.

Calibrate the set of parameters that minimize the difference between the observed and predicted values using blackvolbyrebonato and lsqnonlin.
```

nRates = 10;
CorrFunc = @(i,j,Beta) exp(-Beta*abs(i-j));
objfun = @(x) SwaptionBlackVol(relidx) - blackvolbyrebonato(RateSpec,...
repmat({@(t) ones(size(t)).*(x(1)*t + x(2)).*exp(-x(3)*t) + x(4)},nRates-1,1),···..
CorrFunc(meshgrid(1:nRates-1)',meshgrid(1:nRates-1),x(5)),...
EurExDatesFull(relidx),EurMatFull(relidx),'Period',1);
x0 = [.2 .05 1 .05 .2];
lb = [0 0 .5 0 .01];
ub = [llllll
LMMparams = lsqnonlin(objfun, x0,lb,ub,options)

| Iteration | Func-count | $f(x)$ | step | optimality | CG-iterations |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 6 | 0.212571 |  | 0.493 |  |
| 1 | 12 | 0.0151459 | 0.229951 | 0.0352 | 0 |
| 2 | 18 | 0.0112531 | 0.193839 | 0.00108 | 0 |
| 3 | 24 | 0.00937762 | 0.310582 | 0.00688 | 0 |
| 4 | 30 | 0.00911658 | 0.0790873 | $7.03 e-05$ | 0 |
| 5 | 36 | 0.00911658 | 0.517804 | $7.03 \mathrm{e}-05$ | 0 |

```
```

| 6 | 42 | 0.00907618 | 0.129451 | 0.0028 | 0 |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 7 | 48 | 0.00904774 | 0.0428543 | $6.33 e-05$ | 0 |
| 8 | 54 | 0.00904497 | 0.258902 | 0.00771 | 0 |
| 9 | 60 | 0.00902206 | 0.0115259 | $4.5 e-05$ | 0 |
| 10 | 66 | 0.00901664 | 0.0647254 | 0.000899 | 0 |

Local minimum possible.
lsqnonlin stopped because the final change in the sum of squares relative to its initial value is less than the selected value of the function tolerance.
LMMparams =

```
0.0797
0.2419
0.6073
0.0739
0.0100

Calculate VolFunc for the LMM object.
```

a = LMMparams(1);
b = LMMparams(2);
c = LMMparams(3);
d = LMMparams(4);
Beta = LMMparams(5);
VolFunc = repmat({@(t) ones(size(t)).*(a*t + b).*exp(-c*t) + d},nRates-1,1);

```

Plot the volatility function.
```

figure
fplot(VolFunc{1},[0 20])
title('Volatility Function')

```
CorrelationMatrix = CorrFunc(meshgrid(1:nRates-1)',meshgrid(1:nRates-1), Bet


Inspect the correlation matrix.
```

disp('Correlation Matrix')
fprintf([repmat('%1.3f ',1,length(CorrelationMatrix)) ' \n'],CorrelationMatrix)
Correlation Matrix
1.000 0.990 0.980 0.970 0.961 0.951 0.942 0.932 0.923
0.990 1.000 0.990 0.980 0.970 0.961 0.951 0.942 0.932
0.980 0.990 1.000 0.990 0.980 0.970 0.961 0.951 0.942
0.970 0.980 0.990 1.000 0.990 0.980 0.970 0.961 0.951
0.961 0.970 0.980 0.990 1.000 0.990 0.980 0.970 0.961
0.951 0.961 0.970 0.980 0.990 1.000 0.990 0.980 0.970
0.942 0.951 0.961 0.970 0.980 0.990 1.000 0.990 0.980
0.932 0.942 0.951 0.961 0.970 0.980 0.990 1.000 0.990
0.923 0.932 0.942 0.951 0.961 0.970 0.980 0.990 1.000

```

Create the LMM object and use Monte Carlo simulation to generate the interest-rate paths with LiborMarketModel.simTermStructs.
```

LMM = LiborMarketModel(irdc,VolFunc,CorrelationMatrix,'Period',1);
[LMMZeroRates, ForwardRates] = LMM.simTermStructs(nPeriods,'nTrials',nTrials);
trialIdx = 1;
figure
tmpPlotData = LMMZeroRates(:,:,trialIdx);
tmpPlotData(tmpPlotData == 0) = NaN;
surf(Tenor,SimDates,tmpPlotData)
title(['Evolution of the Zero Curve for Trial:' num2str(trialIdx) ' of LIBOR Market Model'])
xlabel('Tenor (Years)')

```


\section*{Price the European swaption.}

DF = exp(bsxfun(@times,-LMMZeroRates, repmat(Tenor',[nPeriods+1 1])));
SwapRate = (1-DF(exRow,endCol,:))./sum(bsxfun(@times, \(1, \operatorname{DF}(\) exRow, 1 :endCol,:)));
PayoffValue \(=100 * \max (\) SwapRate-InstrumentStrike, 0\() . * \operatorname{sum}(\) bsxfun(@times, \(1, \operatorname{DF}(\) exRow, 1:endCol, : ) ) ) ; RealizedDF \(=\operatorname{prod}(\exp (b s x f u n(@ t i m e s,-L M M Z e r o R a t e s(2: \operatorname{exRow}, 1,:), \operatorname{SimTimes}(1: \operatorname{exRow}-1))), 1)\);
LMM_SwaptionPrice \(=\) mean (RealizedDF.*PayoffValue)

\section*{Compare Interest-Rate Modeling Results}

This example shows how to compare the results for pricing a European swaptions with different interest-rate models.

Compare the results for pricing a European swaptions with interest-rate models using Monte Carlo simulation.
```

disp(' ')
fprintf(' \# of Monte Carlo Trials: %8d\n' , nTrials)
fprintf(' \# of Time Periods/Trial: %8d\n\n' , nPeriods)
fprintf('HW1F European Swaption Price: %8.4f\n', HW1F_SwaptionPrice)
fprintf('LG2F Europesn Swaption Price: %8.4f\n', G2PP_SwaptionPrice)
fprintf(' LMM European Swaption Price: %8.4f\n', LMM_SwaptionPrice)

```
```


# of Monte Carlo Trials: 1000

```
\# of Time Periods/Trial: 5
HW1F European Swaption Price: 2.7649
LG2F Europesn Swaption Price: 2.6186
LMM European Swaption Price: 2.8739

\section*{References}

Brigo, D. and F. Mercurio, Interest Rate Models - Theory and Practice with Smile, Inflation and Credit, Springer Finance, 2006.

Andersen, L. and V. Piterbarg, Interest Rate Modeling, Atlantic Financial Press. 2010.

Hull, J, Options, Futures, and Other Derivatives, Springer Finance, 2003.
Glasserman, P, Monte Carlo Methods in Financial Engineering, Prentice Hall, 2008.

Rebonato, R., K. McKay, and R. White, The Sabr/Libor Market Model: Pricing, Calibration and Hedging for Complex Interest-Rate Derivatives, John Wiley \& Sons, 2010.
See Also \begin{tabular}{l} 
HullWhite1F | LinearGaussian2F | LiborMarketModel | \\
HullWhite1F.simTermStructs | LinearGaussian2F.simTermStructs \\
\\
\\
\\
| LiborMarketModel.simTermStructs | capbylg2f | floorbylg2f | \\
swaptionbylg2f | swaptionbyhw | blackvolbyrebonato | lsqnonlin
\end{tabular}

\section*{Related Examples}

\section*{Pricing Bermudan Swaptions with Monte Carlo Simulation}

This example shows how to price Bermudan swaptions using interest-rate models in Financial Instruments Toolbox \({ }^{\mathrm{TM}}\). Specifically, a Hull-White one factor model, a Linear Gaussian two-factor model, and a LIBOR Market Model are calibrated to market data and then used to generate interest-rate paths using Monte Carlo simulation.

\section*{Zero Curve}

In this example, the ZeroRates for a zero curve is hard-coded. You can also create a zero curve by bootstrapping the zero curve from market data (for example, deposits, futures/forwards, and swaps). The hard-coded data for the zero curve is defined as:
```

Settle = datenum('21-Jul-2008');

```
\% Zero Curve
CurveDates = daysadd(Settle,360*([1 35710 20]),1);
ZeroRates \(=[1.92 .63 .13 .544 .3] ' / 100 ;\)
plot(CurveDates,ZeroRates)
datetick
title(['Zero Curve for ' datestr(Settle)]);
RateSpec = intenvset('Rates',ZeroRates,'EndDates',CurveDates,'StartDate',Se

\section*{Define Swaption Parameters}

For this example we will compute the price of a 10-no-call-1 Bermudan swaption.
```

BermudanExerciseDates = daysadd(Settle,360*(1:9),1);
BermudanMaturity = datenum('21-Jul-2018');
BermudanStrike = .045;

```

Black's Model and the Swaption Volatility Matrix

Black's model is often used to price and quote European exercise interest-rate options, that is, caps, floors and swaptions. In the case of swaptions, Black's model is used to imply a volatility given the current observed market price. The following matrix shows the Black implied volatility for a range of swaption exercise dates (columns) and underlying swap maturities (rows).
```

SwaptionBlackVol = [22 21 19 17 15 13 12
21 19 17 16 15 13 11
20}181816151412121
19 17 15 14 13 12 10
18}1614141312111
15}1413131212111
13 13 12 11 11 10 9]/100;
ExerciseDates = [1:5 7 10];
Tenors = [1:5 7 10];
EurExDatesFull = repmat(daysadd(Settle,ExerciseDates*360,1)',...
length(Tenors),1);
EurMatFull = reshape(daysadd(EurExDatesFull,...
repmat(360*Tenors,1,length(ExerciseDates)),1),size(EurExDatesFull));

```

\section*{Selecting the Calibration Instruments}

Selecting the instruments to calibrate the model to is one of the tasks in calibration. For Bermudan swaptions, it is typical to calibrate to European swaptions that are co-terminal with the Bermudan swaption to be priced. In this case, all swaptions having an underlying tenor that matures before the maturity of the swaption to be priced are used in the calibration.
```

% Find the swaptions that expire on or before the maturity date of the
% sample swaption
relidx = find(EurMatFull <= BermudanMaturity);

```

\section*{Compute Swaption Prices Using Black's Model}

Swaption prices are computed using Black's Model. The swaption prices are then used to compare the model's predicted values. To compute the swaption prices using Black's model:
```

% Compute Swaption Prices using Black's model
SwaptionBlackPrices = zeros(size(SwaptionBlackVol));

```
```

SwaptionStrike = zeros(size(SwaptionBlackVol));
for iSwaption=1:length(ExerciseDates)
for iTenor=1:length(Tenors)
[~,SwaptionStrike(iTenor,iSwaption)] = swapbyzero(RateSpec,[NaN 0],
'StartDate',EurExDatesFull(iTenor,iSwaption),'LegReset',[1 1]);
SwaptionBlackPrices(iTenor,iSwaption) = swaptionbyblk(RateSpec, 'ca
EurExDatesFull(iTenor,iSwaption), EurMatFull(iTenor,iSwaption),
end
end

```

\section*{Simulation Parameters}

The following parameters will be used; each exercise date is a simulation date.
```

nPeriods = 9;
DeltaTime = 1;
nTrials = 1000;
Tenor = (1:10)';
SimDates = daysadd(Settle,360*DeltaTime*(0:nPeriods),1);
SimTimes = diff(yearfrac(SimDates(1),SimDates));

```

Hull White 1 Factor Model
The Hull-White one-factor model describes the evolution of the short rate and is specified by the following:

The Hull-White model is calibrated using the function SWAPTIONNYHW, which constructs a trinomial tree to price the swaptions. Calibration consists of minimizing the difference between the observed market prices (computed above using the Black's implied swaption volatility matrix) and the model's predicted prices.

In this example, the Optimization Toolbox \({ }^{\mathrm{TM}}\) function LSQNONLIN is used to find the parameter set that minimizes the difference between the observed and predicted values. However, other approaches (for example, simulated
annealing) may be appropriate. Starting parameters and constraints for and are set in the variables \(\mathrm{x} 0, \mathrm{lb}\) and ub ; these could also be varied depending upon the particular calibration approach.

TimeSpec = hwtimespec(Settle,daysadd(Settle, 360*(1:11), 1), 2); HW1Fobjfun \(=@(x)\) SwaptionBlackPrices(relidx) - ...
swaptionbyhw(hwtree (hwvolspec (Settle, '11-Aug-2015', x (2) , '11-Aug-2015'
EurExDatesFull(relidx), 0, EurExDatesFull(relidx), EurMatFull(relidx));
options \(=\) optimset('disp','iter','MaxFunEvals', 1000,'TolFun', 1e-5);
```

% Find the parameters that minimize the difference between the observed and
% predicted prices
x0 = [.1 .01];
lb = [0 0];
ub = [ll 1];
HW1Fparams = lsqnonlin(HW1Fobjfun,x0,lb,ub,options);
HW_alpha = HW1Fparams(1);
HW_sigma = HW1Fparams(2);
% Construct the HullWhite1F model using the HullWhite1F constructor.
HW1F = HullWhite1F(RateSpec,HW_alpha,HW_sigma);
% Use Monte Carlo simulation to generate the interest-rate paths with
% HullWhite1F.simTermStructs.
HW1FSimPaths = HW1F.simTermStructs(nPeriods,'NTRIALS',nTrials,...
'DeltaTime',DeltaTime,'Tenor',Tenor,'antithetic',true);

```
\% Examine one simulation
trialldx = 1;
figure
surf(Tenor, SimDates, HW1FSimPaths(: , , trialIdx))
datetick y keepticks keeplimits
title(['Evolution of the Zero Curve for Trial:' num2str(trialIdx) ' of Hull
xlabel('Tenor (Years)')
\% Price the swaption using the helper function hBermudanSwaption
HW1FBermPrice = hBermudanSwaption(HW1FSimPaths,SimDates,Tenor,BermudanStrik
    BermudanExerciseDates,BermudanMaturity);
\begin{tabular}{lcccc} 
& & Norm of & \begin{tabular}{c} 
First-order \\
optimality
\end{tabular} \\
Iteration & Func-count & \(f(x)\) & 0.555648 & \\
0 & 3 & 0.130818 & 0.0156718 & 0.745 \\
1 & 6 & 0.126276 & 0.00920992 & 0.785 \\
2 & 9 & 0.126201 & 0.00528726 & 0.0447 \\
3 & 12 & 0.126201 & 0.00012361 & 0.000109 \\
4 & 15 & \\
Local minimum possible. \\
lsqnonlin stopped because the final change in the sum of squares relative t
\end{tabular}

\section*{Linear Gaussian 2 Factor Model}

The Linear Gaussian two-factor model (called the G2++ by Brigo and Mercurio) is also a short rate model, but involves two factors. Specifically:
where is a two-dimensional Brownian motion with correlation
and is a function chosen to match the initial zero curve.
The function SWAPTIONBYLG2F is used to compute analytic values of the swaption price for model parameters and consequently can be used to calibrate the model. Calibration consists of minimizing the difference between the observed market prices and the model's predicted prices.
```

% Calibrate the set of parameters that minimize the difference between the
% observed and predicted values using SWAPTIONBYLG2F and LSQNONLIN.
G2PPobjfun = @(x) SwaptionBlackPrices(relidx) - ...
swaptionbylg2f(RateSpec,x(1),x(2),x(3),x(4),x(5),SwaptionStrike(relidx)
EurExDatesFull(relidx),EurMatFull(relidx),'Reset',1);
x0 = [.2 .1 .02 .01 -.5];
lb = [00 0 0 0 -1];
ub = [[11 1 1 1 1 1 1];
LG2Fparams = lsqnonlin(G2PPobjfun,x0,lb,ub,options);
LG2f_a = LG2Fparams(1);
LG2f_b = LG2Fparams(2);
LG2f_sigma = LG2Fparams(3);
LG2f_eta = LG2Fparams(4);
LG2f_rho = LG2Fparams(5);
% Create the G2PP object and use Monte Carlo simulation to generate the
% interest-rate paths with LinearGaussian2F.simTermStructs.
G2PP = LinearGaussian2F(RateSpec,LG2f_a,LG2f_b,LG2f_sigma,LG2f_eta,LG2f_rho
G2PPSimPaths = G2PP.simTermStructs(nPeriods,'NTRIALS',nTrials,...
'DeltaTime',DeltaTime,'Tenor',Tenor,'antithetic',true);
% Examine one simulation
trialIdx = 1;
figure
surf(Tenor,SimDates,G2PPSimPaths(:,:,trialIdx))
datetick y keepticks keeplimits
title(['Evolution of the Zero Curve for Trial:' num2str(trialIdx) ' of G2++
xlabel('Tenor (Years)')
\% Price the swaption using the helper function hBermudanSwaption LG2FBermPrice $=$ hBermudanSwaption(G2PPSimPaths,SimDates, Tenor, BermudanStrik

```
\begin{tabular}{ccccc} 
& & Norm of & First-order \\
Iteration & Func-count & \(f(x)\) & step & optimality \\
0 & 6 & 12.1928 & & 67.1 \\
1 & 12 & 1.36663 & 0.0974259 & 8.54 \\
2 & 18 & 1.36663 & 0.112377 & 8.54
\end{tabular}
```

    24 0.442322 0.0280943 1.3
    30 0.236944 0.0561887
0.130779 0.0840413 7.66
0.0394584 0.0145003 6.79
0.0275889 0.037242 0.755
0.0261953 0.112377 0.693
0.0234048 0.206006 0.142
0.0225717 0.140345 0.116
0.02254 0.0245608 1.45
0.0225305 0.018807 1.36
Local minimum possible.
lsqnonlin stopped because the final change in the sum of squares relative
its initial value is less than the selected value of the function tolerance

```

\section*{LIBOR Market Model}

The LIBOR Market Model (LMM) differs from short rate models in that it evolves a set of discrete forward rates. Specifically, the lognormal LMM specifies the following diffusion equation for each forward rate
where
is the volatility function for each rate and is an N dimensional geometric Brownian motion with:

The LMM relates the drifts of the forward rates based on no-arbitrage arguments.

The choice with the LMM is how to model volatility and correlation and how to estimate the parameters of these models for volatility and correlation. In practice, you may use a combination of historical data (for example, observed correlation between forward rates) and current market data. For this example, only swaption data is used. Further, many different parameterizations of the volatility and correlation exist. For this example, two relatively straightforward parameterizations are used.

One of the most popular functional forms in the literature for volatility is:
where adjusts the curve to match the volatility for the forward rate. For this example, all of the Phi's will be taken to be 1 .

For the correlation, the following functional form will be used:

Once the functional forms have been specified, these parameters need to be estimated using market data. One useful approximation, initially developed by Rebonato, is the following, which computes the Black volatility for a European swaption, given an LMM with a set of volatility functions and a correlation matrix.
where

This calculation is done using BLACKVOLBYREBONATO to compute analytic values of the swaption price for model parameters, and consequently, is then used to calibrate the model. Calibration consists of minimizing the difference between the observed implied swaption Black volatilities and the predicted Black volatilities.
nRates = 10;

CorrFunc \(=\) @(i,j,Beta) \(\exp (-B e t a * a b s(i-j)) ;\)
```

objfun = @(x) SwaptionBlackVol(relidx) - blackvolbyrebonato(RateSpec,...
repmat({@(t) ones(size(t)).*(x(1)*t + x(2)).*exp(-x(3)*t) + x(4)},nRate
CorrFunc(meshgrid(1:nRates-1)',meshgrid(1:nRates-1),x(5)),···
EurExDatesFull(relidx),EurMatFull(relidx),'Period',1);
x0 = [l.2 .05 1 . 05 .2];
lb = [00 0 .5 0 .01];
ub = [lllllll}
LMMparams = lsqnonlin(objfun,x0,lb,ub,options);
% Calculate VolFunc for the LMM object.
a = LMMparams(1);
b = LMMparams(2);
c = LMMparams(3);
d = LMMparams(4);
Beta = LMMparams(5);
VolFunc = repmat({@(t) ones(size(t)).*(a*t + b).*exp(-c*t) + d},nRates-1,1)
% Plot the volatility function
figure
fplot(VolFunc{1},[0 20])
title('Volatility Function')
% Inspect the correlation matrix
CorrelationMatrix = CorrFunc(meshgrid(1:nRates-1)',meshgrid(1:nRates - 1),Bet
disp('Correlation Matrix')
fprintf([repmat('%1.3f ',1,length(CorrelationMatrix)) ' \n'],CorrelationMat
% Create the LMM object and use Monte Carlo simulation to generate the
% interest-rate paths with LiborMarketModel.simTermStructs.
LMM = LiborMarketModel(RateSpec,VolFunc,CorrelationMatrix,'Period',1);
[LMMZeroRates, ForwardRates] = LMM.simTermStructs(nPeriods,'nTrials',nTrial
% Examine one simulation
trialIdx = 1;
figure

```
```

tmpPlotData = LMMZeroRates(:,:,trialIdx);
tmpPlotData(tmpPlotData == 0) = NaN;
surf(Tenor,SimDates,tmpPlotData)
title(['Evolution of the Zero Curve for Trial:' num2str(trialIdx) ' of LIBO
xlabel('Tenor (Years)')
\% Price the swaption using the helper function hBermudanSwaption LMMTenor = 1:10;
LMMBermPrice $=$ hBermudanSwaption(LMMZeroRates,SimDates, LMMTenor, . 045, Bermud

```
\begin{tabular}{ccccc} 
& & & Norm of & First-order \\
Iteration & Func-count & \(f(x)\) & step & optimality \\
0 & 6 & 0.251221 & & 0.496 \\
1 & 12 & 0.0206191 & 0.276585 & 0.0333 \\
2 & 18 & 0.0165669 & 0.220027 & 0.00194 \\
3 & 24 & 0.0146667 & 0.380455 & 0.00835 \\
4 & 30 & 0.0143206 & 0.0563957 & 0.00428 \\
5 & 36 & 0.0142098 & 0.0315698 & 0.00123 \\
6 & 42 & 0.014196 & 0.0399949 & \(9.95 e-05\) \\
7 & 48 & 0.0141888 & 0.232111 & 0.00302 \\
8 & 54 & 0.014175 & 0.0226522 & \(5.02 e-05\) \\
9 & 60 & 0.0141697 & 0.0580277 & 0.000427
\end{tabular}

Local minimum possible.
lsqnonlin stopped because the final change in the sum of squares relative its initial value is less than the selected value of the function tolerance
```

Correlation Matrix
1.000 0.990 0.980 0.970 0.961 0.951 0.942 0.932 0.923
0.990 1.000 0.990 0.980 0.970 0.961 0.951 0.942 0.932
0.980 0.990 1.000 0.990 0.980 0.970 0.961 0.951 0.942
0.970 0.980 0.990 1.000 0.990 0.980 0.970 0.961 0.951
0.961 0.970 0.980 0.990 1.000 0.990 0.980 0.970 0.961
0.951 0.961 0.970 0.980 0.990 1.000 0.990 0.980 0.970
0.942 0.951 0.961 0.970 0.980 0.990 1.000 0.990 0.980
0.932 0.942 0.951 0.961 0.970 0.980 0.990 1.000 0.990

```
```

0.923 0.932 0.942 0.951 0.961 0.970 0.980 0.990 1.000

```

\section*{Results}
```

disp(' ')
fprintf(' \# of Monte Carlo Trials: %8d\n' , nTrials)
fprintf(' \# of Time Periods/Trial: %8d\n\n' , nPeriods)
fprintf('HW1F Bermudan Swaption Price: %8.4f\n', HW1FBermPrice)
fprintf('LG2F Bermudan Swaption Price: %8.4f\n', LG2FBermPrice)
fprintf(' LMM Bermudan Swaption Price: %8.4f\n', LMMBermPrice)

```
    \# of Monte Carlo Trials: 1000
    \# of Time Periods/Trial: 9
HW1F Bermudan Swaption Price: 3.8606
LG2F Bermudan Swaption Price: 3.5496
    LMM Bermudan Swaption Price: 3.6988

\section*{Bibliography}

This example is based on the following books, papers and journal articles:
1 Andersen, L. and V. Piterbarg (2010). Interest Rate Modeling, Atlantic Financial Press.

2 Brigo, D. and F. Mercurio (2001). Interest Rate Models - Theory and Practice with Smile, Inflation and Credit (2nd ed. 2006 ed.). Springer Verlag.

3 Glasserman, P. (2003). Monte Carlo Methods in Financial Engineering. Springer.

4 Hull, J. (2008). Options, Futures, and Other Derivatives. Prentice Hall.

5 Rebonato, R., K. McKay, and R. White (2010). The Sabr/Libor Market Model: Pricing, Calibration and Hedging for Complex Interest-Rate Derivatives. John Wiley \& Sons.

\section*{Graphical Representation of Trees}

\author{
In this section... \\ "Introduction" on page 2-139 \\ "Observing Interest Rates" on page 2-139 \\ "Observing Instrument Prices" on page 2-143
}

\section*{Introduction}

You can use the function treeviewer to display a graphical representation of a tree, allowing you to examine interactively the prices and rates on the nodes of the tree until maturity. To get started with this process, first load the data file deriv.mat included in this toolbox.
```

load deriv.mat

```

Note treeviewer price tree diagrams follow the convention that increasing prices appear on the upper branch of a tree and, consequently, decreasing prices appear on the lower branch. Conversely, for interest rate displays, decreasing interest rates appear on the upper branch (prices are rising) and increasing interest rates on the lower branch (prices are falling).

For information on the use of treeviewer to observe interest rate movement, see "Observing Interest Rates" on page 2-139. For information on using treeviewer to observe the movement of prices, see "Observing Instrument Prices" on page 2-143.

\section*{Observing Interest Rates}

If you provide the name of an interest rate tree to the treeviewer function, it displays a graphical view of the path of interest rates. For example, here is the treeviewer representation of all the rates along both the up and down branches of HJMTree.
treeviewer(HJMTree)


The example in "Isolating a Specific Node for a CRRTree" on page 3-19 used bushpath to find the path of forward rates along an HJM tree by taking the first branch up and then two branches down the rate tree.
```

FRates = bushpath(HJMTree.FwdTree, [1 2 2])

```

FRates =
1.0356
1.0364
1.0526
1.0674

With the treeviewer function you can display the identical information by clicking along the same sequence of nodes, as shown next.


Next is a treeviewer representation of interest rates along several branches of BDTTree.
treeviewer(BDTTree)


Note When using treeviewer with recombining trees, such as BDT, BK, and HW, you must click each node in succession from the beginning to the end. Because these trees can recombine, treeviewer is unable to complete the path automatically.

The example in "Isolating a Specific Node for a CRRTree" on page 3-19 used treepath to find the path of interest rates taking the first branch up and then two branches down the rate tree.

FRates = treepath(BDTTree.FwdTree, [lllll
FRates =
1.1000
1.0979
1.1377
1.1606

You can display the identical information by clicking along the same sequence of nodes, as shown next.


\section*{Observing Instrument Prices}

To use treeviewer to display a tree of instrument prices, provide the name of an instrument set along with the name of a price tree in your call to treeviewer, for example:
load deriv.mat
[Price, PriceTree] = hjmprice(HJMTree, HJMInstSet); treeviewer(PriceTree, HJMInstSet)

With treeviewer you select each instrument individually in the instrument portfolio for display.


You can use an analogous process to view instrument prices based on the BDT interest rate tree included in deriv.mat.
load deriv.mat
[BDTPrice, BDTPriceTree] = bdtprice(BDTTree, BDTInstSet);
treeviewer(BDTPriceTree, BDTInstSet)


\section*{Valuation Date Prices}

You can use treeviewer instrument-by-instrument to observe instrument prices through time. For the first 4\% bond in the HJM instrument portfolio, treeviewer indicates a valuation date price of 98.72 , the same value obtained by accessing the PriceTree structure directly.


As a further example, look at the sixth instrument in the price vector, the 3\% cap. At the valuation date, its value obtained directly from the structure is 6.2831. Use treeviewer on this instrument to confirm this price.


\section*{Additional Observation Times}

The second node represents the first-rate observation time, t0bs \(=1\). This node displays two states, one representing the branch going up and the other one representing the branch going down.

Examine the prices of the node corresponding to the up branch.
```

ans =
100.1563
99.7309
0.1007
100.1563
100.3782
3.2594
0.1007
3.5597

```
PriceTree.PBush\{2\}(:,:,1)

As before, you can use treeviewer, this time to examine the price for the \(4 \%\) bond on the up branch. treeviewer displays a price of 100.2 for the first node of the up branch, as expected.


Now examine the corresponding down branch.
PriceTree.PBush\{2\}(:, : , 2)
ans =
96.3041
94.1986

0
96.3041
100.3671
8.6342

0
\(-0.3923\)

Use treeviewer once again, now to observe the price of the \(4 \%\) bond on the down branch. The displayed price of 96.3 conforms to the price obtained from direct access of the PriceTree structure. You may continue this process as far along the price tree as you want.


\section*{Equity Derivatives}
- "Understanding Equity Trees" on page 3-2
- "Supported Equity Derivatives" on page 3-22
- "Supported Energy Derivatives" on page 3-35
- "Pricing European and American Spread Options" on page 3-41
- "Hedging Strategies Using Spread Options" on page 3-57
- "Pricing Swing Options using the Longstaff-Schwartz Method" on page 3-66
- "Pricing Equity Derivatives Using Trees" on page 3-76
- "Equity Derivatives Using Closed-Form Solutions" on page 3-95
- "Pricing European Call Options Using Different Equity Models" on page 3-106

\section*{Understanding Equity Trees}

\author{
In this section... \\ "Introduction" on page 3-2 \\ "Building Equity Binary Trees" on page 3-3 \\ "Building Implied Trinomial Trees" on page 3-8 \\ "Examining Equity Trees" on page 3-16 \\ "Differences Between CRR and EQP Tree Structures" on page 3-21
}

\section*{Introduction}

Financial Instruments Toolbox software supports three types of recombining tree models to represent the evolution of stock prices:
- Cox-Ross-Rubinstein (CRR) model
- Equal probabilities (EQP) model
- Leisen-Reimer (LR) model
- Implied trinomial tree (ITT) model

For a discussion of recombining trees, see "Rate and Price Trees" on page 2-35.
The CRR, EQP, LR, and ITT models are examples of discrete time models. A discrete time model divides time into discrete bits; prices can only be computed at these specific times.

The CRR model is one of the most common methods used to model the evolution of stock processes. The strength of the CRR model lies in its simplicity. It is a good model when dealing with many tree levels. The CRR model yields the correct expected value for each node of the tree and provides a good approximation for the corresponding local volatility. The approximation becomes better as the number of time steps represented in the tree is increased.

The EQP model is another discrete time model. It has the advantage of building a tree with the exact volatility in each tree node, even with small
numbers of time steps. It also provides better results than CRR in some given trading environments, for example, when stock volatility is low and interest rates are high. However, this additional precision causes increased complexity, which is reflected in the number of calculations required to build a tree.

The LR model is another discrete time model. It has the advantage of producing estimates close to the Black-Scholes model using only a small number of steps, while also minimizing the oscillation.

The ITT model is a CRR-style implied trinomial tree which takes advantage of prices quoted from liquid options in the market with varying strikes and maturities to build a tree that more accurately represents the market. An ITT model is commonly used to price exotic options in such a way that they are consistent with the market prices of standard options.

\section*{Building Equity Binary Trees}

The tree of stock prices is the fundamental unit representing the evolution of the price of a stock over a given period of time. The MATLAB functions crrtree, eqptree, and lrtree create CRR trees, EQP trees, and LR trees, respectively. These functions create an output tree structure along with information about the parameters used for creating the tree.

The functions crrtree, eqptree, and lrtree take three structures as input arguments:
- The stock parameter structure StockSpec
- The interest-rate term structure RateSpec
- The tree time layout structure TimeSpec

\section*{Calling Sequence for Equity Binary Trees}

The calling syntax for crrtree is:
CRRTree = crrtree (StockSpec, RateSpec, TimeSpec)
Similarly, the calling syntax for eqptree is:
EQPTree = eqptree (StockSpec, RateSpec, TimeSpec)

And, the calling syntax for lrtree is:
```

LRTree = lrtree(StockSpec, RateSpec, TimeSpec, Strike)

```

All three functions require the structures StockSpec, RateSpec, and TimeSpec as input arguments:
- StockSpec is a structure that specifies parameters of the stock whose price evolution is represented by the tree. This structure, created using the function stockspec, contains information such as the stock's original price, its volatility, and its dividend payment information.
- RateSpec is the interest-rate specification of the initial rate curve. Create this structure with the function intenvset.
- TimeSpec is the tree time layout specification. Create these structures with the functions crrtimespec, eqptimespec, and lrtimespec. The structures contain information regarding the mapping of relevant dates into the tree structure, plus the number of time steps used for building the tree.

\section*{Specifying the Stock Structure for Equity Binary Trees}

The structure StockSpec encapsulates the stock-specific information required for building the binary tree of an individual stock's price movement.

You generate StockSpec with the function stockspec. This function requires two input arguments and accepts up to three additional input arguments that depend on the existence and type of dividend payments.

The syntax for calling stockspec is:
```

StockSpec = stockspec(Sigma, AssetPrice, DividendType, ...
DividendAmounts, ExDividendDates)

```
where:
- Sigma is the decimal annual volatility of the underlying security.
- AssetPrice is the price of the stock at the valuation date.
- DividendType is a string specifying the type of dividend paid by the stock. Allowed values are cash, constant, or continuous.
- DividendAmounts has a value that depends on the specification of DividendType. For DividendType cash, DividendAmounts is a vector of cash dividends. For DividendType constant, it is a vector of constant annualized dividend yields. For DividendType continuous, it is a scalar representing a continuously annualized dividend yield.
- ExDividendDates also has a value that depends on the nature of DividendType. For DividendType cash or constant, ExDividendDates is vector of dividend dates. For DividendType continuous, ExDividendDates is ignored.

\section*{Stock Structure Example Using a Binary Tree}

Consider a stock with a price of \(\$ 100\) and an annual volatility of \(15 \%\). Assume that the stock pays three cash \(\$ 5.00\) dividends on dates January 01, 2003; July 01, 2003; and January 01, 2004. You specify these parameters in MATLAB as:
```

Sigma = 0.15;
AssetPrice = 100;
DividendType = 'cash';
DividendAmounts = [5; 5; 5];
ExDividendDates = {'jan-01-2004', 'july-01-2005', 'jan-01-2006'};
StockSpec = stockspec(Sigma, AssetPrice, DividendType, ...
DividendAmounts, ExDividendDates)

```

StockSpec =
FinObj: 'StockSpec'
Sigma: 0.1500
AssetPrice: 100
DividendType: 'cash'
DividendAmounts: [3x1 double]
ExDividendDates: [3x1 double]

\section*{Specifying the Interest-Rate Term Structure for Equity Binary Trees}

The RateSpec structure defines the interest rate environment used when building the stock price binary tree. "Modeling the Interest-Rate Term Structure" on page 2-47 explains how to create these structures using the
function intenvset, given the interest rates, the starting and ending dates for each rate, and the compounding value.

\section*{Specifying the Tree-Time Term Structure for Equity Binary Trees}

The TimeSpec structure defines the tree layout of the binary tree:
- It maps the valuation and maturity dates to their corresponding times.
- It defines the time of the levels of the tree by dividing the time span between valuation and maturity into equally spaced intervals. By specifying the number of intervals, you define the granularity of the tree time structure.

The syntax for building a TimeSpec structure is:
```

TimeSpec = crrtimespec(ValuationDate, Maturity, NumPeriods)
TimeSpec = eqptimespec(ValuationDate, Maturity, NumPeriods)
TimeSpec = lrtimespec(ValuationDate, Maturity, NumPeriods)

```
where:
- ValuationDate is a scalar date marking the pricing date and first observation in the tree (location of the root node). You enter ValuationDate either as a serial date number (generated with datenum) or a date string.
- Maturity is a scalar date marking the maturity of the tree, entered as a serial date number or a date string.
- NumPeriods is a scalar defining the number of time steps in the tree; for example, NumPeriods \(=10\) implies 10 time steps and 11 tree levels ( 0,1 , \(2, \ldots, 9,10)\).

\section*{TimeSpec Example Using a Binary Tree}

Consider building a CRR tree, with a valuation date of January 1, 2003, a maturity date of January 1, 2008, and 20 time steps. You specify these parameters in MATLAB as:
```

ValuationDate = 'Jan-1-2003';
Maturity = 'Jan-1-2008';
NumPeriods = 20;

```
```

TimeSpec = crrtimespec(ValuationDate, Maturity, NumPeriods)
TimeSpec =
FinObj: 'BinTimeSpec'
ValuationDate: 731582
Maturity: 733408
NumPeriods: 20
Basis: 0
EndMonthRule: 1
tObs: [1x21 double]
dObs: [1x21 double]

```

Two vector fields in the TimeSpec structure are of particular interest: dObs and tObs. These two fields represent the observation times and corresponding dates of all tree levels, with dObs (1) and tObs(1), respectively, representing the root node (ValuationDate), and dObs(end) and tObs (end) representing the last tree level (Maturity).

Note There is no relationship between the dates specified for the tree and the implied tree level times, and the maturities specified in the interest rate term structure. The rates in RateSpec are interpolated or extrapolated as required to meet the time distribution of the tree.

\section*{Examples of Binary Tree Creation}

You can now use the StockSpec and TimeSpec structures described previously to build an equal probability tree (EQPTree), a CRR tree (CRRTree), or a LR tree (LRTree). First, you must define the interest rate term structure. For this example, assume that the interest rate is fixed at \(10 \%\) annually between the valuation date of the tree (January 1, 2003) until its maturity.
```

ValuationDate = 'Jan-1-2003';
Maturity = 'Jan-1-2008';
Rate = 0.1;
RateSpec = intenvset('Rates', Rate, 'StartDates', ...
ValuationDate, 'EndDates', Maturity, 'Compounding', -1);

```
```

To build a CRRTree, enter:
CRRTree = crrtree(StockSpec, RateSpec, TimeSpec)
CRRTree =
FinObj: 'BinStockTree'
Method: 'CRR'
StockSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
tObs: [1\times21 double]
dObs: [1x21 double]
STree: {1x21 cell}
UpProbs: [1x20 double]
To build an EQPTree, enter:
EQPTree = eqptree(StockSpec, RateSpec, TimeSpec)
EQPTree =
FinObj: 'BinStockTree'
Method:
Stock'
TimeSpec:

```

\section*{Building Implied Trinomial Trees}

The tree of stock prices is the fundamental unit representing the evolution of the price of a stock over a given period of time. The MATLAB function itttree creates an output tree structure along with the information about the parameters used to create the tree.

The function itttree takes four structures as input arguments:
- The stock parameter structure StockSpec
- The interest-rate term structure RateSpec
- The tree time layout structure TimeSpec
- The stock option specification structure StockOptSpec

\section*{Calling Sequence for Implied Trinomial Trees}

The calling syntax for itttree is:
ITTTree = itttree (StockSpec,RateSpec,TimeSpec,StockOptSpec)
- StockSpec is a structure that specifies parameters of the stock whose price evolution is represented by the tree. This structure, created using the function stockspec, contains information such as the stock's original price, its volatility, and its dividend payment information.
- RateSpec is the interest-rate specification of the initial rate curve. Create this structure with the function intenvset.
- TimeSpec is the tree time layout specification. Create these structures with the function itttimespec. This structure contains information regarding the mapping of relevant dates into the tree structure, plus the number of time steps used for building the tree.
- StockOptSpec is a structure containing parameters of European stock options instruments. Create this structure with the function stockoptspec.

\section*{Specifying the Stock Structure for Implied Trinomial Trees}

The structure StockSpec encapsulates the stock-specific information required for building the trinomial tree of an individual stock's price movement.

You generate StockSpec with the function stockspec. This function requires two input arguments and accepts up to three additional input arguments that depend on the existence and type of dividend payments.

The syntax for calling stockspec is:
```

StockSpec = stockspec(Sigma, AssetPrice, DividendType, ...
DividendAmounts, ExDividendDates)

```
where:
- Sigma is the decimal annual volatility of the underlying security.
- AssetPrice is the price of the stock at the valuation date.
- DividendType is a string specifying the type of dividend paid by the stock. Allowed values are cash, constant, or continuous.
- DividendAmounts has a value that depends on the specification of DividendType. For DividendType cash, DividendAmounts is a vector of cash dividends. For DividendType constant, it is a vector of constant annualized dividend yields. For DividendType continuous, it is a scalar representing a continuously annualized dividend yield.
- ExDividendDates also has a value that depends on the nature of DividendType. For DividendType cash or constant, ExDividendDates is vector of dividend dates. For DividendType continuous, ExDividendDates is ignored.

\section*{Stock Structure Example Using an Implied Trinomial Tree}

Consider a stock with a price of \(\$ 100\) and an annual volatility of \(12 \%\). Assume that the stock is expected to pay a dividend yield of \(6 \%\). You specify these parameters in MATLAB as:
```

So=100;
DividendYield = 0.06;
Sigma=.12;
StockSpec = stockspec(Sigma, So, 'continuous', DividendYield)
StockSpec =
FinObj: 'StockSpec'
Sigma: 0.1200
AssetPrice: 100
DividendType: 'continuous'
DividendAmounts: 0.0600
ExDividendDates: []

```

\section*{Specifying the Interest-Rate Term Structure for Implied Trinomial Trees}

The structure RateSpec defines the interest rate environment used when building the stock price binary tree. "Modeling the Interest-Rate Term Structure" on page 2-47 explains how to create these structures using the function intenvset, given the interest rates, the starting and ending dates for each rate, and the compounding value.

\section*{Specifying the Tree-Time Term Structure for Implied Trinomial Trees}

The TimeSpec structure defines the tree layout of the trinomial tree:
- It maps the valuation and maturity dates to their corresponding times.
- It defines the time of the levels of the tree by dividing the time span between valuation and maturity into equally spaced intervals. By specifying the number of intervals, you define the granularity of the tree time structure.

The syntax for building a TimeSpec structure is:
TimeSpec = itttimespec(ValuationDate, Maturity, NumPeriods)
where:
- ValuationDate is a scalar date marking the pricing date and first observation in the tree (location of the root node). You enter ValuationDate either as a serial date number (generated with datenum) or a date string.
- Maturity is a scalar date marking the maturity of the tree, entered as a serial date number or a date string.
- NumPeriods is a scalar defining the number of time steps in the tree; for example, NumPeriods = 10 implies 10 time steps and 11 tree levels ( 0,1 , \(2, \ldots, 9,10)\).

\section*{TimeSpec Example Using an Implied Trinomial Tree}

Consider building an ITT tree, with a valuation date of January 1, 2006, a maturity date of January 1, 2008, and four time steps. You specify these parameters in MATLAB as:
```

ValuationDate = '01-01-2006';
EndDate = '01-01-2008';
NumPeriods = 4;
TimeSpec = itttimespec(ValuationDate, EndDate, NumPeriods)
TimeSpec =
FinObj: 'ITTTimeSpec'
ValuationDate: 732678
Maturity: 733408
NumPeriods: 4
Basis: 0
EndMonthRule: 1
tObs: [0 0.5000 1 1.5000 2]
dObs: [732678 732860 733043 733225 733408]

```

Two vector fields in the TimeSpec structure are of particular interest: dObs and tObs. These two fields represent the observation times and corresponding dates of all tree levels, with dObs (1) and tObs (1), respectively, representing the root node (ValuationDate), and dObs(end) and tObs (end) representing the last tree level (Maturity).

\section*{Specifying the Option Stock Structure for Implied Trinomial Trees}

The StockOptSpec structure encapsulates the option-stock-specific information required for building the implied trinomial tree. You generate StockOptSpec with the function stockoptspec. This function requires five input arguments. An optional sixth argument InterpMethod, specifying the interpolation method, can be included. The syntax for calling stockoptspec is:
[StockOptSpec] = stockoptspec(OptPrice, Strike, Settle, Maturity, OptSpec)
where:
- Optprice is a NINST-by- 1 vector of European option prices.
- Strike is a NINST-by-1 vector of strike prices.
- Settle is a scalar date marking the settlement date.
- Maturity is a NINST-by-1 vector of maturity dates.
- OptSpec is a NINST-by-1 cell array of strings 'call' or 'put'.

\section*{Option Stock Structure Example Using an Implied Trinomial Tree}

Consider the following data quoted from liquid options in the market with varying strikes and maturity. You specify these parameters in MATLAB as:
```

Settle = '01/01/06';
Maturity = ['07/01/06';
'07/01/06';
07/01/06';
07/01/06';
'01/01/07';
'01/01/07';
'01/01/07';
'01/01/07';
'07/01/07';
07/01/07';
'07/01/07';
'07/01/07';
'01/01/08';
'01/01/08';
'01/01/08';
'01/01/08'];
Strike = [113;
101;
100;
88;
128;
112;
100;
78;
144;
112;
100;
69;

```
```

    162;
    112;
    100;
    61];
    OptPrice =
0;
4.807905472659144;
1.306321897011867;
0.048039195057173;
0;
2.310953054191461;
1.421950392866235;
0.020414826276740;
0;
5.091986935627730;
1.346534812295291;
0.005101325584140;
0;
8.047628153217246;
1.219653432150932;
0.001041436654748];
OptSpec = { 'call';
'call';
'put';
'put';
'call';
'call';
'put';
'put';
'call';
'call';
'put';
'put';
'call';
'call';
'put';
'put'};

```
```

StockOptSpec = stockoptspec(OptPrice, Strike, Settle, Maturity, OptSpec)
StockOptSpec =
FinObj: 'StockOptSpec'
OptPrice: [16x1 double]
Strike: [16x1 double]
Settle: 732678
Maturity: [16x1 double]
OptSpec: {16x1 cell}
InterpMethod: 'price'

```

Note The algorithm for building the ITT tree requires specifying option prices for all tree nodes. The maturities of those options correspond to those of the tree levels, and the strike to the prices on the tree nodes. The types of option are Calls for the nodes above the central nodes, and Puts for those below and including the central nodes.

Clearly, all these options will not be available in the market, hence making interpolation and extrapolation necessary to obtain the node option prices. The degree to which the tree reflects the market will unavoidably be tied to the results of these interpolations and extrapolations. Keeping in mind that extrapolation is less accurate than interpolation, and more so the further away the extrapolated points are from the data points, the function itttree issues a warning with a list of the options for which extrapolation was necessary.

In some cases, it may be desirable to view a list of ideal option prices to form an idea of the ranges needed. This can be achieved by calling the function itttree specifying only the first three input arguments. The second output argument is a structure array containing the list of ideal options needed.

\section*{Creating an Implied Trinomial Tree}

You can now use the StockSpec, TimeSpec, and StockOptSpec structures described in "Stock Structure Example Using an Implied Trinomial Tree" on page 3-10, "TimeSpec Example Using an Implied Trinomial Tree" on page \(3-11\), and "Option Stock Structure Example Using an Implied Trinomial Tree" on page 3-13 to build an implied trinomial tree (ITT). First, you must define
the interest rate term structure. For this example, assume that the interest rate is fixed at \(8 \%\) annually between the valuation date of the tree (January 1,2006 ) until its maturity.
```

Rate = 0.08;
ValuationDate = '01-01-2006';
EndDate = '01-01-2008';
RateSpec = intenvset('StartDates', ValuationDate, 'EndDates', EndDate, ...
'ValuationDate', ValuationDate, 'Rates', Rate, 'Compounding', -1);

```

To build an ITTTree, enter:
```

ITTTree = itttree(StockSpec, RateSpec, TimeSpec, StockOptSpec)

```
```

ITTTree =

```


\section*{Examining Equity Trees}

Financial Instruments Toolbox software uses equity binary and implied trinomial trees to represent prices of equity options and of underlying stocks. At the highest level, these trees have structures wrapped around them. The structures encapsulate information required to interpret information in the tree.

To examine an equity binary or trinomial tree, load the data in the MAT-file deriv.mat into the MATLAB workspace.
load deriv.mat
Display the list of variables loaded from the MAT-file with the whos command.
\begin{tabular}{llrl} 
Name & Size & Bytes \begin{tabular}{l} 
Class
\end{tabular} Atributes \\
& & & \\
BDTInstSet & \(1 \times 1\) & 15956 & struct \\
BDTTree & \(1 \times 1\) & 5138 & struct \\
BKInstSet & \(1 \times 1\) & 15946 & struct \\
BKTree & \(1 \times 1\) & 5904 & struct \\
CRRInstSet & \(1 \times 1\) & 12434 & struct \\
CRRTree & \(1 \times 1\) & 5058 & struct \\
EQPInstSet & \(1 \times 1\) & 12434 & struct \\
EQPTree & \(1 \times 1\) & 5058 & struct \\
HJMInstSet & \(1 \times 1\) & 5948 & struct \\
HJMTree & \(1 \times 1\) & 15946 & struct \\
HWInstSet & \(1 \times 1\) & 5904 & struct \\
HWTree & \(1 \times 1\) & 12438 & struct \\
ITTInstSet & \(1 \times 1\) & 8862 & struct \\
ITTTree & \(1 \times 1\) & 10282 & struct \\
ZeroInstSet & \(1 \times 1\) & 1580 & struct
\end{tabular}

\section*{Examining a CRRTree}

You can examine in some detail the contents of the CRRTree structure contained in this file.

CRRTree
```

CRRTree =
FinObj: 'BinStockTree
Method: 'CRR'
StockSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
tObs: [0 1 2 2 3 4]
dObs: [731582 731947 732313 732678 733043]
STree: {[100] [110.5171 90.4837] [122.1403 100 81.8731] [1x4 double] [1x5 double]}
UpProbs: [0.7309 0.7309 0.7309 0.7309]

```

The Method field of the structure indicates that this is a CRR tree, not an EQP tree.

The fields StockSpec, TimeSpec and RateSpec hold the original structures passed into the function crrtree. They contain all the context information required to interpret the tree data.

The fields tObs and dObs are vectors containing the observation times and dates, the times and dates of the levels of the tree. In this particular case, tObs reveals that the tree has a maturity of 4 years (tObs (end) \(=4\) ) and that it has four time steps (the length of tobs is five).

The field dObs shows the specific dates for the tree levels, with a granularity of 1 day. This means that all values in tObs that correspond to a given day from 00:00 hours to 24:00 hours are mapped to the corresponding value in dObs. You can use the function datestr to convert these MATLAB serial dates into their string representations.

The field UpProbs is a vector representing the probabilities for up movements from any node in each level. This vector has 1 element per tree level. All nodes for a given level have the same probability of an up movement. In the specific case being examined, the probability of an up movement is 0.7309 for all levels, and the probability for a down movement is \(0.2691(1-0.7309)\).

Finally, the field STree contains the actual stock tree. It is represented in MATLAB as a cell array with each cell array element containing a vector of prices corresponding to a tree level. The prices are in descending order, that is, CRRTree.STree\{3\} (1) represents the topmost element of the third level of the tree, and CRRTree.STree\{3\} (end) represents the bottom element of the same level of the tree.

\section*{Examining an ITTTree}

You can examine in some detail the contents of the ITTTree structure contained in this file.
```

ITTTree

```
```

ITTTree =

```

```

RateSpec: [1x1 struct]
tObs: [0 1 1 2 3 4]
dObs: [732678 733043 733408 733773 734139]
STree: {1x5 cell}
Probs: {[3\times1 double] [3\times3 double] [3\times5 double] [3\times7 double]}

```

The fields StockSpec, StockOptSpec, TimeSpec, and RateSpec hold the original structures passed into the function itttree. They contain all the context information required to interpret the tree data.

The fields tObs and dObs are vectors containing the observation times and dates, the times and dates of the levels of the tree. In this particular case, tObs reveals that the tree has a maturity of 4 years (tObs (end) \(=4\) ) and that it has four time steps (the length of tObs is five).

The field dObs shows the specific dates for the tree levels, with a granularity of 1 day. This means that all values in tObs that correspond to a given day from 00:00 hours to 24:00 hours are mapped to the corresponding value in dObs. You can use the function datestr to convert these MATLAB serial dates into their string representations.

The field Probs is a vector representing the probabilities for movements from any node in each level. This vector has three elements per tree node. In the specific case being examined, at tObs= 1 , the probability for an up movement is 0.4675 , and the probability for a down movement is 0.1934 .

Finally, the field STree contains the actual stock tree. It is represented in MATLAB as a cell array with each cell array element containing a vector of prices corresponding to a tree level. The prices are in descending order, that is, ITTTree.STree\{4\} (1) represents the top element of the fourth level of the tree, and ITTTree.STree \(\{4\}\) (end) represents the bottom element of the same level of the tree.

\section*{Isolating a Specific Node for a CRRTree}

The function treepath can isolate a specific set of nodes of a binary tree by specifying the path used to reach the final node. As an example, consider the nodes touched by starting from the root node, then following a down movement, then an up movement, and finally a down movement. You use a vector to specify the path, with 1 corresponding to an up movement and 2
corresponding to a down movement. An up-down-up path is then represented as [lll \(\left.\begin{array}{lll}2 & 1 & 2\end{array}\right]\). To obtain the values of all nodes touched by this path, enter:
```

SVals = treepath(CRRTree.STree, [l2 1 2])
SVals =
100.0000
90.4837
100.0000
90.4837

```

The first value in the vector SVals corresponds to the root node, and the last value corresponds to the final node reached by following the path indicated.

\section*{Isolating a Specific Node for an ITTTree}

The function trintreepath can isolate a specific set of nodes of a trinomial tree by specifying the path used to reach the final node. As an example, consider the nodes touched by starting from the root node, then following an up movement, then a middle movement, and finally a down movement. You use a vector to specify the path, with 1 corresponding to an up movement, 2 corresponding to a middle movement, and 3 corresponding to a down movement. An up-down-middle-down path is then represented as [ \(\left.\begin{array}{llll}1 & 3 & 2 & 3\end{array}\right]\). To obtain the values of all nodes touched by this path, enter:
```

pathSVals = trintreepath(ITTTree, [$$
\begin{array}{llll}{1}&{3}&{2}&{3}\end{array}
$$])

```
pathSVals =
50.0000
66.3448
50.0000
50.0000
37.6819

The first value in the vector pathSVals corresponds to the root node, and the last value corresponds to the final node reached by following the path indicated.

\section*{Differences Between CRR and EQP Tree Structures}

In essence, the structures representing CRR trees and EQP trees are similar. If you create a CRR or an EQP tree using identical input arguments, only a few of the tree structure fields differ:
- The Method field has a value of 'CRR' or 'EQP' indicating the method used to build the structure.
- The prices in the STree cell array have the same structure, but the prices within the cell array are different.
- For EQP, the structure field UpProb always holds a vector with all elements set to 0.5 , while for CRR, these probabilities are calculated based on the input arguments passed when building the tree.

\section*{Supported Equity Derivatives}

\section*{In this section...}
"Asian Option" on page 3-22
"Barrier Option" on page 3-24
"Basket Option" on page 3-25
"Compound Option" on page 3-26
"Lookback Option" on page 3-27
"Digital Option" on page 3-29
"Rainbow Option" on page 3-30
"Vanilla Option" on page 3-31
"Spread Option" on page 3-33

\section*{Asian Option}

An Asian option is a path-dependent option with a payoff linked to the average value of the underlying asset during the life (or some part of the life) of the option. They are similar to lookback options in that there are two types of Asian options: fixed (average price option) and floating (average strike option). Fixed Asian options have a specified strike, while floating Asian options have a strike equal to the average value of the underlying asset over the life of the option.

There are four Asian option types, each with its own characteristic payoff formula:
- Fixed call (average price option): \(\max \left(0, S_{a v}-X\right)\)
- Fixed put (average price option): \(\max \left(0, X-S_{a v}\right)\)
- Floating call (average strike option): \(\max \left(0, S-S_{a v}\right)\)
- Floating put (average strike option): \(\max \left(0, S_{a v}-S\right)\)
where:
\(S_{a v}\) is the average price of underlying asset.
\(S\) is the price of the underlying asset.
\(X\) is the strike price (applicable only to fixed Asian options).
\(S_{a v}\) is defined using either a geometric or an arithmetic average.
The following functions support Asian options.
\begin{tabular}{l|l}
\hline Function & Purpose \\
\hline asianbycrr & Price Asian options from a CRR binomial tree. \\
\hline asianbyeqp & Price Asian options from an EQP binomial tree. \\
\hline asianbyitt & \begin{tabular}{l} 
Price Asian options using an implied trinomial tree \\
(ITT).
\end{tabular} \\
\hline instasian & Construct an Asian option. \\
\hline asianbyls & \begin{tabular}{l} 
Price European or American Asian options using the \\
Longstaff-Schwartz model.
\end{tabular} \\
\hline asiansensbyls & \begin{tabular}{l} 
Calculate prices and sensitivities of European or \\
American Asian options using the Longstaff-Schwartz \\
model.
\end{tabular} \\
\hline asianbykv & \begin{tabular}{l} 
Price European geometric Asian options using the \\
Kemna Vorst model.
\end{tabular} \\
\hline asiansensbykv & \begin{tabular}{l} 
Calculate prices and sensitivities of European \\
geometric Asian options using the Kemna Vorst \\
model.
\end{tabular} \\
\hline asianbylevy & \begin{tabular}{l} 
Price European arithmetic Asian options using the \\
Levy model.
\end{tabular} \\
\hline asiansensbylevy & \begin{tabular}{l} 
Calculate prices and sensitivities of European \\
arithmetic Asian options using the Levy model.
\end{tabular} \\
\hline
\end{tabular}

\section*{Barrier Option}

A barrier option is similar to a vanilla put or call option, but its life either begins or ends when the price of the underlying stock passes a predetermined barrier value. There are four types of barrier options.

\section*{Up Knock-In}

This option becomes effective when the price of the underlying stock passes above a barrier that is above the initial stock price. Once the barrier has knocked in, it will not knock out even if the price of the underlying instrument moves below the barrier again.

\section*{Up Knock-Out}

This option terminates when the price of the underlying stock passes above a barrier that is above the initial stock price. Once the barrier has knocked out, it will not knock in even if the price of the underlying instrument moves below the barrier again.

\section*{Down Knock-In}

This option becomes effective when the price of the underlying stock passes below a barrier that is below the initial stock price. Once the barrier has knocked in, it will not knock out even if the price of the underlying instrument moves above the barrier again.

\section*{Down Knock-Out}

This option terminates when the price of the underlying stock passes below a barrier that is below the initial stock price. Once the barrier has knocked out, it will not knock in even if the price of the underlying instrument moves above the barrier again.

\section*{Rebates}

If a barrier option fails to exercise, the seller may pay a rebate to the buyer of the option. Knock-outs may pay a rebate when they are knocked out, and knock-ins may pay a rebate if they expire without ever knocking in.

The following functions support barrier options.
\begin{tabular}{l|l}
\hline Function & Purpose \\
\hline barrierbycrr & Price barrier options from a CRR binomial tree. \\
\hline barrierbyeqp & Price barrier options from an EQP binomial tree. \\
\hline barrierbyitt & \begin{tabular}{l} 
Price barrier options using an implied trinomial tree \\
(ITT).
\end{tabular} \\
\hline instbarrier & Construct a barrier option. \\
\hline
\end{tabular}

\section*{Basket Option}

A basket option is an option on a portfolio of several underlying equity assets. Payout for a basket option depends on the cumulative performance of the collection of the individual assets. A basket option tends to be cheaper than the corresponding portfolio of plain vanilla options for these reasons:
- If the basket components correlate negatively, movements in the value of one component neutralize opposite movements of another component. Unless all the components correlate perfectly, the basket option is cheaper than a series of individual options on each of the assets in the basket.
- A basket option minimizes transaction costs because an investor has to purchase only one option instead of several individual options.

The payoff for a basket option is as follows:
- For a call: \(\max \left(\sum W i * S i-K ; 0\right)\)
- For a put: \(\max \left(\sum K-W i * S i ; 0\right)\)
where:
\(S i\) is the price of asset \(i\) in the basket.
\(W i\) is the quantity of asset \(i\) in the basket.
\(K\) is the strike price.

Financial Instruments Toolbox software supports Longstaff-Schwartz and Nengiu Ju models for pricing basket options. The Longstaff-Schwartz model supports both European, Bermuda, and American basket options. The Nengiu Ju model only supports European basket options. If you want to price either an American or Bermuda basket option, use the functions for the Longstaff-Schwartz model. To price a European basket option, use either the functions for the Longstaff-Schwartz model or the Nengiu Ju model.
\begin{tabular}{l|l}
\hline Function & Purpose \\
\hline basketbyls & \begin{tabular}{l} 
Price basket options using the Longstaff-Schwartz \\
model.
\end{tabular} \\
\hline basketsensbyls & \begin{tabular}{l} 
Calculate price and sensitivities for basket options \\
using the Longstaff-Schwartz model.
\end{tabular} \\
\hline basketbyju & \begin{tabular}{l} 
Price European basket options using the Nengjiu Ju \\
approximation model.
\end{tabular} \\
\hline basketsensbyju & \begin{tabular}{l} 
Calculate European basket options price and \\
sensitivity using the Nengjiu Ju approximation \\
model.
\end{tabular} \\
\hline basketstockspec & Specify a basket stock structure. \\
\hline
\end{tabular}

\section*{Compound Option}

A compound option is basically an option on an option; it gives the holder the right to buy or sell another option. With a compound option, a vanilla stock option serves as the underlying instrument. Compound options thus have two strike prices and two exercise dates.

There are four types of compound options:
- Call on a call
- Put on a put
- Call on a put
- Put on a call

Note The payoff formulas for compound options are too complex for this discussion. If you are interested in the details, consult the paper by Mark Rubinstein entitled "Double Trouble," published in Risk 5 (1991).

Consider the third type, a call on a put. It gives the holder the right to buy a put option. In this case, on the first exercise date, the holder of the compound option pay the first strike price and receives a put option. The put option gives the holder the right to sell the underlying asset for the second strike price on the second exercise date.

The following functions support compound options.
\begin{tabular}{l|l}
\hline Function & Purpose \\
\hline compoundbycrr & Price compound options from a CRR binomial tree. \\
\hline compoundbyeqp & Price compound options from an EQP binomial tree. \\
\hline compoundbyitt & \begin{tabular}{l} 
Price compound options using an implied trinomial \\
tree (ITT).
\end{tabular} \\
\hline instcompound & Construct a compound option. \\
\hline
\end{tabular}

\section*{Lookback Option}

A lookback option is a path-dependent option based on the maximum or minimum value the underlying asset achieves during the entire life of the option.

Financial Instruments Toolbox software supports two types of lookback options: fixed and floating. Fixed lookback options have a specified strike price, while floating lookback options have a strike price determined by the asset path. Consequently, there are a total of four lookback option types, each with its own characteristic payoff formula:
- Fixed call: \(\max \left(0, S_{\max }-X\right)\)
- Fixed put: \(\max \left(0, X-S_{\min }\right)\)
- Floating call: \(\max \left(0, S-S_{\min }\right)\)
- Floating put: \(\max \left(0, S_{\max }-S\right)\)
where:
\(S_{\text {max }}\) is the maximum price of underlying stock found along the particular path followed to the node.
\(S_{\text {min }}\) is the minimum price of underlying stock found along the particular path followed to the node.
\(S\) is the price of the underlying stock on the node.
\(X\) is the strike price (applicable only to fixed lookback options).
The following functions support lookback options.
\begin{tabular}{l|l}
\hline Function & Purpose \\
\hline lookbackbycrr & Price lookback options from a CRR binomial tree. \\
\hline lookbackbyeqp & Price lookback options from an EQP binomial tree. \\
\hline lookbackbyitt & \begin{tabular}{l} 
Price lookback options using an implied trinomial \\
tree (ITT).
\end{tabular} \\
\hline instlookback & \begin{tabular}{l} 
Construct a lookback option based on an equity tree \\
model.
\end{tabular} \\
\hline lookbackbycvgsg & \begin{tabular}{l} 
Calculate prices of European lookback fixed and \\
floating strike options using the Conze-Viswanathan \\
and Goldman-Sosin-Gatto models. For more \\
information, see "Lookback Option" on page 3-38.
\end{tabular} \\
\hline lookbacksensbycvgs©alculate prices and sensitivities of European \\
fixed and floating strike lookback options using \\
the Conze-Viswanathan and Goldman-Sosin-Gatto \\
models. For more information, see "Lookback Option" \\
on page 3-38.
\end{tabular}
\begin{tabular}{l|l}
\hline Function & Purpose \\
\hline lookbackbyls & \begin{tabular}{l} 
Calculate prices of lookback fixed and floating strike \\
options using the Longstaff-Schwartz model. For \\
more information, see "Lookback Option" on page
\end{tabular} \\
\hline \(3-38\).
\end{tabular}

\section*{Digital Option}

A digital option is an option whose payoff is characterized as having only two potential values: a fixed payout, when the option is in the money or a zero payout otherwise. This is the case irrespective of how far the asset price at maturity is above (call) or below (put) the strike.

Digital options are attractive to sellers because they guarantee a known maximum loss in the event that the option is exercised. This overcomes a fundamental problem with the vanilla options, where the potential loss is unlimited. Digital options are attractive to buyers because the option payoff is a known constant amount, and this amount can be adjusted to provide the exact quantity of protection required.

Financial Instruments Toolbox supports four types of digital options:
- Cash-or-nothing option - Pays some fixed amount of cash if the option expires in the money.
- Asset-or-nothing option - Pays the value of the underlying security if the option expires in the money.
- Gap option - One strike decides if the option is in or out of money; another strike decides the size the size of the payoff.
- Supershare - Pays out a proportion of the assets underlying a portfolio if the asset lies between a lower and an upper bound at the expiry of the option.

The following functions calculate pricing and sensitivity for digital options.
\begin{tabular}{l|l}
\hline Function & Purpose \\
\hline cashbybls & \begin{tabular}{l} 
Calculate the price of cash-or-nothing digital \\
options using the Black-Scholes model.
\end{tabular} \\
\hline assetbybls & \begin{tabular}{l} 
Calculate the price of asset-or-nothing digital \\
options using the Black-Scholes model.
\end{tabular} \\
\hline gapbybls & \begin{tabular}{l} 
Calculate the price of gap digital options \\
using the Black-Scholes model.
\end{tabular} \\
\hline supersharebybls & \begin{tabular}{l} 
Calculate the price of supershare digital \\
options using the Black-Scholes model.
\end{tabular} \\
\hline cashsensbybls & \begin{tabular}{l} 
Calculate the price and sensitivities of \\
cash-or-nothing digital options using the \\
Black-Scholes model.
\end{tabular} \\
\hline assetsensbybls & \begin{tabular}{l} 
Calculate the price and sensitivities of \\
asset-or-nothing digital options using the \\
Black-Scholes model.
\end{tabular} \\
\hline gapsensbybls & \begin{tabular}{l} 
Calculate the price and sensitivities of gap \\
digital options using the Black-Scholes \\
model.
\end{tabular} \\
\hline supersharesensbybls & \begin{tabular}{l} 
Calculate the price and sensitivities of \\
supershare digital options using the \\
Black-Scholes model.
\end{tabular} \\
\hline
\end{tabular}

\section*{Rainbow Option}

A rainbow option payoff depends on the relative price performance of two or more assets. A rainbow option gives the holder the right to buy or sell the best or worst of two securities, or options that pay the best or worst of two assets.

Rainbow options are popular because of the lower premium cost of the structure relative to the purchase of two separate options. The lower cost reflects the fact that the payoff is generally lower than the payoff of the two separate options.

Financial Instruments Toolbox supports two types of rainbow options:
- Minimum of two assets - The option holder has the right to buy(sell) one of two risky assets, whichever one is worth less.
- Maximum of two assets - The option holder has the right to buy(sell) one of two risky assets, whichever one is worth more.

The following rainbow options speculate/hedge on two equity assets.
\begin{tabular}{l|l}
\hline Function & Purpose \\
\hline minassetbystulz & \begin{tabular}{l} 
Calculate the European rainbow option price \\
on minimum of two risky assets using the \\
Stulz option pricing model.
\end{tabular} \\
\hline minassetsensbystulz & \begin{tabular}{l} 
Calculate the European rainbow option prices \\
and sensitivities on minimum of two risky \\
assets using the Stulz pricing model.
\end{tabular} \\
\hline maxassetbystulz & \begin{tabular}{l} 
Calculate the European rainbow option price \\
on maximum of two risky assets using the \\
Stulz option pricing model.
\end{tabular} \\
\hline maxassetsensbystulz & \begin{tabular}{l} 
Calculate the European rainbow option prices \\
and sensitivities on maximum of two risky \\
assets using the Stulz pricing model.
\end{tabular} \\
\hline
\end{tabular}

\section*{Vanilla Option}

A vanilla option is a category of options that includes only the most standard components. A vanilla option has an expiration date and straightforward strike price. American-style options and European-style options are both categorized as vanilla options.

The payoff for a vanilla option is as follows:
- For a call: \(\max (S t-K, 0)\)
- For a put: \(\max (K-S t, 0)\)
where:
\(S t\) is the price of the underlying asset at time \(t\).
\(K\) is the strike price.
The following functions support specifying or pricing a vanilla option.
\begin{tabular}{ll}
\hline Function & Purpose \\
\hline optstockbycrr & \begin{tabular}{l} 
Calculate the price of a European, Bermuda, \\
or American stock option using a CRR tree.
\end{tabular} \\
optstockbyeqp & \begin{tabular}{l} 
Calculate the price of a European, Bermuda, \\
or American stock option using an EQP tree.
\end{tabular} \\
optstockbyitt & \begin{tabular}{l} 
Calculate the price of a European, Bermuda, \\
or American stock option using an ITT tree.
\end{tabular} \\
optstockbylr & \begin{tabular}{l} 
Calculate the price of a European, Bermuda, or \\
American stock option using the Leisen-Reimer \\
(LR) binomial tree model.
\end{tabular} \\
optstockbybls & \begin{tabular}{l} 
Price options using the Black-Scholes option \\
pricing model.
\end{tabular} \\
optstocksensbybls & \begin{tabular}{l} 
Calculate option prices and sensitivities using \\
the Black-Scholes option pricing model.
\end{tabular} \\
optstockbyrgw & \begin{tabular}{l} 
Calculate American call option prices using \\
the Roll-Geske-Whaley option pricing model.
\end{tabular} \\
optstocksensbyrgw & \begin{tabular}{l} 
Calculate American call option prices and \\
sensitivities using the Roll-Geske-Whaley \\
option pricing model.
\end{tabular} \\
optstockbybjs & \begin{tabular}{l} 
Price American options using the \\
Bjerksund-Stensland 2002 option pricing \\
model.
\end{tabular} \\
optstocksensbybjs & \begin{tabular}{l} 
Calculate American option prices and \\
sensitivities using the Bjerksund-Stensland \\
2002 option pricing model.
\end{tabular} \\
optstockbyls & \begin{tabular}{l} 
Price vanilla options using the \\
Longstaff-Schwartz model.
\end{tabular} \\
instoptstock & \begin{tabular}{l} 
Calculate vanilla option prices and sensitivities \\
using the Longstaff-Schwartz model.
\end{tabular} \\
\hline Specify a European or Bermuda option.
\end{tabular}

\section*{Bermuda Put and Call Schedule}

A Bermuda option resembles a hybrid of American and European options. You exercise it on predetermined dates only, usually monthly. In Financial Instruments Toolbox software, you indicate the relevant information for a Bermuda option in two input matrices:
- Strike - Contains the strike price values for the option.
- ExerciseDates - Contains the schedule when you can exercise the option.

\section*{Spread Option}

A spread option is an option written on the difference of two underlying assets. For example, a European call on the difference of two assets X1 and \(X 2\) would have the following pay off at maturity:
\[
\max (X 1-X 2-K, 0)
\]
where:
\(K\) is the strike price.
The following functions support spread options.
\begin{tabular}{l|l}
\hline Function & Purpose \\
\hline spreadbykirk & \begin{tabular}{l} 
Price European spread options using the Kirk pricing \\
model.
\end{tabular} \\
\hline spreadsensbykirk & \begin{tabular}{l} 
Calculate European spread option prices and \\
sensitivities using the Kirk pricing model.
\end{tabular} \\
\hline spreadbybjs & \begin{tabular}{l} 
Price European spread options using the \\
Bjerksund-Stensland pricing model.
\end{tabular} \\
\hline spreadsensbybjs & \begin{tabular}{l} 
Calculate European spread option prices and \\
sensitivities using the Bjerksund-Stensland pricing \\
model.
\end{tabular} \\
\hline spreadbyfd & \begin{tabular}{l} 
Price European or American spread options using the \\
Alternate Direction Implicit (ADI) finite difference \\
method.
\end{tabular} \\
\hline
\end{tabular}
\begin{tabular}{l|l}
\hline Function & Purpose \\
\hline spreadsensbyfd & \begin{tabular}{l} 
Calculate price and sensitivities of European or \\
American spread spread options using the Alternate \\
Direction Implicit (ADI) finite difference method.
\end{tabular} \\
\hline spreadbyls & \begin{tabular}{l} 
Price European or American spread options using \\
Monte Carlo simulations.
\end{tabular} \\
\hline spreadsensbyls & \begin{tabular}{l} 
Calculate price and sensitivities for European \\
or American spread options using Monte Carlo \\
simulations.
\end{tabular} \\
\hline
\end{tabular}

\section*{Supported Energy Derivatives}

\author{
In this section... \\ "Asian Option" on page 3-35 \\ "Vanilla Option" on page 3-36 \\ "Spread Option" on page 3-37 \\ "Lookback Option" on page 3-38
}

\section*{Asian Option}

An Asian option is a path-dependent option with a payoff linked to the average value of the underlying asset during the life (or some part of the life) of the option. They are similar to lookback options in that there are two types of Asian options: fixed (average price option) and floating (average strike option). Fixed Asian options have a specified strike, while floating Asian options have a strike equal to the average value of the underlying asset over the life of the option.

There are four Asian option types, each with its own characteristic payoff formula:
- Fixed call (average price option): \(\max \left(0, S_{a v}-X\right)\)
- Fixed put (average price option): \(\max \left(0, X-S_{a v}\right)\)
- Floating call (average strike option): \(\max \left(0, S-S_{a v}\right)\)
- Floating put (average strike option): \(\max \left(0, S_{a v}-S\right)\)
where:
\(S_{a v}\) is the average price of underlying asset.
\(S\) is the price of the underlying asset.
\(X\) is the strike price (applicable only to fixed Asian options).
\(S_{a v}\) is defined using either a geometric or an arithmetic average.
The following functions support Asian options.
\begin{tabular}{l|l}
\hline Function & Purpose \\
\hline asianbyls & \begin{tabular}{l} 
Price European or American Asian options using the \\
Longstaff-Schwartz model.
\end{tabular} \\
\hline asiansensbyls & \begin{tabular}{l} 
Calculate prices and sensitivities of European or \\
American Asian options using the Longstaff-Schwartz \\
model.
\end{tabular} \\
\hline asianbykv & \begin{tabular}{l} 
Price European geometric Asian options using the \\
Kemna Vorst model.
\end{tabular} \\
\hline asiansensbykv & \begin{tabular}{l} 
Calculate prices and sensitivities of European \\
geometric Asian options using the Kemna Vorst \\
model.
\end{tabular} \\
\hline asianbylevy & \begin{tabular}{l} 
Price European arithmetic Asian options using the \\
Levy model.
\end{tabular} \\
\hline asiansensbylevy & \begin{tabular}{l} 
Calculate prices and sensitivities of European \\
arithmetic Asian options using the Levy model.
\end{tabular} \\
\hline
\end{tabular}

\section*{Vanilla Option}

A vanilla option is a category of options that includes only the most standard components. A vanilla option has an expiration date and straightforward strike price. American-style options and European-style options are both categorized as vanilla options.

The payoff for a vanilla option is as follows:
- For a call: \(\max (S t-K, 0)\)
- For a put: \(\max (K-S t, 0)\)
where:
\(S t\) is the price of the underlying asset at time \(t\).
\(K\) is the strike price.
The following functions support specifying or pricing a vanilla option.
\begin{tabular}{ll}
\hline Function & Purpose \\
\hline optstockbyls & \begin{tabular}{l} 
Price European, Bermudan, or American \\
vanilla options using the Longstaff-Schwartz \\
model.
\end{tabular} \\
optstocksensbyls & \begin{tabular}{l} 
Calculate European, Bermudan, or American \\
vanilla option prices and sensitivities using \\
the Longstaff-Schwartz model.
\end{tabular} \\
\hline
\end{tabular}

\section*{Spread Option}

A spread option is an option written on the difference of two underlying assets. For example, a European call on the difference of two assets X1 and \(X 2\) would have the following pay off at maturity:
\[
\max (X 1-X 2-K, 0)
\]
where:
\(K\) is the strike price.
The following functions support spread options.
\begin{tabular}{l|l}
\hline Function & Purpose \\
\hline spreadbykirk & \begin{tabular}{l} 
Price European spread options using the Kirk pricing \\
model.
\end{tabular} \\
\hline spreadsensbykirk & \begin{tabular}{l} 
Calculate European spread option prices and \\
sensitivities using the Kirk pricing model.
\end{tabular} \\
\hline spreadbybjs & \begin{tabular}{l} 
Price European spread options using the \\
Bjerksund-Stensland pricing model.
\end{tabular} \\
\hline spreadsensbybjs & \begin{tabular}{l} 
Calculate European spread option prices and \\
sensitivities using the Bjerksund-Stensland pricing \\
model.
\end{tabular} \\
\hline
\end{tabular}
\begin{tabular}{l|l}
\hline Function & Purpose \\
\hline spreadbyfd & \begin{tabular}{l} 
Price European or American spread options using the \\
Alternate Direction Implicit (ADI) finite difference \\
method.
\end{tabular} \\
\hline spreadsensbyfd & \begin{tabular}{l} 
Calculate price and sensitivities of European or \\
American spread options using the Alternate \\
Direction Implicit (ADI) finite difference method.
\end{tabular} \\
\hline spreadbyls & \begin{tabular}{l} 
Price European or American spread options using \\
Monte Carlo simulations.
\end{tabular} \\
\hline spreadsensbyls & \begin{tabular}{l} 
Calculate price and sensitivities for European \\
or American spread options using Monte Carlo \\
simulations.
\end{tabular} \\
\hline
\end{tabular}

For more information on using spread options, see "Pricing European and American Spread Options" on page 3-41.

\section*{Lookback Option}

A lookback option is a path-dependent option based on the maximum or minimum value the underlying asset (e.g. electricity, stock) achieves during the entire life of the option. Basically the holder of the option can 'look back' over time to determine the payoff. This type of option provides price protection over a selected period, reduces uncertainties with the timing of market entry, moderates the need for the ongoing management, and therefore, is usually more expensive than vanilla options.

Lookback call options give the holder the right to buy the underlying asset at the lowest price. Lookback put options give the right to sell the underlying asset at the highest price.

Financial Instruments Toolbox software supports two types of lookback options: fixed and floating. The difference is related to how the strike price is set in the contract.Fixed lookback options have a specified strike price, , and the option pays out the maximum of the difference between the highest (lowest) observed price of the underlying during the life of the option and the strike. Floating lookback options have a strike price determined at maturity, and it is set at the lowest (highest) price of the underlying reached during the
life of the option. This means that for a floating strike lookback call (put), the holder has the right to buy (sell) the underlying asset at its lowest (highest) price observed during the life of the option. Consequently, there are a total of four lookback option types, each with its own characteristic payoff formula:
- Fixed call: \(\max \left(0, S_{\max }-X\right)\)
- Fixed put: \(\max \left(0, X-S_{\min }\right)\)
- Floating call: \(\max \left(0, S-S_{\min }\right)\)
- Floating put: \(\max \left(0, S_{\max }-S\right)\)
where:
\(S_{\max }\) is the maximum price of underlying asset.
\(S_{\min }\) is the minimum price of underlying asset.
\(S\) is the price of the underlying asset at maturity.
\(X\) is the strike price.
The following functions support lookback options.
\begin{tabular}{l|l}
\hline Function & Purpose \\
\hline lookbackbycvgsg & \begin{tabular}{l} 
Calculate prices of European lookback fixed and \\
floating strike options using the Conze-Viswanathan \\
and Goldman-Sosin-Gatto models.
\end{tabular} \\
\hline lookbacksensbycvg & \begin{tabular}{l} 
Galculate prices and sensitivities of European \\
fixed and floating strike lookback options using \\
the Conze-Viswanathan and Goldman-Sosin-Gatto \\
models.
\end{tabular} \\
\hline
\end{tabular}
\begin{tabular}{l|l}
\hline Function & Purpose \\
\hline lookbackbyls & \begin{tabular}{l} 
Calculate prices of lookback fixed and floating strike \\
options using the Longstaff-Schwartz model.
\end{tabular} \\
\hline lookbacksensbyls & \begin{tabular}{l} 
Calculate prices and sensitivities of lookback fixed and \\
floating strike options using the Longstaff-Schwartz \\
model.
\end{tabular} \\
\hline
\end{tabular}

Lookback options as well as Asian options are instruments used in the electricity market to manage purchase timing risk. Electricity purchasers cover part of their expected electricity consumption on the forward market to avoid the volatility and limited liquidity of the spot market. Using Asian options as a hedging tool is a passive approach to solving the purchase timing problem. An Asian option instrument diminishes the wrong timing risk but it also reduces any potential benefit to the buyer from falling prices. On the other hand, lookback options allow the purchasers to buy electricity at the lowest price, but as mentioned before, this instrument is more expensive than Asian and vanilla options.

\section*{Pricing European and American Spread Options}

This example shows how to price and calculate sensitivities for European and American spread options using various techniques. First, the price and sensitivities for an European spread option is calculated using closed form solutions. Then, price and sensitivities for an American spread option is calculated using finite difference and Monte Carlo simulations. Finally, further analysis is conducted on spread options with different range of inputs.

Spread options are options on the difference of two underlying asset prices. For example, a call option on the spread between two assets will have the following payoff at maturity:
where is the price of the first underlying asset, is the price of the second underlying asset, and is the strike price. At maturity, if the spread is greater than the strike price, the option holder will exercise the option and gain the difference between the spread and the strike price. If the spread is less than 0 , the option holder will not exercise the option, and the payoff is 0 . Spread options are frequently traded in the energy market. Two examples are:
- Crack spreads: Options on the spread between refined petroleum products and crude oil. The spread represents the refinement margin made by the oil refinery by "cracking" the crude oil into a refined petroleum product.
- Spark spreads: Options on the spread between electricity and some type of fuel. The spread represents the margin of the power plant, which takes fuel to run its generator to produce electricity.

\section*{Overview of the Pricing Methods}

There are several methods to price spread options, as discussed in [1]. This example uses the closed form, finite difference, and Monte Carlo simulations to price spread options. The advantages and disadvantages of each method is discussed below:
- Closed form solutions/approximations of partial differential equations (PDE) are advantageous because they are very fast, and extend well to
computing sensitivities (Greeks). However, closed form solutions are not always available, for example for American spread options.
- The finite difference method is a numerical procedure to solve PDEs by discretizing the price and time variables into a grid. A detailed analysis of this method can be found in [2]. It can handle cases where closed form solutions are not available. Also, finite difference extends well to calculating sensitivities because it outputs a grid of option prices for a range of underlying prices and times. However, it is slower than the closed form solutions.
- Monte Carlo simulation uses random sampling to simulate movements of the underlying asset prices. It handles cases where closed solutions do not exist. However, it usually takes a long time to run, especially if sensitivities need to be calculated.

\section*{Pricing an European Spread Option}

The following example demonstrates the pricing of a crack spread option.
A refiner is concerned about its upcoming maintenance schedule and needs to protect against decreasing crude oil prices and increasing heating oil prices. During the maintenance the refiner needs to continue providing customers with heating oil to meet their demands. The refiner's strategy is to use spread options to manage its hedge.

On January 2013, the refiner buys a \(1: 1\) crack spread option by purchasing heating oil futures and selling crude oil futures. CLF14 WTI crude oil futures is at \(\$ 100\) per barrel and HOF14 heating oil futures contract is at \(\$ 2.6190\) per gallon.
```

clear;

```
\% Price, volatility, and dividend of heating oil
Price1gallon = 2.6190; \(\quad \%\) \$/gallon
Price1 = Price1gallon*42; \% \$/barrel
Vol1 = 0.10;
Div1 \(=0.03\);
\% Price, volatility, and dividend of WTI crude oil
```

Price2 = 100; % \$/barrel
Vol2 = 0.15;
Div2 = 0.02;
% Correlation of underlying prices
Corr = 0.3;
% Option type
OptSpec = 'call';
% Strike
Strike = 5;
% Settlement date
Settle = '01-Jan-2013';
% Maturity
Maturity = '01-Jan-2014';
% Risk free rate
RiskFreeRate = 0.05;
The pricing functions take an interest rate term structure and stock structure as inputs. Also, we need to specify which outputs we are interested in.
\% Define RateSpec
Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
'EndDates', Maturity, 'Rates', RiskFreeRate, 'Compounding', ...
Compounding, 'Basis', Basis);
\% Define StockSpec for the two assets
StockSpec1 = stockspec(Vol1, Price1, 'Continuous', Div1);
StockSpec2 = stockspec(Vol2, Price2, 'Continuous', Div2);
\% Specify price and sensitivity outputs
OutSpec = \{'Price', 'Delta', 'Gamma'\};

```

The Financial Instruments Toolbox \({ }^{\text {TM }}\) contains two types of closed form approximations for calculating price and sensitivities of European spread options: the Kirk's approximation (spreadbykirk, spreadsensbykirk) and the Bjerksund and Stensland model (spreadbybjs, spreadsensbybjs) [3].

The function spreadsensbykirk calculates prices and sensitivities for an European spread option using the Kirk's approximation.
```

% Kirk's approximation
[PriceKirk, DeltaKirk, GammaKirk] = ...
spreadsensbykirk(RateSpec, StockSpec1, StockSpec2, Settle, ...
Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec)
PriceKirk =
8.3636
DeltaKirk =
0.6108 -0.5590
GammaKirk =
0.0225 0.0249

```

The function spreadsensbybjs calculates the prices and sensitivities for an European spread option using the Bjerksund and Stensland model. In [3], Bjerksund and Stensland explains that the Kirk's approximation tends to underprice the spread option when the strike is close to zero, and overprice when the strike is further away from zero. In comparison, the model by Bjerksund and Stensland has higher precision.
\% Bjerksund and Stensland model
[PriceBJS, DeltaBJS, GammaBJS] = ...
spreadsensbybjs(RateSpec, StockSpec1, StockSpec2, Settle, ... Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec)

PriceBJS =
8.3662

DeltaBJS =
\(0.6115-0.5597\)

GammaBJS =
\(0.0225 \quad 0.0248\)

A comparison of the calculated prices show that the two closed form models produce similar results for price and sensitivities. In addition to delta and gamma, the functions can also calculate theta, vega, lambda, and rho.
```

% Comparison of calculated prices
fprintf('Comparison of prices:\n\n');
fprintf('Kirk: %f\n', PriceKirk);
fprintf('BJS: %f\n\n', PriceBJS);
% Comparison of Delta
fprintf('Comparison of delta:\n\n');
fprintf('Kirk: %f %f\n', DeltaKirk(1), DeltaKirk(2));
fprintf('BJS: %f %f\n\n', DeltaBJS(1), DeltaBJS(2));
% Comparison of Gamma
fprintf('Comparison of gamma:\n\n');
fprintf('Kirk: %f %f\n', GammaKirk(1), GammaKirk(2));
fprintf('BJS: %f %f\n\n', GammaBJS(1), GammaBJS(2));

```

\section*{Comparison of prices:}
```

Kirk:
8.363641
BJS:
8.366158

```

Comparison of delta:
```

Kirk: 0.610790 -0.558959
BJS: 0.611469 -0.559670

```

Comparison of gamma:
\begin{tabular}{lll} 
Kirk: & 0.022533 & 0.024850 \\
BJS: & 0.022495 & 0.024819
\end{tabular}

\section*{Pricing an American Spread Option}

Although the closed form approximations are fast and well suited for pricing European spread options, they cannot price American spread options. Using the finite difference method and the Monte Carlo method, an American spread option can be priced. In this example, an American spread option is priced with the same attributes as the above crack spread option.

The finite difference method numerically solves a PDE by discretizing the underlying price and time variables into a grid. The Financial Instrument Toolbox \({ }^{\text {TM }}\) contains the functions spreadbyfd and spreadsensbyfd, which calculate prices and sensitivities for European and American spread options using the finite difference method. For the finite difference method, the composition of the grid has a large impact on the quality of the output and the execution time. Generally, a finely discretized grid will result in outputs that are closer to the theoretical value, but it comes at the cost of longer execution times. The composition of the grid is controlled using optional paramters PriceGridSize, TimeGridSize, AssetPriceMin and AssetPriceMax.
\% To indicate that we are pricing an American option, add an optional input \% of |AmericanOpt| with a value of 1 to the argument of the function.
\% Finite difference method for American spread option
[PriceFD, DeltaFD, GammaFD, PriceGrid, AssetPrice1, ... AssetPrice2] = ... spreadsensbyfd(RateSpec, StockSpec1, StockSpec2, Settle, ... Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec, ... 'PriceGridSize', [500 500], 'TimeGridSize', 100, ...
```

    'AssetPriceMin', [0 0], 'AssetPriceMax', [2000 2000], ...
    'AmericanOpt', 1);
    % Display price and sensitivities
PriceFD
DeltaFD
GammaFD
PriceFD =
8.5463
DeltaFD =
0.6306 -0.5777
GammaFD =
0.0233 0.0259

```

The function spreadsensbyfd also returns a grid that contains the option prices for a range of underlying prices and times. The grid of option prices at time zero, which is the option prices at the settle date, can be plotted for a range of underlying prices.
```

% Plot option prices
figure;
mesh(AssetPrice1, AssetPrice2, PriceGrid(:, :, 1)');
title('American Spread Option Prices for Range of Underlying Prices');
xlabel('Price of underlying asset 1');
ylabel('Price of underlying asset 2');
zlabel('Price of spread option');

```

An American style option can be priced by Monte Carlo methods using the least square method of Longstaff and Schwartz [4]. The Financial Instruments Toolbox \({ }^{\mathrm{TM}}\) contains the functions spreadbyls and spreadsensbyls, which calculate prices and sensetivities of European and American options using simulations. The Monte Carlo simulation method in spreadsensbyls generates multiple paths of simulations according to a geometric Brownian motion (GBM) for the two underlying asset prices. Similar to the finite difference method where the granularity of the grid determined the quality of the output and the execution time, the quality of output and execution time of the Monte Carlo simluation depends on the number of paths (NumTrials) and the number of time periods per path (NumPeriods). Also, note that the results obtained by Monte Carlo simulations are not deterministic. Each run will have different results depending on the simulation outcomes.
```

% To indicate that we are pricing an American option using the Longstaff
% and Schwartz method, add an optional input of |AmericanOpt| with a value
% of 1 to the argument of the function.
% Monte Carlo method for American spread option
[PriceMC, DeltaMC, GammaMC] = ...
spreadsensbyls(RateSpec, StockSpec1, StockSpec2, Settle, ...
Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec, ...
'NumTrials', 1000, 'Antithetic', true, 'AmericanOpt', 1)
PriceMC =
8.4999
DeltaMC =
0.6325 -0.5931
GammaMC =
-0.0873 0.0391

```

The results of the two models are compared. The prices and sensitivities calcluated by the Longstaff and Schwartz method will vary at each run, depending on the outcome of the simulations. It is important to note again that the quality of the results from the finite difference method and the Monte Carlo simulation depend on the optional input parameters. For example, increasing the number of paths (NumTrials) for the SPREADSENSBYLS function will result in more precise results at the cost of longer execution times.
```

% Comparison of calculated prices
fprintf('Comparison of prices:\n\n');
fprintf('Finite Difference: %f\n', PriceFD);
fprintf('Monte Carlo: %f\n\n', PriceMC);
% Comparison of Delta
fprintf('Comparison of delta:\n\n');
fprintf('Finite Difference: %f %f\n', DeltaFD(1), DeltaFD(2));
fprintf('Monte Carlo : %f %f\n\n', DeltaMC(1), DeltaMC(2));
% Comparison of Gamma
fprintf('Comparison of gamma:\n\n');
fprintf('Finite Difference: %f %f\n', GammaFD(1), GammaFD(2));
fprintf('Monte Carlo : %f %f\n\n', GammaMC(1), GammaMC(2));
Comparison of prices:
Finite Difference: 8.546285
Monte Carlo: 8.499894
Comparison of delta:
Finite Difference: 0.630606 -0.577686
Monte Carlo : 0.632549 -0.593106
Comparison of gamma:
Finite Difference: 0.023273 0.025852
Monte Carlo : -0.087340 0.039120

```

\section*{Comparing Results for a Range of Strike Prices}

As discussed earlier, the Kirk's approximation tends to overprice spread options when the strike is further away from zero. To confirm this, a spread option will be priced with the same attributes as before, but for a range of strike prices.
```

% Specify outputs
OutSpec = {'Price', 'Delta'};
% Range of strike prices
Strike = [-25; -15; -5; 0; 5; 15; 25];

```

The results from the Kirk's approximation and the Bjerkland and Stensland model will be compared against the numerical approximation from the finite difference method. Since spreadsensbyfd can only price one option at a time, it is called in a loop for each strike value. The Monte Carlo simulation (spreadsensbyls) with a large number of trial paths could also be used as a benchmark, but the finite difference will be used for this example.
```

% Kirk's approximation
[PriceKirk, DeltaKirk] = ...
spreadsensbykirk(RateSpec, StockSpec1, StockSpec2, Settle, ...
Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec);
% Bjerkland and Stensland model
[PriceBJS, DeltaBJS] = ...
spreadsensbybjs(RateSpec, StockSpec1, StockSpec2, Settle, ...
Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec);
% Finite difference
PriceFD = zeros(numel(Strike), 1);
DeltaFD = zeros(numel(Strike), 2);
for i=1:numel(Strike)
[PriceFD(i), DeltaFD(i,:)] = ...
spreadsensbyfd(RateSpec, StockSpec1, StockSpec2, Settle, ...
Maturity, OptSpec, Strike(i), Corr, 'OutSpec', OutSpec, ...
'PriceGridSize', [500 500], 'TimeGridSize', 100, ...
'AssetPriceMin', [0 0], 'AssetPriceMax', [2000 2000]);
end
% Comparison of calculated prices

```
```

fprintf('Prices for range of strikes:\n\n');
fprintf('Kirk \tBJS \tFD \n');
for i=1:numel(Strike)
fprintf('%f\t%f\t%f\n', PriceKirk(i), PriceBJS(i), PriceFD(i));
end
Prices for range of strikes:
Kirk BJS FD
32.707787 32.672353 32.676040
23.605307 23.577099 23.580307
15.236908 15.228510 15.230919
11.560332 11.560332 11.562023
8.363641 8.366158 8.367212
3.689909 3.678862 3.680493
1.243753 1.219079 1.221866

```

The difference in prices between the closed form and finite difference method is plotted below. It is clear that as the strike moves further away from 0 , the difference between the Kirk's approximation and finite difference (red line) increases, while the difference between the Bjerkland and Stensland model and finite difference (blue line) stays at the same level. As stated in [3], the Kirk's approximation is overpricing the spread option when the strike is far away from 0 .
```

% Plot of difference in price against the benchmark
figure;
plot(PriceKirk-PriceFD, 'Color', 'red');
hold on;
plot(PriceBJS-PriceFD, 'Color', 'blue');
hold off;
title('Difference in Price Against Finite Difference');
legend('Kirk', 'BJS', 'Location', 'EastOutside');
xlabel('Strike');
set(gca, 'XTickLabel', Strike);
ylabel('Difference in Price');

```

Next, the difference in delta between the closed form models and finite difference is plotted. The top plot shows the difference in delta for the first asset, and the bottom plot shows the difference in delta for the second asset. As seen from the small increments in the \(y\)-axis of order \(10 \mathrm{e}-3\), it can be seen that all three models (Kirk, BJS, finite difference) produce similar values for delta.
```

% Plot of difference in delta of first asset against the benchmark
figure;
subplot(2, 1, 1);
plot(DeltaKirk(:,1)-DeltaFD(:,1), 'Color', 'red');
hold on;
plot(DeltaBJS(:,1)-DeltaFD(:,1), 'Color', 'blue');
hold off;
title('Difference in Delta (Asset 1) Against FD');
legend('Kirk', 'BJS', 'Location', 'EastOutside');
xlabel('Strike');
set(gca, 'XTickLabel', Strike);
ylabel('Difference in Delta');
% Plot of difference in delta of second asset against the benchmark
subplot(2, 1, 2);
plot(DeltaKirk(:,2)-DeltaFD(:,2), 'Color', 'red');
hold on;
plot(DeltaBJS(:,2)-DeltaFD(:,2), 'Color', 'blue');
hold off;
title('Difference in Delta (Asset 2) Against FD');
legend('Kirk', 'BJS', 'Location', 'EastOutside');
xlabel('Strike');
set(gca, 'XTickLabel', Strike);
ylabel('Difference in Delta');

```

\section*{Analyzing Prices and Vega at Different Levels of Volatility}

To further show the type of analysis that can be conducted using these models, the above spread option will now be priced at different levels of volatility for the first asset. The price and vega will be compared at three levels of
volatility for the first asset: \(0.1,0.3\), and 0.5 . The Bjerkland and Stensland model will be used for this analysis.
```

% Strike
Strike = 5;
% Specify output
OutSpec = {'Price', 'Vega'};
% Different levels of volatility for asset 1
Vol1 = [0.1, 0.3, 0.5];
StockSpec1 = stockspec(Vol1, Price1, 'Continuous', Div1);
% Bjerkland and Stensland model
[PriceBJS, VegaBJS] = ...
spreadsensbybjs(RateSpec, StockSpec1, StockSpec2, Settle, ...
Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec);
% Display price
fprintf('\nPrices for different vol levels in asset 1:\n\n');
for i=1:numel(Vol1)
fprintf('%f\n', PriceBJS(i));
end
% Display vega for first asset
fprintf('\nAsset 1 vega for different vol levels in asset 1:\n\n');
for i=1:numel(Vol1)
fprintf('%f\n', VegaBJS(i,1));
end
fprintf('\n');
% Display vega for second asset
fprintf('\nAsset 2 vega for different vol levels in asset 1:\n\n');
for i=1:numel(Vol1)
fprintf('%f\n', VegaBJS(i,2));
end
fprintf('\n');

```
```

Prices for different vol levels in asset 1:
8.366158
14.209112
21.795746
Asset 1 vega for different vol levels in asset 1:
15.534849
36.212192
38.794348
Asset 2 vega for different vol levels in asset 1:
29.437036
7.133657
-0.557852

```

The change in the price and vega with respect to the volatility of the first asset is plotted below. It can be observed that as the volatility of the first asset increases, the price of the spread option also increases. Also, the changes in vega indicate that the price of the spread option becomes more sensitive to the volatility of the first asset and less sensitive to the volatility of the second asset.
figure;
```

% Plot price for BJS model
subplot(2, 1, 1);
plot(PriceBJS, 'Color', 'red');
title('Price (BJS)');
legend('Price', 'Location', 'EastOutside');
xlabel('Vol of Asset 1');
set(gca, 'XTick', 1:3);
set(gca, 'XTickLabel', Vol1);
ylabel('Price');
% Plot of vega for BJS model

```
```

subplot(2, 1, 2);
plot(VegaBJS(:,1), 'Color', 'red');
hold on;
plot(VegaBJS(:,2), 'Color', 'blue');
hold off;
title('Vega (BJS)');
legend('Asset 1', 'Asset 2', 'Location', 'EastOutside');
xlabel('Vol of Asset 1');
set(gca, 'XTick', 1:3);
set(gca, 'XTickLabel', Vol1);
set(gca, 'YLim', [-1 40]);
ylabel('Vega');

```

\section*{Summary}

In this example, European and American spread options were priced and analyzed using various techniques. The Financial Instruments Toolbox \({ }^{\text {TM }}\) provides functions for two types of closed form solutions (Kirk, BJS), the finite difference method, and the Monte Carlo simulation method. The closed form solutions are well suited for pricing and sensitivity calculation of European spread options because they are fast. However, they cannot price American spread options. The finite difference method and Monte Carlo method can price both European and American options. However, they are not as fast in pricing European spread options as compared to closed form solutions.

\section*{References}
[1] Carmona, Rene, Durrleman, Valdo, Pricing and Hedging Spread Options, SIAM Review, Vol. 45, No. 4, pp. 627-685, Society for Industrial and Applied Mathematics, 2003.
[2] Wilmott, Paul, Dewynne, Jeff, Howison, Sam, Option Pricing, Oxford Financial Press, 1993.
[3] Bjerksund, Petter, Stensland, Gunnar, Closed form spread option valuation, Department of Finance, NHH, 2006.
[4] Longstaff, Francis A, Schwartz, Eduardo S, Valuing American Options by Simulation: A Simple Least-Squares Approach, Anderson Graduate School of Management, UC Los Angeles, 2001.

\section*{Hedging Strategies Using Spread Options}

This example shows different hedging strategies to minimize exposure in the Energy market using Crack Spread Options.

\section*{Understanding Crack Spread Options}

In the petroleum industry, refiners are concerned about the difference between their input costs (crude oil) and output prices (refined products gasoline, heating oil, diesel fuel, etc). The differential between these two underlying commodities is referred to as a Crack Spread. It represents the profit margin between crude oil and the refined products.

A Spread option is an option on the spread where the holder has the right, but not the obligation, to enter into a spot or forward spread contract. Crack Spread Options are often used to protect against declines in the crack spread or to monetise volatility or price expectations on the spread.

\section*{Example 1: Protecting Margins using a 1:1 Crack Spread Option}

A marketer is interested in protecting his gasoline margin since current prices are strong. A crack spread option strategy will be used to mantain profits for the following season. In March the June WTI crude oil futures is at \(\$ 91.10\) per barrel and RBOB gasoline futures contract is at \(\$ 2.72\) per gallon. The marketer's strategy is a long crack call involving purchasing RBOB gasoline futures and selling crude oil futures.
```

OldFormat = get(0, 'format');
format bank
% Price and volatility of RBOB gasoline
Price1gallon = 2.72; % \$/gallon
Price1 = Price1gallon * 42; % \$/barrel
Vol1 = 0.39;
% Price and volatility of WTI crude oil
Price2 = 91.10; % \$/barrel
Vol2 = 0.34;
% Assume the following data

```
```

% Spread Option
Strike = 20;
OptSpec = 'call';
Settle = '01-March-2013';
Maturity = '01-June-2013';
Corr = 0.45; % Correlation of underlying commodities

```

Define RateSpec and StockSpec
```

% Define RateSpec
Rate = 0.035;
Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
'EndDates', Maturity, 'Rates', Rate, 'Compounding', ...
Compounding, 'Basis', Basis);

```
\% Define StockSpec for the two assets
StockSpec1 = stockspec(Vol1, Price1);
StockSpec2 = stockspec(Vol2, Price2);

Price the Crack Spread Option
\% Use the function 'spreadbybjs' in the Financial Instruments Toolbox(TM)
\% price the spread option using the Bjerksund and Stensland model.
Price = spreadbybjs(RateSpec, StockSpec1, StockSpec2, Settle, ...
                                    Maturity, OptSpec, Strike, Corr)
Price =
\[
9.91
\]

The 1:1 implied current crack spread between these two underlyings is \(\$ 23.14\) per barrel.
```

CrackSpread = Price1 - Price2 % \$/barrel

```

CrackSpread =
23.14

Suppose that by expiration day, June crude oil prices decrease to \(\$ 90.34\) per barrel and gasoline prices rise to \(\$ 2.89\) per gallon.
\% The price changes cause the marketer's profit margin (the new implied cra \% spread) to increase from \$23.14/barrel to \$31.04/barrel: NewCrackSpread \(=(2.89\) * 42) - 90.34

NewCrackSpread =
31.04

Since the marketer purchased a long crack call on the \(\$ 20\) call, the option is now in the money by \(\$ 11.04\)
(NewCrackSpread - Strike)
ans =
11.04

The marketer paid \(\$ 9.91\) from the long crack call, this protects the margin by \(\$ 1.13\).
(NewCrackSpread - Strike - Price)
ans =
1.13

This strategy provides the marketer protection during spread increase scenarios.

\section*{Example 2: Creating a Floor with Crack Spread Options}

A refiner is interested in covering its fixed and operating costs, but still profit from a favorable move in the market. In March the May WTI crude oil futures is at \(\$ 99.43\) per barrel and RBOB gasoline futures contract is at \(\$ 3.04\) per gallon. The refiner believes that the spread between those commodities of \(\$ 28.25\) per barrel is favorable. Of this, \(\$ 11\) corresponds to operating and fixed costs, and \(\$ 17.25\) is the net refining margin. The refiner's strategy is to sell the crack spread by selling 10 RBOB gasoline futures and buying 10 crude oil futures.
```

% Price and volatility of RBOB gasoline
Price1gallon = 3.04; % \$/gallon
Price1 = Price1gallon * 42; % \$/barrel
Vol1 = 0.35;
Div1 = 0.0783;
% Price and volatility of WTI crude oil
Price2 = 99.43; % \$/barrel
Vol2 = 0.38;
Div2 = 0.0571;

```
\% The refiner purchases 10 May RBOB gasoline crack spread puts with a strik
\% price of \(\$ 25\).
\% Assume the following data
\% Spread Option
Strike = 25;
OptSpec = 'put';
Settle = '01-March-2013';
Maturity = '01-May-2013';
Corr \(=0.30 ; \quad\) \% Correlation of underlying commodities

Define RateSpec and StockSpec
```

% Define RateSpec
Rate = 0.035;
Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
'EndDates', Maturity, 'Rates', Rate, 'Compounding', ...
Compounding, 'Basis', Basis);
% Define StockSpec for the two assets
StockSpec1 = stockspec(Vol1, Price1, 'Continuous', Div1);
StockSpec2 = stockspec(Vol2, Price2, 'Continuous', Div2);
Price the Crack Spread Option
% Use the function 'spreadbyfd' in the Financial Instruments Toolbox(TM)
% to price the American spread option using the finite difference method.
Price = spreadbyfd(RateSpec, StockSpec1, StockSpec2, Settle, ...
Maturity, OptSpec, Strike, Corr, 'AmericanOpt', 1)
Price =
6.61
By expiration, if option is exercised, the refiner would have hedged the cost of purchasing 10000 barrels of crude oil with the revenue of selling 10000 barrels of RBOB gasoline. The futures contract represents 1000 barrels of crude oil and 42000 gallons of gasoline.

```
```

CostOfHedge = Price * 10000 % Option premium

```
CostOfHedge = Price * 10000 % Option premium
% The hedge cost $66386 to implement and guarantees that neither a fall in
% RBOB gasoline prices or an increase in WTI crude oil prices will diminish
% the refining margin below $25.
ProfitMargin = 14 * 10000 %$
CrackingMargin = ProfitMargin - CostOfHedge
```

```
% This strategy allows a cracking margin of $73613.
CostOfHedge =
    66122.24
ProfitMargin =
    140000.00
CrackingMargin =
    73877.76
Another strategy for the refiner could be to buy the \(\$ 22\) puts at a price of \(\$ 5.38\).
StrikeNew = 22;
PriceNew = spreadbyfd(RateSpec, StockSpec1, StockSpec2, Settle, ...
                                    Maturity, OptSpec, StrikeNew, Corr, 'AmericanOpt',
% This time the hedge would have cost $53823, but it also guarantees a $11
% per barrel or $56176 cracking margin.
NewCostOfHedge = PriceNew * 10000 % Option premium
NewProfitMargin = 11 * 10000
CrackingMargin = NewProfitMargin - NewCostOfHedge
PriceNew =
5.36
NewCostOfHedge =
```

53570.97

```
NewProfitMargin =
    110000.00
CrackingMargin =
56429.03
```


## Example 3: Using Collars to Reduce the Cost of Hedging

A refiner is concerned about its cost of hedging and decides to use a collar strategy. In April the crack spread is trading at $\$ 4.23$ per barrel. The refiner is not convinced to lock in this margin, but also wants to protect against price changes causing the refinery margin to decrease less than $\$ 4$ per barrel.

```
% Price and volatility of heating oil
Price1gallon = 2.52; % $/gallon
Price1 = Price1gallon * 42; % $/barrel
Vol1 = 0.38;
Div1 = 0.0762;
% Price and volatility of WTI crude oil
Price2 = 101.61; % $/barrel
Vol2 = 0.34;
Div2 = 0.1169;
```

\% To accomplish the collar strategy the refiner sells a call spread option \% with a strike of $\$ 4.50$ and uses the premium income to offset the cost of \% purchasing a put spread option with a strike of $\$ 4$. This allows the \% refiner to benefit if market prices move up and protects it if market \% prices move down.
\% Assume the following data

```
Strike = [4.50;4];
OptSpec = {'call';'put'};
Settle = '01-April-2013';
Maturity = '01-June-2013';
Corr = 0.35; % Correlation of underlying commodities
Define RateSpec and StockSpec
```

```
% Define RateSpec
```

% Define RateSpec
Rate = 0.035;
Rate = 0.035;
Compounding = -1;
Compounding = -1;
Basis = 1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
'EndDates', Maturity, 'Rates', Rate, 'Compounding', ...
'EndDates', Maturity, 'Rates', Rate, 'Compounding', ...
Compounding, 'Basis', Basis);
Compounding, 'Basis', Basis);
% Define StockSpec for the two assets
StockSpec1 = stockspec(Vol1, Price1, 'Continuous', Div1);
StockSpec2 = stockspec(Vol2, Price2, 'Continuous', Div2);
Price the Crack Spread Options
\% Use the function 'spreadbybjs' in the Financial Instruments Toolbox(TM)
% price the spread options using the Bjerksund and Stensland model.
Price = spreadbybjs(RateSpec, StockSpec1, StockSpec2, Settle, ...
Maturity, OptSpec, Strike, Corr)
% The collar strategy allows the refiner to reduced the cost of the hedge
% to \$0.63:
% CostOfHedge = Premium of Call - Premium of Put
CostOfHedge = Price(1) - Price(2)

```

Price =
```

CostOfHedge =
0.63

```

The refiner is protected if the crack spread narrows to less than \(\$ 4\). If the crack spread widens to more than \(\$ 4.50\), the refiner will not benefit over this amount if he has hedged \(100 \%\) of all its market exposure.
```

set(0, 'format', OldFormat);

```

\section*{Pricing Swing Options using the Longstaff-Schwartz Method}

This example shows how to price a swing option using a Monte Carlo simulation and the Longstaff-Schwartz method [6]. A risk-neutral simulation of the underlying natural gas price is conducted using a mean-reverting model. The simulation results are used to price a swing option based on the Longstaff-Schwartz method. This approach uses a regression technique to approximate the continuation value of the option. A comparison is made between a polynomial and spline basis to fit the regression. Finally, the resulting prices are analyzed against lower and upper price boundaries derived from standard European and American options.

\section*{Overview of Swing Options}

Swing options are popular financial instruments in the energy market, which provide flexibility in the volume of the delivered asset. In order for energy consumers to protect themselves against fluctuations in energy prices, they want to lock in a price by purchasing a forward contract, called the baseload forward contract. However, consumers do not know exactly how much energy will be used in the future, and energy commodities such as electricity and gas cannot easily be stored. Therefore, the consumer wants the flexibility to change the amount of energy that is delivered at each delivery date. Swing options provide this flexibility. Thus, the full contract is composed of two parts: the baseload forward contract, and the swing option component.

Swing options are generally over-the-counter (OTC) contracts that can be highly customized. Therefore, there are many different types of constraints and penalties (see [5] for more details). In this example, a swing option is priced where the only constraint is the daily volume, which is known as the Daily Contract Quantity (DCQ). When a swing right is exercised, the volume cannot go below the minimum DCQ (minDCQ), or go above the maximum DCQ (maxDCQ).

There are several methods to price swing options, such as finite differences, simulation, and dynamic programming based on trees [5]. This example uses the simulation based approach with the Longstaff-Schwartz method. The benefit of the simulation based approach is that the dynamics used to simulate the underlying asset price is separated from the pricing algorithm.

In the finite difference and tree based methods, the pricing algorithm must be changed in order to consider pricing with a different underlying price dynamic.

\section*{Risk-Neutral Simulation of Natural Gas Price}

In this example, natural gas is used as the underlying asset with the following mean-reverting dynamic [8]:
where is a standard Brownian motion. Applying Ito's Lemma to the logarithm of the price leads to an Orstein-Uhlenbeck process:
where , , and is defined as:
is the mean-reversion level that determines the value at which the simulated values will revert to in the long run. is the mean-reversion speed that determines how fast this reversion occurs. is the volatility of . We first proceed by simulating the logarithm of the price. Afterwards, the exponential of the simulated values are taken to obtain the prices.

The length of the simulation is for a one year period, with the initial price of 3.9 dollars per MMBtu. The Monte Carlo simulation is conducted for 1,000 trials, with daily periods. In practice, these parameters are calibrated against market data. In this example, , and . The HWV object from the Financial Toolbox \({ }^{\mathrm{TM}}\) is used to simulate the mean-reverting dynamics of the natural gas price.
```

% Settlement date
Settle = '01-Jun-2014';
% Maturity Date
Maturity = '01-Jun-2015';
% Actual/Actual basis
Basis = 0;

```
```

% Initial log(price in \$/MMBtu)
X0 = log(3.9);
% Volatility of log(price)
Sigma = 0.59;
% Number of trials in the Monte Carlo simulation
NumTrials =1000;
% Number of periods (daily)
NumPeriods = daysdif(Settle, Maturity, Basis);
% Daily time step
dt = 1/NumPeriods;
% Mean reversion speed of log(price)
Kappa = 1.2;
% Mean reversion level of log(price)
Theta = 1.7;
% Create HWV object
hwvobj = hwv(Kappa, Theta, Sigma, 'StartState', XO);
The simulation is run and plotted below.

```
```

% Set random number generator seed

```
% Set random number generator seed
savedState = rng(0, 'twister');
savedState = rng(0, 'twister');
% Simulate gas prices
% Simulate gas prices
[Paths, Times] = hwvobj.simBySolution(NumPeriods, 'NTRIALS', NumTrials, ...
[Paths, Times] = hwvobj.simBySolution(NumPeriods, 'NTRIALS', NumTrials, ...
    'DeltaTime', dt);
    'DeltaTime', dt);
Paths = squeeze(exp(Paths));
Paths = squeeze(exp(Paths));
% Restore random number generator state
% Restore random number generator state
rng(savedState);
rng(savedState);
% Plot paths
% Plot paths
figure;
```

figure;

```
```

plot(Times, Paths);
title('Natural Gas Risk-Neutral Price Simulation');
xlabel('Time');
ylabel('Price');

```

In this example, natural gas is used as the underlying asset with a mean-reverting dynamic. However, the Longstaff-Schwartz algorithm can be used for other underlying assets, such as electricity, with any underlying price dynamic.

\section*{Pricing the Swing Option}

We consider a swing option with five swing rights at the strike of \(\$ 4.69 / \mathrm{MMBtu}\), which can be exercised daily between the day after the settlement date and the maturity date. The Daily Contract Quantity (DCQ) is \(10,000 \mathrm{MMBtu}\), which is the average amount of natural gas that the consumer expects to purchase on a given day. The consumer has the flexibility to reduce the purchase amount (downswing) in one day to the minimum DCQ of 2,500 MMBtu, or increase the purchase (upswing) to \(15,000 \mathrm{MMBtu}\). The continuously compounded annual risk-free rate is \(1 \%\).

RATESPEC is used to represent the interest-rate term structure. For the sake of simplicity, we consider a flat interest-rate term structure in this example. The values of RATESPEC can be modified to accomodate any interest-rate curve. The function HSWINGBYLS in this example assumes a daily exercise if the ExerciseDates input is empty.
```

% Define RateSpec
rfrate = 0.01;
Compounding = -1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
'EndDates', Maturity, 'Rates', rfrate, ...
'Compounding', Compounding, 'Basis', Basis);
% Daily exercise
% hswingbyls assumes daily exercise for empty ExerciseDates
ExerciseDates = [];

```
```

% Number of swings
NumSwings = 5;
% Daily Contract Quantity in MMBtu
DCQ = 10000;
% Minimum DCQ constraint in MMBtu
minDCQ = 2500;
% Maximum DCQ constraint in MMBtu
maxDCQ = 15000;
% Strike
Strike = 4.69;

```

The Longstaff-Schwartz method is a backward iteration algorithm, which steps backward in time from the maturity date. At each exercise date, the algorithm approximates the continuation value, which is the value of the option if it is not exercised. This is done by fitting a regression against the values of the simulated prices and the discounted future value of the option at the next exercise date. The future value of the option is known as the algorithm moves backward in time. The continuation value is compared to the sum of the payoff from immediate exercise (a downswing or upswing) and the continuation value of a swing option with one less swing right. If this sum is smaller, the option holder's optimal strategy is to not exercise on that date. The function HSWINGBYLS in this example uses this method to determine the optimal exercise strategy and the price for swing options [1,2,7].

As discussed earlier, the only constraint considered in this example is the minimum and maximum DCQ. In this case, the optimal early exercise strategy is of a bang-bang type. This means that when it is optimal to upswing or downswing at a certain exercise date, the option holder should always exercise at the maximum or minimum DCQ to maximize profit. The bang-bang exercise would not be the optimal strategy if, for example, there is a terminal penalty based on volume. The pricing algorithm would then need to additionally keep track of all possible volume levels, which significantly adds to the runtime performance cost.

First, the swing option is priced using a 3rd order polynomial to fit the regression of the Longstaff-Schwartz method. The function HSWINGBYLS
also generates a plot of the regression between the underlying price and the continuation value at the exercise date before maturity.
```

% Price swing option using 3rd order polynomial to fit Longstaff-Schwartz
% regression
tic;
useSpline = false;
SwingPrice = hswingbyls(Paths, Times, RateSpec, Settle, Maturity, ...
Strike, ExerciseDates, NumSwings, DCQ, minDCQ, maxDCQ, useSpline, ...
[], true)
lsPolyTime = toc;
SwingPrice =
5.6943e+04

```

The above plot of the regression fit shows that the 3rd order polynomial does not fit the continuation value perfectly, especially near the hinge and at the extreme points. We now use the CSAPS function to fit the regression using a cubic smoothing spline with a smoothing parameter of 0.7 . The Curve Fitting Toolbox \({ }^{\text {TM }}\) is required to run the remainder of the example.
```

% Price swing option using smoothed splines to fit Longstaff-Schwartz
% regression
tic;
useSpline = true;
smoothingParam = 0.7;
SwingPriceSpline = hswingbyls(Paths, Times, RateSpec, Settle, Maturity, ..
Strike, ExerciseDates, NumSwings, DCQ, minDCQ, maxDCQ, useSpline, ...
smoothingParam, true)
lsSplineTime = toc;

```

SwingPriceSpline =
\(6.0729 e+04\)

The plot of the regression shows that the cubic smoothing spline has a better fit against the data, thus obtaining a more accurate value for the continuation values. However, the below comparison shows that using a cubic smoothing spline takes longer than a 3rd order polynomial.
```

% Print comparison of running times
fprintf('Comparison of running times:\n\n');
fprintf('3rd order polynomial: %.2f sec\n', lsPolyTime);
fprintf('Spline : %.2f sec\n\n', lsSplineTime);
Comparison of running times:
3rd order polynomial: 14.26 sec
Spline : 35.26 sec

```

Also, it is important to note that the price represents solely the optionality component. Hence, the price of the baseload forward contract is not included in the above calculated price. Because we used a fixed strike price, the baseload contract has a non-zero value, which can be calculated by:
where are the exercise dates (see [3] for more details). The full price of the contract including the baseload and the swing option is calculated below using the swing option price from the smoothed cubic spline.
```

% Obtain discount factors
RS2 = intenvset(RateSpec, 'StartTimes', 0, 'EndTimes', Times(2:end));
D = intenvget(RS2,'Disc');
% Calculate baseload price
BaseLoadPrice = DCQ.*mean(Paths(2:end,:)-Strike,2)'*D;
% Calculate full contract price, based on results from cubic spline LS
FullContractPrice = BaseLoadPrice + SwingPriceSpline

```

\section*{FullContractPrice =}
\(1.2479 \mathrm{e}+05\)

\section*{Price Bounds}

A lower bound for the swing option is a strip of European options, and the upper bound is a strip of American options [4]. Compared to European options, swing options have an early exercise premium at each exercise date, thus the price should be higher. The price is lower than the American option strips, because only a single swing right can be exercised at each exercise date. More than one strip can be exercised in a single day using American options.

The prices for the strips of the lower and upper bounds are calculated below to check that the swing option prices are within these bounds. The European strip prices are calculated against the last five excercise dates.
```

% Obtain discount factor for the last NumSwings exercise dates
D = D(end-NumSwings+1:end);
% European lower bound
idx = size(Paths, 1):-1:(size(Paths, 1) - NumSwings + 1);
putEuro = D'*mean(max(Strike - Paths(idx,:), 0),2);
callEuro = D'*mean(max(Paths(idx,:) - Strike, 0),2);
lowerBound = ((DCQ-minDCQ).*putEuro+(maxDCQ-DCQ).*callEuro);
% American upper bound
[putAmer, callAmer] = hamericanPrice(Paths, Times, RateSpec, Strike);
upperBound = NumSwings.*((DCQ-minDCQ).*putAmer+(maxDCQ-DCQ).**allAmer);
% Print price and lower/upper bounds
fprintf('Comparison to lower and upper bounds:\n\n');
fprintf('Lower bound (European) : %.2f\n', lowerBound);
fprintf('Swing Option Price : %.2f\n', SwingPriceSpline);
fprintf('Upper bound (American) : %.2f\n\n', upperBound);
Comparison to lower and upper bounds:

```
```

Lower bound (European) : 44412.14
Swing Option Price : 60729.00
Upper bound (American) : 68181.42

```

The prices calculated using the Longstaff-Schwartz algorithm are within the lower and upper bounds. The below plot shows a comparison between the swing option and the upper and lower bounds as the number of swings increases, ceteris paribus. When the number of swings is one, the swing option is equivalent to an American option. In the case of daily exericse opportunity (NumSwings = 365), the swing option is equivalent to the strip of European options with daily maturity.

\section*{Conclusion}

The example shows the use of the Longstaff-Schwartz method to price a swing option where the underlying asset follows a mean-reverting dynamic. A 3rd order polynomial and a smoothed cubic spline is used to fit the regression in the Longstff-Schwartz algorithm to approximate the continuation value. It was shown that the smoothed cubic spline fits the data better at the cost of slower performance. Finally, the resulting swing option prices were checked against the lower bound of a strip of European options and an upper bound of a strip of American options.

\section*{References}
[1] Boogert, A., de Jong, C., Gas storage valuation using a Monte Carlo method. Journal of Derivatives, 15(3):81-98, 2008.
[2] Dorr, Uwe, Valuation of Swing Options and Estimation of Exercise strategies by Monte Carlo Techniques, Oxford, 2002.
[3] Hull, John C., Options, Futures, and Other Derivatives, Sixth Edition, Pearson Education, Inc., 2006.
[4] Jaillet, P., Ronn, E. I., Tompaidis, S., Valuation of commodity-based swing options, Management Science, 50(7):909-921, 2004.
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[6] Longstaff, Francis A., Schwartz, Eduardo S, Valuing American Options by Simulation: A Simple Least-Squares Approach, The Review of Financial Studies, 14(1):113-147, 2001.
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[8] Schwartz, Eduardo S, The Stochastic Behavior of Commodity Prices:
Implications for Valuation and Hedging, The Journal of Finance, 52(3):923-973, 1997.

\section*{Pricing Equity Derivatives Using Trees}

\author{
In this section... \\ "Computing Instrument Prices" on page 3-76 \\ "Computing Prices Using CRR" on page 3-77 \\ "Computing Prices Using EQP" on page 3-80 \\ "Computing Prices Using ITT" on page 3-82 \\ "Examining Output from the Pricing Functions" on page 3-85 \\ "Computing Instrument Sensitivities" on page 3-89 \\ "Graphical Representation of Equity Derivative Trees" on page 3-94
}

\section*{Computing Instrument Prices}

The portfolio pricing functions crrprice, eqpprice, and ittprice calculate the price of any set of supported instruments based on a binary equity price tree or an implied trinomial price tree. These functions are capable of pricing the following instrument types:
- Vanilla stock options
- American and European puts and calls
- Exotic options
- Asian
- Barrier
- Compound
- Lookback
- Stock options (Bermuda put and call schedules)

The syntax for calling the function crrprice is:
[Price, PriceTree] = crrprice(CRRTree, InstSet, Options)
The syntax for eqpprice is:
```

[Price, PriceTree] = eqpprice(EQPTree, InstSet, Options)

```

The syntax for ittprice is:

Price = ittprice(ITTTree, ITTInstSet, Options)
These functions require two input arguments: the equity price tree and the set of instruments, InstSet, and allow a third optional argument.

\section*{Required Arguments}

CRRTree is a CRR equity price tree created using crrtree. EQPTree is an equal probability equity price tree created using eqptree. ITTTree is an ITT equity price tree created using itttree. See "Building Equity Binary Trees" on page 3-3 and "Building Implied Trinomial Trees" on page 3-8 to learn how to create these structures.

InstSet is a structure that represents the set of instruments to be priced independently using the model.

\section*{Optional Argument}

You can enter a third optional argument, Options, used when pricing barrier options. For more specific information, see Appendix B, "Derivatives Pricing Options".

These pricing functions internally classify the instruments and call the appropriate individual instrument pricing function for each of the instrument types. The CRR pricing functions are asianbycrr, barrierbycrr, compoundbycrr, lookbackbycrr, and optstockbycrr. A similar set of functions exists for EQP and ITT pricing. You can also use these functions directly to calculate the price of sets of instruments of the same type. See the reference pages for these individual functions for further information.

\section*{Computing Prices Using CRR}

Consider the following example, which uses the portfolio and stock price data in the MAT-file deriv.mat included in the toolbox. Load the data into the MATLAB workspace.
load deriv.mat

Use the MATLAB whos command to display a list of the variables loaded from the MAT-file.
\begin{tabular}{llrl} 
Name & Size & Bytes & Class Attributes \\
& & \\
BDTInstSet & \(1 \times 1\) & 15956 & struct \\
BDTTree & \(1 \times 1\) & 5138 & struct \\
BKInstSet & \(1 \times 1\) & 15946 & struct \\
BKTree & \(1 \times 1\) & 5904 & struct \\
CRRInstSet & \(1 \times 1\) & 12434 & struct \\
CRRTree & \(1 \times 1\) & 5058 & struct \\
EQPInstSet & \(1 \times 1\) & 12434 & struct \\
EQPTree & \(1 \times 1\) & 5058 & struct \\
HJMInstSet & \(1 \times 1\) & 5948 & struct \\
HJMTree & \(1 \times 1\) & 5838 & struct \\
HWInstSet & \(1 \times 1\) & 5946 & struct \\
HWTree & \(1 \times 1\) & 12438 & struct \\
ITTInstSet & \(1 \times 1\) & 8862 & struct \\
ITTTree & \(1 \times 1\) & 10282 & struct \\
ZeroInstSet & \(1 \times 1\) & 1580 & struct
\end{tabular}

CRRTree and CRRInstSet are the required input arguments to call the function crrprice.

Use instdisp to examine the set of instruments contained in the variable CRRInstSet.
instdisp(CRRInstSet)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline Index & Type & OptSpec & Strike & Settle & ExerciseDates & AmericanOpt & Name & Quantity & & \\
\hline 1 & OptStock & call & 105 & 01-Jan-2003 & 01-Jan-2005 & 1 & Call1 & 10 & & \\
\hline 2 & OptStock & & 105 & 01-Jan-2003 & 01-Jan-2006 & 0 & Put1 & 5 & & \\
\hline Index & Type & OptSpec & Strike S & Settle & ExerciseDates & AmericanOpt & Barrier & Spec Barrier & Rebate & Name \\
\hline 3 & Barrier & call & 1050 & 01-Jan-2003 & 01-Jan-2006 & 1 & ui & 102 & 0 & Barrier1 \\
\hline
\end{tabular}
\begin{tabular}{lllllllll} 
Index & Type & UOptSpec UStrike USettle & UExerciseDates & UAmericanOpt COptSpec CStrike CSettle \\
4 & Compound call & 130 & \(01-J a n-2003\) & \(01-J a n-2006\) & 1 & put & 5 & \(01-J a n-2003\)
\end{tabular}


Note Because of space considerations, the compound option above (Index 4) has been condensed to fit the page. The instdisp command displays all compound option fields on your computer screen.

The instrument set contains eight instruments:
- Two vanilla options (Call1, Put1)
- One barrier option (Barrier1)
- One compound option (Compound1)
- Two lookback options (Lookback1, Lookback2)
- Two Asian options (Asian1, Asian2)

Each instrument has a corresponding index that identifies the instrument prices in the price vector returned by crrprice.

Now use crrprice to calculate the price of each instrument in the instrument set.
```

Price = crrprice(CRRTree, CRRInstSet)

```
Price =
8.2863
2.5016
12.1272
3.3241
7.6015
11.7772

> 4.1797
> 3.4219

\section*{Computing Prices Using EQP}

Load the data into the MATLAB workspace.
load deriv.mat

Use the MATLAB whos command to display a list of the variables loaded from the MAT-file.
\begin{tabular}{llrl} 
Name & Size & Bytes & Class
\end{tabular} Attributes

EQPTree and EQPInstSet are the input arguments required to call the function eqpprice.

Use the command instdisp to examine the set of instruments contained in the variable EQPInstSet.
```

instdisp(EQPInstSet)

```
\begin{tabular}{llllllll} 
Index & Type & OptSpec Strike Settle & ExerciseDates & AmericanOpt Name Quantity \\
1 & OptStock call & 105 & \(01-\) Jan-2003 & \(01-\) Jan-2005 & 1 & Call1 10
\end{tabular}


Note Because of space considerations, the compound option above (Index 4) has been condensed to fit the page. The instdisp command displays all compound option fields on your computer screen.

The instrument set contains eight instruments:
- Two vanilla options (Call1, Put1)
- One barrier option (Barrier1)
- One compound option (Compound1)
- Two lookback options (Lookback1, Lookback2)
- Two Asian options (Asian1, Asian2)

Each instrument has a corresponding index that identifies the instrument prices in the price vector returned by eqpprice.

Now use eqpprice to calculate the price of each instrument in the instrument set.

Price = eqpprice(EQPTree, EQPInstSet)

Price =
8.4791
2.6375
12.2632
3.5091
8.7941
12.9577
4.7444
3.9178

\section*{Computing Prices Using ITT}

Consider the following example, which uses the portfolio and stock price data in the MAT-file deriv.mat included in the toolbox. Load the data into the MATLAB workspace.
load deriv.mat
Use the MATLAB whos command to display a list of the variables loaded from the MAT-file.
\begin{tabular}{llrl} 
Name & Size & Bytes & Class \\
& Attributes \\
BDTInstSet & \(1 \times 1\) & 15956 & struct \\
BDTTree & \(1 \times 1\) & 5138 & struct \\
BKInstSet & \(1 \times 1\) & 15946 & struct \\
BKTree & \(1 \times 1\) & 5904 & struct \\
CRRInstSet & \(1 \times 1\) & 12434 & struct \\
CRRTree & \(1 \times 1\) & 5058 & struct \\
EQPInstSet & \(1 \times 1\) & 12434 & struct \\
EQPTree & \(1 \times 1\) & 5058 & struct \\
HJMInstSet & \(1 \times 1\) & 15948 & struct \\
HJMTree & \(1 \times 1\) & 5838 & struct \\
HWInstSet & \(1 \times 1\) & 15946 & struct \\
HWTree & \(1 \times 1\) & 5904 & struct \\
ITTInstSet & \(1 \times 1\) & 12438 & struct \\
ITTTree & \(1 \times 1\) & 8812 & struct \\
ZeroInstSet & \(1 \times 1\) & 10282 & struct \\
ZeroRateSpec & \(1 \times 1\) & 1580 & struct
\end{tabular}

ITTTree and ITTInstSet are the input arguments required to call the function ittprice. Use the command instdisp to examine the set of instruments contained in the variable ITTInstSet.
```

instdisp(ITTInstSet)

```
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline Index & Type & OptSpec & Strike & Settle & ExerciseDates & AmericanOpt & Name Quanti & ity & & \\
\hline 1 & OptStock & call & 105 & 01-Jan-2003 & 01-Jan-2005 & 1 & Call1 10 & & & \\
\hline 2 & OptStock & & 105 & 01-Jan-2003 & 01-Jan-2006 & 0 & Put1 5 & & & \\
\hline Index & Type & OptSpec S & Strike S & Settle & ExerciseDates & AmericanOpt B & BarrierSpec B & Barrier & Rebate & Name \\
\hline 3 & Barrier & call 1 & 1050 & 01-Jan-2003 & 01-Jan-2006 & 1 u & ui & 102 & 0 & Barrier1 \\
\hline Index & Type & UOptSpec & c UStrik & ke USettle & UExerciseDat & tes UAmericano & Opt COptSpec & CStrike & CSett & \\
\hline 4 & Compound & call & 130 & 01-Jan-2003 & 01-Jan-2006 & 1 & put & 5 & 01-Ja & -2003 \\
\hline
\end{tabular}


The instrument set contains eight instruments:
- Two vanilla options (Call1, Put1)
- One barrier option (Barrier1)
- One compound option (Compound1)
- Two lookback options (Lookback1, Lookback2)
- Two Asian options (Asian1, Asian2)

Each instrument has a corresponding index that identifies the instrument prices in the price vector returned by ittprice.

Now use ittprice to calculate the price of each instrument in the instrument set.
```

Price = ittprice(ITTTree, ITTInstSet)

```

Price =
1.650
10.68
2.407
3.229
0.542
6.184
3.205
6.607

\section*{Examining Output from the Pricing Functions}

The prices in the output vector Price correspond to the prices at observation time zero (tObs \(=0\) ), which is defined as the valuation date of the equity tree. The instrument indexing within Price is the same as the indexing within InstSet.

In the CRR example, the prices in the Price vector correspond to the instruments in this order.
```

InstNames = instget(CRRInstSet, 'FieldName','Name')
InstNames =
Call1
Put1
Barrier1
Compound1
Lookback1
Lookback2
Asian1
Asian2

```

Consequently, in the Price vector, the fourth element, 3.3241, represents the price of the fourth instrument (Compound1), and the sixth element, 11.7772, represents the price of the sixth instrument (Lookback2).

In the ITT example, the prices in the Price vector correspond to the instruments in this order.
```

InstNames = instget(ITTInstSet, 'FieldName','Name')
InstNames =
Call1
Put1
Barrier1
Compound1
Lookback1
Lookback2
Asian1
Asian2

```

Consequently, in the Price vector, the first element, 1.650, represents the price of the first instrument (Call1), and the eight element, 6.607, represents the price of the eighth instrument (Asian2).

\section*{Price Tree Output for CRR}

If you call a pricing function with two output arguments, for example:
```

[Price, PriceTree] = crrprice(CRRTree, CRRInstSet)

```
you generate a price tree structure along with the price information.
This price tree structure PriceTree holds all pricing information.
```

PriceTree =
FinObj: 'BinPriceTree'
PTree: {[8x1 double] [8x2 double] [8x3 double] [8x4 double] [8x5 double]}
tObs: [00 1 2 2 3 4]
dObs: [731582 731947 732313 732678 733043]

```

The first field of this structure, FinObj, indicates that this structure represents a price tree. The second field, PTree, is the tree holding the prices of the instruments in each node of the tree. Finally, the third and fourth fields, tObs and dObs, represent the observation time and date of each level of PTree, with tObs using units in terms of compounding periods.

Using the command-line interface, you can directly examine PriceTree. PTree, the field within the PriceTree structure that contains the price tree with the price vectors at every state. The first node represents tObs \(=0\), corresponding to the valuation date.
```

PriceTree.PTree{1}
ans =
8.2863
2.5016
12.1272
3.3241
7.6015
11.7772
4.1797
3.4219

```

With this interface, you can observe the prices for all instruments in the portfolio at a specific time.

The function eqpprice also returns a price tree that you can examine in the same way.

\section*{Price Tree Output for ITT}

If you call a pricing function with two output arguments, for example:
```

[Price, PriceTree] = ittprice(ITTTree, ITTInstSet)

```
you generate a price tree structure along with the price information.
This price tree structure PriceTree holds all pricing information.
```

FinObj: 'TrinPriceTree'
PTree: {[8x1 double] [8x3 double] [8x5 double] [8x7 double] [8x9 double]}
tObs: [0 1 1 2 3 4]
dObs: [732678 733043 733408 733773 734139]

```

The first field of this structure, FinObj, indicates that this structure represents a trinomial price tree. The second field, PTree is the tree holding the prices of the instruments in each node of the tree. Finally, the third and fourth fields, tObs and dObs, represent the observation time and date of each level of PTree, with tObs using units in terms of compounding periods.

Using the command-line interface, you can directly examine PriceTree. PTree, the field within the PriceTree structure that contains the price tree with the price vectors at every state. The first node represents tObs \(=0\), corresponding to the valuation date.

PriceTree. PTree\{1\}
ans =
1.6506
10.6832
2.4074
3.2294
0.5426
6.1845
3.2052
6.6074

With this interface, you can observe the prices for all instruments in the portfolio at a specific time.

\section*{Prices for Lookback and Asian Options for Equity Trees}

Lookback options and Asian options are path dependent, and, as such, there are no unique prices for any node except the root node. Consequently, the corresponding values for lookback and Asian options in the price tree are set to NaN, the only exception being the root node. This becomes apparent if you examine the prices in the second node (tobs \(=1\) ) of the CRR price tree:

PriceTree.PTree\{2\}
\begin{tabular}{rr}
11.9176 & 0 \\
0.9508 & 7.1914 \\
16.4600 & 2.6672 \\
2.5896 & 5.0000 \\
NaN & NaN \\
NaN & NaN \\
NaN & NaN \\
NaN & NaN
\end{tabular}

Examining the prices in the second node (tobs \(=1\) ) of the ITT price tree displays:

PriceTree. PTree\{2\}
ans =
\begin{tabular}{rrr}
3.9022 & 0 & 0 \\
6.3736 & 13.3743 & 22.1915 \\
5.6914 & 0 & 0 \\
2.7663 & 3.8594 & 5.0000 \\
NaN & NaN & NaN \\
NaN & NaN & NaN \\
NaN & NaN & NaN \\
NaN & NaN & NaN
\end{tabular}

\section*{Computing Instrument Sensitivities}

Sensitivities can be reported either as dollar price changes or percentage price changes. The delta, gamma, and vega sensitivities that the toolbox computes are dollar sensitivities.

The functions crrsens, eqpsens, and ittsens compute the delta, gamma, and vega sensitivities of instruments using a stock tree. They also optionally return the calculated price for each instrument. The sensitivity functions require the same two input arguments used by the pricing functions (CRRTree and CRRInstSet for CRR, EQPTree and EQPInstSet for EQP, andITTTree and ITTInstSet for ITT).

As with the instrument pricing functions, the optional input argument Options is also allowed. You would include this argument if you want a sensitivity function to generate a price for a barrier option as one of its outputs and want to control the method that the toolbox uses to perform the pricing operation. See Appendix B, "Derivatives Pricing Options" or the derivset function for more information.

For path-dependent options (lookback and Asian), delta and gamma are computed by finite differences in calls to crrprice, eqpprice, and ittprice. For the other options (stock option, barrier, and compound), delta and gamma are computed from the CRR, EQP, and ITT trees and the corresponding option price tree. (See Chriss, Neil, Black-Scholes and Beyond, pp. 308-312.)

\section*{CRR Sensitivities Example}

The calling syntax for the sensitivity function is:
[Delta, Gamma, Vega, Price] = crrsens(CRRTree, InstSet, Options)
Using the example data in deriv.mat, calculate the sensitivity of the instruments.
```

load deriv.mat
[Delta, Gamma, Vega, Price] = crrsens(CRRTree, CRRInstSet);

```

You can conveniently examine the sensitivities and the prices by arranging them into a single matrix.
```

format bank
All = [Delta, Gamma, Vega, Price]
All =

```
\begin{tabular}{rrrr}
0.59 & 0.04 & 53.45 & 8.29 \\
-0.31 & 0.03 & 67.00 & 2.50 \\
0.69 & 0.03 & 67.00 & 12.13 \\
-0.12 & -0.01 & -98.08 & 3.32 \\
-0.40 & -45926.32 & 88.18 & 7.60 \\
-0.42 & -112143.15 & 119.19 & 11.78 \\
0.60 & 45926.32 & 49.21 & 4.18 \\
0.82 & 112143.15 & 41.71 & 3.42
\end{tabular}

As with the prices, each row of the sensitivity vectors corresponds to the similarly indexed instrument in CRRInstSet. To view the per-dollar sensitivities, divide each dollar sensitivity by the corresponding instrument price.
```

All = [Delta ./ Price, Gamma ./ Price, Vega ./ Price, Price]

```
\begin{tabular}{rrrr}
0.07 & 0.00 & 6.45 & 8.29 \\
-0.12 & 0.01 & 26.78 & 2.50 \\
0.06 & 0.00 & 5.53 & 12.13 \\
-0.04 & -0.00 & -29.51 & 3.32 \\
-0.05 & -6041.77 & 11.60 & 7.60 \\
-0.04 & -9522.02 & 10.12 & 11.78 \\
0.14 & 10987.98 & 11.77 & 4.18 \\
0.24 & 32771.92 & 12.19 & 3.42
\end{tabular}

\section*{ITT Sensitivities Example}

The calling syntax for the sensitivity function is:
[Delta, Gamma, Vega, Price] = ittsens(ITTTree, ITTInstSet, Options)

Using the example data in deriv.mat, calculate the sensitivity of the instruments.
load deriv.mat
```

warning('off', 'fininst:itttree:Extrapolation');
[Delta, Gamma, Vega, Price] = ittsens(ITTTree, ITTInstSet);

```

You can conveniently examine the sensitivities and the prices by arranging them into a single matrix.
format bank
All = [Delta, Gamma, Vega, Price]

All =
\begin{tabular}{rrrr}
0.24 & 0.03 & 19.35 & 1.65 \\
-0.43 & 0.02 & 49.69 & 10.68 \\
0.35 & 0.04 & 12.29 & 2.41 \\
-0.07 & 0.00 & 6.73 & 3.23 \\
0.63 & 142945.66 & 38.90 & 0.54 \\
0.60 & 22703.21 & 68.92 & 6.18 \\
0.32 & -142945.66 & 18.48 & 3.21 \\
0.67 & -22703.21 & 17.75 & 6.61
\end{tabular}

As with the prices, each row of the sensitivity vectors corresponds to the similarly indexed instrument in ITTInstSet.

Note In this example, the extrapolation warnings are turned off before calculating the sensitivities to avoid displaying many warnings on the Command Window as the sensitivities are calculated.

If the extrapolation warnings are turned on
```

warning('on', 'fininst:itttree:Extrapolation');

```
and ittsens is rerun, the extrapolation warnings scroll as the command executes:
[Delta, Gamma, Vega, Price] = ittsens(ITTTree, ITTInstSet)

Warning: The option set specified in StockOptSpec was too narrow for the generated tree. This makes extrapolation necessary. The list of options outside of the
range of those specified in StockOptSpec are:
```

Option Type: 'call' Maturity: 01-Jan-2007 Strike=66.3529
Option Type: 'put' Maturity: 01-Jan-2007 Strike=50.0061
Option Type: 'put' Maturity: 01-Jan-2008 Strike=50.0061
Option Type: 'put' Maturity: 31-Dec-2008 Strike=50.0061
Option Type: 'call' Maturity: 01-Jan-2010 Strike=155.0141
Option Type: 'put' Maturity: 01-Jan-2010 Strike=50.006
> In itttree>InterpOptPrices at 675
In itttree at 277
In stocktreesens>stocktreedeltagamma_PD at 127

```
```

In stocktreesens at 83
In ittsens at 81

```

Warning: The option set specified in StockOptSpec was too narrow for the generated tree. This made extrapolation necessary. Below is a list of the options that were outside of the range of those specified in StockOptSpec.
```

Option Type: 'call' Maturity: 01-Jan-2007 Strike=66.3367
Option Type: 'put' Maturity: 01-Jan-2007 Strike=37.6773
Option Type: 'call' Maturity: 01-Jan-2008 Strike=66.3367
Option Type: 'put' Maturity: 01-Jan-2008 Strike=28.3951
Option Type: 'call' Maturity: 31-Dec-2008 Strike=66.3367
Option Type: 'call' Maturity: 01-Jan-2010 Strike=66.3367
Option Type: 'put' Maturity: 01-Jan-2010 Strike=16.1276
> In itttree>InterpOptPrices at 675
In itttree at 277
In stocktreesens>stocktreedeltagamma_PD at 131
In stocktreesens at 83
In ittsens at 81

```
Warning: The option set specified in StockOptSpec was too narrow for the generated tree.
This made extrapolation necessary. Below is a list of the options that were outside of the
range of those specified in StockOptSpec.
\begin{tabular}{llll} 
Option Type: 'call' & Maturity: 01-Jan-2007 & Strike=67.2897 \\
Option Type: 'put' & Maturity: 01-Jan-2007 & Strike=37.1528 \\
Option Type: 'put' & Maturity: 01-Jan-2008 & Strike=27.6066 \\
Option Type: 'put' & Maturity: 31-Dec-2008 & Strike=20.5132 \\
Option Type: 'call' & Maturity: 01-Jan-2010 & Strike=164.0157 \\
Option Type: 'put' Maturity: 01-Jan-2010 & Strike=15.2424
\end{tabular}
> In itttree>Interp0ptPrices at 675
    In itttree at 277
    In stocktreesens>stocktreevega at 191
    In stocktreesens at 92
    In ittsens at 81

These warnings are a consequence of having to extrapolate to find the option price of the tree nodes. In this example, the set of inputs options was too
narrow for the shift in the tree nodes introduced by the disturbance used to calculate the sensitivities. As a consequence extrapolation for some of the nodes was needed. Since the input data is quite close the extrapolated data, the error introduced by extrapolation is fairly low.

\section*{Graphical Representation of Equity Derivative Trees}

You can use the function treeviewer to display a graphical representation of a tree, allowing you to examine interactively the prices and rates on the nodes of the tree until maturity. The graphical representations of CRR, EQP, and LR trees are equivalent to Black-Derman-Toy (BDT) trees, given that they are all binary recombining trees. The graphical representations of ITT trees are equivalent to Hull-White (HW) trees, given that they are all trinomial recombining trees. See "Graphical Representation of Trees" on page 2-139 for an overview on the use of treeviewer with CRR trees, EQP trees, LR trees, and ITT trees and their corresponding option price trees. Follow the instructions for BDT trees.

\title{
Equity Derivatives Using Closed-Form Solutions
}

\author{
In this section... \\ "Introduction" on page 3-95 \\ "Black-Scholes Model" on page 3-95 \\ "Black Model" on page 3-97 \\ "Roll-Geske-Whaley Model" on page 3-97 \\ "Bjerksund-Stensland 2002 Model" on page 3-98 \\ "Pricing Using the Black-Scholes Model" on page 3-99 \\ "Pricing Using the Black Model" on page 3-101 \\ "Pricing Using the Roll-Geske-Whaley Model" on page 3-102 \\ "Pricing Using the Bjerksund-Stensland Model" on page 3-103
}

\section*{Introduction}

Financial Instruments Toolbox software supports four types of closed-form solutions and analytical approximations to calculate price and sensitivities (greeks) of vanilla options:
- Black-Scholes model
- Black model
- Roll-Geske-Whaley model
- Bjerksund-Stensland 2002 model

\section*{Black-Scholes Model}

The Black-Scholes model is one of the most commonly used models to price European calls and puts. It serves as a basis for many closed-form solutions used for pricing options. The standard Black-Scholes model is based on the following assumptions:
- There are no dividends paid during the life of the option.
- The option can only be exercised at maturity.
- The markets operate under a Markov process in continuous time.
- No commissions are paid.
- The risk-free interest rate is known and constant.
- Returns on the underlying stocks are log-normally distributed.

Note The Black-Scholes model implemented in Financial Instruments Toolbox software allows dividends. The following three dividend methods are supported:
- Cash dividend
- Continuous dividend yield
- Constant dividend yield

However, not all Black-Scholes closed-form pricing functions support all three dividend methods. For more information on specifying the dividend methods, see stockspec.

Closed-form solutions based on a Black-Scholes model support the following tasks.
\begin{tabular}{l|l}
\hline Task & Function \\
\hline \begin{tabular}{l} 
Price European options with different \\
dividends using the Black-Scholes option \\
pricing model.
\end{tabular} & optstockbybls \\
\hline \begin{tabular}{l} 
Calculate European option prices and \\
sensitivities using the Black-Scholes option \\
pricing model.
\end{tabular} & optstocksensbybls \\
\hline \begin{tabular}{l} 
Calculate implied volatility on European \\
options using the Black-Scholes option \\
pricing model.
\end{tabular} & impvbybls \\
\hline \begin{tabular}{l} 
Price European simple chooser options \\
using Black-Scholes model.
\end{tabular} & chooserbybls \\
\hline
\end{tabular}

For an example using the Black-Scholes model, see "Pricing Using the Black-Scholes Model" on page 3-99.

\section*{Black Model}

Use the Black model for pricing European options on physical commodities, forwards or futures. The Black model supported by Financial Instruments Toolbox software is a special case of the Black-Scholes model. The Black model uses a forward price as an underlier in place of a spot price. The assumption is that the forward price at maturity of the option is log-normally distributed.

Closed-form solutions for a Black model support the following tasks.
\begin{tabular}{l|l}
\hline Task & Function \\
\hline \begin{tabular}{l} 
Price European options on futures using the \\
Black option pricing model.
\end{tabular} & optstockbyblk \\
\hline \begin{tabular}{l} 
Calculate European option prices and \\
sensitivities on futures using the Black \\
option pricing model.
\end{tabular} & optstocksensbyblk \\
\hline \begin{tabular}{l} 
Calculate implied volatility for European \\
options using the Black option pricing \\
model.
\end{tabular} & impvbyblk \\
\hline
\end{tabular}

For an example using the Black model, see "Pricing Using the Black Model" on page 3-101.

\section*{Roll-Geske-Whaley Model}

Use the Roll-Geske-Whaley approximation method to price American call options paying a single cash dividend. This model is based on the modification of the observed stock price for the present value of the dividend and also supports a compound option to account for the possibility of early exercise. The Roll-Geske-Whaley model has drawbacks due to an escrowed dividend price approach which may lead to arbitrage. For further explanation, see Options, Futures, and Other Derivatives by John Hull.

Closed-form solutions for a Roll-Geske-Whaley model support the following tasks.
\begin{tabular}{l|l}
\hline Task & Function \\
\hline \begin{tabular}{l} 
Price American call options with a single \\
cash dividend using the Roll-Geske-Whaley \\
option pricing model.
\end{tabular} & optstockbyrgw \\
\hline \begin{tabular}{l} 
Calculate American call prices and \\
sensitivities using the Roll-Geske-Whaley \\
option pricing model.
\end{tabular} & optstocksensbyrgw \\
\hline \begin{tabular}{l} 
Calculate implied volatility for American \\
call options using the Roll-Geske-Whaley \\
option pricing model.
\end{tabular} & impvbyrgw \\
\hline
\end{tabular}

For an example using the Roll-Geske-Whaley model, see "Pricing Using the Roll-Geske-Whaley Model" on page 3-102.

\section*{Bjerksund-Stensland 2002 Model}

Use the Bjerksund-Stensland 2002 model for pricing American puts and calls with continuous dividend yield. This model works by dividing the time to maturity of the option in two separate parts, each with its own flat exercise boundary (trigger price). The Bjerksund-Stensland 2002 method is a generalization of the Bjerksund and Stensland 1993 method and is considered to be computationally efficient. For further explanation, see Closed Form Valuation of American Options by Bjerksund and Stensland.

Closed-form solutions for a Bjerksund-Stensland 2002 model support the following tasks.
\begin{tabular}{l|l}
\hline Task & Function \\
\hline \begin{tabular}{l} 
Price American options with \\
continuous dividend yield using the
\end{tabular} & optstockbybjs \\
\hline
\end{tabular}
\begin{tabular}{l|l}
\hline Task & Function \\
\hline \begin{tabular}{l} 
Bjerksund-Stensland 2002 option pricing \\
model.
\end{tabular} & \\
\hline \begin{tabular}{l} 
Calculate American options prices and \\
sensitivities using the Bjerksund-Stensland \\
2002 option pricing model.
\end{tabular} & optstocksensbybjs \\
\hline \begin{tabular}{l} 
Calculate implied volatility for American \\
options using the Bjerksund-Stensland 2002 \\
option pricing model.
\end{tabular} & impvbybjs \\
\hline
\end{tabular}

For an example using the Bjerksund-Stensland 2002 model, see "Pricing Using the Bjerksund-Stensland Model" on page 3-103.

\section*{Pricing Using the Black-Scholes Model}

Consider a European stock option with an exercise price of \(\$ 40\) on January 1, 2008 that expires on July 1, 2008. Assume the underlying stock pays dividends of \(\$ 0.50\) on March 1 and June 1. The stock is trading at \(\$ 40\) and has a volatility of \(30 \%\) per annum. The risk-free rate is \(4 \%\) per annum. Using this data, calculate the price of a call and a put option on the stock using the Black-Scholes option pricing model:
```

Strike = 40;
AssetPrice = 40;
Sigma = .3;
Rates = 0.04;
Settle = 'Jan-01-08';
Maturity = 'Jul-01-08';
Div1 = 'March-01-2008';
Div2 = 'Jun-01-2008';
Create RateSpec and StockSpec:
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates',...
Maturity, 'Rates', Rates, 'Compounding', -1);
StockSpec = stockspec(Sigma, AssetPrice, {'cash'}, 0.50,{Div1,Div2});

```

Define two options, one call and one put:
OptSpec = \{'call'; 'put'\};
Calculate the price of the European options:
```

Price = optstockbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike)
Price =

```
3.2063
3.4027

The first element of the Price vector represents the price of the call (\$3.21); the second is the price of the put (\$3.40). Use the function optstocksensbybls to compute six sensitivities for the Black-Scholes model: delta, gamma, vega, lambda, rho, and theta and the price of the option.

The selection of output parameters and their order is determined by the optional input parameter OutSpec. This parameter is a cell array of strings, each one specifying a desired output parameter. The order in which these output parameters are returned by the function is the same as the order of the strings contained in OutSpec.

As an example, consider the same options as the previous example. To calculate their Delta, Rho, Price, and Gamma, build the cell array OutSpec as follows:
```

OutSpec = {'delta', 'rho', 'price', 'gamma'};
[Delta, Rho, Price, Gamma] =optstocksensbybls(RateSpec, StockSpec, Settle,...
Maturity, OptSpec, Strike, 'OutSpec', OutSpec)
Delta =
0.5328
-0.4672
Rho =

```

\section*{Pricing Using the Black Model}

Consider two European call options on a futures contract with exercise prices of \(\$ 20\) and \(\$ 25\) that expire on September 1, 2008. Assume that on May 1, 2008 the contract is trading at \(\$ 20\) and has a volatility of \(35 \%\) per annum. The risk-free rate is \(4 \%\) per annum. Using this data, calculate the price of the call futures options using the Black model:
```

Strike = [20; 25];
AssetPrice = 20;
Sigma = .35;
Rates = 0.04;
Settle = 'May-01-08';
Maturity = 'Sep-01-08';
Create RateSpec and StockSpec:

```
```

RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1);
'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1);
StockSpec = stockspec(Sigma, AssetPrice);
```

StockSpec = stockspec(Sigma, AssetPrice);

```

Define the call option:
OptSpec = \{'call'\};

Calculate price and all sensitivities of the European futures options:
```

OutSpec = {'All'}
[Delta, Gamma, Vega, Lambda, Rho, Theta, Price] = optstocksensbyblk(RateSpec,...
StockSpec, Settle, Maturity, OptSpec, Strike, 'OutSpec', OutSpec);
Price =
1.5903
0.3037

```

The first element of the Price vector represents the price of the call with an exercise price of \(\$ 20(\$ 1.59)\); the second is the price of the call with an exercise price of \(\$ 25\) ( \(\$ 2.89\) ).

The function impvbyblk is used to compute the implied volatility using the Black option pricing model. Assuming that the previous European call futures are trading at \(\$ 1.5903\) and \(\$ 0.3037\), you can calculate their implied volatility:

Volatility = impvbyblk(RateSpec, StockSpec, Settle, Maturity,... OptSpec, Strike, Price);

As expected, you get volatilities of \(35 \%\). If the call futures were trading at \(\$ 1.50\) and \(\$ 0.50\) in the market, the implied volatility would be \(33 \%\) and \(42 \%\) :
```

Volatility = impvbyblk(RateSpec, StockSpec, Settle, Maturity,...

```
OptSpec, Strike, [1.50;0.5])
Volatility =
0.3301
0.4148

\section*{Pricing Using the Roll-Geske-Whaley Model}

Consider two American call options, with exercise prices of \(\$ 110\) and \(\$ 100\) on June 1, 2008, that expire on June 1, 2009. Assume the underlying stock pays dividends of \(\$ 0.001\) on December 1, 2008. The stock is trading at \(\$ 80\) and has a volatility of \(20 \%\) per annum. The risk-free rate is \(6 \%\) per annum. Using this
data, calculate the price of the American calls using the Roll-Geske-Whaley option pricing model:
```

AssetPrice = 80;
Settle = 'Jun-01-2008';
Maturity = 'Jun-01-2009';
Strike = [110; 100];
Rate = 0.06;
Sigma = 0.2;
DivAmount = 0.001;
DivDate = 'Dec-01-2008';
Create RateSpec and StockSpec:
StockSpec = stockspec(Sigma, AssetPrice, {'cash'}, DivAmount, DivDate);
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rate, 'Compounding', -1);
Calculate the call prices:

```
```

Price = optstockbyrgw(RateSpec, StockSpec, Settle, Maturity, Strike)

```
Price = optstockbyrgw(RateSpec, StockSpec, Settle, Maturity, Strike)
Price =
    0.8398
    2.0236
```

The first element of the Price vector represents the price of the call with an exercise price of $\$ 110(\$ 0.84)$; the second is the price of the call with an exercise price of $\$ 100(\$ 2.02)$.

## Pricing Using the Bjerksund-Stensland Model

Consider four American stock options (two calls and two puts) with an exercise price of $\$ 100$ that expire on July 1, 2008. Assume the underlying stock pays a continuous dividend yield of $4 \%$ as of January 1, 2008. The stock has a volatility of $20 \%$ per annum and the risk-free rate is $8 \%$ per annum. Using this data, calculate the price of the American calls and puts assuming
the following current prices of the stock: $\$ 80, \$ 90$ (for the calls) and $\$ 100$ and $\$ 110$ (for the puts):

```
Settle = 'Jan-1-2008';
Maturity = 'Jul-1-2008';
Strike = 100;
AssetPrice = [80; 90; 100; 110];
DivYield = 0.04;
Rate = 0.08;
Sigma = 0.20;
Create RateSpec and StockSpec:
StockSpec = stockspec(Sigma, AssetPrice, {'continuous'}, DivYield);
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rate, 'Compounding', -1);
```

Define the option type:

```
OptSpec = {'call'; 'call'; 'put'; 'put'};
```

Compute the option prices:
Price $=$ optstockbybjs(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike)
Price =
0.4144
2.1804
4.7253
1.7164

The first two elements of the Price vector represent the price of the calls ( $\$ 0.41$ and $\$ 2.18$ ), the last two elements represent the price of the put options (\$4.72 and \$1.72). Use the function optstocksensbybjs to compute six sensitivities for the Bjerksund-Stensland model: delta, gamma, vega, lambda, rho, and theta and the price of the option. The selection of output parameters and their order is determined by the optional input parameter OutSpec. This parameter is a cell array of strings, each one specifying a
desired output parameter. The order in which these output parameters are returned by the function is the same as the order of the strings contained in OutSpec. As an example, consider the same options as the previous example. To calculate their delta, gamma, and price, build the cell array OutSpec as follows:

```
OutSpec = {'delta', 'gamma', 'price'};
```

The outputs of optstocksensbybjs will be in the same order as in OutSpec.

```
[Delta ,Gamma, Price]= optstocksensbybjs(RateSpec, StockSpec, Settle,...
Maturity, OptSpec, Strike, 'OutSpec', OutSpec)
Delta =
    0.0843
    0.2912
    0.4803
    0.2261
Gamma =
    0.0136
    0 . 0 2 6 7
    0.0304
    0.0217
Price =
    0.4144
    2.1804
    4.7253
    1.7164
```

For more information on the Bjerksund-Stensland model, see "Closed-Form Solutions Modeling" on page C-9.

## Pricing European Call Options Using Different Equity Models

This example illustrates how the Financial Instruments Toolbox ${ }^{\text {TM }}$ can be used to price European vanilla call options using different equity models.

The example compares call option prices using the Cox-Ross-Rubinstein model, the Leisen-Reimer model and the Black-Scholes closed formula.

## Define the Call Instrument

Consider a European call option, with an exercise price of $\$ 30$ on January 1, 2010. The option expires on Sep 1, 2010. Assume that the underlying stock provides no dividends. The stock is trading at $\$ 25$ and has a volatility of $35 \%$ per annum. The annualized continuously compounded risk-free rate is $1.11 \%$ per annum.

```
% Option
Settle = 'Jan-01-2010';
Maturity = 'Sep-01-2010';
Strike = 30;
OptSpec ='call';
% Stock
AssetPrice = 25;
Sigma = .35;
```


## Create the Interest Rate Term Structure

```
StartDates = '01 Jan 2010';
EndDates = '01 Jan 2013';
Rates = 0.0111;
ValuationDate ='01 Jan 2010';
Compounding = -1;
RateSpec = intenvset('Compounding',Compounding,'StartDates', StartDates,..
    'EndDates', EndDates, 'Rates', Rates,'ValuationDate',
```


## Create the Stock Structure

Suppose we wish to create two scenarios. The first one assumes that AssetPrice is currently $\$ 25$, the option is out of the money (OTM). The second scenario assumes that the option is at the money (ATM), and therefore AssetPriceATM $=30$.

AssetPriceATM = 30;
StockSpec = stockspec(Sigma, AssetPrice);
StockSpecATM = stockspec(Sigma, AssetPriceATM);

## Price the Options Using the Black-Scholes Closed Formula

Use the function 'optstockbybls' in the Financial Instruments Toolbox to compute the price of the European call options.

```
% Price the option with AssetPrice = 25
PriceBLS = optstockbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, St
% Price the option with AssetPrice = 30
PriceBLSATM = optstockbybls(RateSpec, StockSpecATM, Settle, Maturity, OptSp
```


## Build the Cox-Ross-Rubinstein Tree

```
% Create the time specification of the tree
```

% Create the time specification of the tree
NumPeriods = 15;
CRRTimeSpec = crrtimespec(ValuationDate, Maturity, NumPeriods);
% Build the tree
CRRTree = crrtree(StockSpec, RateSpec, CRRTimeSpec);
CRRTreeATM = crrtree(StockSpecATM, RateSpec, CRRTimeSpec);

```

\section*{Build the Leisen-Reimer Tree}
```

% Create the time specification of the tree

```
% Create the time specification of the tree
LRTimeSpec = lrtimespec(ValuationDate, Maturity, NumPeriods);
LRTimeSpec = lrtimespec(ValuationDate, Maturity, NumPeriods);
% Use the default method 'PP1' (Peizer-Pratt method 1 inversion)to build
% Use the default method 'PP1' (Peizer-Pratt method 1 inversion)to build
% the tree
% the tree
LRTree = lrtree(StockSpec, RateSpec, LRTimeSpec, Strike);
LRTree = lrtree(StockSpec, RateSpec, LRTimeSpec, Strike);
LRTreeATM = lrtree(StockSpecATM, RateSpec, LRTimeSpec, Strike);
```

LRTreeATM = lrtree(StockSpecATM, RateSpec, LRTimeSpec, Strike);

```
```

Price the Options Using the Cox-Ross-Rubinstein (CRR) Model
PriceCRR = optstockbycrr(CRRTree, OptSpec, Strike, Settle, Maturity);
PriceCRRATM = optstockbycrr(CRRTreeATM, OptSpec, Strike, Settle, Maturity);
Price the Options Using the Leisen-Reimer (LR) Model
PriceLR = optstockbylr(LRTree, OptSpec, Strike, Settle, Maturity);
PriceLRATM = optstockbylr(LRTreeATM, OptSpec, Strike, Settle, Maturity);

```

\section*{Compare BLS, CRR and LR Results}
```

sprintf('PriceBLS: \t%f\nPriceCRR: \t%f\nPriceLR:\t%f\n', PriceBLS, ...

```
sprintf('PriceBLS: \t%f\nPriceCRR: \t%f\nPriceLR:\t%f\n', PriceBLS, ...
    PriceCRR, PriceLR)
    PriceCRR, PriceLR)
sprintf('\t== ATM ==\nPriceBLS ATM: \t%f\nPriceCRR ATM: \t%f\nPriceLR ATM:\
sprintf('\t== ATM ==\nPriceBLS ATM: \t%f\nPriceCRR ATM: \t%f\nPriceLR ATM:\
    PriceCRRATM, PriceLRATM)
    PriceCRRATM, PriceLRATM)
ans =
PriceBLS: 1.275075
PriceCRR: 1.294979
PriceLR: 1.275838
ans =
    == ATM ==
PriceBLS ATM: 3.497891
PriceCRR ATM: 3.553938
PriceLR ATM: 3.498571
```


## Convergence of CRR and LR Models to a BLS Solution

The following tables compare call option prices using the CRR and LR models against the results obtained with the Black-Scholes formula.

While the CRR binomial model and the Black-Scholes model converge as the number of time steps gets large and the length of each step gets small, this convergence, except for at the money options, is anything but smooth or uniform.

The tables below show that the Leisen-Reimer model reduces the size of the error with even as few steps of 45 .

| Strike $=30$, | Asset Price $=30$ |  |
| :---: | :---: | :---: |
| \#Steps | LR | CRR |
| 15 | 3.4986 | 3.5539 |
| 25 | 3.4981 | 3.5314 |
| 45 | 3.4980 | 3.5165 |
| 65 | 3.4979 | 3.5108 |
| 85 | 3.4979 | 3.5077 |
| 105 | 3.4979 | 3.5058 |
| 201 | 3.4979 | 3.5020 |
| 501 | 3.4979 | 3.4996 |
| 999 | 3.4979 | 3.4987 |
| Strike = 30, | Asset Price $=25$ |  |
| \#Steps | LR | CRR |
| 15 | 1.2758 | 1.2950 |
| 25 | 1.2754 | 1.2627 |
| 45 | 1.2751 | 1.2851 |
| 65 | 1.2751 | 1.2692 |
| 85 | 1.2751 | 1.2812 |
| 105 | 1.2751 | 1.2766 |
| 201 | 1.2751 | 1.2723 |
| 501 | 1.2751 | 1.2759 |
| 999 | 1.2751 | 1.2756 |

Analyze the Effect of the Number of Periods on the Price of the Options

The following graphs show how convergence changes as the number of steps in the binomial calculation increases, as well as the impact on convergence to changes to the stock price. Observe that the Leisen-Reimer model removes the oscillation and produces estimates close to the Black-Scholes model using only a small number of steps.

```
NPoints = 300;
```

```
% Cox-Ross-Rubinstein
NumPeriodCRR = 5 : 1 : NPoints;
NbStepCRR = length(NumPeriodCRR);
PriceCRR = nan(NbStepCRR, 1);
PriceCRRATM = PriceCRR;
```

for $i=1$ : NbStepCRR
CRRTimeSpec = crrtimespec(ValuationDate, Maturity, NumPeriodCRR(i));
CRRT = crrtree(StockSpec, RateSpec, CRRTimeSpec);
PriceCRR(i) = optstockbycrr(CRRT, OptSpec, Strike,ValuationDate, Maturi
CRRTATM $=$ crrtree(StockSpecATM, RateSpec, CRRTimeSpec);
PriceCRRATM(i) = optstockbycrr(CRRTATM, OptSpec, Strike,ValuationDate,
end;
\% Now with Leisen-Reimer
NumPeriodLR $=5$ : 2 : NPoints;
NbStepLR $=$ length(NumPeriodLR);
PriceLR = nan(NbStepLR, 1);
PriceLRATM = PriceLR;
for i = 1 : NbStepLR
LRTimeSpec = lrtimespec(ValuationDate, Maturity, NumPeriodLR(i));
LRT = lrtree(StockSpec, RateSpec, LRTimeSpec, Strike);
PriceLR(i) = optstockbylr(LRT, OptSpec, Strike,ValuationDate, Maturity)
LRTATM = lrtree(StockSpecATM, RateSpec, LRTimeSpec, Strike);
PriceLRATM(i) = optstockbylr(LRTATM, OptSpec, Strike, ValuationDate, Mat
end;

First scenario: Out of the Money call option

```
% For Cox-Ross-Rubinstein
plot(NumPeriodCRR, PriceCRR);hold on;
plot(NumPeriodCRR, PriceBLS*ones(NbStepCRR,1),'Color',[0 0.9 0], 'linewidth
% For Leisen-Reimer
plot(NumPeriodLR, PriceLR, 'Color',[0.9 0 0], 'linewidth', 1.5);
% Concentrate in the area of interest by clipping on the Y axis at 5x the
% LR Price:
YLimDelta = 5*abs(PriceLR(1) - PriceBLS);
set(gca, 'ylim', [PriceBLS-YLimDelta PriceBLS+YLimDelta]);
% Annotate Plot
titleString = sprintf('\nConvergence of CRR and LR models to a BLS Solution
title(titleString)
ylabel('Option Price')
xlabel('Number of Steps')
legend('CRR', 'BLS', 'LR', 'Location', 'NorthEast')
Second scenario: At the Money call option
```

```
% For Cox-Ross-Rubinstein
```

% For Cox-Ross-Rubinstein
figure;
figure;
plot(NumPeriodCRR, PriceCRRATM);hold on;
plot(NumPeriodCRR, PriceCRRATM);hold on;
plot(NumPeriodCRR, PriceBLSATM*ones(NbStepCRR,1),'Color',[0 0.9 0], 'linewi
plot(NumPeriodCRR, PriceBLSATM*ones(NbStepCRR,1),'Color',[0 0.9 0], 'linewi
% For Leisen-Reimer
% For Leisen-Reimer
plot(NumPeriodLR, PriceLRATM, 'Color',[0.9 0 0], 'linewidth', 1.5);
plot(NumPeriodLR, PriceLRATM, 'Color',[0.9 0 0], 'linewidth', 1.5);
% Concentrate in the area of interest by clipping on the Y axis at 5x the
% Concentrate in the area of interest by clipping on the Y axis at 5x the
% LR Price:
% LR Price:
YLimDelta = 5*abs(PriceLRATM(1) - PriceBLSATM);
YLimDelta = 5*abs(PriceLRATM(1) - PriceBLSATM);
set(gca, 'ylim', [PriceBLSATM-YLimDelta PriceBLSATM+YLimDelta]);
set(gca, 'ylim', [PriceBLSATM-YLimDelta PriceBLSATM+YLimDelta]);
% Annotate Plot
% Annotate Plot
titleString = sprintf('\nConvergence of CRR and LR models to a BLS Solution
titleString = sprintf('\nConvergence of CRR and LR models to a BLS Solution
title(titleString)
title(titleString)
ylabel('Option Price')

```
ylabel('Option Price')
```

```
xlabel('Number of Steps')
legend('CRR', 'BLS', 'LR', 'Location', 'NorthEast')
```


## Hedging Portfolios

- "Hedging" on page 4-2
- "Hedging Functions" on page 4-3
- "Pricing and Hedging a Portfolio Using the Black-Karasinski Model" on page 4-16
- "Specifying Constraints with ConSet" on page 4-31
- "Hedging with Constrained Portfolios" on page 4-36


## Hedging

Hedging is an important consideration in modern finance. Whether or not to hedge, how much portfolio insurance is adequate, and how often to rebalance a portfolio are important considerations for traders, portfolio managers, and financial institutions alike.

If there were no transaction costs, financial professionals would prefer to rebalance portfolios continually, thereby minimizing exposure to market movements. However, in practice, the transaction costs associated with frequent portfolio rebalancing may be expensive. Therefore, traders and portfolio managers must carefully assess the cost required to achieve a particular portfolio sensitivity (for example, maintaining delta, gamma, and vega neutrality). Thus, the hedging problem involves the fundamental tradeoff between portfolio insurance and the cost of such insurance coverage.

## Hedging Functions

In this section...<br>"Introduction" on page 4-3<br>"Hedging with hedgeopt" on page 4-4<br>"Self-Financing Hedges with hedgeslf" on page 4-12

## Introduction

Financial Instruments Toolbox software offers two functions for assessing the fundamental hedging tradeoff, hedgeopt and hedgeslf.

The first function, hedgeopt, addresses the most general hedging problem. It allocates an optimal hedge to satisfy either of two goals:

- Minimize the cost of hedging a portfolio given a set of target sensitivities.
- Minimize portfolio sensitivities for a given set of maximum target costs.
hedgeopt allows investors to modify portfolio allocations among instruments according to either of the goals. The problem is cast as a constrained linear least-squares problem. For additional information about hedgeopt, see "Hedging with hedgeopt" on page 4-4.

The second function, hedgeslf, attempts to allocate a self-financing hedge among a portfolio of instruments. In particular, hedgeslf attempts to maintain a constant portfolio value consistent with reduced portfolio sensitivities (that is, the rebalanced portfolio is hedged against market moves and is closest to being self-financing). If hedgeslf cannot find a self-financing hedge, it rebalances the portfolio to minimize overall portfolio sensitivities. For additional information on hedgeslf, see "Self-Financing Hedges with hedgeslf" on page 4-12.

The examples in this section consider the delta, gamma, and vega sensitivity measures. In this toolbox, when you work with interest-rate derivatives, delta is the price sensitivity measure of shifts in the forward yield curve, gamma is the delta sensitivity measure of shifts in the forward yield curve, and vega is the price sensitivity measure of shifts in the volatility process. See bdtsens
or hjmsens for details on the computation of sensitivities for interest-rate derivatives.

For equity exotic options, the underlying instrument is the stock price instead of the forward yield curve. Consequently, delta now represents the price sensitivity measure of shifts in the stock price, gamma is the delta sensitivity measure of shifts in the stock price, and vega is the price sensitivity measure of shifts in the volatility of the stock. See crrsens, eqpsens, or ittsens for details on the computation of sensitivities for equity derivatives.

For examples showing the computation of sensitivities for interest-rate based derivatives, see "Computing Instrument Sensitivities" on page 2-56. Likewise, for examples showing the computation of sensitivities for equity exotic options, see "Computing Instrument Sensitivities" on page 3-89.

Note The delta, gamma, and vega sensitivities that the toolbox calculates are dollar sensitivities.

## Hedging with hedgeopt

Note The numerical results in this section are displayed in the MATLAB bank format. Although the calculations are performed in floating-point double precision, only two decimal places are displayed.

To illustrate the hedging facility, consider the portfolio HJMInstSet obtained from the example file deriv.mat. The portfolio consists of eight instruments: two bonds, one bond option, one fixed-rate note, one floating-rate note, one cap, one floor, and one swap.

Both hedging functions require some common inputs, including the current portfolio holdings (allocations), and a matrix of instrument sensitivities. To create these inputs, load the example portfolio into memory
load deriv.mat;
compute price and sensitivities

```
[Delta, Gamma, Vega, Price] = hjmsens(HJMTree, HJMInstSet);
Warning: Not all cash flows are aligned with the tree. Result will
be approximated.
```

and extract the current portfolio holdings.
Holdings = instget(HJMInstSet, 'FieldName', 'Quantity');
For convenience place the delta, gamma, and vega sensitivity measures into a matrix of sensitivities.

Sensitivities = [Delta Gamma Vega];
Each row of the Sensitivities matrix is associated with a different instrument in the portfolio, and each column with a different sensitivity measure.

To summarize the portfolio information
disp([Price Holdings Sensitivities])

| 98.72 | 100.00 | -272.65 | 1029.90 | 0.00 |
| ---: | ---: | ---: | ---: | ---: |
| 97.53 | 50.00 | -347.43 | 1622.69 | -0.04 |
| 0.05 | -50.00 | -8.08 | 643.40 | 34.07 |
| 98.72 | 80.00 | -272.65 | 1029.90 | 0.00 |
| 100.55 | 8.00 | -1.04 | 3.31 | 0 |
| 6.28 | 30.00 | 294.97 | 6852.56 | 93.69 |
| 0.05 | 40.00 | -47.16 | 8459.99 | 93.69 |
| 3.69 | 10.00 | -282.05 | 1059.68 | 0.00 |

The first column above is the dollar unit price of each instrument, the second is the holdings of each instrument (the quantity held or the number of contracts), and the third, fourth, and fifth columns are the dollar delta, gamma, and vega sensitivities, respectively.

The current portfolio sensitivities are a weighted average of the instruments in the portfolio.

```
TargetSens = Holdings' * Sensitivities
```


## Maintaining Existing Allocations

To illustrate using hedgeopt, suppose that you want to maintain your existing portfolio. The first form of hedgeopt minimizes the cost of hedging a portfolio given a set of target sensitivities. If you want to maintain your existing portfolio composition and exposure, you should be able to do so without spending any money. To verify this, set the target sensitivities to the current sensitivities.
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price,... Holdings, [], [], [], TargetSens)

Sens =

```
    -61910.22 788946.21 4852.91
```

Cost $=$

0

Quantity' =
100.00
50.00
$-50.00$
80.00
8.00
30.00
40.00
10.00

Portfolio composition and sensitivities are unchanged, and the cost associated with doing nothing is zero. The cost is defined as the change in portfolio value. This number cannot be less than zero because the rebalancing cost is defined as a nonnegative number.

If Value0 and Value1 represent the portfolio value before and after rebalancing, respectively, the zero cost can also be verified by comparing the portfolio values.

```
ValueO = Holdings' * Price
Value0 =
```

23674.62

Value1 = Quantity * Price
Value1 $=$
23674.62

## Partially Hedged Portfolio

Building on the example in "Maintaining Existing Allocations" on page $4-6$, suppose you want to know the cost to achieve an overall portfolio dollar sensitivity of [-23000-3300 3000], while allowing trading only in instruments 2 , 3 , and 6 (holding the positions of instruments $1,4,5,7$, and 8 fixed). To find the cost, first set the target portfolio dollar sensitivity.

TargetSens = [-23000 -3300 3000];
Then, specify the instruments to be fixed.
FixedInd = [lllll 14578$] ;$
Finally, call hedgeopt
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price,... Holdings, FixedInd, [], [], TargetSens);
and again examine the results.

```
Sens =
    -23000.00 -3300.00 3000.00
Cost =
```

19174.02

## Quantity' =

100.00
-141.03
137.26
80.00
8.00
$-57.96$
40.00
10.00

Recompute Value1, the portfolio value after rebalancing.

```
Value1 = Quantity * Price
```

Value1 =
4500.60

As expected, the cost, $\$ 19174.02$, is the difference between Value0 and Value1, $\$ 23674.62$ - $\$ 4500.60$. Only the positions in instruments 2, 3, and 6 have been changed.

## Fully Hedged Portfolio

The example in "Partially Hedged Portfolio" on page 4-7 illustrates a partial hedge, but perhaps the most interesting case involves the cost associated with a fully hedged portfolio (simultaneous delta, gamma, and vega neutrality). In this case, set the target sensitivity to a row vector of 0s and call hedgeopt again. The following example uses data from "Hedging with hedgeopt" on page 4-4.

TargetSens = [0 0 0 ];
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price, ... Holdings, FixedInd, [], [], TargetSens);

Examining the outputs reveals that you have obtained a fully hedged portfolio

```
Sens =
    -0.00 -0.00 -0.00
```

but at an expense of over $\$ 20,000$.
Cost =
23055.90

The positions required to achieve a fully hedged portfolio

```
Quantity' =
```

    100.00
    -182.36
    -19.55
    80.00
        8.00
    -32.97
    40.00
    10.00
    result in the new portfolio value
Value1 = Quantity * Price

Value1 =
618.72

## Minimizing Portfolio Sensitivities

The examples in "Fully Hedged Portfolio" on page 4-8 illustrate how to use hedgeopt to determine the minimum cost of hedging a portfolio given a set of target sensitivities. In these examples, portfolio target sensitivities are treated as equality constraints during the optimization process. You tell hedgeopt what sensitivities you want, and it tells you what it will cost to get those sensitivities.

A related problem involves minimizing portfolio sensitivities for a given set of maximum target costs. For this goal, the target costs are treated as inequality constraints during the optimization process. You tell hedgeopt the most you are willing spend to insulate your portfolio, and it tells you the smallest portfolio sensitivities you can get for your money.

To illustrate this use of hedgeopt, compute the portfolio dollar sensitivities along the entire cost frontier. From the previous examples, you know that spending nothing replicates the existing portfolio, while spending $\$ 23,055.90$ completely hedges the portfolio.

Assume, for example, you are willing to spend as much as $\$ 50,000$, and want to see what portfolio sensitivities will result along the cost frontier. Assume that the same instruments are held fixed, and that the cost frontier is evaluated from $\$ 0$ to $\$ 50,000$ at increments of $\$ 1000$.

MaxCost = [0:1000:50000];
Now, call hedgeopt.
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price, ... Holdings, FixedInd, [], MaxCost);

With this data, you can plot the required hedging cost versus the funds available (the amount you are willing to spend)

```
plot(MaxCost/1000, Cost/1000, 'red'), grid
xlabel('Funds Available for Rebalancing ($1000''s)')
ylabel('Actual Rebalancing Cost ($1000''s)')
title ('Rebalancing Cost Profile')
```



## Rebalancing Cost Profile

and the portfolio dollar sensitivities versus the funds available.

```
figure
plot(MaxCost/1000, Sens(:,1), '-red')
hold('on')
plot(MaxCost/1000, Sens(:,2), '-.black')
plot(MaxCost/1000, Sens(:,3), '--blue')
grid
xlabel('Funds Available for Rebalancing ($1000''s)')
ylabel('Delta, Gamma, and Vega Portfolio Dollar Sensitivities')
title ('Portfolio Sensitivities Profile')
legend('Delta', 'Gamma', 'Vega', O)
```



## Funds Available for Rebalancing

## Self-Financing Hedges with hedgeslf

The figures Rebalancing Cost Profile on page 4-11 and Funds Available for Rebalancing on page 4-12 indicate that there is no benefit because the funds available for hedging exceed $\$ 23,055.90$, the point of maximum expense required to obtain simultaneous delta, gamma, and vega neutrality. You can also find this point of delta, gamma, and vega neutrality using hedgeslf.
[Sens, Value1, Quantity] = hedgeslf(Sensitivities, Price,... Holdings, FixedInd);

Sens =

$$
\begin{aligned}
& -0.00 \\
& -0.00 \\
& -0.00
\end{aligned}
$$

```
Quantity =
    100.00
    -182.36
    -19.55
    80.00
    8.00
    -32.97
    40.00
    10.00
```

Similar to hedgeopt, hedgeslf returns the portfolio dollar sensitivities and instrument quantities (the rebalanced holdings). However, in contrast, the second output parameter of hedgeslf is the value of the rebalanced portfolio, from which you can calculate the rebalancing cost by subtraction.

```
Value0 - Value1
ans =
23055.90
```

In this example, the portfolio is clearly not self-financing, so hedgeslf finds the best possible solution required to obtain zero sensitivities.

There is, in fact, a third calling syntax available for hedgeopt directly related to the results shown above for hedgeslf. Suppose, instead of directly specifying the funds available for rebalancing (the most money you are willing to spend), you want to simply specify the number of points along the cost frontier. This call to hedgeopt samples the cost frontier at 10 equally spaced points between the point of minimum cost (and potentially maximum exposure) and the point of minimum exposure (and maximum cost).

```
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price,...
Holdings, FixedInd, 10);
```

Sens $=$

| -32784.46 | 2231.83 | -49694.33 |
| :--- | :--- | :--- |
| -29141.74 | 1983.85 | -44172.74 |
| -25499.02 | 1735.87 | -38651.14 |

```
    -21856.30
    1487.89
    1239.91 -27607.96
    -18213.59
    991.93
    743.94 -16564.78
    -10928.15
        495.96 -11043.18
    -7285.43
            247.98
                            -5521.59
    -3642.72
            0.00
                            -0.00
                            0.00
Cost =
            0.00
            2561.77
            5123.53
            7685.30
            10247.07
            12808.83
            15370.60
            17932.37
            20494.14
            23055.90
Now plot this data.
```

```
figure
```

figure
plot(Cost/1000, Sens(:,1), '-red')
plot(Cost/1000, Sens(:,1), '-red')
hold('on')
hold('on')
plot(Cost/1000, Sens(:,2), '-.black')
plot(Cost/1000, Sens(:,2), '-.black')
plot(Cost/1000, Sens(:,3), '--blue')
plot(Cost/1000, Sens(:,3), '--blue')
grid
grid
xlabel('Rebalancing Cost (\$1000''s)')
xlabel('Rebalancing Cost (\$1000''s)')
ylabel('Delta, Gamma, and Vega Portfolio Dollar Sensitivities')
ylabel('Delta, Gamma, and Vega Portfolio Dollar Sensitivities')
title ('Portfolio Sensitivities Profile')
title ('Portfolio Sensitivities Profile')
legend('Delta', 'Gamma', 'Vega', O)

```
legend('Delta', 'Gamma', 'Vega', O)
```



## Rebalancing Cost

In this calling form, hedgeopt calls hedgeslf internally to determine the maximum cost needed to minimize the portfolio sensitivities (\$23,055.90), and evenly samples the cost frontier between $\$ 0$ and $\$ 23,055.90$.

Note that both hedgeopt and hedgeslf cast the optimization problem as a constrained linear least squares problem. Depending on the instruments and constraints, neither function is guaranteed to converge to a solution. In some cases, the problem space may be unbounded, and additional instrument equality constraints, or user-specified constraints, may be necessary for convergence. See "Hedging with Constrained Portfolios" on page 4-36 for additional information.

## Pricing and Hedging a Portfolio Using the Black-Karasinski Model

This example illustrates how MATLAB® can be used to create a portfolio of interest rate derivatives securities, and price it using the Black-Karasinski interest rate model. The example also shows some hedging strategies to minimize exposure to market movements.

## Create the Interest Rate Term Structure Based on Reported Data

The structure RateSpec is an interest rate term structure that defines the initial rate specification from which the tree rates are derived. Use the information of annualized zero coupon rates in the table below to populate the RateSpec structure.

| From |  | To | Rate |
| :---: | :---: | :---: | :---: |
| 27 Feb 2007 | 27 | Feb 2008 | 0.0493 |
| 27 Feb 2007 | 27 | Feb 2009 | 0.0459 |
| 27 Feb 2007 | 27 | Feb 2010 | 0.0450 |
| 27 Feb 2007 | 27 | Feb 2012 | 0.0446 |
| 27 Feb 2007 | 27 | Feb 2014 | 0.0445 |
| 27 Feb 2007 | 27 | Feb 2017 | 0.0450 |
| Feb 2007 | 27 | Feb 2027 | . 04 |

This data could be retrieved from the Federal Reserve Statistical Release page by using the Datafeed Toolbox ${ }^{\mathrm{TM}}$. In this case, the Datafeed Toolbox ${ }^{\mathrm{TM}}$ will connect to FRED® and pull back the rates of the following treasury notes.

| Terms <br> $=======$ | Symbol <br> $======$ |  |
| :---: | :--- | :--- |
| 1 | $=$ | DGS1 |
| 2 | $=$ | DGS2 |
| 3 | $=$ | DGS3 |
| 5 | $=$ | DGS5 |
| 7 | $=$ | DGS7 |
| 10 | $=$ | DGS10 |
| 20 | $=$ | DGS20 |

Create connection object:

$$
c=\text { fred; }
$$

Create symbol fetch list:


Define Terms:

```
Terms = [ 1; ... % 1 Year
    2; ... % 2 Year
    3; ... % 3 Year
    5; ... % 5 Year
    7; ... % 7 Year
    10; ... % 10 Year
    20]; % 20 Year
```

Set StartDate to Feb 27, 2007:

```
StartDate = datenum('Feb-27-2007');
FredRet = fetch(c,FredNames,StartDate);
```

Set ValuationDate based on StartDate:
ValuationDate = StartDate;
EndDates = [];

Rates =[];

Create EndDates:
for idx = 1:length(FredRet)
\%Pull the rates associated with Feb 27, 2007. All the Fred Rates come

```
%back as percents
Rates = [Rates; ...
        FredRet(idx).Data(1,2) / 100];
    %Determine the EndDates by adding the Term to the year of the
    %StartDate
    EndDates = [EndDates; ...
        round(datenum(...
            year(StartDate)+ Terms(idx,1), ...
            month(StartDate),...
            day(StartDate)))];
end
```

Use the function 'intenvset' to create the RateSpec with the following data:
Compounding = 1;
StartDate = '27-Feb-2007';
Rates $=$ [0.0493; 0.0459; 0.0450; 0.0446; 0.0446; 0.0450; 0.0473];
EndDates = \{'27-Feb-2008'; '27-Feb-2009';'27-Feb-2010'; '27-Feb-2012';...
'27-Feb-2014' ; '27-Feb-2017'; '27-Feb-2027'\};
ValuationDate = StartDate;
RateSpec $=$ intenvset('Compounding', Compounding,'StartDates', StartDate,...
EndDates', EndDates, 'Rates', Rates,'ValuationDate',
RateSpec =
FinObj: 'RateSpec'
Compounding: 1
Disc: [7x1 double]
Rates: [7x1 double]
EndTimes: [7x1 double]
StartTimes: [7x1 double]
EndDates: [7x1 double]
StartDates: 733100
ValuationDate: 733100
Basis: 0
EndMonthRule: 1

## Specify the Volatility Model

Create the structure VolSpec that specifies the volatility process with the following data.

```
Volatility = [0.011892; 0.01563; 0.02021; 0.02125; 0.02165; 0.02065; 0.0180
Alpha = [0.0001];
VolSpec = bkvolspec(ValuationDate, EndDates, Volatility, EndDates(end), Alp
VolSpec =
                    FinObj: 'BKVolSpec'
    ValuationDate: 733100
        VolDates: [7x1 double]
        VolCurve: [7x1 double]
        AlphaCurve: 1.0000e-04
        AlphaDates: 740405
    VolInterpMethod: 'linear'
```


## Specify the Time Structure of the Tree

The structure TimeSpec specifies the time structure for an interest rate tree. This structure defines the mapping between the observation times at each level of the tree and the corresponding dates.

TimeSpec = bktimespec(ValuationDate, EndDates)

TimeSpec =

FinObj: 'BKTimeSpec'
ValuationDate: 733100
Maturity: [7x1 double]
Compounding: -1
Basis: 0
EndMonthRule: 1

## Create the BK Tree

Use the previously computed values for RateSpec, VolSpec and TimeSpec to create the BK tree.

```
BKTree = bktree(VolSpec, RateSpec, TimeSpec)
```

```
BKTree =
```

```
        FinObj: 'BKFwdTree'
        VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tObs: [[00 1 2 2 3 5 7 10]
        CFlowT: {1x7 cell}
        Probs: {1x6 cell}
    Connect: {1x6 cell}
    FwdTree: {1x7 cell}
```

        dObs: [733100 733465733831734196734926735657 736753]
    
## Observe the Interest Rate Tree

Visualize the interest rate evolution along the tree by looking at the output structure BKTree. The function 'bktree' returns an inverse discount tree, which you can convert into an interest rate tree with the 'cvtree' function.

```
BKTreeR = cvtree(BKTree)
BKTreeR =
```

```
    FinObj: 'BKRateTree'
```

    FinObj: 'BKRateTree'
    VolSpec: [1x1 struct]
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tObs: [0 1 2 3 5 7 10]
        tObs: [0 1 2 3 5 7 10]
        dObs: [733100 733465 733831 734196 734926 735657 736753]
    ```
        dObs: [733100 733465 733831 734196 734926 735657 736753]
```

```
    CFlowT: {1x7 cell}
    Probs: {1x6 cell}
Connect: {1x6 cell}
RateTree: {1x7 cell}
```

Look at the upper, middle and lower branch paths of the tree:

```
OldFormat = get(0, 'format');
format short
%Rate at root node:
RateRoot = trintreepath(BKTreeR, 0)
%Rates along upper branch:
RatePathUp = trintreepath(BKTreeR, [\begin{array}{llllll}{1}&{1}&{1}&{1}&{1}&{1}\end{array}])
```

\%Rates along middle branch:
RatePathMiddle = trintreepath(BKTreeR, [2 2222 2])
\%Rates along lower branch:
RatePathDown = trintreepath(BKTreeR, $\left.\begin{array}{lllll}3 & 3 & 3 & 3 & 3\end{array}\right]$ )
RateRoot =
0.0481
RatePathUp =
0.0481
0.0425
0.0446
0.0478
0.0510
0.0554
0.0619

```
RatePathMiddle =
    0.0481
    0.0416
    0.0423
    0.0430
    0.0436
    0.0448
    0.0483
RatePathDown =
    0.0481
    0.0408
    0.0401
    0.0387
    0.0373
    0.0363
    0.0377
```

You can also display a graphical representation of the tree to examine interactively the rates on the nodes of the tree until maturity. The function treeviewer displays the structure of the rate tree in the left window. The tree visualization in the right window is blank, but by selecting Table/Diagram and clicking on the nodes you can examine the rates along the paths.

```
treeviewer(BKTreeR);
```


## Create an Instrument Portfolio

Create a portfolio consisting of two bonds instruments and an option on the 5\% Bond.

```
% Two Bonds
CouponRate = [0.04;0.05];
Settle = '27 Feb 2007';
```

```
Maturity = {'27 Feb 2009';'27 Feb 2010'};
Period = 1;
% American Option on the 5% Bond
OptSpec = {'call'};
Strike = 98;
ExerciseDates = '27 Feb 2010';
AmericanOpt = 1;
InstSet = instadd('Bond', CouponRate, Settle, Maturity, Period);
InstSet = instadd(InstSet,'OptBond', 2, OptSpec, Strike, ExerciseDates, Ame
% Assign Names and Holdings
Holdings = [10; 15;3];
Names = {'4% Bond'; '5% Bond'; 'Option 98'};
InstSet = instsetfield(InstSet, 'Index',1:3, 'FieldName', {'Quantity'}, 'Da
InstSet = instsetfield(InstSet, 'Index',1:3, 'FieldName', {'Name'}, 'Data',
Examine the set of instruments contained in the variable InstSet.
instdisp(InstSet)
\begin{tabular}{lllllll} 
Index & Type CouponRate Settle & Maturity & Period & Basis & EndMonthRu \\
1 & Bond 0.04 & \(27-\) Feb-2007 & \(27-\) Feb-2009 & 1 & 0 & 1 \\
2 & Bond 0.05 & \(27-\) Feb-2007 & \(27-\) Feb-2010 & 1 & 0 & 1
\end{tabular}
\begin{tabular}{llllllll} 
Index & Type & UnderInd & OptSpec & Strike & ExerciseDates & AmericanOpt Quantity \\
3 & OptBond 2 & call & 98 & \(27-\) Feb-2010 & 1 & 3 & 0
\end{tabular}
```


## Price the Portfolio Using the BK Model

Calculate the price of each instrument in the portfolio.

```
[Price, PTree] = bkprice(BKTree, InstSet)
```

$$
98.8843
$$

101.3476
3.3476

```
PTree =
    FinObj: 'BKPriceTree'
        PTree: {1x8 cell}
        AITree: {1x8 cell}
        tObs: [llllllllll}
    Connect: {1x6 cell}
        Probs: {1x6 cell}
```

The prices in the output vector Price correspond to the prices at observation time zero ( $\mathrm{tObs}=0$ ), which is defined as the Valuation Date of the interest-rate tree.

In the Price vector, the first element, 98.884, represents the price of the first instrument ( $4 \%$ Bond); the second element, 101.347, represents the price of the second instrument ( $5 \%$ Bond), and 3.347 represents the price of the American call option.

You can also display a graphical representation of the price tree to examine the prices on the nodes of the tree until maturity.
treeviewer(PTree,InstSet);

## Add More Instruments to the Existing Portfolio

Add instruments to the existing portfolio: cap, floor, floating rate note, vanilla swap and a puttable and callable bond.

```
% Cap
StrikeC =0.035;
InstSet = instadd(InstSet,'Cap', StrikeC, Settle, '27 Feb 2010');
```

```
% Floor
StrikeF =0.05;
InstSet = instadd(InstSet,'Floor', StrikeF, Settle, '27 Feb 2009');
% Floating Rate Note
InstSet = instadd(InstSet,'Float', 30, Settle, '27 Feb 2009');
% Vanilla Swap
    LegRate =[0.04 5];
    InstSet = instadd(InstSet,'Swap', LegRate, Settle, '27 Feb 2010');
% Puttable and Callable Bonds
InstSet = instadd(InstSet,'OptEmBond', CouponRate, Settle, '27 Feb 2010',
    Strike, '27 Feb 2010','AmericanOpt', 1, 'Period', 1);
% Process Names and Holdings
Holdings = [15 ;5 ;8; 7; 9; 4];
Names = {'3.5% Cap';'5% Floor';'30BP Float';'4%/5BP Swap'; 'PuttBond'; 'Cal
InstSet = instsetfield(InstSet, 'Index',4:9, 'FieldName', {'Quantity'}, 'Da
InstSet = instsetfield(InstSet, 'Index',4:9, 'FieldName', {'Name'}, 'Data',
Examine the set of instruments contained in the variable InstSet.
instdisp(InstSet)
\begin{tabular}{lllllll} 
Index & Type CouponRate Settle & Maturity & Period & Basis EndMonthRu \\
1 & Bond 0.04 & \(27-\) Feb-2007 & \(27-\) Feb-2009 & 1 & 0 & 1 \\
2 & Bond 0.05 & \(27-\) Feb-2007 & \(27-\) Feb-2010 & 1 & 0 & 1
\end{tabular}
\begin{tabular}{llllllll} 
Index & Type & UnderInd & OptSpec & Strike & ExerciseDates & AmericanOpt Quantity & N \\
3 & OptBond 2 & call & 98 & \(27-\) Feb-2010 & 1 & 3 & 0
\end{tabular}
\begin{tabular}{llllllll} 
Index & Type Strike Settle & Maturity & & CapReset Basis Principal Qu \\
4 & Cap 0.035 & \(27-\) Feb-2007 & 27-Feb-2010 & 1 & 0 & 100 & 15
\end{tabular}
\begin{tabular}{lllllll} 
Index & Type Strike Settle & Maturity & FloorReset Basis Principal \\
5 & Floor 0.05 & \(27-\) Feb-2007 & \(27-\) Feb-2009 & 1 & 0 & 100
\end{tabular}
Index Type Spread Settle Maturity FloatReset Basis Principal
```



## Hedging

The idea behind hedging is to minimize exposure to market movements. As the underlying changes, the proportions of the instruments forming the portfolio may need to be adjusted to keep the sensitivities within the desired range.

Calculate sensitivities using the BK model
[Delta, Gamma, Vega, Price] = bksens(BKTree, InstSet);
Get the current portfolio holdings.
Holdings = instget(InstSet, 'FieldName', 'Quantity');
Create a matrix of sensitivities
Sensitivities = [Delta Gamma Vega];
Each row of the Sensitivities matrix is associated with a different instrument in the portfolio, and each column with a different sensitivity measure.

Display the information

```
format bank
disp([Price Holdings Sensitivities])
```

| 98.88 | 10.00 | -185.46 | 528.44 | 0.01 |
| ---: | ---: | ---: | ---: | ---: |
| 101.35 | 15.00 | -277.50 | 1044.93 | 0.03 |
| 3.35 | 3.00 | -223.50 | 11845.22 | 0.03 |
| 2.77 | 15.00 | 250.00 | 2926.95 | -0.03 |


| 0.75 | 5.00 | -132.97 | 11564.31 | 0.01 |
| ---: | ---: | ---: | ---: | ---: |
| 100.56 | 8.00 | -0.80 | 2.02 | 0.00 |
| -1.53 | 7.00 | -272.07 | 1027.74 | 0.03 |
| 98.60 | 9.00 | -168.94 | 21705.60 | 0.03 |
| 98.00 | 4.00 | -54.00 | -10800.29 | 0 |

The first column above is the dollar unit price of each instrument, the second is the number of contracts of each instrument, and the third, fourth, and fifth columns are the dollar delta, gamma, and vega sensitivities.

The current portfolio sensitivities are a weighted average of the instruments in the portfolio.

```
TargetSens = Holdings' * Sensitivities
```


## TargetSens =

$$
\begin{array}{lll}
-7249.76 & 317579.35 & 0.86
\end{array}
$$

## Obtain a Neutral Sensitivity Portfolio

Suppose you want to obtain a delta, gamma and vega neutral portfolio. The function hedgeslf finds the reallocation in a portfolio of financial instruments closest to being self-financing (maintaining constant portfolio value).
[Sens, Value1, Quantity]= hedgeslf(Sensitivities, Price,Holdings)

Sens =

$$
0.00
$$

0
-0.00

Value1 =

```
Quantity =
    10.00
    5.59
    -3.21
    8.63
    14.68
    9.74
    -4.31
    12.48
    8.80
```

The function hedgeslf returns the portfolio dollar sensitivities (Sens), the value of the rebalanced portfolio (Value1) and the new allocation for each instrument (Quantity). If Value0 and Value1 represent the portfolio value before and after rebalancing, you can verify the cost by comparing the portfolio values

```
ValueO = Holdings' * Price
```

```
Value0 =
```

    4637.56
    In this example, the portfolio is fully hedged (simultaneous delta, gamma, and vega neutrality) and self-financing (the values of the portfolio before and after balancing (Value0 and Value1) are the same.

## Adding Constraints to Hedge a Portfolio

Suppose that you want to place upper and lower bounds on the individual instruments in the portfolio. Let's say that you want to bound the position of all instruments to within +/- 11 contracts.

Applying these constraints disallows the current positions in the fifth and eighth instruments. All other instruments are currently within the upper/lower bounds.

```
% Specify the lower and upper bounds
LowerBounds = [llllllllll
UpperBounds = [[ 11 1.11
% Use the function portcons to build the constraints
ConSet = portcons('AssetLims', LowerBounds, UpperBounds);
% Apply the constraints to the portfolio
[Sens, Value, Quantity1] = hedgeslf(Sensitivities, Price, Holdings, [], Con
Sens =
    -0.00
    -0.00
    0.00
Value =
    4637.56
Quantity1 =
    2.56
    11.00
    -6.59
    3.60
    -4.67
    11.00
    -11.00
    1 1 . 0 0
    1 1 . 0 0
```

Observe that hedgeslf enforces the bounds on the fifth and eighth instruments, and the portfolio continues to be fully hedged and self-financing.

```
set(O, 'format', OldFormat);
```


## Specifying Constraints with ConSet

In this section...<br>"Introduction" on page 4-31<br>"Setting Constraints" on page 4-31<br>"Portfolio Rebalancing" on page 4-34

## Introduction

Both hedgeopt and hedgeslf accept an optional input argument, ConSet, that allows you to specify a set of linear inequality constraints for instruments in your portfolio. The examples in this section are brief. For additional information regarding portfolio constraint specifications, refer to "Analyzing Portfolios".

## Setting Constraints

For the first example of setting constraints, return to the fully hedged portfolio example that used hedgeopt to determine the minimum cost of obtaining simultaneous delta, gamma, and vega neutrality (target sensitivities all 0). Recall that when hedgeopt computes the cost of rebalancing a portfolio, the input target sensitivities you specify are treated as equality constraints during the optimization process. The situation is reproduced next for convenience.

```
TargetSens = [0 0 0];
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price,...
Holdings, FixedInd, [], [], TargetSens);
```

The outputs provide a fully hedged portfolio
Sens =

$$
\begin{array}{lll}
-0.00 & -0.00 & -0.00
\end{array}
$$

at an expense of over $\$ 23,000$.
Cost $=$
23055.90

The positions required to achieve this fully hedged portfolio are

```
Quantity' =
```

100.00
$-182.36$
-19.55
80.00
8.00
-32.97
40.00
10.00

Suppose now that you want to place some upper and lower bounds on the individual instruments in your portfolio. You can specify these constraints, along with a variety of general linear inequality constraints, with Financial Toolbox function portcons.

As an example, assume that, in addition to holding instruments $1,4,5,7$, and 8 fixed as before, you want to bound the position of all instruments to within +/- 180 contracts (for each instrument, you cannot short or long more than 180 contracts). Applying these constraints disallows the current position in the second instrument (short 182.36). All other instruments are currently within the upper/lower bounds.

You can generate these constraints by first specifying the lower and upper bounds vectors and then calling portcons.

```
LowerBounds = [-180 -180 -180 -180 -180 -180 -180 -180];
UpperBounds = [[ 180 180 180 180 180 180 180
ConSet = portcons('AssetLims', LowerBounds, UpperBounds);
```

To impose these constraints, call hedgeopt with ConSet as the last input.

```
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price,...
```

Holdings, FixedInd, [], [], TargetSens, ConSet);

Examine the outputs and see that they are all set to NaN, indicating that the problem, given the constraints, is not solvable. Intuitively, the results mean
that you cannot obtain simultaneous delta, gamma, and vega neutrality with these constraints at any price.

To see how close you can get to portfolio neutrality with these constraints, call hedgeslf.
[Sens, Value1, Quantity] = hedgeslf(Sensitivities, Price,... Holdings, FixedInd, ConSet);

Sens =
-352.43
21.99
-498.77

Value1 =
855.10

Quantity =
100.00
-180.00
-37.22
80.00
8.00
$-31.86$
40.00
10.00
hedgeslf enforces the lower bound for the second instrument, but the sensitivity is far from neutral. The cost to obtain this portfolio is

Value0 - Value1
ans $=$
22819.52

## Portfolio Rebalancing

As a final example of user-specified constraints, rebalance the portfolio using the second hedging goal of hedgeopt. Assume that you are willing to spend as much as $\$ 20,000$ to rebalance your portfolio, and you want to know what minimum portfolio sensitivities you can get for your money. In this form, recall that the target cost $(\$ 20,000)$ is treated as an inequality constraint during the optimization process.

For reference, startup hedgeopt without any user-specified linear inequality constraints.
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price,... Holdings, FixedInd, [], 20000);

Sens =
$\begin{array}{lll}-4345.36 & 295.81 & -6586.64\end{array}$
Cost $=$
20000.00

Quantity' =
100.00
-151.86
-253.47
80.00
8.00
$-18.18$
40.00
10.00

This result corresponds to the $\$ 20,000$ point along the Portfolio Sensitivities Profile shown in the figure Rebalancing Cost on page 4-15.

Assume that, in addition to holding instruments 1, 4, 5, 7, and 8 fixed as before, you want to bound the position of all instruments to within $+/-150$ contracts (for each instrument, you cannot short more than 150 contracts and you cannot long more than 150 contracts). These bounds disallow the current
position in the second and third instruments (-151.86 and -253.47). All other instruments are currently within the upper/lower bounds.

As before, you can generate these constraints by first specifying the lower and upper bounds vectors and then calling portcons.

```
LowerBounds = [-150 -150 -150 -150 -150 -150 -150 -150];
UpperBounds = [[\begin{array}{llllllll}{150}&{150}&{150}&{150}&{150}&{150}&{150}&{150];}\end{array}]
ConSet = portcons('AssetLims', LowerBounds, UpperBounds);
```

To impose these constraints, again call hedgeopt with ConSet as the last input.
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price,... Holdings,FixedInd, [], 20000, [], ConSet);

Sens =
$\begin{array}{lll}-8818.47 & 434.43 & -4010.79\end{array}$
Cost =
19876.89

Quantity' =
100.00
-150.00
-150.00
80.00
8.00
$-28.32$
40.00
10.00

With these constraints, hedgeopt enforces the lower bound for the second and third instruments. The cost incurred is $\$ 19,876.89$.

## Hedging with Constrained Portfolios

In this section...<br>"Overview" on page 4-36<br>"Example: Fully Hedged Portfolio" on page 4-36<br>"Example: Minimize Portfolio Sensitivities" on page 4-39<br>"Example: Under-Determined System" on page 4-40<br>"Example: Portfolio Constraints with hedgeslf" on page 4-42

## Overview

Both hedging functions cast the optimization as a constrained linear least-squares problem. (See the function lsqlin in the Optimization Toolbox documentation for details.) In particular, lsqlin attempts to minimize the constrained linear least squares problem

$$
\left.\begin{array}{rl}
\min _{x} \frac{1}{2}\|C x-d\|_{2}^{2} \text { such that } & A \cdot x
\end{array}\right)
$$

where $C, A$, and $A e q$ are matrices, and $d, b, b e q, l b$, and $u b$ are vectors. For Financial Instruments Toolbox software, $x$ is a vector of asset holdings (contracts).

Depending on the constraint and the number of assets in the portfolio, a solution to a particular problem may or may not exist. Furthermore, if a solution is found, it may not be unique. For a unique solution to exist, the least squares problem must be sufficiently and appropriately constrained.

## Example: Fully Hedged Portfolio

Recall that hedgeopt allows you to allocate an optimal hedge by one of two goals:

- Minimize the cost of hedging a portfolio given a set of target sensitivities.
- Minimize portfolio sensitivities for a given set of maximum target costs.

As an example, reproduce the results for the fully hedged portfolio example.

```
TargetSens = [0 0 0];
FixedInd = [1 4 4 5 7 8];
```

[Sens,Cost, Quantity] = hedgeopt(Sensitivities, Price,...
Holdings, FixedInd, [], [], TargetSens);

Sens =
$\begin{array}{lll}-0.00 & -0.00 & -0.00\end{array}$

Cost =
23055.90

## Quantity' =

98.72
-182.36
-19.55
80.00
8.00
-32.97
40.00
10.00

This example finds a unique solution at a cost of just over $\$ 23,000$. The matrix C (formed internally by hedgeopt and passed to lsqlin) is the asset Price vector expressed as a row vector.

C = Price' $=[98.7297 .530 .0598 .72100 .556 .280 .05$ 3.69]
The vector $d$ is the current portfolio value Value $0=23674.62$. The example maintains, as closely as possible, a constant portfolio value subject to the specified constraints.

## Additional Constraints

In the absence of any additional constraints, the least squares objective involves a single equation with eight unknowns. This is an under-determined system of equations. Because such systems generally have an infinite number of solutions, you need to specify additional constraints to achieve a solution with practical significance.

The additional constraints can come from two sources:

- User-specified equality constraints
- Target sensitivity equality constraints imposed by hedgeopt

The example in "Fully Hedged Portfolio" on page $4-8$ specifies five equality constraints associated with holding assets $1,4,5,7$, and 8 fixed. This reduces the number of unknowns from eight to three, which is still an under-determined system. However, when combined with the first goal of hedgeopt, the equality constraints associated with the target sensitivities in TargetSens produce an additional system of three equations with three unknowns. This additional system guarantees that the weighted average of the delta, gamma, and vega of assets 2,3 , and 6 , together with the remaining assets held fixed, satisfy the overall portfolio target sensitivity needs in TargetSens.

Combining the least-squares objective equation with the three portfolio sensitivity equations provides an overall system of four equations with three unknown asset holdings. This is no longer an under-determined system, and the solution is as shown.

If the assets held fixed are reduced, for example, FixedInd = [ $\left.\begin{array}{llll}1 & 4 & 5 & 7\end{array}\right]$, hedgeopt returns a no cost, fully hedged portfolio (Sens = [lll 0000 and Cost $=0$ ).
 hedgeopt always returns a no cost, fully hedged portfolio. In these cases, insufficient constraints result in an under-determined system. Although hedgeopt identifies no cost, fully hedged portfolios, there is nothing unique about them. These portfolios have little practical significance.

Constraints must be sufficient and appropriately defined. Additional constraints having no effect on the optimization are called dependent constraints. As a simple example, assume that parameter $Z$ is constrained such that $Z \leq 1$. Furthermore, assume you somehow add another constraint that effectively restricts $Z \leq 0$. The constraint $Z \leq 1$ now has no effect on the optimization.

## Example: Minimize Portfolio Sensitivities

To illustrate using hedgeopt to minimize portfolio sensitivities for a given maximum target cost, specify a target cost of $\$ 20,000$ and determine the new portfolio sensitivities, holdings, and cost of the rebalanced portfolio.

```
MaxCost = 20000;
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price,...
Holdings, [1 4 5 7 8], [], MaxCost);
Sens =
    -4345.36 295.81 -6586.64
Cost =
```

    20000.00
    Quantity' =
100.00
$-151.86$
-253.47
80.00
8.00
$-18.18$
40.00
10.00

This example corresponds to the $\$ 20,000$ point along the cost axis in the figures Rebalancing Cost Profile on page 4-11, Funds Available for Rebalancing on page 4-12, and Rebalancing Cost on page 4-15.

When minimizing sensitivities, the maximum target cost is treated as an inequality constraint; in this case, MaxCost is the most you are willing to spend to hedge a portfolio. The least-squares objective matrix C is the matrix transpose of the input asset sensitivities

C = Sensitivities'
a 3-by-8 matrix in this example, and d is a 3-by-1 column vector of zeros, [0 0 0]'.

Without any additional constraints, the least-squares objective results in an under-determined system of three equations with eight unknowns. By holding assets $1,4,5,7$, and 8 fixed, you reduce the number of unknowns from eight to three. Now, with a system of three equations with three unknowns, hedgeopt finds the solution shown.

## Example: Under-Determined System

Reducing the number of assets held fixed creates an under-determined system with meaningless solutions. For example, see what happens with only four assets constrained.

FixedInd = [11 4 5 7];
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price,... Holdings, FixedInd, [], MaxCost);

Sens =
-0.00
-0.00
-0.00

Cost $=$
20000.00

Quantity' =
100.00
-149.31
$-14.91$
80.00

$$
\begin{array}{r}
8.00 \\
-34.64 \\
40.00 \\
-32.60
\end{array}
$$

You have spent $\$ 20,000$ (all the funds available for rebalancing) to achieve a fully hedged portfolio.

With an increase in available funds to $\$ 50,000$, you still spend all available funds to get another fully hedged portfolio.

```
MaxCost = 50000;
```

[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price,...
Holdings, FixedInd, [], MaxCost);
Sens =
-0.00
0.00
0.00

Cost =
50000.00

Quantity' =
100.00
-473.78
-60.51
80.00
8.00
-18.20
40.00
385.60

All solutions to an under-determined system are meaningless. You buy and sell various assets to obtain zero sensitivities, spending all available funds every time. If you reduce the number of fixed assets any further, this problem is insufficiently constrained, and you find no solution (the outputs are all NaN ).

Note also that no solution exists whenever constraints are inconsistent. Inconsistent constraints create an infeasible solution space; the outputs are all NaN .

## Example: Portfolio Constraints with hedgeslf

The other hedging function, hedgeslf, attempts to minimize portfolio sensitivities such that the rebalanced portfolio maintains a constant value (the rebalanced portfolio is hedged against market moves and is closest to being self-financing). If a self-financing hedge is not found, hedgeslf tries to rebalance a portfolio to minimize sensitivities.

From a least-squares systems approach, hedgeslf first attempts to minimize cost in the same way that hedgeopt does. If it cannot solve this problem (a no cost, self-financing hedge is not possible), hedgeslf proceeds to minimize sensitivities like hedgeopt. Thus, the discussion of constraints for hedgeopt is directly applicable to hedgeslf as well.

To illustrate this hedging facility using equity exotic options, consider the portfolio CRRInstSet obtained from the example MAT-file deriv.mat. The portfolio consists of eight option instruments: two stock options, one barrier, one compound, two lookback, and two Asian.

The hedging functions require inputs that include the current portfolio holdings (allocations) and a matrix of instrument sensitivities. To create these inputs, start by loading the example portfolio into memory
load deriv.mat;
Next, compute the prices and sensitivities of the instruments in this portfolio.
[Delta, Gamma, Vega, Price] = crrsens(CRRTree, CRRInstSet);
Extract the current portfolio holdings (the quantity held or the number of contracts).

```
Holdings = instget(CRRInstSet, 'FieldName', 'Quantity');
```

For convenience place the delta, gamma, and vega sensitivity measures into a matrix of sensitivities.

## Sensitivities = [Delta Gamma Vega];

Each row of the Sensitivities matrix is associated with a different instrument in the portfolio and each column with a different sensitivity measure.
disp([Price Holdings Sensitivities])

| 8.29 | 10.00 | 0.59 | 0.04 | 53.45 |
| ---: | ---: | ---: | ---: | ---: |
| 2.50 | 5.00 | -0.31 | 0.03 | 67.00 |
| 12.13 | 1.00 | 0.69 | 0.03 | 67.00 |
| 3.32 | 3.00 | -0.12 | -0.01 | -98.08 |
| 7.60 | 7.00 | -0.40 | -45926.32 | 88.18 |
| 11.78 | 9.00 | -0.42 | -112143.15 | 119.19 |
| 4.18 | 4.00 | 0.60 | 45926.32 | 49.21 |
| 3.42 | 6.00 | 0.82 | 112143.15 | 41.71 |

The first column contains the dollar unit price of each instrument, the second contains the holdings of each instrument, and the third, fourth, and fifth columns contain the delta, gamma, and vega dollar sensitivities, respectively.

Suppose that you want to obtain a delta, gamma and vega neutral portfolio using hedgeslf.
[Sens, Value1, Quantity]= hedgeslf(Sensitivities, Price, ... Holdings)

Sens =
0.00
-0.00
0.00

Value1 =
313.93

Quantity =
10.00
7.64
hedgeslf returns the portfolio dollar sensitivities (Sens), the value of the rebalanced portfolio (Value1) and the new allocation for each instrument (Quantity).

If Value0 and Value1 represent the portfolio value before and after rebalancing, respectively, you can verify the cost by comparing the portfolio values.

Value0= Holdings' * Price
Value0 =
313.93

In this example, the portfolio is fully hedged (simultaneous delta, gamma, and vega neutrality) and self-financing (the values of the portfolio before and after balancing (Value0 and Value1) are the same.

Suppose now that you want to place some upper and lower bounds on the individual instruments in your portfolio. By using Financial Toolbox function portcons, you can specify these constraints, along with a variety of general linear inequality constraints.

As an example, assume that, in addition to holding instrument 1 fixed as before, you want to bound the position of all instruments to within +/20 contracts (for each instrument, you cannot short or long more than 20 contracts). Applying these constraints disallows the current position in the fourth instrument (long 26.13). All other instruments are currently within the upper/lower bounds.

You can generate these constraints by first specifying the lower and upper bounds vectors and then calling portcons.

```
LowerBounds = [l-20 -20 -20 -20 -20 -20 -20 -20];
UpperBounds =[[\begin{array}{lllllllll}{20}&{20}&{20}&{20}&{20}&{20}&{20}&{20}\end{array}];
ConSet = portcons('AssetLims', LowerBounds, UpperBounds);
```

To impose these constraints, call hedgeslf with ConSet as the last input.
[Sens, Cost, Quantity1] = hedgeslf(Sensitivities, Price, ... Holdings, 1, ConSet)

Sens =

- 0.00
0.00
0.00

Cost =
313.93

Quantity1 =
10.00
5.28
10.98
20.00
20.00
-6.99
$-20.00$
9.39

Observe that hedgeslf enforces the upper bound on the fourth instrument, and the portfolio continues to be fully hedged and self-financing.

## Mortgage-Backed Securities

- "What Are Mortgage-Backed Securities?" on page 5-2
- "Fixed-Rate Mortgage Pool" on page 5-3
- "Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model" on page 5-17
- "Pricing Mortgage Backed Securities Using the Black-Derman-Toy Model" on page 5-32
- "Using Collateralized Mortgage Obligations (CMOs)" on page 5-38
- "Create PAC and Sequential CMO" on page 5-53


## What Are Mortgage-Backed Securities?

Mortgage-backed securities (MBSs) are a type of investment that represents ownership in a group of mortgages. Principal and interest from the individual mortgages are used to pay principal and interest on the MBS.

Ownership in a group of mortgages is typically represented by a pass-through certificate (PC). Most pass-through certificates are issued by the Government National Mortgage Agency, a branch of the United States government, or by one of two private corporations: Fannie Mae or Freddie Mac. With these certificates, homeowners' payments pass from the originating bank through the issuing agency to holders of the certificates. These agencies also frequently guarantee that the certificate holder receives timely payment of principal and interest from the PCs.

## Fixed-Rate Mortgage Pool

```
In this section...
"Introduction" on page 5-3
"Inputs to Functions" on page 5-4
"Generating Prepayment Vectors" on page 5-4
"Mortgage Prepayments" on page 5-6
"Risk Measurement" on page 5-8
"Mortgage Pool Valuation" on page 5-9
"Computing Option-Adjusted Spread" on page 5-10
"Prepayments with Fewer Than 360 Months Remaining" on page 5-13
"Pools with Different Numbers of Coupons Remaining" on page 5-15
```


## Introduction

Financial Instruments Toolbox software supports calculations involved with generic fixed-rate mortgage pools and balloon mortgages. Pass-through certificates typically have embedded call options in the form of prepayment. Prepayment is an excess payment applied to the principal of a PC. These accelerated payments reduce the effective life of a PC.

The toolbox comes with a standard Bond Market Association (PSA) prepayment model and can generate multiples of standard prepayment speeds. The Public Securities Association provides a set of uniform practices for calculating the characteristics of mortgage-backed securities when there is an assumed prepayment function.

Alternatively, aside from the standard PSA implementation in this toolbox, you can supply your own projected prepayment vectors. At this time, however, custom prepayment functionality that incorporates pool-specific information and interest rate forecasts are not available in this toolbox. If you plan to use custom prepayment vectors in your calculations, you presumably already own such a suite in MATLAB.

## Inputs to Functions

Because of the generic, all-purpose nature of the toolbox pass-through functions, you can fine-tune them to conform to a particular mortgage. Most functions require at least this set of inputs:

- Gross coupon rate
- Settlement date
- Issue (effective) date
- Maturity date

Typical optional inputs include standard prepayment speed (or customized vector), net coupon rate (if different from gross coupon rate), and payment delay in number of days.

All calculations are based on expected payment dates and actual cash flow to the investor. For example, when GrossRate and CouponRate differ as inputs to mbsdurp, the function returns a modified duration based on CouponRate. (A notable exception is mbspassthrough, which returns interest quantities based on the GrossRate.)

## Generating Prepayment Vectors

You can generate PSA multiple prepayment vectors quickly. To generate prepayment vectors of 100 and 200 PSA, type

PSASpeed = [100, 200];
[CPR, SMM] = psaspeed2rate(PSASpeed);
This function computes two prepayment values: conditional prepayment rate (CPR) and single monthly mortality (SMM) rate. CPR is the percentage of outstanding principal prepaid in 1 year. SMM is the percentage of outstanding principal prepaid in 1 month. In other words, CPR is an annual version of SMM.

Since the entire 360-by- 2 array is too long to show in this document, observe the SMM (100 and 200 PSA) plots, spaced one month apart, instead.


Prepayment assumptions form the basis upon which far more comprehensive MBS calculations are based. As an illustration observe the following example, which shows the use of the function mbscfamounts to generate cash flows and timings based on a set of standard prepayments.

Consider three mortgage pools that were sold on the issue date (which starts unamortized). The first two pools "balloon out" in 60 months, and the third is regularly amortized to the end. The prepayment speeds are assumed to be 100, 200, and 200 PSA, respectively.

```
Settle = [datenum('1-Feb-2000');
    datenum('1-Feb-2000');
    datenum('1-Feb-2000')];
Maturity = [datenum('1-Feb-2030')];
IssueDate = datenum('1-Feb-2000');
GrossRate = 0.08125;
CouponRate = 0.075;
Delay = 14;
```

```
PSASpeed = [100, 200];
[CPR, SMM] = psaspeed2rate(PSASpeed);
PrepayMatrix = ones(360,3);
PrepayMatrix(1:60,1:2) = SMM(1:60,1:2);
PrepayMatrix(:,3) = SMM(:,2);
[CFlowAmounts, CFlowDates, TFactors, Factors] = ...
mbscfamounts(Settle, Maturity, IssueDate, GrossRate, ...
CouponRate, Delay, [], PrepayMatrix);
```

The fourth output argument, Factors, indicates the fraction of the balance still outstanding at the beginning of each month. A snapshot of this argument in the MATLAB Variables editor illustrates the 60 -month life of the first two of the mortgages with balloon payments and the continuation of the third mortgage until the end ( 360 months).

| F9 Variable Editor |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 個 |  | $\Leftrightarrow 10$ | \| Stack: | - | o valid plc |
| Factors |  |  |  |  |  |
| \# Factors < 3x361 double> |  |  |  |  |  |
|  | 59 | 60 | 61 | 62 | 63 |
| 1 | 0.7627 | 0.7580 | 0.7533 | 0 | 0 |
| 2 | 0.6021 | 0.5951 | 0.5882 | 0 | 0 |
| 3 | 0.6021 | 0.5951 | 0.5882 | 0.5813 | 0.5746 |
| 1 |  | III |  |  |  |

You can readily see that mbscfamounts is the building block of most fixed rate and balloon pool cash flows.

## Mortgage Prepayments

Prepayment is beneficial to the pass-through owner when a mortgage pool has been purchased at discount. The next example compares mortgage yields (compounded monthly) versus the purchase clean price with constant prepayment speed. The example illustrates that when you have purchased a pool at a discount, prepayment generates a higher yield with decreasing purchase price.

```
Price = [85; 90; 95];
Settle = datenum('15-Apr-2002');
Maturity = datenum('1 Jan 2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
CouponRate = 0.075;
Delay = 14;
Speed = 100;
```

Compute the mortgage and bond-equivalent yields.
[MYield, BEMBSYield] = mbsyield(Price, Settle, Maturity, ... IssueDate, GrossRate, CouponRate, Delay, Speed)

MYield =

> 0.1018
> 0.0918
> 0.0828

BEMBSYield =
0.1040
0.0936
0.0842

If for this same pool of mortgages, there was no prepayment (Speed $=0$ ), the yields would decline to

MYield =
0.0926
0.0861
0.0802

BEMBSYield =
0.0944
0.0877
0.0815

Likewise, if the rate of prepayment doubled (Speed = 200), the yields would increase to

MYield =
0.1124
0.0984
0.0858

BEMBSYield =
0.1151
0.1004
0.0873

For the same prepayment vector, deeper discount pools earn higher yields. For more information, see mbsprice and mbsyield.

## Risk Measurement

Financial Instruments Toolbox software provides the most basic risk measures of a pool portfolio:

- Modified duration
- Convexity
- Average life of pool

Consider the following example, which calculates the Macaulay and modified durations given the price of a mortgage pool.

```
Price = [95; 100; 105];
Settle = datenum('15-Apr-2002');
Maturity = datenum('1-Jan-2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
CouponRate = 0.075;
Delay = 14;
Speed = 100;
```

```
[YearDuration, ModDuration] = mbsdurp(Price, Settle, ...
Maturity, IssueDate, GrossRate, CouponRate, Delay, Speed)
YearDuration =
    6.1341
    6.3882
    6.6339
ModDuration =
    5.8863
    6.1552
    6.4159
```

Using Financial Instruments Toolbox functions, you can obtain modified duration and convexity from either price or yield, as long as you specify a prepayment vector or an assumed prepayment speed. The toolbox risk-measurement functions (mbsdurp, mbsdury, mbsconvp, mbsconvy, and mbswal) adhere to the guidelines listed in the PSA Uniform Practices manual.

## Mortgage Pool Valuation

For accurate valuation of a mortgage pool, you must generate interest rate paths and use them with mortgage pool characteristics to properly value the pool. A widely used methodology is the option-adjusted spread (OAS). OAS measures the yield spread that is not directly attributable to the characteristics of a fixed-income investment.

## Calculating OAS

Prepayment alters the cash flows of an otherwise regularly amortizing mortgage pool. A comprehensive option-adjusted spread calculation typically begins with the generation of a set of paths of spot rates to predict prepayment. A path is collection of $i$ spot-rate paths, with corresponding $j$ cash flows on each of those paths.

The effect of the OAS on pool pricing is shown mathematically in the following equation, where $K$ is the option-adjusted spread.

$$
\text { PoolPrice }=\frac{1}{\text { NumberofPaths }} \times \sum_{i}^{\text {NumberofPaths }} \sum_{j}^{C F_{i j}} \frac{C F_{i j}}{\left(1+\text { zerorates }_{i j}+K\right)^{T_{i j}}}
$$

## Calculating Effective Duration

Alternatively, if you are more interested in the sensitivity of a mortgage pool to interest rate changes, use effective duration, which is a more appropriate measure. Effective duration is defined mathematically with the following equation.

$$
\text { Effective Duration }=\frac{P(y+\Delta y)-P(y-\Delta y)}{2 P(y) \Delta y}
$$

## Calculating Market Price

The toolbox has all the components required to calculate OAS and effective duration if you supply prepayment vectors or assumptions. For OAS, given a prepayment vector, you can generate a set of cash flows with mbscfamounts. Discounting these cash flows with the reference curve and then adding OAS produces the market price. See "Computing Option-Adjusted Spread" on page $5-10$ for a discussion on the computation of option-adjusted spread.

Effective duration is a more difficult issue. While modified duration changes the discounting process (by changing the yield used to discount cash flows), effective duration must account for the change in cash flow because of the change in yield. A possible solution is to recompute prices using mbsprice for a small change in yield, in both the upwards and downwards directions. In this case, you must recompute the prepayment input. Internally, this alters the cash flows of the mortgage pool. Assuming that the OAS stays constant in all yield environments, you can apply a set of discounting factors to the cash flows in up and down yield environments to find the effective duration.

## Computing Option-Adjusted Spread

The option-adjusted spread (OAS) is an amount of extra interest added above (or below if negative) the reference zero curve. To compute the OAS, you must provide the zero curve as an extra input. You can specify the zero curve in any intervals and with any compounding method. (To minimize any error due to interpolation, keep the intervals as regular and frequent as possible.)

You must supply a prepayment vector or specify a speed corresponding to a standard PSA prepayment vector.

One way to compute the appropriate zero curve for an agency is to look at its bond yields and bootstrap them from the shortest maturity onwards. You can do this with Financial Toolbox functions zbtprice and zbtyield.

The following example shows how to calculate an appropriate zero curve followed by computation of the pool's OAS. This example calculates the OAS of a 30 -year fixed rate mortgage with about a 28 -year weighted average maturity left, given an assumption of 0,50 , and 100 PSA prepayment speeds.

Create curve for zerorates.

| Bonds $=$ | $[$ datenum('11/21/2002') |
| ---: | :--- |
|  | datenum('02/20/2003') |
|  | 0 |
|  | datenum('07/31/2004') |
|  | 0.03 |
|  | datenum('08/15/2007') |
|  | 0.035 |
|  | datenum('08/15/2012') |
|  | datenum('02/15/2031') |
| Yields $=$ | 0.04875 |
|  | 0.05375 |
|  | 100 |
|  | 100 |
|  | $0.0162 ;$ |
|  | $0.0163 ;$ |
|  | $0.0211 ;$ |
|  | $0.0328 ;$ |
|  | $0.0420 ;$ |
|  | $0.0501] ;$ |

Since the above is Treasury data and not selected agency data, a term structure of spread is assumed. In this example, the spread declines proportionally from a maximum of 250 basis points at the shortest maturity.

```
Yields = Yields + 0.025 * (1./[1:6]');
```

Get parameters from Bonds matrix.

```
Settle = datenum('20-Aug-2002');
Maturity = Bonds(:,1);
CouponRate = Bonds(:,2);
Face = Bonds(:,3);
Period = Bonds(:,4);
```

```
Basis = Bonds(:,5);
EndMonthRule = Bonds(:,6);
[Prices, AccruedInterest] = bndprice(Yields, CouponRate, ...
Settle, Maturity, Period, Basis, EndMonthRule, [], [], [], [], ...
Face);
```

Use zbtprice to solve for zero rates.

```
[ZeroRatesP, CurveDatesP] = zbtprice(Bonds, Prices, Settle);
ZeroCompounding = 2*ones(size(ZeroRatesP));
ZeroMatrix = [CurveDatesP, ZeroRatesP, ZeroCompounding];
```

Use output from zbtprice to calculate the OAS.

```
Price = 95;
```

Settle = datenum('20-Aug-2002');
Maturity = datenum('2-Jan-2030');
IssueDate = datenum('2-Jan-2000');
GrossRate = 0.08125;
CouponRate = 0.075;
Delay = 14;
Interpolation = 1;
PrepaySpeed = [0; 50; 100];
OAS = mbsprice2oas(ZeroMatrix, Price, Settle, Maturity, ...
IssueDate, GrossRate, CouponRate, Delay, Interpolation, ...
PrepaySpeed)

## OAS =

26.0502
28.6348
31.2222

This example shows that one cash flow set is being discounted and solved for its OAS, as contrasted with the NumberOfPaths set of cash flows as shown in "Mortgage Pool Valuation" on page 5-9. Averaging the sets of cash flows resulting from all simulations into one average cash flow vector and solving
for the OAS, discounts the averaged cash flows to have a present value of today's (average) price.

While this example uses the mortgage pool price (mbsprice2oas) to determine the OAS, you can also use yield to resolve it (mbsyield2oas). Also, there are reverse OAS functions that return prices and yields given OAS (mbsoas2price and mbsoas2yield).

The example also restates earlier examples that show discount securities benefit from higher level of prepayment, keeping everything else unchanged. The relation is reversed for premium securities.

## Prepayments with Fewer Than 360 Months Remaining

When fewer than 360 months remain in the pool, the applicable PSA prepayment vector is "seasoned" by the pool's age. (Elements in the 360 -element prepayment vector that represent past payments are skipped. For example, on a 30 -year mortgage that is 10 months old, only the final 350 prepayments are applied.)

Assume, for example, that you have two 30-year loans, one new and another 10 months old. Both have the same PSA speed of 100 and prepay using the vectors plotted below.


Still within the scope of relative valuation, you could also solve for the percentage of the standard PSA prepayment vector given the pool's arbitrary, user-supplied prepayment vector, such that the PSA speed gives the same Macaulay duration as the user-supplied prepayment vector.

If you supply a custom prepayment vector, you must account for the number of months remaining.

```
Price = 101;
Settle = datenum('1-Jan-2001');
Maturity = datenum('1-Jan-2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
PrepayMatrix = 0.005*ones(348,1);
CouponRate = 0.075;
Delay = 14;
```

ImpliedSpeed = mbsprice2speed(Price, Settle, Maturity, ...
IssueDate, GrossRate, PrepayMatrix, CouponRate, Delay)

## ImpliedSpeed =

104.2543

Examine the prepayment input. The remaining 29 years require 348 monthly elements in the prepayment vector. Suppose then, keeping everything the same, you change Settle to February 14, 2003.

Settle = datenum('14-Feb-2003');
You can use cpncount to count all incoming coupons received after Settle by invoking

NumCouponsRemaining = cpncount(Settle, Maturity, 12, 1, [], ... IssueDate)

NumCouponsRemaining = 323

The input 12 defines the monthly payment frequency, 1 defines the 30/360 basis, and IssueDate defines aging and determination-of-holder date. Thus, you must supply a 323 -element vector to account for a prepayment corresponding to each monthly payment.

## Pools with Different Numbers of Coupons Remaining

Suppose one pool has two remaining coupons, and the other has three. MATLAB software expects the prepayment matrix to be in the following format:

```
V11 V21
V12 V22
NaN V23
```

$V_{i j}$ denotes the single monthly mortality (SMM) rate for pool i during the $j$ th coupon period since Settle.

The use of NaN to pad the prepayment matrix is necessary because MATLAB cannot concatenate vectors of different lengths into a matrix. Also, it can serve as an error check against any unintended operation (any MATLAB operation that would return NaN ).

For example, assume that the 2 month pool has a constant SMM of $0.5 \%$ and the 3 month has a constant SMM of $1 \%$ in every period. The prepayment matrix you would create is depicted below.


Create this input in whatever manner is best for you.

## Summary of Prepayment Data Vector Representation

- When you specify a PSA prepayment speed, MATLAB "seasons" the pool according to its age.
- When you specify your own prepayment matrix, identify the maximum number of coupons remaining using cpncount. Then supply the matrix elements up to the point when cash flow ceases to exist.
- When different length pools must exist in the same matrix, pad the shorter one(s) with NaN. Each column of the prepayment matrix corresponds to a specific pool.


## Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model

This example shows how to model prepayment in MATLAB® using functionality from the Financial Instruments Toolbox ${ }^{\text {TM }}$. Specifically, a variation of the Richard and Roll prepayment model is implemented using a two factor Hull-White interest-rate model and a LIBOR Market Model to simulate future interest-rate paths. A mortgage-backed security is priced with both the custom and default prepayment models.

## Introduction

Prepayment modeling is crucial to the analysis of mortgage-backed securities (MBS). Prepayments by individual mortgage holders affect both the amount and timing of cash flows -- and for collateralized mortgage obligations (e.g.: interest-only securities), prepayment can greatly affect the value of the securities.

## PSA Model

The most basic prepayment model is the Public Securities Association (PSA) model, which assumes a ramp-up phase and then a constant conditional prepayment rate (CPR). The PSA model can be generated in MATLAB using the Financial Instruments Toolbox function PSASPEED2RATE

```
G2PP_CPR = psaspeed2rate([100 200]);
figure
plot(G2PP_CPR)
title('100 and 200 PSA Prepayment Speeds')
xlabel('Months')
ylabel('CPR')
ylim([0 .14])
legend({'100 PSA','200 PSA'}, 'Location', 'Best')
```


## Mortgage-Backed Security

The MBS analyzed in this example matures in 2020 and has the properties outlined in this section. Cash flows are generated for PSA prepayment speeds simply by entering the PSA speed as an input argument.

```
% Parameters for MBS passthrough to be priced
Settle = datenum('15-Dec-2007');
Maturity = datenum('15-Dec-2020');
IssueDate = datenum('15-Dec-2000');
GrossRate = .0475;
CouponRate = .045;
Delay = 14;
Period = 12;
Basis = 4;
% Generate cash flows and dates for baseline case using 100 PSA
[CFlowAmounts, CFlowDates] = mbscfamounts(Settle,Maturity, IssueDate,...
    GrossRate, CouponRate, Delay,100);
CFlowTimes = yearfrac(Settle,CFlowDates);
NumCouponsRemaining = cpncount(Settle, Maturity, Period,Basis, 1, IssueDate
```


## Richard and Roll Model

While prepayment modeling often involves quite complex and sophisticated modeling, often at the loan level, this example will use a slightly modified approach based on the model proposed by Richard and Roll in [6].

The Richard and Roll prepayment model involves the following factors:

- Refinancing incentive
- Seasonality (month of the year)
- Seasoning or age of the mortgage
- Burnout

Richard and Roll propose a multiplicative model of the following:

For the custom model in this example, the Burnout Multiplier, which describes the tendency of prepayment to slow when a significant number of homeowners have already refinanced, is ignored and the first three terms are used.

The refinancing incentive is a function of the ratio of the coupon-rate of the mortgage to the available mortgage rate at that particular point in time. For example, the Office of Thrift Supervision (OTS) proposes the following model:

The refinancing incentive requires a simulation of future interest rates. This will be discussed later in this example.

```
C_M = .1:.1:2;
G2PP_Refi = .2406 - . 1389 * atan(5.952*(1.089 - C_M));
figure
plot(C_M,G2PP_Refi)
xlabel('Coupon/Mortgage Rate')
ylabel('CPR')
title('Refinancing Incentive')
```

Seasoning captures the tendency of prepayment to ramp up at the beginning of a mortgage before leveling off. The OTS models the seasoning multiplier as follows:

```
Seasoning = ones(360,1);
Seasoning(1:29) = (1:29)/30;
figure
plot(Seasoning)
xlim([1 360])
title('Seasoning Multiplier')
xlabel('Months')
```

The seasonality multiplier simply models the seasonal behavior of prepayments -- this data is based on Figure 3 of [6], which applies to the behavior of Ginnie Mae 30-year, single-family MBSs.

```
Seasonality = [ . 94 . 76 . 73 . 96 .98 .92 .99 1.1 1.18 1.21 1.23 .97];
figure
plot(Seasonality)
xlim([[1 12])
set(gca,'xtick',1:12)
set(gca,'xtickLabel',{'Jan','Feb','Mar','Apr','May','Jun','Jul','Aug',...
    'Sep','Oct','Nov','Dec'})
title('Seasonality Multiplier')
```


## G2++ Interest-Rate Model

Since the refinancing incentive requires a simulation of future interest rates, an interest-rate model must be used. One choice is a two-factor additive Gaussian model, referred to as G2++ by Brigo and Mercurio [2].

The G2++ Interest Rate Model is:
where is a two-dimensional Brownian motion with correlation
and is the short rate, and are mean reversion constants and and are volatility constants, and is the market forward rate, or the forward rate observed on the Settle date.

## LIBOR Market Model

The LIBOR Market Model (LMM) differs from short-rate models in that it evolves a set of discrete forward rates. Specifically, the lognormal LMM specifies the following diffusion equation for each forward rate:
where
dW is an N dimensional geometric Brownian motion with:

The LMM relates the drifts of the forward rates based on no-arbitrage arguments. Specifically, under the Spot LIBOR measure, the drifts are expressed as the following:
where
is the time fraction associated with the ith forward rate
and the Spot LIBOR numeraire is defined as the following:

Given the above, the choice with the LMM is how to model volatility and correlation.

The volatility of the rates can be modeled with a stochastic volatility, but for this example a deterministic volatility is used, and so a functional form needs to be specified. One of the most popular functional forms in the literature is the following:
where adjusts the curve to match the volatility for the forward rate.
Similarly, the correlation between the forward rates needs to be specified. This can be estimated from historical data or fitted to option prices. For this example, the following functional form will be used:

Once the volatility and correlation are specified, the parameters need to be calibrated -- this can be done with historical or market data, typically swaptions or caps/floors. For this example, we simply use reasonable estimates for the correlation and volatility parameters.

```
% The volatility function to be used -- and one choice for the parameters
LMMVolFunc = @(a,t) (a(1)*t + a(2)).*exp(-a(3)*t) + a(4);
LMMVolParams = [.13 .04 .7 .08];
% Volatility specification
fplot(@(t) LMMVolFunc(LMMVolParams,t),[0 10])
title(['Volatility Function with parameters ' mat2str(LMMVolParams)])
ylabel('Volatility (%)')
xlabel('Tenor (years)')
```


## Calibration to Market Data

The parameters in the G2++ model can be calibrated to market data. Typically, the parameters are calibrated to observed interest-rate cap, floor and/or swaption data. For now, market cap data is used for calibration.

This data is hardcoded but could be imported into MATLAB with the Database Toolbox ${ }^{\text {TM }}$ or Datafeed Toolbox ${ }^{\text {TM }}$.

```
% Zero Curve -- this data is hardcoded for now but could be bootstrapped
% using the bootstrap method of IRDataCurve
ZeroTimes = [3/12 6/12 1 5 7 10 20 30]';
ZeroRates = [0.033 0.034 0.035 0.040 0.042 0.044 0.048 0.0475]';
ZeroDates = daysadd(Settle,360*ZeroTimes,1);
DiscountRates = zero2disc(ZeroRates,ZeroDates,Settle);
irdc = IRDataCurve('Zero',Settle,ZeroDates,ZeroRates);
figure
plot(ZeroDates,ZeroRates)
datetick
title(['US Zero Curve for ' datestr(Settle)])
```

```
% Cap Data
Reset = 2;
Notional = 100;
CapMaturity = daysadd(Settle,360*[1:5 7 10 15 20 25 30],1);
CapVolatility = [.28 . 30 . 32 . 31 . 30 .27 . 23 .2 . 18 .17 .165]';
% ATM strikes could be computed with SWAPBYZERO
Strike = [0.0353 0.0366 0.0378 0.0390 0.0402 0.0421 0.0439 ...
    0.0456 0.0471 0.0471 0.0471]';
% This could be computed with CAPBYBLK
BlackCapPrices = [0.1532 0.6416 1.3366 2.0290 2.7366 4.2960 6.5992 ...
    9.6787 12.2580 14.0969 15.7873]';
figure
scatter(CapMaturity,CapVolatility)
datetick
title(['ATM Volatility for Caps on ' datestr(Settle)])
%
% To calibrate the model parameters, a parameter set will be found that
% minimizes the sum of the squared differences between the G2++ predicted
% Cap values and the observed Black Cap values. The Optimization Toolbox(TM
% function LSQNONLIN is used in this example, although other approaches
% (e.g. Global Optimization) may also be applicable. The function CAPBYLG2F
% is used to compute analytic values for the caps given parameter values.
%
% Upper and lower bounds for the model parameters are set to be
% relatively constrained. As Brigo and Mercurio discuss, the correlation
% parameter, $$ rho $$, can often be close to -1 when fitting a G2++ model
% to interest-rate cap prices. Therefore, $$ rho $$ is constrained
% to be between -.7 and . }7\mathrm{ to ensure that the parameters represent a truly
% two-factor model. The remaining mean reversion and volatility
% parameters are constrained to be between 0 and .5. Calibration remains a
% complex task, and while the plot below indicates that the best fit
% parameters seem to do a reasonably good job of reproducing the Cap
% prices, it should be noted that the procedure outlined here simply
% represents one approach.
```

```
% Call to LSQNONLIN to calibrate parameters
objfun = @(x) BlackCapPrices - capbylg2f(irdc,x(1),x(2),x(3),x(4),x(5),Stri
x0 = [.5 .05 .1 .01 -.1];
lb = [0 0 0 0 -.7];
ub = [lllllllll
G2PP_Params = lsqnonlin(objfun,x0,lb,ub);
a = G2PP_Params(1);
b = G2PP_Params(2);
sigma = G2PP_Params(3);
eta = G2PP_Params(4);
rho = G2PP_Params(5);
% Compare the results
figure
scatter(CapMaturity,BlackCapPrices)
hold on
scatter(CapMaturity, capbylg2f(irdc,a,b,sigma,eta,rho,Strike,CapMaturity),'
datetick
title('Market and Model Implied Prices')
ylabel('Price ($)')
```

Local minimum possible.
lsqnonlin stopped because the final change in the sum of squares relative
its initial value is less than the default value of the function tolerance.

## G2++ Model Implementation

The LinearGaussian2F model can be used to specify the G2++ model and simulate future paths interest rates.

```
% G2++ model from Brigo and Mercurio with time homogeneous volatility
% parameters
G2PP = LinearGaussian2F(irdc,a,b,sigma,eta,rho);
```


## LIBOR Market Model Implementation

After the volatility and correlation have been calibrated, Monte Carlo simulation is used to evolve the rates forward in time. The LiborMarketModel object is used to simulate the forward rates.

While factor reduction is often used with the LMM to reduce computational complexity, there is no factor reduction in this example.

6M LIBOR rates are chosen to be evolved in this simulation. Since a monthly prepayment vector must be computed, interpolation is used to generate the intermediate rates. Simple linear interpolation is used.

```
numForwardRates = 46;
% Instead of being fit, VolPhi is simply hard-coded
% representative of a declining volatility over time.
VolPhi = linspace(1.2,.8,numForwardRates-1)';
Beta = .08;
CorrFunc = @(i,j,Beta) exp(-Beta*abs(i-j));
CorrMat = CorrFunc(meshgrid(1:numForwardRates-1)',meshgrid(1:numForwardRate
VolFunc = cell(length(VolPhi),1);
for jdx=1:length(VolPhi)
    VolFunc(jdx) = {@(t) VolPhi(jdx)*ones(size(t)).*(LMMVolParams(1)*t + ..
        LMMVolParams(2)).*exp(-LMMVolParams(3)*t) + LMMVolParams(4)};
end
LMM = LiborMarketModel(irdc,VolFunc,CorrMat);
```


## G2++ Monte Carlo Simulation

The various interest-rate paths can be simulated by calling the simTermStructs method.

One limitation to two-factor Gaussian models like this one is that it does permit negative interest rates. This is a concern, particularly in low interest-rate environments. To handle this possibility, any interest-rate paths with negative rates are simply rejected.

```
nPeriods = NumCouponsRemaining;
nTrials = 100;
DeltaTime = 1/12;
%Generate factors and short rates
Tenor = [1/12 1 2 3 4 5 7 10 15 20 30];
G2PP_SimZeroRates = G2PP.simTermStructs(nPeriods,'NTRIALS',nTrials,...
        'Tenor',Tenor,'DeltaTime',DeltaTime);
SimDates = daysadd(Settle,360*DeltaTime*(0:nPeriods),1);
% Tenors that will be recovered for each simulation date. The stepsize is
% included here to facilitate computing a discount factor for each
% simulation path.
% Remove any paths that go negative
NegIdx = squeeze(any(any(G2PP_SimZeroRates < 0,1),2));
G2PP_SimZeroRates(:,:,NegIdx) = [];
nTrials = size(G2PP_SimZeroRates,3);
% Plot evolution of one sample path
trialIdx = 1;
figure
surf(Tenor,SimDates,G2PP_SimZeroRates(:,:,trialIdx))
datetick y keepticks keeplimits
title(['Evolution of the Zero Curve for Trial:' num2str(trialIdx) ' of G2++
xlabel('Tenor (Years)')
```

LIBOR Market Model Simulation

The various interest-rate paths can be simulated by calling the simTermStructs method of the LiborMarketModel object.

```
LMMPeriod = 2; % Semi-annual rates
LMMNumPeriods = NumCouponsRemaining/12*LMMPeriod; % Number of semi-annual
LMMDeltaTime = 1/LMMPeriod;
LMMNTRIALS = 100;
% Simulate
[LMMZeroRates, LMMForwardRates] = LMM.simTermStructs(LMMNumPeriods,'nTrials
ForwardTimes = 1/2:1/2:numForwardRates/2;
LMMSimTimes = 0:1/LMMPeriod:LMMNumPeriods/LMMPeriod;
% Plot evolution of one sample path
trialIdx = 1;
figure
tmpPlotData = LMMZeroRates(:,:,trialIdx);
tmpPlotData(tmpPlotData == 0) = NaN;
surf(ForwardTimes,LMMSimTimes,tmpPlotData)
title(['Evolution of the Zero Curve for Trial:' num2str(trialIdx) ' of LIBO
xlabel('Tenor (Years)')
```


## Compute Mortgage Rates from Simulation

Once the interest-rate paths have been simulated, the mortgage rate needs to be computed -- one approach, discussed by [7], is to compute the mortgage rate from a combination of the 2 -year and 10 -year rates.

For this example, the following is used:

```
% Compute mortgage rates from interest rate paths
TwoYearRates = squeeze(G2PP_SimZeroRates(:,Tenor == 2,:));
TenYearRates = squeeze(G2PP_SimZeroRates(:,Tenor == 7,:));
G2PP_MortgageRates = . 024 + .2*TwoYearRates + .6*TenYearRates;
LMMMOrtgageRates = squeeze(.024 + .2*LMMZeroRates(:,4,:) + .6*LMMZeroRates(
```

```
LMMDiscountFactors = squeeze(cumprod(1./(1 + LMMZeroRates(:,1,:)*.5)));
% Interpolate to get monthly mortgage rates
MonthlySimTimes = 0:1/12:LMMNumPeriods/LMMPeriod;
LMMMonthlyMortgageRates = zeros(nPeriods+1,LMMNTRIALS);
LMMMonthlyDF = zeros(nPeriods+1,LMMNTRIALS);
for trialidx=1:LMMNTRIALS
    LMMMonthlyMortgageRates(:,trialidx) = interp1(LMMSimTimes,LMMMortgageRat
    LMMMonthlyDF(:,trialidx) = interp1(LMMSimTimes,LMMDiscountFactors(:,tria
end
```


## Computing CPR and Generating and Valuing Cash Flows

Once the Mortgage Rates have been simulated, the CPR can be computed from the multiplicative model for each interest-rate path.

```
% Compute Seasoning and Refinancing Multipliers
Seasoning = ones(nPeriods+1,1);
Seasoning(1:30) = 1/30*(1:30);
G2PP_Refi = .2406 - .1389 * atan(5.952*(1.089 - CouponRate./G2PP_MortgageRa
LMM_Refi = .2406 - . 1389 * atan(5.952*(1.089 - CouponRate./LMMMonthlyMortga
% CPR is simply computed by evaluating the multiplicative model
G2PP_CPR = bsxfun(@times,G2PP_Refi,Seasoning.*(Seasonality(month(CFlowDates
LMM_CPR = bsxfun(@times,LMM_Refi,Seasoning.*(Seasonality(month(CFlowDates))
% Compute single monthly mortality (SMM) from CPR
G2PP_SMM = 1 - (1 - G2PP_CPR).^(1/12);
LMM_SMM = 1 - (1 - LMM_CPR).^(1/12);
% Plot CPR's against 100 PSA
CPR_PSA100 = psaspeed2rate(100);
figure
PSA_handle = plot(CPR_PSA100(1:nPeriods),'rx');
hold on
G2PP_handle = plot(G2PP_CPR,'b');
LMM_handle = plot(LMM_CPR,'g');
title('Prepayment Speeds')
legend([PSA_handle(1) G2PP_handle(1) LMM_handle(1)],{'100 PSA','G2PP CPR','
```


## Generate Cash Flows and Compute Present Value

With a vector of single monthly mortalities (SMM) computed for each interest-rate path, cash flows for the MBS can be computed and discounted.

```
% Compute the baseline zero rate at each cash flow time
CFlowZero = interp1(ZeroTimes,ZeroRates,CFlowTimes,'linear','extrap');
% Compute DF for each cash flow time
CFlowDF_Zero = zero2disc(CFlowZero,CFlowDates,Settle);
% Compute the price of the MBS using the zero curve
Price_Zero = CFlowAmounts*CFlowDF_Zero';
% Generate the cash flows for each IR Path
G2PP_CFlowAmounts = mbscfamounts(Settle, ...
    repmat(Maturity,1,nTrials), IssueDate, GrossRate, CouponRate, Delay, []
% Compute the DF for each IR path
G2PP_CFlowDFSim = cumprod(exp(squeeze(-G2PP_SimZeroRates(:,1,:).*DeltaTime)
% Present value the cash flows for each MBS
G2PP_Price_Ind = sum(G2PP_CFlowAmounts.*G2PP_CFlowDFSim',2);
G2PP_Price = mean(G2PP_Price_Ind);
% Repeat for LMM
LMM_CFlowAmounts = mbscfamounts(Settle, ...
    repmat(Maturity,1,LMMNTRIALS), IssueDate, GrossRate, CouponRate, Delay,
% Present value the cash flows for each MBS
LMM_Price_Ind = sum(LMM_CFlowAmounts.*LMMMonthlyDF',2);
LMM_Price = mean(LMM_Price_Ind);
```

The results from the different approaches can be compared. The number of trials for the G2++ model will typically be less than 100 due to the filtering out of any paths that produce negative interest rates.

Additionally, while the number of trials for the G2++ model in this example is set to be 100 , it is often the case that a larger number of simulations need to be run to produce an accurate valuation.

```
fprintf(' # of Monte Carlo Trials: %8d\n' , nTrials)
fprintf(' # of Time Periods/Trial: %8d\n\n' , nPeriods
fprintf(' MBS Price with PSA 100: %8.4f\n' , Price_Ze
fprintf(' MBS Price with Custom G2PP Prepayment Model: %8.4f\n\n', G2PP_Pri
fprintf(' MBS Price with Custom LMM Prepayment Model: %8.4f\n\n', LMM_Price
    # of Monte Carlo Trials: 73
    # of Time Periods/Trial: 156
    MBS Price with PSA 100: 1.0187
MBS Price with Custom G2PP Prepayment Model: 0.9871
MBS Price with Custom LMM Prepayment Model: 0.9993
```


## Conclusion

This example shows how to calibrate and simulate a G2++ interest-rate model and how to use the generated interest-rate paths in a prepayment model loosely based on the Richard and Roll model. This example also provides a useful starting point to using the G2++ and LMM interest-rate models in other financial applications.

## Bibliography

This example is based on the following books, papers and journal articles:
1 Andersen, L. and V. Piterbarg (2010). Interest Rate Modeling, Atlantic Financial Press.

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# Pricing Mortgage Backed Securities Using the Black-Derman-Toy Model 

This example illustrates how the Financial Toolbox ${ }^{\text {TM }}$ and Financial Instruments Toolbox ${ }^{\mathrm{TM}}$ can be used to price a level mortgage backed security using the BDT model.

## Load the BDT Tree Stored in the Data File

load mbsexample.mat

## Observe the Interest Rate Tree

Visualize the interest rate evolution along the tree by looking at the output structure BDTTree. BDTTree returns an inverse discount tree, which you can convert into an interest rate tree with the cvtree function.

```
BDTTreeR = cvtree(BDTTree);
```

Look at the upper branch and lower branch paths of the tree:

```
OldFormat = get(0, 'format');
format short
%Rate at root node:
RateRoot = treepath(BDTTreeR.RateTree, [0])
%Rates along upper branch:
RatePathUp = treepath(BDTTreeR.RateTree, [\begin{array}{lllll}{1}&{1}&{1}&{1}&{1}\end{array}])
%Rates along lower branch:
RatePathDown = treepath(BDTTreeR.RateTree, [2 2 2 2 2])
```

RateRoot =
0.0399
RatePathUp =

> 0.0399
> 0.0397
> 0.0391
> 0.0383
> 0.0373
> 0.0360

RatePathDown =
0.0399
0.0470
0.0550
0.0638
0.0734
0.0841

## Compute the Price Tree for the Non-Prepayable Mortgage

Let's say that we have a 3 year $\$ 10000$ level prepayable loan, with a mortgage interest rate of $4.64 \%$ semi-annually compounded.

```
MortgageAmount = 10000;
CouponRate = 0.0464;
Period = 2;
Settle='01-Jan-2007';
Maturity='01-Jan-2010';
Compounding = BDTTree.TimeSpec.Compounding;
format bank
% Use the function 'amortize.m' in the Financial Toolbox to calculate the
% payment of the loan (MP), the interest and principal components, and the
% outstanding principal balance.
NumPeriods = date2time(Settle,Maturity, Compounding)';
[Principal, InterestPayment, OutstandingBalance, MP] = amortize(CouponRate/
```

```
% Display Principal, Interest and Outstanding balances
PrincipalAmount = Principal'
InterestPaymentAmount = InterestPayment'
OutstandingBalanceAmount =OutstandingBalance'
CFlowAmounts = MP*ones(1,NumPeriods);
% The CFlowDates are the same as the tree level dates
CFlowDates= {'01-Jul-2007' ,'01-Jan-2008' ,'01-Jul-2008' , '01-Jan-2009' ,
% Calculate the price of the non-prepayable mortgage
[PriceNonPrepayableMortgage, PriceTreeNonPrepayableMortgage] = cfbybdt(BDTT
for iLevel = 2:length(PriceTreeNonPrepayableMortgage.PTree)
    PriceTreeNonPrepayableMortgage.PTree{iLevel}(:,:)= PriceTreeNonPrepayab
end
% Look at the price of the mortgage today (tObs = 0)
PriceNonPrepayableMortgage
% The value of the non-prepayable mortgage is $10017.47. This value exceeds
% the $10000 amount borrowed since the homeowner received not only $10000,
% also a prepayment option.
% Look at the value of the mortgage on the last date, right after the last
% mortgage payment, is zero:
PriceTreeNonPrepayableMortgage.PTree{end};
% Visualize the price tree for the non-prepayable mortgage.
treeviewer(PriceTreeNonPrepayableMortgage)
PrincipalAmount =
    1572.59
    1609.07
    1646.40
    1684.60
    1723.68
    1763.67
```

```
InterestPaymentAmount =
    232.00
    195.52
    158.19
    119.99
    80.91
    40.92
OutstandingBalanceAmount =
    8427.41
    6818.34
    5171.94
    3487.35
    1763.67
        0.00
PriceNonPrepayableMortgage =
    10017.47
```


## Compute the Price Tree of the Prepayment Option

```
% The Prepayment option is like a call option on a bond.
```

% The Prepayment option is like a call option on a bond.
%
%
% The exercise price or strike will be equal to the outstanding principal
% The exercise price or strike will be equal to the outstanding principal
% which has been calculated using the function 'amortize.m'.
% which has been calculated using the function 'amortize.m'.
OptSpec = 'call';
OptSpec = 'call';
Strike = [MortgageAmount OutstandingBalance];
Strike = [MortgageAmount OutstandingBalance];
ExerciseDates =[Settle CFlowDates];
ExerciseDates =[Settle CFlowDates];
AmericanOpt = 0;
AmericanOpt = 0;
Maturity = CFlowDates(end);

```
Maturity = CFlowDates(end);
```

```
% Compute the price of the prepayment option:
[PricePrepaymentOption, PriceTreePrepaymentOption] = prepaymentbybdt(BDTTre
                    O, Settle, Maturity,[], [], [], ...
                                [], [], [], [], 0, [], CFlowAmounts);
% Look at the price of the prepayment option today (tObs = 0)
PricePrepaymentOption
% The value of the prepayment option is $17.47 as expected.
% Visualize the price tree for the prepayment option
treeviewer(PriceTreePrepaymentOption)
```

PricePrepaymentOption =
17.47

## Calculate the Price Tree of the Prepayable Mortgage.

\% Compute the price of the prepayable mortgage.

PricePrepayableMortgage = PriceNonPrepayableMortgage - PricePrepaymentOptio

PriceTreePrepayableMortgage = PriceTreeNonPrepayableMortgage;
for iLevel = 1:length(PriceTreeNonPrepayableMortgage.PTree) PriceTreePrepayableMortgage.PTree\{iLevel\} (: : ) = PriceTreeNonPrepayableM PriceTreePrepaymentOption.PTree\{iLevel\}(:, :);
end
\% Look at the price of the prepayable mortgage today (tObs $=0$ ) PricePrepayableMortgage
\% The value of the prepayable mortgage is $\$ 10000$ as expected.

```
% Visualize the price and price tree for the prepayable mortgage
treeviewer(PriceTreePrepayableMortgage)
set(O, 'format', OldFormat);
PricePrepayableMortgage =
```

10000.00

# Using Collateralized Mortgage Obligations (CMOs) 

In this section...<br>"What Are CMOs?" on page 5-38<br>"Prepayment Risk" on page 5-38<br>"CMO Workflow" on page 5-49

## What Are CMOs?

Financial Instruments Toolbox supports collateralized mortgage obligations (CMOs) to provide investors with a greater range of risk and return characteristics than mortgage-backed securities (MBS). In contrast to an MBS, which simply redirects principal and interest cash flows to investors on a pro rata basis, a CMO structures cash flows to different tranches, or slices, to create securities that are better tailored to specific investors.

For example, banks might be primarily concerned with extension risk, or the risk that their investment lengthens in time due to increasing interest rates, given that they typically have short-term deposits as liabilities. Insurance companies and pension funds might be concerned primarily with contraction risk, or the risk that their investment will pay off too soon, with liabilities that have much longer lives. A CMO structure addresses the interest-rate risk of extension or contraction with a blend of short-term and long-term CMO securities, called tranches.

## Prepayment Risk

Prepayment risk is the risk that the term of the security varies according to differing rates of repayment of principal by borrowers (repayments from refinancings, sales, curtailments, or foreclosures). In a CMO, you can structure the principal (and associated coupon) stream from the underlying mortgage pool collateral to allocate prepayment risk. If principal is prepaid faster than expected (for example, if mortgage rates fall and borrowers refinance), then the overall term of the mortgage pool collateral shortens.

You cannot remove prepayment risk, but you can reallocate it among CMO tranches so that some tranches have some protection against this risk, and other tranches will absorb more of this risk. To facilitate this allocation of
prepayment risk, CMOs are structured such that prepayments are allocated among tranches using a fixed set of rules. The most common schemes for prepayment tranching are:

- Sequential tranching, with or without, Z-bond tranching
- Schedule bond tranching
- Planned amortization class (PAC) bonds
- Target amortization class (TAC) bonds

Financial Instruments Toolbox supports these schemes for prepayment tranching for CMOs and tools for pricing and scheduling cash flows between the tranches, as well as analyzing the price and yield for CMOs. Financial Instruments Toolbox functionality for CMOs does not model credit risk. Therefore, this functionality is most appropriate for CMOs where credit risk is not an issue (for example, agency CMOs where the underlying mortgage pool collateral is insured for default by the agency Government-Sponsored Enterprises (GSEs), such as Fannie Mae and Freddie Mac).

## Sequential Tranches Without a Z-Bond

All available principal and interest payments go to the first sequential tranche, until its balance decrements to zero, then to the second, and so on. For example, consider the following example where all principal and interest from the underlying mortgage pool is repaid on tranche A first, then tranche B, then tranche C. Note that interest is paid on each tranche as long as the principal for the tranche has not been retired.


## Sequential Tranches With a Z-Bond

The Z-bond, also called an accural bond, is a type of interest and principal pay rule. The Z-bond tranche supports other sequential pay tranches by not receiving an interest payment. The interest payment that would have accrued to the Z-bond tranche pays off the principal of other bonds, and the principal of the Z-bond tranche increases. The Z-bond tranche starts receiving interest and principal payments only after the other tranches in the CMO have been fully paid. The Z-bond tranche is used in a sequential-pay structure to accelerate the principal repayments of the sequential-pay bonds.

A Z-bond differs from other CMO instruments because it is not tranching principal but interest. The Z-bond receives no cash flows until all other securities have been paid off. In the interim, the interest that is owed to the Z-bond is accrued to its principal. The following chart demonstrates the difference between a Z-bond and a normal sequential pay tranche. Note that the C tranche pays off sooner with the Z-bond, because the interest cash flows to the Z-bond are being used to pay down the principal of the C tranche.


For comparison, the following graphic is the same sequential CMO with no Z-bond.


## PAC Tranches

Planned amortization class (PAC) bonds help reduce the effects of prepayment risk. They are designed to produce more stable cash flows by redirecting prepayments from the underlying mortgage collateral to other classes (tranches) called companion or support classes. PAC bonds have a principal payment rate over a predetermined period of time. The PAC bond payment schedule is determined by two different prepayment rates, which together form a band (also called a collar). Early in the life of the CMO, the prepayment at the lower PSA yields a lower prepayment. Later in its life, the principal in the higher PSA declines enough that it yields a lower prepayment. The PAC tranche receives whichever rate is lower, so it will change prepayment at one PSA for the first part of its life, then switch to the other rate. The ability to stay on this schedule is maintained by a support bond, which absorbs excess prepayments, and receives less prepayments to prevent extension of average life.

However, the PAC is only protected from extension to the amount that prepayments are made on the underlying MBSs. If there is a sustained
period of fast prepayments, then that might completely eliminate a PAC bond's outstanding support class. When the principal of the associated PAC bond is exhausted, the CMO is called a "busted PAC", or "busted collar". Alternatively, in times of slow prepayments, amortization of the support bonds is delayed if there is not enough principal for the currently paying PAC bond. This extends the average life of the class.

A PAC bond protects against both extension and contraction risk by:

- Specifying a schedule of principal payments for the PAC bond
- Including support tranches that are allocated prepayments inside a specified prepayment band

PAC bonds typically specify a band expressed using the PSA model. A PAC bond with a range of 100 to $250 \%$ has this principal schedule.


The principal repayment schedule is the minimum principal payment as Region 1 shows. Note that this is the principal payment schedule as long as the actual prepayment stays within the prepayment band of 100 to $250 \%$ PSA.

For example, for different prepayment speeds of $125 \%, 175 \%$, and $225 \%$ PSA, the actual principal payments are shown in the following graphs. Note that at higher prepayment speeds, the support tranche is allocated principal earlier while the principal timing for the other tranches remains constant.




## TAC Tranches

Target amortization class (TAC) bonds are similar to PAC bonds, but they do not provide protection against extension of average life. Create the schedule of principal payments by using just a single PSA. TAC bonds pay a "targeted" principal payment schedule at a single, constant prepayment speed. As long as the underlying mortgage collateral does not prepay at a rate slower than this speed, the TAC bond payment schedule is met. TAC bonds can protect against increasing prepayments and early retirement of the TAC bond investment. If the principal cash flow from the mortgage collateral exceeds the TAC schedule, the excess is allocated to TAC companion (support) classes. Alternatively, if prepayments fall below the speed necessary to maintain the TAC schedule, the weighted average life of the TAC is extended. The TAC bond does not protect against low prepayment rates.

For example, here is a TAC structure rated for $125 \%, 175 \%$, and $450 \%$ PSA.




Note that for prepayments below $175 \%$ PSA, the TAC bond extends like a normal sequential pay CMO. TAC bonds are appealing because they offer higher yields than comparable PAC bonds. The unaddressed risk from low prepyament rates generally does not concern investors as much as risk from high prepayment rates.

## CMO Workflow

In general, the CMO workflow is:
1 Calculate underlying mortgage cash flows.
2 Define CMO tranches
3 If using a PAC or TAC CMO, calculate the principal schedule.
4 Calculate cash flows for each tranche.
5 Analyze the CMO by computing price, yield, spread of CMO cash flows.

## Calculate Underlying Mortgage Cash Flows

Underlying mortgage pool pass-through cash flows are calculated by the existing function mbspassthrough. The CMO cash flow functions require the principal payments (including prepayments) calculated from existing functions mbspassthrough or mbscfamounts.

```
principal = 10000000;
coupon = 0.06;
terms = 360;
psa = 150;
```

```
[principal_balance, monthly_payments, sched_principal_payments,...
interest_payments, prepayments] = mbspassthrough(principal,...
coupon, terms, terms, psa, []);
principal_payments = sched_principal_payments.' + prepayments.';
```

After determining principal payments for the underlying mortgage collateral, you can generate cash flows for a sequential CMO, with or without a Z-bond, by using cmoseqcf. For a PAC or TAC CMO, the cash flows are generated using cmoschedcf

## Define CMO Tranches

Define CMO tranche; for example, define a CMO with 2 tranches:
TranchePrincipals = [500000; 500000];
TrancheCoupons = [0.06; 0.06];

## If Using a PAC or TAC CMO, Calculate Principal Schedule

Calculate the PAC/TAC principal balance schedule based on a band of PSA speeds. For scheduled CMOs (PAC/TAC), the CMO cash flow functions additionally take in the principal balance schedule calculated by the CMO schedule function cmosched.

```
speed = [100 300];
[balanceSchedule, initialBalance] = cmosched(principal, coupon,...
terms, terms, speed, TranchePrincipals(1));
```


## Calculate Cash Flows for Each Tranche

You can reuse the output from the cash flow generation functions to further divide the cash flows into tranches. For example, the output from cmoschedcf for a PAC tranche can be divided into sequential tranches by passing the principal cash flows of the PAC tranche into the cmoschedcf function. The output of the CMO cash flow functions are the principal and interest cash flows, as well as the principal balance.
[principal_balances, principal_cashflows, interest_cashflows] = cmoschedcf(principal_payments,...
TranchePrincipals, TrancheCoupons, balanceSchedule);

## Analyze CMO by Computing Price, Yield, and Spread of CMO Cash Flows

The outputs from the CMO functions (cmoseqcf and cmoschedcf) are cash flows. The functions used to analyze a CMO are based on these cash flows. To that end, you can use cfbyzero, cfspread, cfyield, and cfprice to compute prices, yield, and spreads for the CMO cash flows. In addition, using the following, you can calculate a weighted average life (WAL) for each tranche in the CMO:

$$
W A L=\sum_{i=1}^{n} \frac{P_{i}}{P} t_{i}
$$

where:
$P$ is the total principal.
$P_{i}$ is the principal repayment of the coupon $i$.
$\frac{P_{i}}{P}$ is the fraction of the principal repaid in coupon $i$.
$t_{i}$ is the time in years from the start to coupon $i$.
Related
Examples

- "Create PAC and Sequential CMO" on page 5-53
- "Fixed-Rate Mortgage Pool" on page 5-3
Concepts
- "What Are Mortgage-Backed Securities?" on page 5-2


## Create PAC and Sequential CMO

This example shows how to use an underlying mortgage-backed security (MBS) pool for a 30 -year fixed-rate mortgage of $6 \%$ to define a PAC bond, and then define a sequential CMO from the PAC bond. Analyze the CMO by comparing the CMO spread to a zero-rate curve for a 30 -year Treasury bond and then calculate the weighted-average life (WAL) for the PAC bond.

## Step 1. Define the underlying mortgage pool.

```
principal = 100000000;
grossrate = 0.06;
coupon = 0.05;
originalTerm = 360;
termRemaining = 360;
speed = 100;
delay = 14;
Settle = datenum('1-Jan-2011');
IssueDate = datenum('1-Jan-2011');
Maturity = addtodate(IssueDate, 360, 'month');
```


## Step 2. Calculate underlying pool cash flow.

[CFlowAmounts, CFlowDates, ~, ~, ~, UnitPrincipal, UnitInterest, ... UnitPrepayment] = mbscfamounts(Settle, Maturity, IssueDate, grossrate, ... coupon, delay, speed, []);

## Step 3. Calculate prepayments.

```
principalPayments = UnitPrincipal * principal;
netInterest = UnitInterest * principal;
prepayments = UnitPrepayment * principal;
dates = CFlowDates' + delay;
```


## Step 4. Generate a plot for underlying MBS payments.

```
area([principalPayments'+prepayments', netInterest'])
title('Underlying MBS Payments');
legend('Principal Payments (incl. Prepayments)', 'Interest Payments')
```



## Step 5. Calculate the PAC schedule.

```
pacSpeed = [80 300];
[balanceSchedule, pacInitBalance] = ...
cmosched(principal, grossrate, originalTerm, termRemaining, ...
pacSpeed, []);
```

Step 6. Generate a plot for the PAC principal balance schedule.

```
figure;
area([pacInitBalance'; balanceSchedule'])
```



## Step 7. Calculate PAC cash flow.

pacTranchePrincipals = [pacInitBalance; principal-pacInitBalance]; pacTrancheCoupons = [0.05; 0.05];
[pacBalances, pacPrincipals, pacInterests] = ...
cmoschedcf(principalPayments+prepayments, ...
pacTranchePrincipals, pacTrancheCoupons, balanceSchedule);

## Step 8. Generate a plot for the PAC CMO tranches.

Generate a plot for the PAC CMO tranches:

```
figure;
area([pacPrincipals' pacInterests']);
title('PAC CMO (PAC and Support Tranches)');
legend('PAC Principal Payments', 'Support Principal Payments', ...
'PAC Interest Payments', 'Support Interest Payments');
```



Step 9. Create sequential CMO from the PAC bond.

```
CMO tranches, A, B, C, and D
seqTranchePrincipals = ...
[20000000; 20000000; 10000000; pacInitBalance-50000000];
seqTrancheCoupons = [0.05; 0.05; 0.05; 0.05];
```

Step 10. Calculate cash flows for each tranche.

```
[seqBalances, seqPrincipals, seqInterests] = ...
cmoseqcf(pacPrincipals(1, :), seqTranchePrincipals, ...
seqTrancheCoupons, false);
```


## Step 11. Generate a plot for the sequential PAC CMO.

Generate a plot for the sequential PAC CMO:

```
figure
area([seqPrincipals' pacPrincipals(2, :)' pacInterests']);
title('Sequential PAC CMO (Sequential PAC and Support Tranches)');
legend('Sequential PAC Principals (A)', 'Sequential PAC Principals (B)',
'Sequential PAC Principals (C)', 'Sequential PAC Principals (D)', ...
'Support Principal Payments', 'PAC Interest Payments', ...
'Support Interest Payments');
```



## Step 12. Create the discount curve.

```
CurveSettle = datenum('1-Jan-2011');
ZeroRates = [lllllllllllllll
CurveTimes = [1/12 3/12 6/12 1 2 3 5 7 10 20 30]';
CurveDates = daysadd(CurveSettle, 360 * CurveTimes, 1);
zeroCurve = intenvset('Rates', ZeroRates, 'StartDates', CurveSettle, ...
'EndDates', CurveDates);
```


## Step 13. Price the CMO cash flows.

The cash flow for the sequential PAC principal A tranche is calculated using the cash flow functions cfbyzero, cfyield, cfprice, and cfspread.

```
cflows = seqPrincipals(1, :)+seqInterests(1, :);
cfdates = dates(2:end)';
price1 = cfbyzero(zeroCurve, cflows, cfdates, Settle, 4)
yield = cfyield(cflows, cfdates, price1, Settle, 'Basis', 4)
price2 = cfprice(cflows, cfdates, yield, Settle, 'Basis', 4)
spread = cfspread(zeroCurve, price2, cflows, cfdates, Settle, 'Basis', 4)
WAL = sum(cflows .* yearfrac(Settle, cfdates, 4)) / sum(cflows)
price1 =
    2.2109e+07
yield =
    0.0090
price2 =
    2.2109e+07
spread =
    8.5711e-13
WAL =
    2.5408
```

The weighted average life (WAL) for the sequential PAC principal A tranche is 2.54 years.
See Also cmoseqcf | cmosched | cmoschedcf | mbscfamounts | cfbyzero | cfyield | cfprice | cfspread | cfbyzero
Related - "Fixed-Rate Mortgage Pool" on page 5-3
Examples
Concepts - "Using Collateralized Mortgage Obligations (CMOs)" on page 5-38

## Debt Instruments

- "Agency Option-Adjusted Spreads" on page 6-2
- "Using Zero-Coupon Bonds" on page 6-7
- "Stepped-Coupon Bonds" on page 6-12
- "Term Structure Calculations" on page 6-15


## Agency Option-Adjusted Spreads

Often bonds are issued with embedded options, which then makes standard price/yield or spread measures irrelevant. For example, a municipality concerned about the chance that interest rates may fall in the future might issue bonds with a provision that allows the bond to be repaid before the bond's maturity. This is a call option on the bond and must be incorporated into the valuation of the bond. Option-adjusted spread (OAS), which adjusts a bond spread for the value of the option, is the standard measure for valuing bonds with embedded options. Financial Instruments Toolbox software supports computing option-adjusted spreads for bonds with single embedded options using the agency model.

The Securities Industry and Financial Markets Association (SIFMA) has a simplified approach to compute OAS for agency issues (Government Sponsored Entities like Fannie Mae and Freddie Mac) termed "Agency OAS". In this approach, the bond has only one call date (European call) and uses Black's model (a variation on Black Scholes, http://en.wikipedia.org/wiki/Black_model) to value the bond option. The price of the bond is computed as follows:

Price $_{\text {Callable }}=$ Price $_{\text {NonCallable }}-$ Price $_{\text {Option }}$
where
Price $_{\text {Callable }}$ is the price of the callable bond.
Price $_{\text {NonCallable }}$ is the price of the noncallable bond, i.e., price of the bond using bndspread.

Price $_{\text {Option }}$ is the price of the option, i.e., price of the option using Black's model.

The Agency OAS is the spread, when used in the previous formula, yields the market price. Financial Instruments Toolbox software supports these functions:

## (Continued)

| Agency OAS <br> Functions | Purpose |
| :--- | :--- |
| agencyoas | Compute the OAS of the callable bond using the <br> Agency OAS model. |
| agencyprice | Price the callable bond OAS using Agency using <br> the OAS model. |

## Computing the Agency OAS for Bonds

To compute the Agency OAS using agencyoas, you must provide the zero curve as the input ZeroData. You can specify the zero curve in any intervals and with any compounding method. You can do this using Financial Toolbox ${ }^{\text {TM }}$ functions zbtprice and zbtyield. Or, you can use IRDataCurve to construct an IRDataCurve object, and then use the getZeroRates to convert to dates and data for use in the ZeroData input.

After creating the ZeroData input for agencyoas, you can then:
1 Assign parameters for CouponRate, Settle, Maturity, Vol, CallDate, and Price.

2 Compute the option-adjusted spread using agencyoas to derive the OAS output.

If you have the Agency OAS for the callable bond, you can use the OAS value as an input to agencyprice to determine the price for a callable bond.

In the following example, the Agency OAS is computed using agencyoas for a range of bond prices and the spread of an identically priced noncallable bond is calculated using bndspread.

```
%% Data
% Bond data -- note that there is only 1 call date
Settle = datenum('20-Jan-2010');
Maturity = datenum('30-Dec-2013');
Coupon = .022;
Vol = .5117;
```

```
CallDate = datenum('30-Dec-2010');
Period = 2;
Basis = 1;
Face = 100;
% Zero Curve data
ZeroTime = [.25 .5 1 2 3 4 5 7 10 20 30]';
ZeroDates = daysadd(Settle,360*ZeroTime,1);
ZeroRates = [.0008 .0017 .0045 . 0102 . 0169 .0224 .0274 .0347 .0414 .0530 .0740]';
ZeroData = [ZeroDates ZeroRates];
CurveCompounding = 2;
CurveBasis = 1;
Price = 94:104;
OAS = agencyoas(ZeroData, Price', Coupon, Settle,Maturity, Vol, CallDate,'Basis',Basis)
Spread = bndspread(ZeroData, Price', Coupon, Settle, Maturity)
plot(OAS,Price)
hold on
plot(Spread,Price,'r')
xlabel('Spread (bp)')
ylabel('Price')
title('AOAS and Spread for an Agency and Equivalent Noncallable Bond')
legend({'Callable Issue','Noncallable Issue'})
OAS =
    163.4942
    133.7306
    103.8735
        73.7505
        43.1094
        11.5608
        -21.5412
        -57.3869
        -98.5675
    -152.5226
    -239.6462
Spread =
```

168.1412
139.7047
111.6123
83.8561
56.4286
29.3227
2.5314
-23.9523
-50.1348

- 76.0226
$-101.6218$

The following plot demonstrates as the price increases, the value of the embedded option in the Agency issue increases, and the value of the issue itself does not increase as much as it would for a noncallable bond, illustrating the negative convexity of this issue:


## Using Zero-Coupon Bonds

In this section...<br>"Introduction" on page 6-7<br>"Measuring Zero-Coupon Bond Function Quality" on page 6-7<br>"Pricing Treasury Notes" on page 6-8<br>"Pricing Corporate Bonds" on page 6-10

## Introduction

A zero-coupon bond is a corporate, Treasury, or municipal debt instrument that pays no periodic interest. Typically, the bond is redeemed at maturity for its full face value. It will be a security issued at a discount from its face value, or it may be a coupon bond stripped of its coupons and repackaged as a zero-coupon bond.

Financial Instruments Toolbox software provides functions for valuing zero-coupon debt instruments. These functions supplement existing coupon bond functions such as bndprice and bndyield that are available in Financial Toolbox software.

## Measuring Zero-Coupon Bond Function Quality

Zero-coupon function quality is measured by how consistent the results are with coupon-bearing bonds. Because the zero coupon's yield is bond-equivalent, comparisons with coupon-bearing bonds are possible.

In the textbook case, where time $(t)$ is measured continuously and the rate ( $r$ ) is continuously compounded, the value of a zero bond is the principal multiplied by $e^{-r t}$. In reality, the rate quoted is continuous and the basis can be variable, requiring a more consistent approach to meet the stricter demands of accurate pricing.

The following two examples

- "Pricing Treasury Notes" on page 6-8
- "Pricing Corporate Bonds" on page 6-10
show how the zero functions are consistent with supported coupon bond functions.


## Pricing Treasury Notes

A Treasury note can be considered to be a package of zeros. The toolbox functions that price zeros require a coupon bond equivalent yield. That yield can originate from any type of coupon paying bond, with any periodic payment, or any accrual basis. The next example shows the use of the toolbox to price a Treasury note and compares the calculated price with the actual price quotation for that day.

```
Settle = datenum('02-03-2003');
MaturityCpn = datenum('05-15-2009');
Period = 2;
Basis = 0;
% Quoted yield.
QYield = 0.03342;
% Quoted price.
QPriceACT = 112.127;
CouponRate = 0.055;
```

Extract the cash flow and compute price from the sum of zeros discounted.

```
[CFlows, CDates] = cfamounts(CouponRate, Settle, MaturityCpn, ...
Period, Basis);
MaturityofZeros = CDates;
```

Compute the price of the coupon bond identically as a collection of zeros by multiplying the discount factors to the corresponding cash flows.

```
PriceofZeros = CFlows * zeroprice(QYield, Settle, ...
MaturityofZeros, Period, Basis)/100;
```

The following table shows the intermediate calculations.

| Cash Flows | Discount Factors | Discounted Cash <br> Flows |
| :--- | :--- | :--- |
| -1.2155 | 1.0000 | -1.2155 |
| 2.7500 | 0.9908 | 2.7246 |
| 2.7500 | 0.9745 | 2.6799 |
| 2.7500 | 0.9585 | 2.6359 |
| 2.7500 | 0.9427 | 2.5925 |
| 2.7500 | 0.9272 | 2.5499 |
| 2.7500 | 0.9120 | 2.5080 |
| 2.7500 | 0.8970 | 2.4668 |
| 2.7500 | 0.8823 | 2.4263 |
| 2.7500 | 0.8678 | 2.3864 |
| 2.7500 | 0.8535 | 2.3472 |
| 2.7500 | 0.8395 | 2.3086 |
| 2.7500 | 0.8257 | 2.2706 |
| 102.7500 | 0.8121 | 83.4451 |
|  |  | Total |

Compare the quoted price and the calculated price based on zeros.
[QPriceACT PriceofZeros]
ans =

## $112.1270 \quad 112.1263$

This example shows that zeroprice can satisfactorily price a Treasury note, a semiannual actual/actual basis bond, as if it were a composed of a series of zero-coupon bonds.

## Pricing Corporate Bonds

You can similarly price a corporate bond, for which there is no corresponding zero-coupon bond, as opposed to a Treasury note, for which corresponding zeros exist. You can create a synthetic zero-coupon bond and arrive at the quoted coupon-bond price when you later sum the zeros.

```
Settle = datenum('02-05-2003');
MaturityCpn = datenum('01-14-2009');
Period = 2;
Basis = 1;
% Quoted yield.
QYield = 0.05974;
% Quoted price.
QPrice30 = 99.382;
CouponRate = 0.05850;
```

Extract cash flow and compute price from the sum of zeros.

```
[CFlows, CDates] = cfamounts(CouponRate, Settle, MaturityCpn, ...
Period, Basis);
Maturity = CDates;
```

Compute the price of the coupon bond identically as a collection of zeros by multiplying the discount factors to the corresponding cash flows.

```
Price30 = CFlows * zeroprice(QYield, Settle, Maturity, Period, ...
Basis)/100;
```

Compare quoted price and calculated price based on zeros.

```
[QPrice30 Price30]
ans =
99.3820 99.3828
```

As a test of fidelity, intentionally giving the wrong basis, say actual/actual (Basis $=0$ ) instead of $30 / 360$, gives a price of 99.3972 . Such a systematic error, if recurring in a more complex pricing routine, quickly adds up to large inaccuracies.

In summary, the zero functions in MATLAB software facilitate extraction of present value from virtually any fixed-coupon instrument, up to any period in time.

## Stepped-Coupon Bonds

In this section...<br>"Introduction" on page 6-12<br>"Cash Flows from Stepped-Coupon Bonds" on page 6-12<br>"Price and Yield of Stepped-Coupon Bonds" on page 6-14

## Introduction

A stepped-coupon bond has a fixed schedule of changing coupon amounts. Like fixed coupon bonds, stepped-coupon bonds could have different periodic payments and accrual bases.

The functions stepcpnprice and stepcpnyield compute prices and yields of such bonds. An accompanying function stepcpncfamounts produces the cash flow schedules pertaining to these bonds.

## Cash Flows from Stepped-Coupon Bonds

Consider a bond that has a schedule of two coupons. Suppose the bond starts out with a $2 \%$ coupon that steps up to $4 \%$ in 2 years and onward to maturity. Assume that the issue and settlement dates are both March 15, 2003. The bond has a 5 year maturity. Use stepcpncfamounts to generate the cash flow schedule and times.

```
Settle = datenum('15-Mar-2003');
Maturity = datenum('15-Mar-2008');
ConvDates = [datenum('15-Mar-2005')];
CouponRates = [0.02, 0.04];
[CFlows, CDates, CTimes] = stepcpncfamounts(Settle, Maturity, ...
ConvDates, CouponRates)
```

Notably, ConvDates has 1 less element than CouponRates because MATLAB software assumes that the first element of CouponRates indicates the coupon schedule between Settle (March 15, 2003) and the first element of ConvDates (March 15, 2005), shown diagrammatically below.

|  | Pay 2\% from March <br> 15,2003 |  | Pay 4\% from March <br> 15,2003 |
| :--- | :--- | :--- | :--- |
| Effective 2\% on March <br> 15,2003 |  | Effective 4\% on March <br> 15,2005 |  |


| Coupon Dates | Semiannual Coupon Payment |
| :--- | :--- |
| 15-Mar-03 | 0 |
| 15-Sep-03 | 1 |
| 15-Mar-04 | 1 |
| 15-Sep-04 | 1 |
| 15-Mar-05 | 1 |
| 15-Sep-05 | 2 |
| 15-Mar-06 | 2 |
| 15-Sep-06 | 2 |
| 15-Mar-07 | 2 |
| 15-Sep-07 | 2 |
| 15-Mar-08 | 102 |

The payment on March 15, 2005 is still a $2 \%$ coupon. Payment of the $4 \%$ coupon starts with the next payment, September 15, 2005. March 15, 2005 is the end of first coupon schedule, not to be confused with the beginning of the second.

In summary, MATLAB takes user input as the end dates of coupon schedules and computes the next coupon dates automatically.

The payment due on settlement (zero in this case) represents the accrued interest due on that day. It is negative if such amount is nonzero. Comparison with cfamounts in Financial Toolbox shows that the two functions operate identically.

## Price and Yield of Stepped-Coupon Bonds

The toolbox provides two basic analytical functions to compute price and yield for stepped-coupon bonds. Using the above bond as an example, you can compute the price when the yield is known.

You can estimate the yield to maturity as a number-of-year weighted average of coupon rates. For this bond, the estimated yield is:

$$
\frac{(2 \times 2)+(4 \times 3)}{5}
$$

or $3.33 \%$. While definitely not exact (due to nonlinear relation of price and yield), this estimate suggests close to par valuation and serves as a quick first check on the function.

```
Yield = 0.0333;
[Price, AccruedInterest] = stepcpnprice(Yield, Settle, ...
Maturity, ConvDates, CouponRates)
```

The price returned is 99.2237 (per $\$ 100$ notional), and the accrued interest is zero, consistent with our earlier assertions.

To validate that there is consistency among the stepped-coupon functions, you can use the above price and see if indeed it implies a $3.33 \%$ yield by using stepcpnyield.

YTM = stepcpnyield(Price, Settle, Maturity, ConvDates, ... CouponRates)

YTM =
0.0333

## Term Structure Calculations

```
In this section...
"Introduction" on page 6-15
"Computing Spot and Forward Curves" on page 6-15
"Computing Spreads" on page 6-17
```


## Introduction

So far, a more formal definition of "yield" and its application has not been developed. In many situations when cash flow is available, discounting factors to the cash flows may not be immediately apparent. In other cases, what is relevant is often a spread, the difference between curves (also known as the term structure of spread).

All these calculations require one main ingredient, the Treasury spot, par-yield, or forward curve. Typically, the generation of these curves starts with a series of on-the-run and selected off-the-run issues as inputs.

MATLAB software uses these bonds to find spot rates one at a time, from the shortest maturity onwards, using bootstrap techniques. All cash flows are used to construct the spot curve, and rates between maturities (for these coupons) are interpolated linearly.

## Computing Spot and Forward Curves

For an illustration of how this works, observe the use of zbtyield (or equivalently zbtprice) on a portfolio of six Treasury bills and bonds.

| Bills | Maturity Date | Current Yield |
| :--- | :--- | :--- |
| 3 month | $4 / 17 / 03$ | 1.15 |
| 6 month | $7 / 17 / 03$ | 1.18 |


| Notes/Bonds | Coupon | Maturity Date | Current Yield |
| :--- | :--- | :--- | :--- |
| 2 year | 1.750 | $12 / 31 / 04$ | 1.68 |
| 5 year | 3.000 | $11 / 15 / 07$ | 2.97 |
| 10 year | 4.000 | $11 / 15 / 12$ | 4.01 |
| 30 year | 5.375 | $2 / 15 / 31$ | 4.92 |

You can specify prices or yields to the bonds above to infer the spot curve. The function zbtyield accepts yields (bond-equivalent yield, to be exact).

To proceed, first assemble the above table into a variable called Bonds. The first column contains maturities, the second contains coupons, and the third contains notionals or face values of the bonds. (Note that bills have zero coupons.)

| Bonds $=$ | $[$ datenum('04/17/2003') | 0 | $100 ;$ |
| ---: | :--- | :--- | :--- |
|  | datenum('07/17/2003') | 0 | $100 ;$ |
|  | datenum('12/31/2004') | 0.0175 | $100 ;$ |
|  | datenum('11/15/2007') | 0.03 | $100 ;$ |
|  | datenum('11/15/2012') | 0.04 | $100 ;$ |
|  | datenum('02/15/2031') | 0.05375 | $100] ;$ |

Then specify the corresponding yields.

$$
\begin{aligned}
\text { Yields }= & {[0.0115 ;} \\
& 0.0118 ; \\
& 0.0168 ; \\
& 0.0297 ; \\
& 0.0401 ; \\
& 0.0492] ;
\end{aligned}
$$

You are now ready to compute the spot curve for each of these six maturities. The spot curve is based upon a settlement date of January 17, 2003.

```
Settle = datenum('17-Jan-2003');
[ZeroRates, CurveDates] = zbtyield(Bonds, Yields, Settle)
```

This gets you the Treasury spot curve for the day.

You can compute the forward curve from this spot curve with zero2fwd.
[ForwardRates, CurveDates] = zero2fwd(ZeroRates, CurveDates, ...
Settle)
Here the notion of forward rates refers to rates between the maturity dates shown above, not to a certain period (forward 3 month rates, for example).


## Computing Spreads

Calculating the spread between specific, fixed forward periods (such as the Treasury-Eurodollar spread) requires an extra step. Interpolate the zero rates (or zero prices, instead) for the corresponding maturities on the interval dates. Then use the interpolated zero rates to deduce the forward rates, and thus the spread of Eurodollar forward curve segments versus the relevant forward segments from Treasury bills.

Additionally, the variety of curve functions (including zero2fwd) helps to standardize such calculations. For instance, by making both rates quoted with quarterly compounding and on an actual/360 basis, the resulting spread structure is fully comparable. This avoids the small inconsistency that occurs when directly comparing the bond-equivalent yield of a Treasury bill to the quarterly forward rates implied by Eurodollar futures.

## Noise in Curve Computations

When introducing more bonds in constructing curves, noise may become a factor and may need some "smoothing" (with splines, for example); this helps obtain a smoother forward curve.

The following spot and forward curves are constructed from 67 Treasury bonds. The fitted and bootstrapped spot curve (bottom right figure) displays comparable stability. The forward curve (upper-left figure) contains significant noise and shows an improbable forward rate structure. The noise is not necessarily bad; it could uncover trading opportunities for a relative-value approach. Yet, a more balanced approach is desired when the bootstrapped forward curve oscillates this much and contains a negative rate as large as $-10 \%$ (not shown in the plot because it is outside the limits).


Implied Forward Curves. The jagged curve comes from direct bootstrapping. The smooth curve shows the effect of smoothing with splines.


Implied Spot Rate Curves.
These curves correspond to the forward curve above.

This example uses termfit, an example function from Financial Toolbox software that also requires the use of Curve Fitting Toolbox ${ }^{\mathrm{TM}}$ software.

## Derivative Securities

- "Interest Rate Swaps" on page 7-2
- "Convertible Bond Valuation" on page 7-10
- "Bond Futures" on page 7-12
- "Managing Interest Rate Risk with Bond Futures" on page 7-18
- "Fitting the Diebold Li Model" on page 7-26


## Interest Rate Swaps

## In this section...

"Swap Pricing Assumptions" on page 7-2
"Swap Pricing Example" on page 7-3
"Portfolio Hedging" on page 7-8

## Swap Pricing Assumptions

Financial Instruments Toolbox software contains the function liborfloat2fixed, which computes a fixed-rate par yield that equates the floating-rate side of a swap to the fixed-rate side. The solver sets the present value of the fixed side to the present value of the floating side without having to line up and compare fixed and floating periods.

## Assumptions on Floating-Rate Input

- Rates are quarterly, for example, that of Eurodollar futures.
- Effective date is the first third Wednesday after the settlement date.
- All delivery dates are spaced 3 months apart.
- All periods start on the third Wednesday of delivery months.
- All periods end on the same dates of delivery months, 3 months after the start dates.
- Accrual basis of floating rates is actual/360.
- Applicable forward rates are estimated by interpolation in months when forward-rate data is not available.


## Assumptions on Fixed-Rate Output

- Design allows you to create a bond of any coupon, basis, or frequency, based upon the floating-rate input.
- The start date is a valuation date, that is, a date when an agreement to enter into a contract by the settlement date is made.
- Settlement can be on or after the start date. If it is after, a forward fixed-rate contract results.
- Effective date is assumed to be the first third Wednesday after settlement, the same date as that of the floating rate.
- The end date of the bond is a designated number of years away, on the same day and month as the effective date.
- Coupon payments occur on anniversary dates. The frequency is determined by the period of the bond.
- Fixed rates are not interpolated. A fixed-rate bond of the same present value as that of the floating-rate payments is created.


## Swap Pricing Example

This example shows the use of the functions in computing the fixed rate applicable to a series of 2 -, 5 -, and 10 -year swaps based on Eurodollar market data. According to the Chicago Mercantile Exchange (http://www.cmegroup.com), Eurodollar data on Friday, October 11, 2002, was as shown in the following table.

Note This example illustrates swap calculations in MATLAB software. Timing of the data set used was not rigorously examined and was assumed to be the proxy for the swap rate reported on October 11, 2002.

## Eurodollar Data on Friday, October 11, 2002

| Month | Year | Settle |
| :--- | :--- | :--- |
| 10 | 2002 | 98.21 |
| 11 | 2002 | 98.26 |
| 12 | 2002 | 98.3 |
| 1 | 2003 | 98.3 |
| 2 | 2003 | 98.31 |
| 3 | 2003 | 98.275 |
| 6 | 2003 | 98.12 |

## Eurodollar Data on Friday, October 11, 2002 (Continued)

| Month | Year | Settle |
| :--- | :--- | :--- |
| 9 | 2003 | 97.87 |
| 12 | 2003 | 97.575 |
| 3 | 2004 | 97.26 |
| 6 | 2004 | 96.98 |
| 9 | 2004 | 96.745 |
| 12 | 2004 | 96.515 |
| 3 | 2005 | 96.33 |
| 6 | 2005 | 96.135 |
| 9 | 2005 | 95.955 |
| 12 | 2005 | 95.78 |
| 3 | 2006 | 95.63 |
| 6 | 2006 | 95.465 |
| 9 | 2006 | 95.315 |
| 12 | 2006 | 95.16 |
| 3 | 2007 | 95.025 |
| 6 | 2007 | 94.88 |
| 9 | 2007 | 94.74 |
| 12 | 2007 | 94.595 |
| 3 | 2008 | 94.48 |
| 6 | 2008 | 94.375 |
| 9 | 2008 | 94.28 |
| 12 | 2008 | 94.185 |
| 3 | 2009 | 94.1 |
| 6 | 2009 | 94.005 |
| 9 | 2009 | 93.925 |
| 12 | 93.865 |  |
|  |  |  |
|  |  |  |

## Eurodollar Data on Friday, October 11, 2002 (Continued)

| Month | Year | Settle |
| :--- | :--- | :--- |
| 3 | 2010 | 93.82 |
| 6 | 2010 | 93.755 |
| 9 | 2010 | 93.7 |
| 12 | 2010 | 93.645 |
| 3 | 2011 | 93.61 |
| 6 | 2011 | 93.56 |
| 9 | 2011 | 93.515 |
| 12 | 2011 | 93.47 |
| 3 | 2012 | 93.445 |
| 6 | 2012 | 93.41 |
| 9 | 2012 | 93.39 |

Using this data, you can compute 1 -, $2-, 3-, 4-, 5-, 7$-, and 10 -year swap rates with the toolbox function liborfloat2fixed. The function requires you to input only Eurodollar data, the settlement date, and tenor of the swap. MATLAB software then performs the required computations.

To illustrate how this function works, first load the data contained in the supplied Excel ${ }^{\circledR}$ worksheet EDdata.xls.
[EDRawData, textdata] = xlsread('EDdata.xls');
Extract the month from the first column and the year from the second column. The rate used as proxy is the arithmetic average of rates on opening and closing.

```
Month = EDRawData(:,1);
Year = EDRawData(:,2);
IMMData = (EDRawData(:,4)+EDRawData(:,6))/2;
EDFutData = [Month, Year, IMMData];
```

Next, input the current date.

```
Settle = datenum('11-Oct-2002');
```

To compute for the 2 year swap rate, set the tenor to 2 .
Tenor = 2;
Finally, compute the swap rate with liborfloat2fixed.
[FixedSpec, ForwardDates, ForwardRates] = ...
liborfloat2fixed(EDFutData, Settle, Tenor)
MATLAB returns a par-swap rate of $2.23 \%$ using the default setting (quarterly compounding and 30/360 accrual), and forward dates and rates data (quarterly compounded).

FixedSpec =

```
    Coupon: 0.0223
    Settle: '16-Oct-2002
    Maturity: '16-Oct-2004'
    Period: 4
            Basis: 1
ForwardDates =
    7 3 1 5 0 5
    7 3 1 5 9 6
    7 3 1 6 8 7
    7 3 1 7 7 8
    7 3 1 8 6 9
    7 3 1 9 6 7
    7 3 2 0 5 8
    7 3 2 1 4 9
ForwardRates =
    0.0178
    0.0168
    0.0171
    0.0189
```

0.0216
0.0250
0.0280
0.0306

In the FixedSpec output, note that the swap rate actually goes forward from the third Wednesday of October 2002 (October 16, 2002), 5 days after the original Settle input (October 11, 2002). This, however, is still the best proxy for the swap rate on Settle, as the assumption merely starts the swap's effective period and does not affect its valuation method or its length.

The correction suggested by Hull and White improves the result by turning on convexity adjustment as part of the input to liborfloat2fixed. (See Hull, J., Options, Futures, and Other Derivatives, 4th Edition, Prentice-Hall, 2000.) For a long swap, for example, 5 years or more, this correction could prove to be large.

The adjustment requires additional parameters:

- StartDate, which you make the same as Settle (the default) by providing an empty matrix [] as input.
- ConvexAdj to tell liborfloat2fixed to perform the adjustment.
- RateParam, which provides the parameters a and S as input to the Hull-White short rate process.
- Optional parameters InArrears and Sigma, for which you can use empty matrices [] to accept the MATLAB defaults.
- FixedCompound, with which you can facilitate comparison with values cited in Table H15 of Federal Reserve Statistical Release by turning the default quarterly compounding into semiannual compounding, with the (default) basis of 30/360.

StartDate = [];
Interpolation = [];
ConvexAdj = 1;
RateParam = [0.03; 0.017];
FixedCompound = 2;
[FixedSpec, ForwardDaates, ForwardRates] = ...
liborfloat2fixed(EDFutData, Settle, Tenor, StartDate, ...

```
Interpolation, ConvexAdj, RateParam, [], [], FixedCompound)
```

This returns $2.21 \%$ as the 2 -year swap rate, quite close to the reported swap rate for that date.

Analogously, the following table summarizes the solutions for 1-, $3-, 5-, 7$-, and 10 -year swap rates (convexity-adjusted and unadjusted).

## Calculated and Market Average Data of Swap Rates on Friday, October 11, 2002

| Swap <br> Sength | Unadjusted | Adjusted | Table H15 | Adjusted <br> Error <br> (Bears) |
| :--- | :--- | :--- | :--- | :--- |
| 1 | $1.80 \%$ | $1.79 \%$ | $1.80 \%$ | -1 |
| 2 | $2.24 \%$ | $2.21 \%$ | $2.22 \%$ | -1 |
| 3 | $2.70 \%$ | $2.66 \%$ | $2.66 \%$ | 0 |
| 4 | $3.12 \%$ | $3.03 \%$ | $3.04 \%$ | -1 |
| 5 | $3.50 \%$ | $3.37 \%$ | $3.36 \%$ | +1 |
| 7 | $4.16 \%$ | $3.92 \%$ | $3.89 \%$ | +3 |
| 10 | $4.87 \%$ | $4.42 \%$ | $4.39 \%$ | +3 |

## Portfolio Hedging

You can use these results further, such as for hedging a portfolio. The liborduration function provides a duration-hedging capability. You can isolate assets (or liabilities) from interest-rate risk exposure with a swap arrangement.

Suppose you own a bond with these characteristics:

- $\$ 100$ million face value
- 7\% coupon paid semiannually
- $5 \%$ yield to maturity
- Settlement on October 11, 2002
- Maturity on January 15, 2010
- Interest accruing on an actual/365 basis

Use of the bnddury function from Financial Toolbox software shows a modified duration of 5.6806 years.

To immunize this asset, you can enter into a pay-fixed swap, specifically a swap in the amount of notional principal ( $N s$ ) such that $N s *$ SwapDuration $+\$ 100 \mathrm{M}^{*} 5.6806=0$ (or Ns $=-100 * 5.6806 /$ SwapDuration).

Suppose again, you choose to use a 5 -, 7 -, or 10 -year swap ( $3.37 \%, 3.92 \%$, and $4.42 \%$ from the previous table) as your hedging tool.

```
SwapFixRate = [0.0337; 0.0392; 0.0442];
Tenor = [5; 7; 10];
Settle = '11-Oct-2002';
PayFixDuration = liborduration(SwapFixRate, Tenor, Settle)
```

This gives a duration of $-3.6835,-4.7307$, and -6.0661 years for $5-, 7-$, and 10 -year swaps. The corresponding notional amount is computed by

Ns = -100*5.6806./PayFixDuration
Ns =
154.2163
120.0786
93.6443

The notional amount entered in pay-fixed side of the swap instantaneously immunizes the portfolio.

## Convertible Bond Valuation

A convertible bond (CB) is a debt instrument that can be converted into a predetermined amount of a company's equity at certain times before the bond's maturity.

Financial Instruments Toolbox software uses a binomial and trinomial tree approach (cbprice) to value convertible bonds. The value of a convertible bond is determined by the uncertainty of the related stock. Once the stock evolution is modeled, backward discounting is computed.

The last column of such trees provides the data to decide which is more profitable: the debt notional (plus interest, if any) or the equivalent number of shares per the notional.

Where debt prevails, the toolbox discounts backward with the risk-free rate plus the spread reflecting the credit risk of the issuer. Where stock prevails, the toolbox discounts with the risk-free rate. The intrinsic value of a convertible bond is the sum of the (probability-adjusted) debt and stock portions from the last node. This is compared to current stock price, to see if voluntary or forced conversion may take place. Otherwise, its value is the intrinsic value. From here, the same discounting process is repeated after adjusting debt portion to be equal to 0 if any conversion takes place. Dividends and coupons are handled discretely, at the date they occur.

The approach is equivalent to solving a one-dimensional partial differential equation such as one described by Tsiveriotis and Fernandes. (See Tsiveriotis, K. and C. Fernandes (1998), "Valuing Convertible Bonds with Credit Risk," The Journal of Fixed Income, 8 (3), 95-102.) Using the same example of bond specifications that they use ( $4 \%$ annual coupon, payable twice a year, callable after 2 years at 110, and redeemable at par in 5 years), the toolbox gives results like theirs.


The figure on the left shows the bond "floor" of the convertible (a $5 \%$ yield, given a $4 \%$ coupon at about $97 \%$ par) when share prices are low.

The change of curvature located at the end of the second year is due to the activation of the embedded (soft) call feature (visible on the surface plot in the right figure).

Finally, there is the flat section when time is nearing expiration and share prices are high, indicating a delta of unity, a characteristic of in-the-money equity options embedded in a bond.

## Bond Futures

In this section...<br>"Supported Bond Futures" on page 7-12<br>"Analysis of Bond Futures" on page 7-14<br>"Managing Present Value with Bond Futures" on page 7-16

## Supported Bond Futures

Bond futures are futures contracts where the commodity for delivery is a government bond. There are established global markets for government bond futures. Bond futures provide a liquid alternative for managing interest-rate risk.

In the U.S. market, the Chicago Mercantile Exchange (CME) offers futures on Treasury bonds and notes with maturities of $2,5,10$, and 30 years. Typically, the following bond future contracts from the CME have maturities of $3,6,9$, and 12 months:

- 30-year U.S. Treasury bond
- 10-year U.S. Treasury bond
- 5-year U.S. Treasury bond
- 2-year U.S. Treasury bond

The short position in a Treasury bond or note future contract must deliver to the long position in one of many possible existing Treasury bonds. For example, in a 30 -year Treasury bond future, the short position must deliver a Treasury bond with at least 15 years to maturity. Because these bonds have different values, the bond future contract is standardized by computing a conversion factor. The conversion factor normalizes the price of a bond to a theoretical bond with a coupon of $6 \%$. The price of a bond future contract is represented as:

InvoicePrice $=$ FutPrice $\times C F+A I$
where:

FutPrice is the price of the bond future.
$C F$ is the conversion factor for a bond to deliver in a futures contract.
$A I$ is the accrued interest.
You can reference these conversion factors at U.S. Treasury Bond Futures Contract. The short position in a futures contract has the option of which bond to deliver and, in the U.S. bond market, when in the delivery month to deliver the bond. The short position typically chooses to deliver the bond known as the Cheapest to Deliver (CTD). The CTD bond most often delivers on the last delivery day of the month.

Financial Instruments Toolbox software supports the following bond futures:

- U.S. Treasury bonds and notes
- German Bobl, Bund, Buxl, and Schatz
- UK gilts
- Japanese government bonds (JGBs)

The functions supporting all bond futures are:

| Function | Purpose |
| :--- | :--- |
| convfactor | Calculates bond conversion factors for U.S. Treasury <br> bonds, German Bobl, Bund, Buxl, and Schatz, U.K. <br> gilts, and JGBs. |
| bndfutprice | Prices bond future given repo rates. |
| bndfutimprepo | Calculates implied repo rates for a bond future given <br> price. |

The functions supporting U.S. Treasury bond futures are:

| Function | Purpose |
| :--- | :--- |
| tfutbyprice | Calculates future prices of Treasury bonds given the <br> spot price. |
| tfutbyyield | Calculates future prices of Treasury bonds given <br> current yield. |
| tfutimprepo | Calculates implied repo rates for the Treasury bond <br> future given price. |
| tfutpricebyrepo | Calculates Treasury bond futures price given the <br> implied repo rates. |
| tfutyieldbyrepo | Calculates Treasury bond futures yield given the <br> implied repo rates. |

## Analysis of Bond Futures

The following example demonstrates analyzing German Euro-Bund futures traded on Eurex. However, convfactor, bndfutprice, and bndfutimprepo apply to bond futures in the U.S., U.K., Germany, and Japan. The workflow for this analysis is:

1 Calculate bond conversion factors.
2 Calculate implied repo rates to find the CTD bond.
3 Price the bond future using the term implied repo rate.

## Calculating Bond Conversion Factors

Use conversion factors to normalize the price of a particular bond for delivery in a futures contract. When using conversion factors, the assumption is that a bond for delivery has a $6 \%$ coupon. Use convfactor to calculate conversion factors for all bond futures from the U.S., Germany, Japan, and U.K.

For example, conversion factors for Euro-Bund futures on Eurex are listed at www. eurexchange.com. The delivery date for Euro-Bund futures is the 10th day of the month, as opposed to bond futures in the U.S., where the short position has the option of choosing when to deliver the bond.

For the $4 \%$ bond, compute the conversion factor with:

```
CF1 = convfactor('10-Sep-2009','04-Jul-2018', .04,.06,3)
CF1 =
    0.8659
```

This syntax for convfactor works fine for bonds with standard coupon periods. However, some deliverable bonds have long or short first coupon periods. Compute the conversion factors for such bonds using the optional input parameters StartDate and FirstCouponDate. Specify all optional input arguments for convfactor as parameter/value pairs:

```
CF2 = convfactor('10-Sep-2009','04-Jan-2019', .0375,'Convention',3,'startdate',...
datenum('14-Nov-2008'))
CF2 =
    0.8426
```


## Calculating Implied Repo Rates to Find the CTD Bond

To determine the availability of the cheapest bond for deliverable bonds against a futures contract, compute the implied repo rate for each bond. The bond with the highest repo rate is the cheapest because it has the lowest initial value, thus yielding a higher return, provided you deliver it with the stated futures price. Use bndfutimprepo to calculate repo rates:

```
% Bond Properties
CouponRate = [.0425;.0375;.035];
Maturity = [datenum('04-Jul-2018');datenum('04-Jan-2019'); datenum('04-Jul-2019')];
CF = [0.882668;0.842556;0.818193];
Price = [105.00;100.89;98.69];
% Futures Properties
FutSettle = '09-Jun-2009';
FutPrice = 118.54;
Delivery = '10-Sep-2009';
% Note that the default for BNDFUTIMPREPO is for the bonds to be
% semi-annual with a day count basis of 0. Since these are German
% bonds, we need to have a Basis of 8 and a Period of 1
```

```
ImpRepo = bndfutimprepo(Price, FutPrice, FutSettle, Delivery, CF, ...
CouponRate, Maturity,'Basis',8,'Period',1)
ImpRepo =
    0.0261
    -0.0022
    -0.0315
```


## Pricing Bond Futures Using the Term Implied Repo Rate

Use bndfutprice to perform price calculations for all bond futures from the U.S., Germany, Japan, and U.K. To price the bond, given a term repo rate:

```
% Assume a term repo rate of .0091;
RepoRate = .0091;
[FutPrice,AccrInt] = bndfutprice(RepoRate, Price(1), FutSettle,...
Delivery, CF(1), CouponRate(1), Maturity(1),...
'Basis',8,'Period',1)
FutPrice =
```

    118.0126
    AccrInt =
0.7918

## Managing Present Value with Bond Futures

The Present Value of a Basis Point (PVBP) is used to manage interest-rate risk. PVBP is a measure that quantifies the change in price of a bond given a one-basis point shift in interest rates. The PVBP of a bond is computed with the following:

$$
P V B P_{\text {Bond }}=\frac{\text { Duration } \times \text { MarketValue }}{100}
$$

The PVBP of a bond futures contract can be computed with the following:
$P V B P_{\text {Futures }}=\frac{P V B P_{\text {cTDBond }}}{\text { CTDConversionFactor }}$
Use bnddurp and bnddury from Financial Toolbox software to compute the modified durations of CTD bonds. For more information, see "Managing Interest Rate Risk with Bond Futures" on page 7-18 and "Fitting the Diebold Li Model" on page 7-26.

## Managing Interest Rate Risk with Bond Futures

This example shows how to hedge the interest rate risk of a portfolio using bond futures.

## Modifying the Duration of a Portfolio with Bond Futures

In managing a bond portfolio, you can use a benchmark portfolio to evaluate performance. Sometimes a manager is constrained to keep the portfolio's duration within a particular band of the duration of the benchmark. One way to modify the duration of the portfolio is to buy and sell bonds, however, there may be reasons why a portfolio manager wishes to maintain the existing composition of the portfolio (e.g., the current holdings reflect fundamental research/views about future returns). Therefore, another option for modifying the duration is to buy and sell bond futures.

Bond futures are futures contracts where the commodity to be delivered is a government bond that meets the standard outlined in the futures contract (e.g., the bond has a specified remaining time to maturity).

Since often many bonds are available, and each bond may have a different coupon, you can use a conversion factor to normalize the payment by the long to the short.

There exist well developed markets for government bond futures. Specifically, the Chicago Board of Trade offers futures on the following:

- 2 Year Note
- 3 Year Note
- 5 Year Note
- 10 Year Note
- 30 Year Bond
http://www.cmegroup.com/trading/interest-rates/
Eurex offers futures on the following:
- Euro-Schatz Futures 1.75 to 2.25
- Euro-Bobl Futures 4.5 to 5.5
- Euro-Bund Futures 8.5 to 10.5
- Euro-Buxl Futures 24.0 to 35
http://www.eurexchange.com
Bond futures can be used to modify the duration of a portfolio. Since bond futures derive their value from the underlying instrument, the duration of a bond futures contract is related to the duration of the underlying bond.

There are two challenges in computing this duration:

- Since there are many available bonds for delivery, the short in the contract has a choice in which bond to deliver.
- Some contracts allow the short flexibility in choosing the delivery date.

Typically, the bond used for analysis is the bond that is cheapest for the short to deliver (CTD).

One approach is to compute duration measures using the CTD's duration and the conversion factor.

For example, the Present Value of a Basis Point (PVBP) can be computed from the following:

Note that these definitions of duration for the futures contract are approximate, and do not account for the value of the delivery options for the short.

If the goal is to modify the duration of a portfolio, use the following:

Note that the contract size is typically for 100,000 face value of a bond -- so the contract size is typically 1000 , as the bond face value is 100 .

The following example assumes an initial duration, portfolio value, and target duration for a portfolio with exposure to the Euro interest rate. The June Euro-Bund Futures contract is used to modify the duration of the portfolio.

Note that typically futures contracts are offered for March, June, September and December.

```
% Assume the following for the portfolio and target
PortfolioDuration = 6.4;
PortfolioValue = 100000000;
BenchmarkDuration = 4.8;
% Deliverable Bunds -- note that these conversion factors may also be
% computed with the MATLAB(R) function CONVFACTOR
BondPrice = [106.46;108.67;104.30];
BondMaturity = datenum({'04-Jan-2018','04-Jul-2018','04-Jan-2019'});
BondCoupon = [.04;.0425;.0375];
ConversionFactor = [.868688;.880218;.839275];
% Futures data -- found from http://www.eurexchange.com
FuturesPrice = 122.17;
FuturesSettle = '23-Apr-2009';
FuturesDelivery = '10-Jun-2009';
% To find the CTD bond we can compute the implied repo rate
ImpliedRepo = bndfutimprepo(BondPrice,FuturesPrice,FuturesSettle,...
    FuturesDelivery,ConversionFactor,BondCoupon,BondMaturity);
% Note that the bond with the highest implied repo rate is the CTD
[CTDImpRepo,CTDIndex] = max(ImpliedRepo);
% Compute the CTD's Duration -- note the period and basis for German Bunds
Duration = bnddurp(BondPrice,BondCoupon,FuturesSettle,BondMaturity,1,8);
ContractSize = 1000;
% Use the formula above to compute the number of contracts to sell
```

```
NumContracts = (BenchmarkDuration - PortfolioDuration)*PortfolioValue./...
    (BondPrice(CTDIndex) *ContractSize*Duration(CTDIndex))*ConversionFactor(
disp(['To achieve the target duration, ' num2str(abs(round(NumContracts)))
    ' Euro-Bund Futures must be sold.'])
To achieve the target duration, }180\mathrm{ Euro-Bund Futures must be sold.
```


## Modifying the Key Rate Durations of a Portfolio with Bond Futures

One of the shortcomings of using duration as a risk measure is that it assumes parallel shifts in the yield curve. While many studies have shown that this explains roughly $85 \%$ of the movement in the yield curve, changes in the slope or shape of the yield curve are not captured by duration -- and therefore, hedging strategies are not successful at addressing these dynamics.

One approach is to use key rate duration -- this is particularly relevant when using bond futures with multiple maturities, like Treasury futures.

The following example uses $2,5,10$ and 30 year Treasury Bond futures to hedge the key rate duration of a portfolio.

Computing key rate durations requires a zero curve. This example uses the zero curve published by the Treasury and found at the following location:
http://www.ustreas.gov/offices/domestic-finance/debt-management/interest-rate/yield.shtn
Note that this zero curve could also be derived using the Interest Rate Curve functionality found in IRDataCurve and IRFunctionCurve.

```
% Assume the following for the portfolio and target, where the duration
% vectors are key rate durations at 2, 5, 10, and 30 years.
PortfolioDuration = [.5 1 2 6];
PortfolioValue = 100000000;
BenchmarkDuration = [.4 .8 1.6 5];
% The following are the CTD Bonds for the 30, 10, 5 and 2 year futures
% contracts -- these were determined using the procedure outlined in the
% previous section.
CTDCoupon = [4.75 3.125 5.125 7.5]'/100;
```

```
CTDMaturity = datenum({'3/31/2011','08/31/2013', '05/15/2016','11/15/2024'})
CTDConversion = [0.9794 0.8953 0.9519 1.1484]';
CTDPrice = [ll07.34 105.91 117.00 144.18]';
ZeroRates = [0.07 0.10 0.31 0.50 0.99 1.38 1.96 2.56 3.03 3.99 3.89]'/100;
ZeroDates = daysadd(FuturesSettle,[30 360 360*2 360*3 360*5 ...
    360*7 360*10 360*15 360*20 360*25 360*30],1);
% Compute the key rate durations for each of the CTD bonds.
CTDKRD = bndkrdur([ZeroDates ZeroRates], CTDCoupon,FuturesSettle,...
    CTDMaturity,'KeyRates',[2 5 10 30]);
% Note that the contract size for the 2 Year Note Future is $200,000
ContractSize = [2000;1000;1000;1000];
NumContracts = (bsxfun(@times,CTDPrice.*ContractSize./CTDConversion,CTDKRD)
    (BenchmarkDuration - PortfolioDuration)'*PortfolioValue;
sprintf(['To achieve the target duration, \n' ...
    num2str(-round(NumContracts(1))) ' 2 Year Treasury Note Futures must be
    num2str(-round(NumContracts(2))) ' }5\mathrm{ Year Treasury Note Futures must be
    num2str(-round(NumContracts(3))) ' }10\mathrm{ Year Treasury Note Futures must
    num2str(-round(NumContracts(4))) ' Treasury Bond Futures must be sold,
ans =
To achieve the target duration,
242 Year Treasury Note Futures must be sold,
4 7 5 \text { Year Treasury Note Futures must be sold,}
6 8 1 0 \text { Year Treasury Note Futures must be sold,}
120 Treasury Bond Futures must be sold,
```


## Improving the Performance of a Hedge with Regression

An additional component to consider in hedging interest rate risk with bond futures, again related to movements in the yield curve, is that typically the yield curve moves more at the short end than at the long end.

Therefore, if a position is hedged with a future where the CTD bond has a maturity that is different than the portfolio this could lead to a situation where the hedge under- or over-compensates for the actual interest rate risk of the portfolio.

One approach is to perform a regression on historical yields at different maturities to determine a Yield Beta, which is a value that represents how much more the yield changes for different maturities.

This example shows how to use this approach with UK Long Gilt futures and historical data on Gilt Yields.

Market data on Gilt futures is found at the following:
http://www.euronext.com
Historical data on gilts is found at the following;
http://www.dmo.gov.uk
Note that while this approach does offer the possibility of improving the performance of a hedge, any analysis using historical data depends on historical relationships remaining consistent.

Also note that an additional enhancement takes into consideration the correlation between different maturities. While this approach is outside the scope of this example, you can use this to implement a minimum variance hedge.

```
% Assume the following for the portfolio and target
PortfolioDuration = 6.4;
PortfolioValue = 100000000;
BenchmarkDuration = 4.8;
% This is the CTD Bond for the Long Gilt Futures contract
CTDBondPrice = 113.40;
CTDBondMaturity = datenum('7-Mar-2018');
CTDBondCoupon = .05;
CTDConversionFactor = 0.9325024;
```

```
% Market data for the Long Gilt Futures contract
FuturesPrice = 120.80;
FuturesSettle = '23-Apr-2009';
FuturesDelivery = '10-Jun-2009';
CTDDuration = bnddurp(CTDBondPrice,CTDBondCoupon,FuturesSettle,CTDBondMatur
ContractSize = 1000;
NumContracts = (BenchmarkDuration - PortfolioDuration)*PortfolioValue./...
    (CTDBondPrice*ContractSize*CTDDuration)*CTDConversionFactor;
disp(['To achieve the target duration with a conventional hedge ' ...
    num2str(-round(NumContracts)) ...
    ' Long Gilt Futures must be sold.'])
To achieve the target duration with a conventional hedge 182 Long Gilt Futu
To improve the accuracy of this hedge, historical data is used to determine a relationship between the standard deviation of the yields. Specifically, standard deviation of yields is plotted and regressed vs bond duration. This relationship is then used to compute a Yield Beta for the hedge.
```

```
% Load data from XLS spreadsheet
```

% Load data from XLS spreadsheet
load ukbonddata_20072008
load ukbonddata_20072008
Duration = bnddury(Yield(1,:)',Coupon,Dates(1,:),Maturity);
Duration = bnddury(Yield(1,:)',Coupon,Dates(1,:),Maturity);
scatter(Duration,100*std(Yield))
scatter(Duration,100*std(Yield))
title('Standard Deviation of Yields for UK Gilts 2007-2008')
title('Standard Deviation of Yields for UK Gilts 2007-2008')
ylabel('Standard Deviation of Yields (%)')
ylabel('Standard Deviation of Yields (%)')
xlabel('Duration')
xlabel('Duration')
annotation(gcf,'textbox',[0.4067 0.685 0.4801 0.0989],...
annotation(gcf,'textbox',[0.4067 0.685 0.4801 0.0989],...
'String',{'Note that the Standard Deviation',...
'String',{'Note that the Standard Deviation',...
'of Yields is greater at shorter maturities.'},...
'of Yields is greater at shorter maturities.'},...
'FitBoxToText','off',...
'FitBoxToText','off',...
'EdgeColor','none');
'EdgeColor','none');
stats = regstats(std(Yield),Duration);
stats = regstats(std(Yield),Duration);
YieldBeta = (stats.beta'*[1 PortfolioDuration]')./(stats.beta'*[1 CTDDurati

```
YieldBeta = (stats.beta'*[1 PortfolioDuration]')./(stats.beta'*[1 CTDDurati
```

Now the Yield Beta is used to compute a new value for the number of contracts to be sold. Note that since the duration of the portfolio was less than the duration of the CTD Gilt, the number of futures to sell is actually greater than in the first case.

```
NumContracts = (BenchmarkDuration - PortfolioDuration)*PortfolioValue./..
    (CTDBondPrice*ContractSize*CTDDuration)*CTDConversionFactor*YieldBeta;
disp(['To achieve the target duration using a Yield Beta-modified hedge,
    num2str(abs(round(NumContracts))) ...
    ' Long Gilt Futures must be sold.'])
```

To achieve the target duration using a Yield Beta-modified hedge, 193 Long

## Bibliography

This example is based on the following books and papers:
[1] Burghardt, G., T. Belton, M. Lane and J. Papa. The Treasury Bond Basis. New York, NY: McGraw-Hill, 2005.
[2] Krgin, D. Handbook of Global Fixed Income Calculations. New York, NY: John Wiley \& Sons, 2002.
[3] CFA Program Curriculum, Level III, Volume 4, Reading 31. CFA Institute, 2009.

## Fitting the Diebold Li Model

This example shows how to construct a Diebold Li model of the US yield curve for each month from 1990 to 2010 and how to forecast future yield curves by fitting an autoregressive model to the time series of each parameter.

The paper can be found here:
http://www.ssc.upenn.edu/~fdiebold/papers/paper49/Diebold-Li.pdf

## Load the Data

The data used are monthly Treasury yields from 1990 through 2010 for tenors of $1 \mathrm{Mo}, 3 \mathrm{Mo}, 6 \mathrm{Mo}, 1 \mathrm{Yr}, 2 \mathrm{Yr}, 3 \mathrm{Yr}, 5 \mathrm{Yr}, 7 \mathrm{Yr}, 10 \mathrm{Yr}, 20 \mathrm{Yr}, 30 \mathrm{Yr}$.

Daily data can be found here:
http://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.
Data is stored in a MATLAB® data file as a MATLAB dataset object.
load Data_USYieldCurve

```
% Extract data for the last day of each month
MonthYearMat = repmat((1990:2010)',1,12)';
EOMDates = lbusdate(MonthYearMat(:),repmat((1:12)',21,1));
MonthlyIndex = find(ismember(get(Dataset,'ObsNames'),datestr(EOMDates)));
Estimationdataset = Dataset(MonthlyIndex,:);
EstimationData = double(Estimationdataset);
```


## Diebold Li Model

Diebold and Li start with the Nelson Siegel model
and rewrite it to be the following:

The above model allows the factors to be interpreted in the following way: Beta 1 corresponds to the long term/level of the yield curve, Beta 2 corresponds to the short term/slope, and Beta3 corresponds to the medium term/curvature.

Diebold and Li advocate setting the time factor, lambda, to maximize the loading on the medium term factor, Beta3, at 30 months. This also transforms the problem from a nonlinear fitting one to a simple linear regression.

```
% Explicitly set the time factor lambda
lambda_t = .0609;
% Construct a matrix of the factor loadings
% Tenors associated with data
TimeToMat = [3 6 9 12 24 36 60 84 120 240 360]';
X = [ones(size(TimeToMat)) (1 - exp(-lambda_t*TimeToMat))./(lambda_t*TimeTo
    ((1 - exp(-lambda_t*TimeToMat))./(lambdà_t*TimeToMat) - exp(-lambda_t*T
% Plot the factor loadings
plot(TimeToMat,X)
title('Factor Loadings for Diebold Li Model with time factor of .0609')
xlabel('Maturity (months)')
ylim([0 1.1])
legend({'Beta1','Beta2','Beta3'},'location','east')
```


## Fit the Model

A DieboldLi object has been developed to facilitate fitting the model from yield data. The DieboldLi object inherits from the IRCurve object, so the getZeroRates, getDiscountFactors, getParYields, getForwardRates and toRateSpec methods are all implemented. Additionally, the method fitYieldsFromBetas has been implemented to estimate the Beta parameters given a lambda parameter for observed market yields.

The DieboldLi object will be used to fit a Diebold Li model for each month from 1990 through 2010.

```
% Preallocate the Betas
Beta = zeros(size(EstimationData,1),3);
```

```
% Loop through and fit each end of month yield curve
for jdx=1:size(EstimationData,1)
    tmpCurveModel = DieboldLi.fitBetasFromYields(EOMDates(jdx),lambda_t*12,
    Beta(jdx,:) = [tmpCurveModel.Beta1 tmpCurveModel.Beta2 tmpCurveModel.Be
end
```

The Diebold Li fits on selected dates are included here

```
PlotSettles = datenum({'30-May-1997','31-Aug-1998','29-Jun-2001','31-Oct-20
figure
for jdx=1:length(PlotSettles)
    subplot(2,2,jdx)
    tmpIdx = find(strcmpi(get(Estimationdataset,'ObsNames'),datestr(PlotSet
    tmpCurveModel = DieboldLi.fitBetasFromYields(PlotSettles(jdx),lambda_t*
        daysadd(PlotSettles(jdx),30*TimeToMat),EstimationData(tmpIdx,:)');
    scatter(daysadd(PlotSettles(jdx),30*TimeToMat),EstimationData(tmpIdx,:)
    hold on
    PlottingDates = (PlotSettles(jdx)+30:30:PlotSettles(jdx)+30*360)';
    plot(PlottingDates,tmpCurveModel.getParYields(PlottingDates),'r-')
    title(['Yield Curve on ' datestr(PlotSettles(jdx))])
    datetick
end
```


## Forecasting

The Diebold Li model can be used to forecast future yield curves. Diebold and Li propose fitting an $\mathrm{AR}(1)$ model to the time series of each Beta parameter. This fitted model can then be used to forecast future values of each parameter, and by extension, future yield curves.

For this example the MATLAB function REGRESS is used to estimate the parameters for an $\operatorname{AR}(1)$ model for each Beta.

The confidence intervals for the regression fit are also used to generate two additional yield curve forecasts that serve as additional possible scenarios for the yield curve.

The MonthsLag variable can be adjusted to make different period ahead forecasts. For example, changing the value from 1 to 6 would change the forecast from a 1 month ahead to 6 month ahead forecast.

```
MonthsLag = 1;
```

```
[tmpBeta,bint] = regress(Beta(MonthsLag+1:end,1),[ones(size(Beta(MonthsLag+
ForecastBeta(1,1) = [1 Beta(end,1)]*tmpBeta;
ForecastBeta_Down(1,1) = [1 Beta(end,1)]*bint(:,1);
ForecastBeta_Up(1,1) = [1 Beta(end,1)]*bint(:,2);
[tmpBeta,bint] = regress(Beta(MonthsLag+1:end,2),[ones(size(Beta(MonthsLag
ForecastBeta(1,2) = [1 Beta(end,2)]*tmpBeta;
ForecastBeta_Down(1,2) = [1 Beta(end,2)]*bint(:,1);
ForecastBeta_Up(1,2) = [1 Beta(end,2)]*bint(:,2);
[tmpBeta,bint] = regress(Beta(MonthsLag+1:end,3),[ones(size(Beta(MonthsLag
ForecastBeta(1,3) = [1 Beta(end,3)]*tmpBeta;
ForecastBeta_Down(1,3) = [1 Beta(end,3)]*bint(:,1);
ForecastBeta_Up(1,3) = [1 Beta(end,3)]*bint(:,2);
```

\% Forecasted yield curve
figure
Settle = daysadd(EOMDates(end),30*MonthsLag);
DieboldLi_Forecast = DieboldLi('ParYield',Settle,[ForecastBeta lambda_t*12]
DieboldLi_Forecast_Up = DieboldLi('ParYield',Settle, [ForecastBeta_Up lambda
DieboldLi_Forecast_Down = DieboldLi('ParYield',Settle, [ForecastBeta_Down la
PlottingDates = (Settle+30:30:Settle+30*360)';
plot(PlottingDates,DieboldLi_Forecast.getParYields(PlottingDates), 'b-')
hold on
plot(PlottingDates,DieboldLi_Forecast_Up.getParYields(PlottingDates), 'r-')
plot(PlottingDates,DieboldLi_Forecast_Down.getParYields(PlottingDates), 'r-'
title(['Diebold Li Forecasted Yield Curves on ' datestr(EOMDates(end))
legend(\{'Forecasted Curve','Additional Scenarios'\},'location','southeast')
datetick

## Bibliography

This example is based on the following paper:
[1] Francis X. Diebold, Canlin Li, Forecasting the term structure of government bond yields, Journal of Econometrics, Volume 130, Issue 2, February 2006, Pages 337-364

## Credit Derivatives

- "Credit Default Swap (CDS)" on page 8-2
- "Counterparty Credit Risk and CVA" on page 8-17
- "First-to-Default Swaps" on page 8-31
- "Credit Default Swap Option" on page 8-40


## Credit Default Swap (CDS)

A credit default swap (CDS) is a contract that protects against losses resulting from credit defaults. The transaction involves two parties, the protection buyer and the protection seller, and also a reference entity, usually a bond. The protection buyer pays a stream of premiums to the protection seller, who in exchange offers to compensate the buyer for the loss in the bond's value if a credit event occurs. The stream of premiums is called the premium leg, and the compensation when a credit event occurs is called the protection leg. Credit events usually include situations in which the bond issuer goes bankrupt, misses coupon payments, or enters a restructuring process. Financial Instruments Toolbox software supports:

## CDS Functions

| Function | Purpose |
| :--- | :--- |
| cdsbootstrap | Compute default probability parameters from <br> CDS market quotes. |
| cdsspread | Compute breakeven spreads for the CDS <br> contracts. |
| cdsprice | Compute the price for the CDS contracts. |

## Bootstrapping a Default Probability Curve

In a typical workflow, pricing a new CDS contract involves first estimating a default probability term structure using cdsbootstrap. This requires market quotes of existing CDS contracts, or quotes of CDS indices (e.g., iTraxx). The estimated default probability curve is then used as input to cdsspread or cdsprice. If the default probability information is already known, cdsbootstrap can be bypassed and cdsspread or cdsprice can be called directly.

The market information in this example is provided in the form of running spreads of CDS contracts maturing on the CDS standard payment dates closest to $1,2,3,5$, and 7 years from the valuation date.

```
Settle = '17-Jul-2009'; % valuation date for the CDS
MarketDates = datenum({'20-Sep-10','20-Sep-11','20-Sep-12','20-Sep-14',...
```

```
'20-Sep-16'});
MarketSpreads = [140 175 210 265 310]';
MarketData = [MarketDates MarketSpreads];
ZeroDates = datenum({'17-Jan-10','17-Jul-10','17-Jul-11','17-Jul-12',...
'17-Jul-13','17-Jul-14'});
ZeroRates = [1.35 1.43 1.9 2.47 2.936 3.311]'/100;
ZeroData = [ZeroDates ZeroRates];
[ProbData,HazData] = cdsbootstrap(ZeroData,MarketData,Settle);
```

The bootstrapped default probability curve is plotted against time, in years, from the valuation date.

```
ProbTimes = yearfrac(Settle,ProbData(:,1));
figure
plot([0; ProbTimes],[0; ProbData(:,2)])
grid on
axis([0 ProbTimes(end,1) O ProbData(end,2)])
xlabel('Time (years)')
ylabel('Cumulative Default Probability')
title('Bootstrapped Default Probability Curve')
```



The associated hazard rates are returned as an optional output. The convention is that the first hazard rate applies from the settlement date to the first market date, the second hazard rate from the first to the second market date, etc., and the last hazard rate applies from the second-to-last market date onwards. The following plot displays the bootstrapped hazard rates, plotted against time, in years, from the valuation date:

```
HazTimes = yearfrac(Settle,HazData(:,1));
figure
stairs([0; HazTimes(1:end-1,1); HazTimes(end,1)+1],...
[HazData(:,2);HazData(end,2)])
grid on
axis([0 HazTimes(end,1)+1 0.9*HazData(1,2) 1.1*HazData(end,2)])
xlabel('Time (years)')
ylabel('Hazard Rate')
title('Bootstrapped Hazard Rates')
```



## Finding Breakeven Spread for New CDS Contract

The breakeven, or running, spread is the premium a protection buyer must pay, with no upfront payments involved, to receive protection for credit events associated to a given reference entity. Spreads are expressed in basis points (bp). There is a notional amount associated to the CDS contract to determine the monetary amounts of the premium payments.

New quotes for CDS contracts can be obtained with cdsspread. After obtaining a default probability curve using cdsbootstrap, you get quotes that are consistent with current market conditions.

In this example, instead of standard CDS payment dates, define new maturity dates. Using the period from 3 and 5 years (CDS standard dates), maturities are defined within this range spaced at quarterly intervals (measuring time from the valuation date):

[^0]```
MarketDates = datenum({'20-Sep-10','20-Sep-11','20-Sep-12','20-Sep-14',\ldots.
'20-Sep-16'});
MarketSpreads = [140 175 210 265 310]';
MarketData = [MarketDates MarketSpreads];
ZeroDates = datenum({'17-Jan-10','17-Jul-10','17-Jul-11','17-Jul-12',...
'17-Jul-13','17-Jul-14'});
ZeroRates = [1.35 1.43 1.9 2.47 2.936 3.311]'/100;
ZeroData = [ZeroDates ZeroRates];
[ProbData,HazData] = cdsbootstrap(ZeroData,MarketData,Settle);
Maturity1 = datestr(daysadd('17-Jul-09',360*(3.25:0.25:5),1));
Spread1 = cdsspread(ZeroData,ProbData,Settle,Maturity1);
figure
scatter(yearfrac(Settle,Maturity1),Spread1,'*')
hold on
scatter(yearfrac(Settle,MarketData(3:4,1)),MarketData(3:4,2))
hold off
grid on
xlabel('Time (years)')
ylabel('Spread (bp)')
title('CDS Spreads')
legend('New Quotes','Market','location','SouthEast')
```

This plot displays the resulting spreads:


To evaluate the effect of the recovery rate parameter, instead of $40 \%$ (default value), use a recovery rate of $35 \%$ :

Spread1Rec35 = cdsspread(ZeroData,ProbData,Settle,Maturity1,...
'RecoveryRate', 0.35) ;

```
figure
plot(yearfrac(Settle,Maturity1),Spread1,...
yearfrac(Settle,Maturity1),Spread1Rec35,'--')
grid on
xlabel('Time (years)')
ylabel('Spread (bp)')
title('CDS Spreads with Different Recovery Rates')
legend('40%','35%','location','SouthEast')
```

The resulting plot shows that smaller recovery rates produce higher premia, as expected, since in the event of default, the protection payments will be higher:


## Valuing an Existing CDS Contract

The current value, or mark-to-market, of an existing CDS contract is the amount of money the contract holder would receive (if positive) or pay (if negative) to unwind this contract. The upfront of the contract is the current value expressed as a fraction of the notional amount of the contract, and it is commonly used to quote market values.

The value of existing CDS contracts is obtained with cdsprice. By default, cdsprice treats contracts as long positions. Whether a contract position is long or short is determined from a protection standpoint, that is, long means protection was bought, and short means protection was sold. In the following example, an existing CDS contract pays a premium that is lower than current market conditions. The price is positive, as expected, since it would be more costly to buy the same type of protection today.

```
Settle = '17-Jul-2009'; % valuation date for the CDS
MarketDates = datenum({'20-Sep-10','20-Sep-11','20-Sep-12','20-Sep-14',...
```

```
'20-Sep-16'});
MarketSpreads = [140 175 210 265 310]';
MarketData = [MarketDates MarketSpreads];
ZeroDates = datenum({'17-Jan-10','17-Jul-10','17-Jul-11','17-Jul-12',...
'17-Jul-13','17-Jul-14'});
ZeroRates = [ll.35 1.43 1.9 2.47 2.936 3.311]'/100;
ZeroData = [ZeroDates ZeroRates];
[ProbData,HazData] = cdsbootstrap(ZeroData,MarketData,Settle);
Maturity2 = '20-Sep-2012';
Spread2 = 196;
[Price,AccPrem,PaymentDates,PaymentTimes,PaymentCF] = cdsprice(ZeroData,...
ProbData,Settle,Maturity2,Spread2);
fprintf('Dirty Price: %8.2f\n',Price);
fprintf('Accrued Premium: %8.2f\n',AccPrem);
fprintf('Clean Price: %8.2f\n',Price-AccPrem);
fprintf('\nPayment Schedule:\n\n');
fprintf('Date \t\t Time Frac \t Amount\n');
for k = 1:length(PaymentDates)
    fprintf('%s \t %5.4f \t %8.2f\n',datestr(PaymentDates(k)),...
        PaymentTimes(k),PaymentCF(k));
end
```

This resulting payment schedule is:
Dirty Price: 41628.50
Accrued Premium: 15244.44
Clean Price: 26384.05

Payment Schedule:

| Date Time | Frac | Amount |
| :--- | :---: | ---: |
| 20-Sep-2009 | 0.1806 | 35388.89 |
| $20-$ Dec-2009 | 0.2528 | 49544.44 |
| 20-Mar-2010 | 0.2500 | 49000.00 |
| 20-Jun-2010 | 0.2556 | 50088.89 |


| 20-Sep-2010 | 0.2556 | 50088.89 |
| :--- | :--- | :--- |
| $20-$ Dec-2010 | 0.2528 | 49544.44 |
| $20-$ Mar-2011 | 0.2500 | 49000.00 |
| $20-$ Jun-2011 | 0.2556 | 50088.89 |
| $20-$ Sep-2011 | 0.2556 | 50088.89 |
| $20-$ Dec-2011 | 0.2528 | 49544.44 |
| $20-$ Mar-2012 | 0.2528 | 49544.44 |
| $20-$ Jun-2012 | 0.2556 | 50088.89 |
| $20-$ Sep-2012 | 0.2556 | 50088.89 |

Additionally, you can use cdsprice to value a portfolio of CDS contracts. In the following example, a simple hedged position with two vanilla CDS contracts, one long, one short, with slightly different spreads is priced in a single call and the value of the portfolio is the sum of the returned prices:

```
[Price2,AccPrem2] = cdsprice(ZeroData,ProbData,Settle,...
repmat(datenum(Maturity2),2,1),[Spread2;Spread2+3],...
'Notional',[1e7; -1e7]);
fprintf('Contract \t Dirty Price \t Acc Premium \t Clean Price\n');
fprintf(' Long \t $ %9.2f \t $ %9.2f \t $ %9.2f \t\n',...
    Price2(1), AccPrem2(1), Price2(1) - AccPrem2(1));
fprintf(' Short \t $ %8.2f \t $ %8.2f \t $ %8.2f \t\n',...
    Price2(2), AccPrem2(2), Price2(2) - AccPrem2(2));
fprintf('Mark-to-market of hedged position: $ %8.2f\n',sum(Price2));
```

This resulting value of the portfolio is:

```
Contract Dirty Price Acc Premium Clean Price
    Long $ 41628.50 $ 15244.44 $ 26384.05
    Short $ -32708.11 $ -15477.78 $ -17230.33
```

Mark-to-market of hedged position: \$ 8920.39

## Converting from Running to Upfront

A CDS market quote is given in terms of a standard spread (usually 100 bp or 500 bp ) and an upfront payment, or in terms of an equivalent running or breakeven spread, with no upfront payment. The functions cdsbootstrap, cdsspread, and cdsprice perform upfront to running or running to upfront conversions.

For example, to convert the market quotes to upfront quotes for a standard spread of 100 bp :

```
Settle = '17-Jul-2009'; % valuation date for the CDS
MarketDates = datenum({'20-Sep-10','20-Sep-11','20-Sep-12','20-Sep-14',\ldots.
'20-Sep-16'});
MarketSpreads = [140 175 210 265 310]';
MarketData = [MarketDates MarketSpreads];
ZeroDates = datenum({'17-Jan-10','17-Jul-10','17-Jul-11','17-Jul-12',...
'17-Jul-13','17-Jul-14'});
ZeroRates = [1.35 1.43 1.9 2.47 2.936 3.311]'/100;
ZeroData = [ZeroDates ZeroRates];
[ProbData,HazData] = cdsbootstrap(ZeroData,MarketData,Settle);
Maturity3 = MarketData(:,1);
Spread3Run = MarketData(:,2);
Spread3Std = 100*ones(size(Maturity3));
Price3 = cdsprice(ZeroData,ProbData,Settle,Maturity3,Spread3Std);
Upfront3 = Price3/10000000; % Standard notional of 10MM
display(Upfront3);
```

This resulting value is:

```
Upfront3 =
    0.0047
    0.0158
    0.0327
    0.0737
    0.1182
```

The conversion can be reversed to convert upfront quotes to market quotes:
ProbData3Upf = cdsbootstrap(ZeroData,[Maturity3 Upfront3 Spread3Std],Settle); Spread3RunFromUpf = cdsspread(ZeroData,ProbData3Upf,Settle,Maturity3); display([Spread3Run Spread3RunFromUpf]);

Comparing the results of this conversion to the original market spread demonstrates the reversal:

```
ans =
    140.0000 140.0000
    175.0000 175.0000
    210.0000 210.0000
    265.0000 265.0000
    310.0000 310.0000
```

Under the flat-hazard rate (FHR) quoting convention, a single market quote is used to calibrate a probability curve. This convention yields a single point in the probability curve, and a single hazard rate value. For example, assume a 4 -year (standard dates) CDS contract with a current FHR-based running spread of 550 bp needs conversion to a CDS contract with a standard spread of 500 bp :

```
Maturity4 = datenum('20-Sep-13');
Spread4Run = 550;
ProbData4Run = cdsbootstrap(ZeroData,[Maturity4 Spread4Run],Settle);
Spread4Std = 500;
Price4 = cdsprice(ZeroData,ProbData4Run,Settle,Maturity4,Spread4Std);
Upfront4 = Price4/10000000;
fprintf('A running spread of %5.2f is equivalent to\n',Spread4Run);
fprintf(' a standard spread of %5.2f with an upfront of %8.7f\n',...
    Spread4Std,Upfront4);
A running spread of 550.00 is equivalent to
    a standard spread of 500.00 with an upfront of 0.0167576
```

To reverse the conversion:

```
ProbData4Upf = cdsbootstrap(ZeroData,[Maturity4 Upfront4 Spread4Std],Settle);
Spread4RunFromUpf = cdsspread(ZeroData,ProbData4Upf,Settle,Maturity4);
fprintf('A standard spread of %5.2f with an upfront of %8.7f\n',...
    Spread4Std,Upfront4);
fprintf(' is equivalent to a running spread of %5.2f\n',Spread4RunFromUpf);
A standard spread of 500.00 with an upfront of 0.0167576
    is equivalent to a running spread of 550.00
```

As discussed in Beumee et. al., 2009 (see "Credit Derivatives" on page C-15), the FHR approach is a quoting convention only, and leads to quotes
inconsistent with market data. For example, calculating the upfront for the 3-year (standard dates) CDS contract with a standard spread of 100 bp using the FHR approach and comparing the results to the upfront amounts previously calculated, demonstrates that the FHR-based approach yields a different upfront amount:

```
Maturity5 = MarketData(3,1);
Spread5Run = MarketData(3,2);
ProbData5Run = cdsbootstrap(ZeroData,[Maturity5 Spread5Run],Settle);
Spread5Std = 100;
Price5 = cdsprice(ZeroData,ProbData5Run,Settle,Maturity5,Spread5Std);
Upfront5 = Price5/10000000;
fprintf('Relative error of FHR-based upfront amount: %3.1f%%\n',...
    ((Upfront5-Upfront3(3))/Upfront3(3))*100);
Relative error of FHR-based upfront amount: -0.8%
```


## Bootstrapping from Inverted Market Curves

The following two examples demonstrate the behavior of bootstrapping with inverted CDS market curves, that is, market quotes with higher spreads for short-term CDS contracts. The first example is handled normally by cdsbootstrap:

```
Settle = '17-Jul-2009'; % valuation date for the CDS
MarketDates = datenum({'20-Sep-10','20-Sep-11','20-Sep-12','20-Sep-14',\ldots.
'20-Sep-16'});
ZeroDates = datenum({'17-Jan-10','17-Jul-10','17-Jul-11','17-Jul-12',...
'17-Jul-13','17-Jul-14'});
ZeroRates = [1.35 1.43 1.9 2.47 2.936 3.311]'/100;
ZeroData = [ZeroDates ZeroRates];
MarketSpreadsInv1 = [750 650 550 500 450]';
MarketDataInv1 = [MarketDates MarketSpreadsInv1];
[ProbDataInv1,HazDataInv1] = cdsbootstrap(ZeroData,MarketDataInv1,Settle)
ProbDataInv1 =
```

    \(1.0 \mathrm{e}+05\) *
    ```
    7.3440 0.0000
    7.3477 0.0000
    7.3513 0.0000
    7.3586 0.0000
    7.3659 0.0000
HazDataInv1 =
    1.0e+05 *
\begin{tabular}{ll}
7.3440 & 0.0000 \\
7.3477 & 0.0000 \\
7.3513 & 0.0000 \\
7.3586 & 0.0000 \\
7.3659 & 0.0000
\end{tabular}
```

In the second example, cdsbootstrap generates a warning:

```
MarketSpreadsInv2 = [800 550 400 250 100]';
MarketDataInv2 = [MarketDates MarketSpreadsInv2];
[ProbDataInv2,HazDataInv2] = cdsbootstrap(ZeroData,MarketDataInv2,Settle);
```

Warning: Found non-monotone default probabilities (negative hazard rates)
A non-monotone bootstrapped probability curve implies negative default probabilities and negative hazard rates for certain time intervals. Extreme market conditions can lead to these types of situations. In these cases, you must assess the reliability and usefulness of the bootstrapped results.

The following plot illustrates these bootstrapped probability curves. The curves are concave, meaning that the marginal default probability decreases with time. This result is consistent with the market information that indicates a higher default risk in the short term. The second bootstrapped curve is non-monotone, as indicated by the warning.

```
ProbTimes = yearfrac(Settle, MarketDates);
figure
plot([0; ProbTimes],[0; ProbDataInv1(:,2)])
hold on
```

```
plot([0; ProbTimes],[0; ProbDataInv2(:,2)],'--')
hold off
grid on
axis([0 ProbTimes(end,1) 0 ProbDataInv1(end,2)])
xlabel('Time (years)')
ylabel('Cumulative Default Probability')
title('Probability Curves for Inverted Spread Curves')
legend('1st instance','2nd instance','location','SouthEast')
```

The resulting plot


Also in line with the previous plot, the hazard rates for these bootstrapped curves are decreasing because the short-term risk is higher. Some bootstrapped parameters in the second curve are negative, as indicated by the warning.

```
HazTimes = yearfrac(Settle, MarketDates);
figure
```

```
stairs([0; HazTimes(1:end-1,1); HazTimes(end,1)+1],...
    [HazDataInv1(:,2);HazDataInv1(end,2)])
hold on
stairs([0; HazTimes(1:end-1,1); HazTimes(end,1)+1],...
    [HazDataInv2(:, 2);HazDataInv2(end,2)],'--')
hold off
grid on
xlabel('Time (years)')
ylabel('Hazard Rate')
title('Hazard Rates for Inverted Spread Curves')
legend('1st instance','2nd instance','location','NorthEast')
```

The resulting plot shows the hazard rates for both bootstrapped curves:


For further discussion on inverted curves, and their relationship to arbitrage, see O’Kane and Turnbull, 2003 ("Credit Derivatives" on page C-15).

## Counterparty Credit Risk and CVA

This example shows how to compute the unilateral credit value (valuation) adjustment (CVA) for a bank holding a portfolio of vanilla interest rate swaps with several counterparties. CVA is the expected loss on an over-the-counter contract or portfolio of contracts due to counterparty default. The CVA for a particular counterparty is defined as the sum over all points in time of the discounted expected exposure at each moment multiplied by the probability that the counterparty defaults at that moment, all multiplied by 1 minus the recovery rate. The CVA formula is:

Where $R$ is the recovery, discEE the discounted expected exposure at time $t$, and PD the default probability distribution.

The expected exposure is computed by first simulating many future scenarios of risk factors for the given contract or portfolio. Risk factors can be interest rates, as in this example, but will differ based on the portfolio and can include FX rates, equity or commodity prices, or anything that will affect the market value of the contracts. Once a sufficient set of scenarios has been simulated, the contract or portfolio can be priced on a series of future dates for each scenario. The result is a matrix, or "cube", of contract values.

These prices are converted into exposures after taking into account collateral agreements that the bank might have in place as well as netting agreements, as in this example, where the values of several contracts may offset each other, lowering their total exposure.

The contract values for each scenario are discounted to compute the discounted exposures. The discounted expected exposures can then be computed by a simple average of the discounted exposures at each simulation date.

Finally, counterparty default probabilities are typically derived from credit default swap (CDS) market quotes and the CVA for the counterparty can be computed according to the above formula. We assume that a counterparty default is independent of its exposure (no wrong-way risk).

For this example we will work with a portfolio of vanilla interest rate swaps with the goal of computing the CVA for a particular counterparty.

## Read Swap Portfolio

The portfolio of swaps is close to zero value at time $\mathrm{t}=0$. Each swap is associated with a counterparty and may or may not be included in a netting agreement.

```
% Read swaps from spreadsheet
swapFile = 'cva-swap-portfolio.xls';
swapData = dataset('XLSFile',swapFile,'Sheet','Swap Portfolio');
swaps = struct(...
    'Counterparty',[],...
    'NettingID',[],...
    'Principal',[],...
    'Maturity',[],...
    'LegRate',[],...
    'LegType',[],...
    'LatestFloatingRate',[],...
    'FloatingResetDates',[]);
```

swaps.Counterparty = swapData.CounterpartyID;
swaps.NettingID = swapData.NettingID;
swaps.Principal = swapData.Principal;
swaps.Maturity = swapData.Maturity;
swaps.LegType = [swapData.LegType ~swapData.LegType];
swaps.LegRate = [swapData.LegRateReceiving swapData.LegRatePaying];
swaps.LatestFloatingRate = swapData.LatestFloatingRate;
swaps.Period = swapData.Period;
swaps.LegReset = ones(size(swaps.LegType));
numSwaps = numel(swaps.Counterparty);

Create RateSpec from the Interest Rate Curve

```
Settle = datenum('14-Dec-2007');
Tenor = [3 6 12 5*12 7*12 10*12 20*12 30*12]';
ZeroRates = [0.033 0.034 0.035 0.040 0.042 0.044 0.048 0.0475]';
ZeroDates = datemnth(Settle,Tenor);
```

```
Compounding = 2;
Basis = 0;
RateSpec = intenvset('StartDates', Settle,'EndDates', ZeroDates,...
    'Rates', ZeroRates,'Compounding',Compounding,'Basis',Basis);
figure;
plot(ZeroDates, ZeroRates, 'o-');
xlabel('Date');
datetick('keeplimits');
ylabel('Zero rate'); grid on;
title('Yield Curve at Settle Date');
```


## Set Changeable Simulation Parameters

We can vary here the number of simulated interest rate scenarios we generate. We set our simulation dates to be more frequent at first, then turning less frequent further in the future.

```
% Number of Monte Carlo simulations
numScenarios = 1000;
```

```
% Compute monthly simulation dates, then quarterly dates later.
simulationDates = datemnth(Settle,0:12);
simulationDates = [simulationDates datemnth(simulationDates(end),3:3:74)]';
numDates = numel(simulationDates);
```


## Compute Floating Reset Dates

For each simulation date, we compute previous floating reset date for each swap.

```
floatDates = cfdates(Settle-360,swaps.Maturity,swaps.Period);
swaps.FloatingResetDates = zeros(numSwaps,numDates);
for i = numDates:-1:1
    thisDate = simulationDates(i);
    floatDates(floatDates > thisDate) = 0;
    swaps.FloatingResetDates(:,i) = max(floatDates,[],2);
end
```


## Setup Hull-White Single Factor Model

The risk factor we will simulate to value our contracts is the zero curve. For this example we will model the interest rate term structure using the one-factor Hull-White model. This is a model of the short rate and is defined as:
where

- : Change in the short rate after a small change in time,
- : Mean reversion rate
- : Volatility of the short rate
- : A Weiner process (a standard normal process)
- : Drift function defined as:
: Instantaneous forward rate at time
: Partial derivative of with respect to time
Once we have simulated a path of the short rate we generate a full yield curve at each simulation date using the formula:
: Zero rate at time for a period of
: Price of a zero coupon bond at time that pays one dollar at time

Each scenario contains the full term structure moving forward through time, modeled at each of our selected simulation dates.

Refer to "Calibrating the Hull-White Model Using Market Data" example in the Financial Instruments Toolbox ${ }^{\text {TM }}$ Users' Guide for more details on Hull-White one-factor model calibration.

```
Alpha = 0.2;
Sigma = 0.015;
hw1 = HullWhite1F(RateSpec,Alpha,Sigma);
```


## Simulate Scenarios

For each scenario, we simulate the future interest rate curve at each valuation date using the Hull-White one-factor interest rate model.
\% Use reproducible random number generator (vary the seed to produce \% different random scenarios). prevRNG = rng(0, 'twister');
dt = diff(yearfrac(Settle,simulationDates,1));
nPeriods = numel(dt);
scenarios = hw1.simTermStructs(nPeriods,...
'nTrials', numScenarios,...
'deltaTime', dt);
\% Restore random number generator state
rng(prevRNG);
\% Compute the discount factors through each realized interest rate
\% scenario.
dfactors = ones(numDates, numScenarios);
for $i=2$ numbates
tenorDates = datemnth(simulationDates(i-1),Tenor);
rateAtNextSimDate = interp1(tenorDates, squeeze(scenarios(i-1,:,:)),...
simulationDates(i), 'linear', 'extrap');
\% Compute D(t1,t2)
dfactors(i,:) = zero2disc(rateAtNextSimDate,...
repmat(simulationDates(i),1,numScenarios), simulationDates(i-1), -1,3

```
end
dfactors = cumprod(dfactors,1);
```


## Inspect a Scenario

Create a surface plot of the yield curve evolution for a particular scenario.

```
i = 20;
figure;
surf(Tenor, simulationDates, scenarios(:,:,i))
axis tight
datetick('y','mmmyy');
xlabel('Tenor (Months)');
ylabel('Observation Date');
zlabel('Rates');
set(gca,'View',[-49 32]);
title(sprintf('Scenario %d Yield Curve Evolution\n',i));
```


## Compute Mark to Market Swap Prices

For each scenario the swap portfolio is priced at each future simulation date. Prices are computed using a price approximation function, hswapapprox. It is common in CVA applications to use simplified approximation functions when pricing contracts due to the performance requirements of these Monte Carlo simulations.

Since the simulation dates do not correspond to the swaps cash flow dates (where the floating rates are reset) we estimate the latest floating rate with the 1 -year rate (all swaps have period 1 year) interpolated between the nearest simulated rate curves.

The swap prices are then aggregated into a "cube" of values which contains all future contract values at each simulation date for each scenario. The resulting cube of contract prices is a 3 dimensional matrix where each row represents a simulation date, each column an contract, and each "page" a different simulated scenario.

[^1]```
% approximation function here to improve performance.
values = hcomputeMTMValues(swaps,simulationDates,scenarios,Tenor);
```


## Inspect Scenario Prices

Create a plot of the evolution of all swap prices for a particular scenario.

```
i = 32;
figure;
plot(simulationDates, values(:,:,i));
datetick;
ylabel('Mark-To-Market Price');
title(sprintf('Swap prices along scenario %d', i));
```


## Visualize Simulated Portfolio Values

We plot the total portfolio value for each scenario of our simulation. As each scenario moves forward in time the values of the contracts will move up or down depending on how the modeled interest rate term structure changes. As the swaps get closer to maturity, their values will begin to approach zero since the aggregate value of all remaining cash flows will decrease after each cash flow date.

```
% View portfolio value over time
figure;
totalPortValues = squeeze(sum(values, 2));
plot(simulationDates,totalPortValues);
title('Total MTM Portfolio Value for All Scenarios');
datetick('x','mmmyy')
ylabel('Portfolio Value ($)')
xlabel('Simulation Dates')
```


## Compute Exposure by Counterparty

The exposure of a particular contract (i) at time $t$ is the maximum of the contract value $(\mathrm{Vi})$ and 0 :

And the exposure for a particular counterparty is simply a sum of the individual contract exposures:

In the presence of netting agreements, however, contracts are aggregated together and can offset each other. Therefore the total exposure of all contracts in a netting agreement is:

We compute these exposures for the entire portfolio as well as each counterparty at each simulation date using the creditexposures function.

Unnetted contracts are indicated using a NaN in the NettingID vector. Exposure of an unnetted contract is equal to the market value of the contract if it has positive value, otherwise it is zero.

Contracts included in a netting agreement have their values aggregated together and can offset each other. See the references for more details on computing exposure from mark-to-market contract values.
[exposures, expcpty] = creditexposures(values,swaps.Counterparty,... 'NettingID', swaps.NettingID);

We plot the total portfolio exposure for each scenario in our simulation. Similar to the plot of contract values, the exposures for each scenario will approach zero as the swaps mature.

```
% View portfolio exposure over time
figure;
totalPortExposure = squeeze(sum(exposures,2));
plot(simulationDates,totalPortExposure);
title('Portfolio Exposure for All Scenarios');
datetick('x','mmmyy')
ylabel('Exposure ($)')
xlabel('Simulation Dates')
```


## Exposure Profiles

Several exposure profiles are useful when analyzing the potential future exposure of a bank to a counterparty. Here we compute several (non-discounted) exposure profiles per counterparty as well as for the entire portfolio.

- PFE : Potential Future Exposure : A high percentile (95\%) of the distribution of exposures at any particular future date. Also called Peak Exposure (PE)
- MPFE : Maximum Potential Future Exposure : The maximum PFE across all dates
- EE : Expected Exposure : The mean (average) of the distribution of exposures at each date
- EPE : Expected Positive Exposure : Weighted average over time of the expected exposure
- EffEE : Effective Expected Exposure : The maximum expected exposure at any time, t , or previous time
- EffEPE : Effective Expected Positive Exposure: The weighted average of the effective expected exposure

For further definitions, see for example Basel II document in references.

```
% Compute entire portfolio exposure
portExposures = sum(exposures,2);
% Compute exposure profiles for each counterparty and entire portfolio
cpProfiles = exposureprofiles(simulationDates,exposures);
portProfiles = exposureprofiles(simulationDates,portExposures);
```

We visualize the exposure profiles, first for the entire portfolio, then for a particular counterparty.

```
% Visualize portfolio exposure profiles
figure;
plot(simulationDates,portProfiles.PFE,...
    simulationDates,portProfiles.MPFE * ones(numDates,1),...
```

```
    simulationDates,portProfiles.EE,...
    simulationDates,portProfiles.EPE * ones(numDates,1),...
    simulationDates,portProfiles.EffEE,...
    simulationDates,portProfiles.EffEPE * ones(numDates,1));
legend({'PFE (95%)','Max PFE','Exp Exposure (EE)','Time-Avg EE (EPE)',...
    'Max past EE (EffEE)','Time-Avg EffEE (EffEPE)'})
datetick('x','mmmyy')
title('Portfolio Exposure Profiles');
ylabel('Exposure ($)')
xlabel('Simulation Dates')
```

Visualize exposure profiles for a particular counterparty

```
cpIdx = find(expcpty == 5);
figure;
plot(simulationDates,cpProfiles(cpIdx).PFE,...
    simulationDates,cpProfiles(cpIdx).MPFE * ones(numDates,1),...
    simulationDates,cpProfiles(cpIdx).EE,...
    simulationDates,cpProfiles(cpIdx).EPE * ones(numDates,1),...
    simulationDates,cpProfiles(cpIdx).EffEE,...
    simulationDates,cpProfiles(cpIdx).EffEPE * ones(numDates,1));
legend({'PFE (95%)','Max PFE','Exp Exposure (EE)','Time-Avg EE (EPE)',...
    'Max past EE (EffEE)','Time-Avg EffEE (EffEPE)'})
datetick('x','mmmyy','keeplimits')
title(sprintf('Counterparty %d Exposure Profiles',cpIdx));
ylabel('Exposure ($)')
xlabel('Simulation Dates')
```


## Discounted Exposures

We compute the discounted expected exposures using the discount factors from each simulated interest rate scenario. The discount factor for a given valuation date in a given scenario is the product of the incremental discount
factors from one simulation date to the next, along the interest rate path of that scenario.

```
% Get discounted exposures per counterparty, for each scenario
discExp = zeros(size(exposures));
for i = 1:numScenarios
    discExp(:,:,i) = bsxfun(@times,dfactors(:,i),exposures(:,:,i));
end
```

```
% Discounted expected exposure
```

% Discounted expected exposure
discProfiles = exposureprofiles(simulationDates,discExp,...
discProfiles = exposureprofiles(simulationDates,discExp,...
'ProfileSpec','EE');

```
    'ProfileSpec','EE');
```

We plot the discounted expected exposures for the aggregate portfolio as well as for each counterparty.
\% Aggregate the discounted EE for each counterparty into a matrix discEE = [discProfiles.EE];
\% Portfolio discounted EE figure;
plot(simulationDates, sum(discEE,2))
datetick('x','mmmyy', 'keeplimits')
title('Discounted Expected Exposure for Portfolio');
ylabel('Discounted Exposure (\$)')
xlabel('Simulation Dates')
\% Counterparty discounted EE
figure;
plot(simulationDates,discEE)
datetick('x','mmmyy', 'keeplimits')
title('Discounted Expected Exposure for Each Counterparty');
ylabel('Discounted Exposure (\$)')
xlabel('Simulation Dates')

Calibrating Probability of Default Curve for Each Counterparty

The default probability for a given counterparty is implied by the current market spreads of the counterparty's CDS. We use the function cdsbootstrap to generate the cumulative probability of default at each simulation date.

```
% Import CDS market information for each counterparty
CDS = dataset('XLSfile',swapFile,'Sheet','CDS Spreads') %#ok<NOPTS>
CDSDates = datenum(CDS.Date);
CDSSpreads = double(CDS(:,2:end));
ZeroData = [RateSpec.EndDates RateSpec.Rates];
% Calibrate default probabilities for each counterparty
DefProb = zeros(length(simulationDates), size(CDSSpreads,2));
for i = 1:size(DefProb,2)
    probData = cdsbootstrap(ZeroData, [CDSDates CDSSpreads(:,i)],...
            Settle, 'probDates', simulationDates);
    DefProb(:,i) = probData(:,2);
end
```

\% We plot of the cumulative probability of default for each counterparty.
figure;
plot(simulationDates, DefProb)
title('Default Probability Curve for Each Counterparty');
xlabel('Date');
grid on;
ylabel('Cumulative Probability')
datetick('x','mmmyy')
ylabel('Probability of Default')
xlabel('Simulation Dates')
CDS =

| Date | cp1 | cp2 | cp3 | cp4 | cp5 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| '3/20/2008' $^{\prime}$ | 140 | 85 | 115 | 170 | 140 |
| '3/20/2009' $^{\prime}$ | 185 | 120 | 150 | 205 | 175 |
| $'^{\prime} / 20 / 2010^{\prime}$ | 215 | 170 | 195 | 245 | 210 |
| $'^{\prime} / 20 / 2011^{\prime}$ | 275 | 215 | 240 | 285 | 265 |
| $'^{\prime} / 20 / 2012^{\prime}$ | 340 | 255 | 290 | 320 | 310 |

## CVA Computation

The Credit Value (Valuation) Adjustment (CVA) formula is:

Where $R$ is the recovery, discEE the discounted expected exposure at time $t$, and PD the default probability distribution. This assumes the exposure is independent of default (no wrong-way risk), and it also assumes the exposures were obtained using risk-neutral probabilities.

Here we approximate the integral with a finite sum over the valuation dates as:
where $t \_1$ is todays date, $t \_2, \ldots, t \_n$ the future valuation dates.
We assume CDS info corresponds to counterparty with index cpIdx. The computed CVA is the present market value of our credit exposure to counterparty cpIdx. For this example we set the recovery rate at $40 \%$.

```
Recovery = 0.4;
CVA = (1-Recovery) * sum(discEE(2:end,:) .* diff(DefProb));
for i = 1:numel(CVA)
    fprintf('CVA for counterparty %d = $%.2f\n',i,CVA(i));
end
```

figure;
bar(CVA);
title('CVA for each counterparty');
xlabel('Counterparty');
ylabel('CVA \$');
grid on;
CVA for counterparty 1 = \$2228.36
CVA for counterparty 2 = \$2487.60

```
CVA for counterparty 3 = $920.39
CVA for counterparty 4 = $5478.50
CVA for counterparty 5 = $5859.30
```


## References

1 Pykhtin, Michael, and Steven Zhu, A Guide to Modeling Counterparty Credit Risk, GARP, July/August 2007, issue 37, pp. 16-22.

2 Pykhtin, Michael, and Dan Rosen, Pricing Counterparty Risk at the Trade Level and CVA, 2010.

3 Basel II: http://www.bis.org/publ/bcbs128.pdf page 256

## First-to-Default Swaps

This example shows how to price first-to-default (FTD) swaps under the homogeneous loss assumption.

A first-to-default swap is an instrument that pays a predetermined amount when (and if) the first of a basket of credit instruments defaults. The credit instruments in the basket are usually bonds. If we assume that the loss amount following a credit event is the same for all credits in the basket, we are under the homogeneous loss assumption. This assumption makes models simpler, because any default in the basket triggers the same payment amount. This example is an implementation of the pricing methodology for these instruments, as described in O'Kane [2]. There are two steps in the methodology: a) Compute the survival probability for the basket numerically; b) Use this survival curve and standard single-name credit-default swap (CDS) functionality to find FTD spreads and to price existing FTD swaps.

## Fit Probability Curves to Market Data

Given CDS market quotes for each issuer in the basket, use cdsbootstrap to calibrate individual default probability curves for each issuer.

```
% Interest rate curve
ZeroDates = datenum({'17-Jan-10','17-Jul-10','17-Jul-11','17-Jul-12',...
'17-Jul-13','17-Jul-14'});
ZeroRates = [1.35 1.43 1.9 2.47 2.936 3.311]'/100;
ZeroData = [ZeroDates ZeroRates];
% CDS spreads
% Each row in MarketSpreads corresponds to a different issuer; each
% column to a different maturity date (corresponding to MarketDates)
MarketDates = datenum({'20-Sep-10','20-Sep-11','20-Sep-12', '20-Sep-14',...
'20-Sep-16'});
MarketSpreads = [
    160 195 230 285 330;
    130 165 205 260 305;
    150 180 210 260 300;
    165 200 225 275 295];
% Number of issuers equals number of rows in MarketSpreads
nIssuers = size(MarketSpreads,1);
```

```
% Settlement date
Settle = datenum('17-Jul-2009');
```

In practice, the time axis is discretized and the FTD survival curve is only evaluated at grid points. We use one point every three months. To request that cdsbootstrap returns default probability values over the specific grid points we want, use the optional argument 'ProbDates'. We add the original standard CDS market dates to the grid, otherwise the default probability information on those dates would be interpolated using the two closest dates on the grid, and the prices on market dates would be inconsistent with the original market data.

```
ProbDates = union(MarketDates,daysadd(Settle,360*(0.25:0.25:8),1));
nProbDates = length(ProbDates);
DefProb = zeros(nIssuers,nProbDates);
for ii=1:nIssuers
    MarketData = [MarketDates MarketSpreads(ii,:)'];
    ProbData = cdsbootstrap(ZeroData,MarketData,Settle,...
        'ProbDates',ProbDates);
    DefProb(ii,:) = ProbData(:,2)';
end
```

These are the calibrated default probability curves for each credit in the basket.

```
figure
plot(ProbDates',DefProb)
datetick
title('Individual Default Probability Curves')
ylabel('Cumulative Probability')
xlabel('Date')
```


## Determine Latent Variable Thresholds

Latent variables are used in different credit risk contexts, with different interpretations. In some contexts, a latent variable is a proxy for a change
in the value of assets, and the domain of this variable is binned, with each bin corresponding to a credit rating. The bins limits, or thresholds, are determined from credit migration matrices. In our context, the latent variable is associated to a time to default, and the thresholds determine bins in a discretized time grid where defaults may occur.

Formally, if the time to default of a particular issuer is denoted by , and we know its default probability function, a latent variable and corresponding thresholds satisfy
or

These relationships make latent variable approaches convenient for both simulations and analytical derivations. Both and are functions of time.

The choice of a distribution for the variable determines the thresholds In the standard latent variable model, the variable is chosen to follow a standard normal distribution, from which
where is the cumulative standard normal distribution.
Use the previous formula to determine the default-time thresholds, or simply default thresholds, corresponding to the default probabilities previously obtained for the credits in the basket.

DefThresh = norminv(DefProb);
Derive Survival Curve for the Basket
Following O'Kane [2], we use a one-factor latent variable model to derive expressions for the survival probability function of the basket.

Given parameters for each issuer , and given independent standard normal variables and, the one-factor latent variable model assumes that the latent variable associated to issuer satisfies

This induces a correlation between issuers and of . All latent variables share the common factor as a source of uncertainty, but each latent variable also has an idiosyncratic source of uncertainty . The larger the coefficient , the more the latent variable resembles the common factor .

Using the latent variable model, we derive an analytic formula for the survival probability of the basket.

The probability that issuer survives past time , in other words, that its default time is greater than is
where is the default threshold computed above for issuer, for the -th date in the discretization grid.

Conditional on the value of the one-factor, the probability that all issuers survive past time is
where the product is justified because all the 's are independent. Therefore, conditional on , the 's are independent.

The unconditional probability of no defaults by time is the integral over all values of of the previous conditional probability
with the standard normal density.
By evaluating this one-dimensional integral for each point in the grid, we get a discretization of the survival curve for the whole basket, which is the FTD survival curve.

The latent variable model can also be used to simulate default times, which is the back engine of many pricing methodologies for credit instruments. Loeffler and Posch [1], for example, estimate the survival probability of a basket via simulation. In each simulated scenario a time to default is determined for each issuer. With some bookkeeping, the probability of having the first default on each bucket of the grid can be estimated from the simulation. The simulation approach is also discussed in O'Kane [2]. Simulation is very flexible and applicable to many credit instruments. However, analytic approaches are preferred, when available, because they are much faster and more accurate than simulation.

To compute the FTD survival probabilities in our example, we set all betas to the square root of a target correlation. Then we loop over all dates in the time grid to compute the one dimensional integral that gives the survival probability of the basket.

Regarding implementation, the conditional survival probability as a function of a scalar $Z$ would be
condProb=@(Z)prod(normcdf((-DefThresh(:,jj)+beta*Z)./sqrt(1-beta.^2)));
However, the integration function we use requires that the function handle of the integrand accepts vectors. Although a loop around the scalar version of the conditional probability would work, it is far more efficient to vectorize the conditional probability using bsxfun.

```
beta = sqrt(0.25)*ones(nIssuers,1);
FTDSurvProb = zeros(size(ProbDates));
for jj=1:nProbDates
    % vectorized conditional probability as a function of Z
    vecCondProb = @(Z)prod(normcdf(bsxfun(@rdivide,...
        -repmat(DefThresh(:,jj),1,length(Z))+bsxfun(@times,beta,Z),...
        sqrt(1-beta.^2))));
    % truncate domain of normal distribution to [-5,5] interval
    FTDSurvProb(jj) = integral(@(Z)vecCondProb(Z).*normpdf(Z),-5,5);
end
FTDDefProb = 1-FTDSurvProb;
```

Compare the FTD probability to the default probabilities of the individual issuers.

```
figure
plot(ProbDates',DefProb)
datetick
hold on
plot(ProbDates,FTDDefProb,'LineWidth',3)
datetick
hold off
title('FTD and Individual Default Probability Curves')
ylabel('Cumulative Probability')
xlabel('Date')
```


## Find FTD Spreads and Price Existing FTD Swaps

Under the assumption that all instruments in the basket have the same recovery rate, or homogeneous loss assumption (see O'Kane in References), we get the spread for the FTD swap using the cdsspread function, but passing the FTD probability data just computed.

```
Maturity = MarketDates;
ProbDataFTD = [ProbDates, FTDDefProb];
FTDSpread = cdsspread(ZeroData,ProbDataFTD,Settle,Maturity);
```

Compare the FTD spreads with the individual spreads.

```
figure
plot(MarketDates,MarketSpreads')
datetick
hold on
plot(MarketDates,FTDSpread,'LineWidth',3)
hold off
title('FTD and Individual CDS Spreads')
ylabel('FTD Spread (bp)')
xlabel('Maturity Date')
```

An existing FTD swap can be priced with cdsprice, using the same FTD probability.

```
Maturity0 = MarketDates(1); % Assume maturity on nearest market date
SpreadO = 540; % Spread of existing FTD contract
% Assume default values of recovery and notional
FTDPrice = cdsprice(ZeroData,ProbDataFTD,Settle,Maturity0,Spread0);
fprintf('Price of existing FTD contract: %g\n',FTDPrice)
Price of existing FTD contract: 17644.7
```


## Analyze Sensitivity to Correlation

To illustrate the sensitivity of the FTD spreads to model parameters, we calculate the market spreads for a range of correlation values.

```
corr = [lllllllll}00.010.10 0.25 0.5 0.75 0.90 0.99 1];
FTDSpreadByCorr = zeros(length(Maturity),length(corr));
FTDSpreadByCorr(:,1) = sum(MarketSpreads)';
FTDSpreadByCorr(:,end) = max(MarketSpreads)';
for ii = 2:length(corr)-1
    beta = sqrt(corr(ii))*ones(nIssuers,1);
    FTDSurvProb = zeros(length(ProbDates));
    for jj=1:nProbDates
        % vectorized conditional probability as a function of Z
        condProb = @(Z)prod(normcdf(bsxfun(@rdivide,...
                -repmat(DefThresh(:,jj),1,length(Z))+bsxfun(@times,beta,Z), ...
                sqrt(1-beta.^2))));
            % truncate domain of normal distribution to [-5,5] interval
            FTDSurvProb(jj) = integral(@(Z)condProb(Z).*normpdf(Z), -5,5);
    end
    FTDDefProb = 1-FTDSurvProb;
    ProbDataFTD = [ProbDates, FTDDefProb];
    FTDSpreadByCorr(:,ii) = cdsspread(ZeroData,ProbDataFTD,Settle,Maturity);
end
```

The FTD spreads lie in a band between the sum and the maximum of individual spreads. As the correlation increases to one, the FTD spreads decrease towards the maximum of the individual spreads in the basket (all
credits default together). As the correlation decreases to zero, the FTD spreads approach the sum of the individual spreads (independent credits).

```
figure
legends = cell(1,length(corr));
plot(MarketDates,FTDSpreadByCorr(:,1),'k:')
legends{1} = 'Sum of Spreads';
datetick
hold on
for ii=2:length(corr)-1
    plot(MarketDates,FTDSpreadByCorr(:,ii),'LineWidth',3*corr(ii))
    legends{ii} = ['Corr ' num2str(corr(ii)*100) '%'];
end
plot(MarketDates,FTDSpreadByCorr(:,end),'k-.')
legends{end} = 'Max of Spreads';
hold off
title('FTD Spreads for Different Correlations')
ylabel('FTD Spread (bp)')
xlabel('Maturity Date')
legend(legends,'Location','NW')
```

For short maturities and small correlations, the basket is effectively independent (the FTD spread is very close to the sum of individual spreads). The correlation effect becomes more significant for longer maturities.

Here is an alternative visualization of the dependency of FTD spreads on correlation.

```
figure
surf(corr,MarketDates,FTDSpreadByCorr)
datetick('y')
set(gca,'YDir','reverse')
view(-40,10)
title('FTD Spreads for Different Correlations and Maturities')
xlabel('Correlation')
ylabel('Maturity Date')
zlabel('FTD Spread (bp)')
```


## References

[1] Loeffler, Gunter and Peter Posch, Credit risk modeling using Excel and VBA, Wiley Finance, 2007.
[2] O'Kane, Dominic, Modelling single-name and multi-name Credit Derivatives, Wiley Finance, 2008.

## Credit Default Swap Option

In this section...<br>"Pricing a Single-Name CDS Option" on page 8-40<br>"Pricing a CDS Index Option" on page 8-42<br>"References" on page 8-47

A credit default swap (CDS) option, or credit default swaption, is a contract that provides the holder with the right, but not the obligation, to enter into a credit default swap in the future. CDS options can either be payer swaptions or receiver swaptions. If a payer swaption, the option holder has the right to enter into a CDS where they pay premiums; and, if a receiver swaption, the option holder receives premiums. Financial Instruments Toolbox software provides cdsoptprice for pricing payer and receiver credit default swaptions. Also, with some additional steps, cdsoptprice can be used for pricing multi-name CDS index options.

## Pricing a Single-Name CDS Option

This example shows how to price a single-name CDS option using cdsoptprice. The function cdsoptprice is based on the Black's model as described in O'Kane (2008). The optional knockout argument for cdsoptprice supports two variations of the mechanics of a CDS option. CDS options can be knockout or non-knockout options.

- A knockout option cancels with no payments if there is a credit event before the option expiry date.
- A non-knockout option does not cancel if there is a credit event before the option expiry date. In this case, the option holder of a non-knockout payer swaption can take delivery of the underlying long protection CDS on the option expiry date and exercise the protection, delivering a defaulted obligation in return for par. This portion of protection from option initiation to option expiry is known as the front-end protection (FEP). While this distinction does not affect the receiver swaption, the price of a non-knockout payer swaption is obtained by adding the value of the FEP to the knockout payer swaption price.

Define the CDS instrument.

```
Settle = datenum('12-Jun-2012');
OptionMaturity = datenum('20-Sep-2012');
CDSMaturity = datenum('20-Sep-2017');
OptionStrike = 200;
SpreadVolatility = .4;
```

Define the zero rate.

```
Zero_Time = [.5 1 2 3 4 5]';
Zero_Rate = [.5 .75 1.5 1.7 1.9 2.2]'/100;
Zero_Dates = daysadd(Settle,360*Zero_Time,1);
ZeroData = [Zero_Dates Zero_Rate]
ZeroData =
    1.0e+05 *
    7.3522 0.0000
    7.3540 0.0000
    7.3576 0.0000
    7.3613 0.0000
    7.3649 0.0000
    7.3686 0.0000
```

Define the market data.

```
Market_Time = [1 2 3 5 7 10]';
Market_Rate = [100 120 145 220 245 270]';
Market_Dates = daysadd(Settle,360*Market_Time,1);
MarketData = [Market_Dates Market_Rate];
ProbData = cdsbootstrap(ZeroData, MarketData, Settle)
```

ProbData =

```
1.0e+05 *
\begin{tabular}{ll}
7.3540 & 0.0000 \\
7.3576 & 0.0000 \\
7.3613 & 0.0000 \\
7.3686 & 0.0000 \\
7.3759 & 0.0000 \\
7.3868 & 0.0000
\end{tabular}
```

Define the CDS option.

```
[Payer,Receiver] = cdsoptprice(ZeroData, ProbData, Settle, OptionMaturity,
    CDSMaturity, OptionStrike, SpreadVolatility, 'Knockout', true);
fprintf(' Payer: %.Of Receiver: %.Of (Knockout)\n',Payer,Receiver);
[Payer,Receiver] = cdsoptprice(ZeroData, ProbData, Settle, OptionMaturity,
    CDSMaturity, OptionStrike, SpreadVolatility, 'Knockout', false);
fprintf(' Payer: %.Of Receiver: %.Of (Non-Knockout)\n',Payer,Receiver
Payer: 196 Receiver: 23 (Knockout)
Payer: 224 Receiver: 23 (Non-Knockout)
```


## Pricing a CDS Index Option

This example shows how to price CDS index options by using cdsoptprice with the forward spread adjustment. Unlike a single-name CDS, a CDS portfolio index contains multiple credits. When one or more of the credits default, the corresponding contingent payments are made to the protection buyer but the contract still continues with reduced coupon payments. Considering the fact that the CDS index option does not cancel when some of the underlying credits default before expiry, one might attempt to price CDS index options using the Black's model for non-knockout single-name CDS option. However, Black's model in this form is not appropriate for pricing CDS index options because it does not capture the exercise decision correctly when the strike spread ( $K$ ) is very high, nor does it ensure put-call parity when $(K)$ is not equal to the contractual spread (O'Kane, 2008).

However, with the appropriate modifications, Black's model for single-name CDS options used in cdsoptprice can provide a good approximation for CDS index options. While there are some variations in the way the Black's model
is modified for CDS index options, they usually involve adjusting the forward spread $F$, the strike spread $K$, or both. Here we describe the approach of adjusting the forward spread only. In the Black's model for single-name CDS options, the forward spread $F$ is defined as:

$$
F=S\left(t, t_{E}, T\right)=\frac{S(t, T) R P V 01(t, T)-S\left(t, t_{E}\right) R P V 01\left(t, t_{E}\right)}{R P V 01\left(t, t_{E}, T\right)}
$$

where
$S$ is the spread.
RPV01 is the risky present value of a basis point (see cdsrpv01).
$t$ is the valuation date.
$t_{E}$ is the option expiry date.
$T$ is the CDS maturity date.
To capture the exercise decision correctly for CDS index options, we use the knockout form of the Black's model and adjust the forward spread to incorporate the FEP as follows:

$$
F_{A d j}=F+\frac{F E P}{R P V 01\left(t, t_{E}, T\right)}
$$

where the $F E P$ can be expressed as

$$
F E P=S\left(t, t_{E}\right) R P V 01\left(t, t_{E}\right)
$$

In cdsoptprice, forward spread adjustment can be made with the AdjustedForwardSpread parameter. When computing the adjusted forward spread, we can compute the spreads using cdsspread and the RPV01s using cdsrpv01.

Set up the data for the CDS index, its option, and zero curve. The underlying is a 5 year CDS index maturing on 20 -Jun- 2017 and the option expires on

20-Jun-2012. Note that a flat index spread is assumed when bootstrapping the default probability curve.

```
% CDS index and option data
Recovery = .4;
Basis = 2;
Period = 4;
CDSMaturity = datenum('20-Jun-2017');
ContractSpread = 100;
IndexSpread = 140;
BusDayConvention = 'follow';
Settle = datenum('13-Apr-2012');
OptionMaturity = datenum('20-Jun-2012');
OptionStrike = 140;
SpreadVolatility = .69;
% Zero curve data
MM_Time = [1 2 3 6]';
MM_Rate = [l0.004111 0.00563 0.00757 0.01053]';
MM_Dates = daysadd(Settle,30*MM_Time,1);
Swap_Time = [llllllllllllll}
```



```
    0.02012 0.02132 0.02237 0.02408 0.02564 0.02612 0.02524]';
Swap_Dates = daysadd(Settle,360*Swap_Time,1);
ZeroBasis = 1;
ZeroCompounding = 1;
```

\% Bootstrap the default probability curve assuming a flat index spread.
ZeroData = [MM_Dates MM_Rate; Swap_Dates Swap_Rate];
MarketDates = (Settle+1:30:CDSMaturity)';
MarketData $=$ [MarketDates repmat(IndexSpread,length(MarketDates),1)];
ProbData $=$ cdsbootstrap(ZeroData, MarketData, Settle);

Compute the spot and forward RPV01s, which will be used later in the computation of the FEP and the adjusted forward spread. For this purpose, we can use cdsrpv01.

```
% RPV01(t,T)
RPV01_CDSMaturity = cdsrpv01(ZeroData,ProbData,Settle,CDSMaturity,\ldots.
    'CleanRPV01',false)
```

```
% RPV01(t,t_E,T)
RPV01_OptionExpiryForward = cdsrpv01(ZeroData,ProbData,Settle,CDSMaturity,...
    StartDate',OptionMaturity,'CleanRPV01',false)
% RPV01(t,t_E) = RPV01(t,T) - RPV01(t,t_E,T)
RPV01_OptionExpiry = RPV01_CDSMaturity - RPV01_OptionExpiryForward
RPV01_CDSMaturity =
    4.8523
RPV01_OptionExpiryForward =
    4.5975
RPV01_OptionExpiry =
    0.2549
Compute the spot spreads using cdsspread.
\% S(t,t_E)
Spread_OptionExpiry = cdsspread(ZeroData,ProbData,Settle,OptionMaturity,...
    'Period', Period,'Basis',Basis,'BusDayConvention', BusDayConvention, ...
    'PayAccruedPremium', true, 'ZeroCompounding', ...
    ZeroCompounding,'ZeroBasis',ZeroBasis,'recoveryrate',Recovery)
% S(t,T)
Spread_CDSMaturity = cdsspread(ZeroData,ProbData,Settle,CDSMaturity,...
    'Period',Period, 'Basis',Basis, 'BusDayConvention' ,BusDayConvention, ...
    'PayAccruedPremium',true,'ZeroCompounding',...
    ZeroCompounding,'ZeroBasis',ZeroBasis,'recoveryrate',Recovery)
Spread OptionExpiry =
    140.0005
```

Spread_CDSMaturity =
140.0000

The spot spreads and RPV01s are then used to compute the forward spread.

```
% F = S(t,t_E,T)
ForwardSpread = (Spread_CDSMaturity.*RPV01_CDSMaturity - ...
    Spread_OptionExpiry.*RPV01_OptionExpiry)./RPV01_OptionExpiryForward
ForwardSpread =
```

    139.9999
    Compute the front-end protection (FEP).

```
FEP = Spread_OptionExpiry * (cdsrpv01(ZeroData,ProbData,Settle,CDSMaturity)...
    - cdsrpv01(ZeroData,ProbData,Settle,CDSMaturity,'StartDate',OptionMaturity))
FEP =
```

26.3461

Compute the adjusted forward spread.
AdjustedForwardSpread = ForwardSpread + FEP./RPV01_OptionExpiryForward
AdjustedForwardSpread =
145.7305

Compute the option prices using cdsoptprice with the adjusted forward spread. Note again that the Knockout parameter should be set to be true because the FEP was already incorporated into the adjusted forward spread.

```
[Payer,Receiver] = cdsoptprice(ZeroData, ProbData, Settle, OptionMaturity, ...
    CDSMaturity, OptionStrike, SpreadVolatility,'Knockout',true,...
    'AdjustedForwardSpread', AdjustedForwardSpread, 'PayAccruedPremium',true) ;
fprintf(' Payer: %.Of Receiver: %.Of \n',Payer,Receiver);
Payer: 92 Receiver: 66
```


## References

O’Kane, D., Modelling Single-name and Multi-name Credit Derivatives, Wiley, 2008.

## See Also cdsoptprice | cdsspread | cdsrpv01

## Interest-Rate Curve Objects

- "Interest-Rate Curve Objects" on page 9-2
- "Creating Interest-Rate Curve Objects" on page 9-4
- "Creating an IRDataCurve Object" on page 9-6
- "Bootstrapping a Swap Curve" on page 9-13
- "Dual Curve Bootstrapping" on page 9-16
- "Creating an IRFunctionCurve Object" on page 9-22
- "Fitting Interest Rate Curve Functions" on page 9-34
- "Converting an IRDataCurve or IRFunctionCurve Object" on page 9-40


## Interest-Rate Curve Objects

## In this section...

"Class Structure" on page 9-2
"Workflow Using Interest-Rate Curve Objects" on page 9-3

## Class Structure

Financial Instruments Toolbox class structure supports interest-rate curve objects. The class structure supports five classes.

| Class Name | Description |
| :--- | :--- |
| "@IRCurve" on page A-4 | Base abstract class for interest-rate curves. <br> IRCurve is an abstract class; you cannot <br> create instances of it directly. You can create <br> IRFunctionCurve and IRDataCurve objects <br> that are derived from this class. |
| "@IRDataCurve" on page <br> A-7 | Creates a representation of an interest-rate <br> curve with dates and data. IRDataCurve is <br> constructed directly by specifying dates and <br> corresponding interest rates or discount factors, <br> or you can bootstrap an IRDataCurve object <br> from market data. |
| "@IRFunctionCurve" on <br> page A-12 | Creates a representation of an interest-rate <br> curve with a function. IRFunctionCurve is <br> constructed directly by specifying a function <br> handle, or you can fit a function to market data <br> using methods of the IRFunctionCurve object. |
| "@IRBootstrapOptions" | The IRBootstrapOptions object lets you specify <br> options relating to the bootstrapping of an <br> IRDataCurve object. |
| on page A-2 | The IRFitOptions object lets you specify <br> options relating to the fitting process for an <br> IRFunctionCurve object. |
| AIRFitOptions" on page |  |

## Workflow Using Interest-Rate Curve Objects

The supported workflow model for using interest-rate curve objects is:
1 Create an interest-rate curve based on an IRDataCurve object or an IRFunctionCurve object.

- To create an IRDataCurve object:
- Use vectors of dates and data with interpolation methods.
- Use bootstrapping based on market instruments.
- To create an IRFunctionCurve object:
- Specify a function handle.
- Fit a function using the Nelson-Siegel model, Svensson model, or smoothing spline model.
- Fit a custom function.

2 Use methods of the IRDataCurve or IRFunctionCurve objects to extract forward, zero, discount factor, or par yield curves for the interest-rate curve object.

3 Convert an interest-rate curve from an IRDataCurve or IRFunctionCurve object to a RateSpec structure. This RateSpec structure is identical to the RateSpec produced by the Financial Instruments Toolbox function intenvset. Using the RateSpec for an interest-rate curve object, you can then use Financial Instruments Toolbox functions to model an interest-rate structure and price.

## Creating Interest-Rate Curve Objects

Depending on your data and purpose for analysis, you can create an interest-rate curve object by using an IRDataCurve or IRFunctionCurve object.

To create an IRDataCurve object, you can:

- Use the IRDataCurve constructor.
- Use the IRDataCurve method bootstrap.

Using an IRDataCurve object, you can use the following methods to determine:

- Forward rate curve - getForwardRates
- Zero rate curve - getZeroRates
- Discount rate curve - getDiscountFactors
- Par yield curve - getParYields

Alternatively, to create an IRFunctionCurve object, you can:

- Use the IRFunctionCurve constructor and directly specify a function handle.
- Use IRFunctionCurve methods:
- fitNelsonSiegel fits a "Fitting IRFunctionCurve Object Using Nelson-Siegel Method" on page 9-23 to market data for bonds.
- fitSvensson fits a "Fitting IRFunctionCurve Object Using Svensson Method" on page 9-25 to market data for bonds.
- fitSmoothingSpline fits a "Fitting IRFunctionCurve Object Using Smoothing Spline Method" on page 9-27 function to market data for bonds.
- fitFunction custom fits an interest-rate curve object to market data for bonds.

Using an IRFunctionCurve object, you can use the following method to determine:

- Forward rate curve - getForwardRates
- Zero rate curve - getZeroRates
- Discount rate curve - getDiscountFactors
- Par yield curve - getParYields

In addition, you can convert an IRDataCurve or IRFunctionCurve to a RateSpec structure. For more information, see "Converting an IRDataCurve or IRFunctionCurve Object" on page 9-40.

# Creating an IRDataCurve Object 

In this section...<br>"IRDataCurve Constructor with Dates and Data" on page 9-6<br>"IRDataCurve Bootstrapping Based on Market Instruments" on page 9-7

## IRDataCurve Constructor with Dates and Data

Use the IRDataCurve constructor with vectors of dates and data to create an interest-rate curve object. When constructing the IRDataCurve object, you can also use optional inputs to define how the interest-rate curve is constructed from the dates and data.

## Example

In this example, you create the vectors for Dates and Data for an interest-rate curve:

```
Data = [2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100;
Dates = daysadd(today,[360 2*360 3*360 5*360 7*360 10*360 20*360 30*360],1);
```

Use the IRDataCurve constructor to build interest-rate objects based on the constant and pchip interpolation methods:

```
irdc_const = IRDataCurve('Forward',today,Dates,Data,'InterpMethod','constant');
irdc_pchip = IRDataCurve('Forward',today,Dates,Data,'InterpMethod','pchip');
```

Plot the forward and zero rate curves for the two IRDataCurve objects based on constant and pchip interpolation methods:

```
PlottingDates = daysadd(today,180:10:360*30,1);
plot(PlottingDates,irdc_const.getForwardRates(PlottingDates),'b')
hold on
plot(PlottingDates,irdc_pchip.getForwardRates(PlottingDates), 'r')
plot(PlottingDates,irdc_const.getZeroRates(PlottingDates), 'g')
plot(PlottingDates,irdc_pchip.getZeroRates(PlottingDates), 'yellow')
legend({'Constant Forward Rates','PCHIP Forward Rates','Constant Zero Rates',...
'PCHIP Zero Rates'},'location','SouthEast')
title('Interpolation methods for IRDataCurve objects')
```

The plot demonstrates the relationship of the forward and zero rate curves.


## IRDataCurve Bootstrapping Based on Market Instruments

Use the bootstrapping method, based on market instruments, to create an interest-rate curve object. When bootstrapping, you also have the option to define a range of interpolation methods (linear, spline, constant, and pchip).

## Example 1

In this example, you bootstrap a swap curve from deposits, Eurodollar Futures and swaps. The input market data for this example is hard-coded and specified as two cell arrays of data; one cell array indicates the type of instrument and the other contains the Settle, Maturity values and a market
quote for the instrument. For deposits and swaps, the quote is a rate; for the EuroDollar futures, the quote is a price. Although bonds are not used in this example, a bond would also be quoted with a price.

```
InstrumentTypes = {'Deposit';'Deposit';'Deposit';'Deposit';'Deposit'; ...
    'Futures';'Futures'; ...
    'Futures';'Futures';'Futures'; ...
    'Futures';'Futures';'Futures'; ...
    'Futures';'Futures';'Futures'; ...
    'Futures';'Futures';'Futures'; ...
    'Futures';'Futures';'Futures'; ...
    'Swap';'Swap';'Swap';'Swap';'Swap';'Swap';'Swap'};
Instruments = [datenum('08/10/2007'),datenum('08/17/2007'),.0532063; ...
    datenum('08/10/2007'), datenum('08/24/2007'),.0532000; ...
datenum('08/10/2007'), datenum('09/17/2007'),.0532000; ...
datenum('08/10/2007'), datenum('10/17/2007'),.0534000; ...
datenum('08/10/2007'), datenum('11/17/2007'),.0535866; ...
datenum('08/08/2007'),datenum('19-Dec-2007'),9485; ...
datenum('08/08/2007'), datenum('19-Mar-2008'),9502; ...
datenum('08/08/2007'), datenum('18-Jun-2008'),9509.5; ...
datenum('08/08/2007'), datenum('17-Sep-2008'),9509; ...
datenum('08/08/2007'), datenum('17-Dec-2008'),9505.5; ...
datenum('08/08/2007'),datenum('18-Mar-2009'),9501; ...
datenum('08/08/2007'), datenum('17-Jun-2009'),9494.5; ...
datenum('08/08/2007'),datenum('16-Sep-2009'),9489; ...
datenum('08/08/2007'), datenum('16-Dec-2009'),9481.5; ...
datenum('08/08/2007'), datenum('17-Mar-2010'),9478; ...
datenum('08/08/2007'), datenum('16-Jun-2010'),9474; ...
datenum('08/08/2007'), datenum('15-Sep-2010'),9469.5; ...
datenum('08/08/2007'), datenum('15-Dec-2010'),9464.5; ...
datenum('08/08/2007'), datenum('16-Mar-2011'),9462.5; ...
datenum('08/08/2007'), datenum('15-Jun-2011'),9456.5; ...
datenum('08/08/2007'), datenum('21-Sep-2011'),9454; ...
datenum('08/08/2007'), datenum('21-Dec-2011'),9449.5; ...
datenum('08/08/2007'),datenum('08/08/2014'),.0530; ...
datenum('08/08/2007'),datenum('08/08/2017'),.0545; ...
datenum('08/08/2007'), datenum('08/08/2019'),.0551; ...
datenum('08/08/2007'), datenum('08/08/2022'),.0559; ...
datenum('08/08/2007'),datenum('08/08/2027'),.0565; ...
```

```
datenum('08/08/2007'),datenum('08/08/2032'),.0566; ...
datenum('08/08/2007'),datenum('08/08/2037'),.0566];
```

The bootstrap method is called as a static method of the "@IRDataCurve" on page A-7 class. Inputs to this method include the curve Type (zero or forward), Settle date, InstrumentTypes, and Instrument data. The bootstrap method also supports optional arguments, including an interpolation method, compounding, basis, and an options structure for bootstrapping. For example, you are passing in an "@IRBootstrapOptions" on page A-2 object which includes information for the ConvexityAdjustment to forward rates.

```
IRsigma = .01;
CurveSettle = datenum('08/10/2007');
bootModel = IRDataCurve.bootstrap('Forward', CurveSettle, ...
InstrumentTypes, Instruments,'InterpMethod','pchip',...
'Compounding',-1,'IRBootstrapOptions',...
IRBootstrapOptions('ConvexityAdjustment',@(t) .5*IRsigma^2.*t.^2))
bootModel =
IRDataCurve
```

```
    Type: Forward
    Settle: 733264 (10-Aug-2007)
    Compounding: -1
    Basis: O (actual/actual)
InterpMethod: pchip
    Dates: [29x1 double]
    Data: [29x1 double]
```

The bootstrap method uses an Optimization Toolbox function to solve for any bootstrapped rates.

Plot the forward and zero curves:

```
PlottingDates = (CurveSettle+20:30:CurveSettle+365*25)';
TimeToMaturity = yearfrac(CurveSettle,PlottingDates);
BootstrappedForwardRates = bootModel.getForwardRates(PlottingDates);
BootstrappedZeroRates = bootModel.getZeroRates(PlottingDates);
```

```
figure
hold on
plot(TimeToMaturity,BootstrappedForwardRates,'r')
plot(TimeToMaturity,BootstrappedZeroRates,'g')
title('Bootstrapped Curve')
xlabel('Time')
legend({'Forward','Zero'})
```

The plot demonstrates the forward and zero rate curves for the market data.


## Example 2

In this example, you bootstrap a swap curve from deposits, Eurodollar futures and swaps. The input market data for this example is hard-coded and specified as two cell arrays of data; one cell array indicates the type of instrument and the other cell array contains the Settle, Maturity values and a market quote for the instrument. This example of bootstrapping also demonstrates the use of an InstrumentBasis for each Instrument type:

```
InstrumentTypes = {'Deposit';'Deposit';...
'Futures';'Futures';'Futures';'Futures';'Futures';'Futures';...
'Swap';'Swap';'Swap';'Swap';};
Instruments = [datenum('08/10/2007'),datenum('09/17/2007'),.0532000; ...
datenum('08/10/2007'), datenum('11/17/2007'),.0535866; ...
datenum('08/08/2007'), datenum('19-Dec-2007'),9485; ...
datenum('08/08/2007'), datenum('19-Mar-2008'),9502; ...
datenum('08/08/2007'),datenum('18-Jun-2008'),9509.5; ...
datenum('08/08/2007'), datenum('17-Sep-2008'),9509; ...
datenum('08/08/2007'), datenum('17-Dec-2008'),9505.5; ...
datenum('08/08/2007'), datenum('18-Mar-2009'),9501; ...
datenum('08/08/2007'),datenum('08/08/2014'),.0530; ...
datenum('08/08/2007'),datenum('08/08/2019'),.0551; ...
datenum('08/08/2007'), datenum('08/08/2027'),.0565; ...
datenum('08/08/2007'),datenum('08/08/2037'),.0566];
CurveSettle = datenum('08/10/2007');
```

The bootstrap method is called as a static method of the "@IRDataCurve" on page A-7 class. Inputs to this method include the curve Type (zero or forward), Settle date, InstrumentTypes, and Instrument data. The bootstrap method also supports optional arguments, including an interpolation method, compounding, basis, and an options structure for bootstrapping. In this example, you are passing an additional Basis value for each instrument type:
bootModel=IRDataCurve.bootstrap('Forward', CurveSettle, InstrumentTypes, ...
Instruments, 'InterpMethod', 'pchip', 'InstrumentBasis', [repmat (2, 8, 1) ; repmat (0, 4, 1)] )
bootModel =

IRDataCurve

```
    Type: Forward
    Settle: 733264 (10-Aug-2007)
    Compounding: 2
    Basis: O (actual/actual)
InterpMethod: pchip
    Dates: [12x1 double]
```


## Data: [12x1 double]

The bootstrap method uses an Optimization Toolbox function to solve for any bootstrapped rates.

Plot the par yields curve using the getParYields method:

```
PlottingDates = (datenum('08/11/2007'):30:CurveSettle+365*25)';
plot(PlottingDates,bootModel.getParYields(PlottingDates),'r')
datetick
```

The plot demonstrates the par yields curve for the market data.


## Bootstrapping a Swap Curve

This example shows how to bootstrap an interest rate curve, often referred to as a swap curve, using the new IRDataCurve object. The static bootstrap method takes as inputs a cell array of market instruments (which can be deposits, interest rate futures, swaps, and bonds) and bootstraps an interest rate curve of either the forward or the zero curve. It is also possible to specify multiple interpolation methods, including piecewise constant, linear, and Piecewise Cubic Hermite Interpolating Polynomial (PCHIP)

## Obtain Data

A curve is bootstrapped from market data. In this example, we will bootstrap a swap curve from deposits, Eurodollar Futures and swaps.

For this example, we have hard-coded the input market data, which is simply specified as 2 cell arrays of data, one which indicates the type of instrument and a second cell array containing the Settle, Maturity, and Market Quote for the instrument. For deposits and swaps, the quote is a rate and for the EuroDollar Futures, the quote is a price. Although bonds are not used in this example, a bond would be quoted with a price.

```
InstrumentTypes = {'Deposit';'Deposit';'Deposit';'Deposit';'Deposit'; ...
    'Futures';'Futures'; ...
    'Futures';'Futures';'Futures'; ...
    'Futures';'Futures';'Futures'; ...
    'Futures';'Futures';'Futures'; ...
    'Futures';'Futures';'Futures'; ...
    'Futures';'Futures';'Futures'; ...
    'Swap';'Swap';'Swap';'Swap';'Swap';'Swap';'Swap'};
Instruments = [datenum('08/10/2007'),datenum('08/17/2007'),.0532063; ...
    datenum('08/10/2007'), datenum('08/24/2007'),.0532000; ...
    datenum('08/10/2007'), datenum('09/17/2007'),.0532000; ..
    datenum('08/10/2007'), datenum('10/17/2007'),.0534000; ..
    datenum('08/10/2007'), datenum('11/17/2007'),.0535866; ...
    datenum('08/08/2007'), datenum('19-Dec-2007'),9485; ...
    datenum('08/08/2007'), datenum('19-Mar-2008'),9502; ...
    datenum('08/08/2007'), datenum('18-Jun-2008'),9509.5; ...
    datenum('08/08/2007'), datenum('17-Sep-2008'),9509; ...
```

```
datenum('08/08/2007'), datenum('17-Dec-2008'),9505.5; ...
datenum('08/08/2007'),datenum('18-Mar-2009'),9501; ...
datenum('08/08/2007'), datenum('17-Jun-2009'),9494.5; ...
datenum('08/08/2007'), datenum('16-Sep-2009'),9489; ...
datenum('08/08/2007'), datenum('16-Dec-2009'),9481.5; ...
datenum('08/08/2007'), datenum('17-Mar-2010'),9478; ...
datenum('08/08/2007'), datenum('16-Jun-2010'),9474; ...
datenum('08/08/2007'), datenum('15-Sep-2010'),9469.5; ...
datenum('08/08/2007'), datenum('15-Dec-2010'),9464.5; ...
datenum('08/08/2007'), datenum('16-Mar-2011'),9462.5; ...
datenum('08/08/2007'), datenum('15-Jun-2011'),9456.5; ...
datenum('08/08/2007'), datenum('21-Sep-2011'),9454; ....
datenum('08/08/2007'), datenum('21-Dec-2011'),9449.5; ...
datenum('08/08/2007'), datenum('08/08/2014'),.0530; ...
datenum('08/08/2007'), datenum('08/08/2017'),.0545; ...
datenum('08/08/2007'), datenum('08/08/2019'),.0551; ...
datenum('08/08/2007'), datenum('08/08/2022'),.0559; ...
datenum('08/08/2007'),datenum('08/08/2027'),.0565; ...
datenum('08/08/2007'), datenum('08/08/2032'),.0566; ...
datenum('08/08/2007'), datenum('08/08/2037'),.0566];
```


## Construct the Curve via Bootstrapping

The bootstrap method is called as a static method of the IRDataCurve class. Inputs to this method include the curve type (Zero or Forward), settle date, instrument types, instrument data and optional arguments including an interpolation method, compounding and an options structure for bootstrapping. Note that in this example, we are passing in an IRBootstrapOptions object which includes information for the convexity adjustment to forward rates.

```
IRsigma = .01;
CurveSettle = datenum('08/10/2007');
bootModel = IRDataCurve.bootstrap('Forward', CurveSettle, ...
    InstrumentTypes, Instruments,'InterpMethod','pchip',...
    'Compounding',-1,'IRBootstrapOptions',...
    IRBootstrapOptions('ConvexityAdjustment',@(t) .5*IRsigma^2.*t.^2));
```


## Plot

We can now plot both the forward and zero curves

```
PlottingDates = (CurveSettle+20:30:CurveSettle+365*25)';
TimeToMaturity = yearfrac(CurveSettle,PlottingDates);
BootstrappedForwardRates = bootModel.getForwardRates(PlottingDates);
BootstrappedZeroRates = bootModel.getZeroRates(PlottingDates);
figure
hold on
plot(TimeToMaturity,BootstrappedForwardRates,'r')
plot(TimeToMaturity,BootstrappedZeroRates,'g')
title('Bootstrapped Curve')
xlabel('Time')
legend({'Forward','Zero'})
```


## Bibliography

This example draws from the following papers and journal articles:
[1] Hagan, P., West, G. (2006), "Interpolation Methods for Curve Construction", Applied Mathematical Finance, Vol 13, No. 2
[2] Ron, Uri(2000), "A Practical Guide to Swap Curve Construction", Working Papers 00-17, Bank of Canada.

## Dual Curve Bootstrapping

This example shows how to bootstrap a forward curve using a different curve for discounting.

## Define the Data

Data is needed for both the forward and discount curve. For this particular example, it is assumed that the data is provided for EONIA (the discount curve) and EURIBOR (the forward curve). However, this approach can be used in any case where the curve to be built is different than the curve used for discounting cash flows. While the data in this example is hardcoded, it could also be imported into MATLAB with Datafeed Toolbox ${ }^{\mathrm{TM}}$ or Database Toolbox ${ }^{\mathrm{TM}}$.

```
Settle = datenum('20-Aug-2013');
% Deposit data
EONIADepositRates = [.0007 .00067]';
EONIADepositMat = datenum({'3-Sep-2013','20-Sep-2013'});
EONIADepositBasis = 2; % act/360
EONIADepositPeriod = 0;
```

\% FRA
EONIAFRARates $=[.00025$. 0003 . 00043 .00054]';
EONIAFRAStartDate $=$ datenum(\{'11-Sep-2013', '9-Oct-2013', '13-Nov-2013', '11-Dec-2013'\});
EONIAFRAEndDate = datenum(\{'9-Oct-2013', '13-Nov-2013', '11-Dec-2013', '11-Jan-2013'\});
EONIAFRABasis $=2$; $\%$ act $/ 360$
EONIAFRAPeriod $=0$;

```
% Swap data
EONIASwapRates = [.0003 .001 .002 .004 .008 .012 .0155 .018 .0193 .02]';
EONIASwapMat = datemnth(Settle,12*[2:5 7 10 15 20 25 30]');
EONIASwapBasis = 5; % 30/360 ISDA
EONIASwapPeriod = 1;
% EURIBOR Deposit data
EURIBORDepositRates = [.0022 .0021 .002 .0019]';
EURIBORDepositMat = datenum({'3-Sep-2013','20-Sep-2013','21-Oct-2013','20-Nov-2013'});
EURIBORDepositBasis = 2; % act/360
```

```
EURIBORDepositPeriod = 0;
% EURIBOR Futures
EURIBORFRARates = [9982 9978 9976 9975]';
EURIBORFRAStartDate = datenum({'18-Dec-2013','19-Mar-2014','18-Jun-2014','17-Sep-2014'});
EURIBORFRAEndDate = datenum({'18-Mar-2014','19-Jun-2014','18-Sep-2014','17-Dec-2014'});
EURIBORFRABasis = 2; % act/360
EURIBORFRAPeriod = 4
% EURIBOR Swap data
EURIBORSwapRates = [.0026 . 0044 . 0062 . 0082 .012 .015 . 018 . 02 .021 .0215]';
EURIBORSwapMat = datemnth(Settle,12*[2:5 7 10 15 20 25 30]');
EURIBORSwapBasis = 5; % 30/360 ISDA
EURIBORSwapPeriod = 1;
```


## Construct a EONIA Discount Curve

Build the EONIA curve. This is essentially the same as the single curve case.

```
CurveType = 'zero';
CurveCompounding = 1;
CurveBasis = 3; % act/365
nEONIADeposits = length(EONIADepositMat);
nEONIAFRA = length(EONIAFRAEndDate);
nEONIASwaps = length(EONIASwapMat);
EONIAInstrumentTypes = [repmat({'deposit'},nEONIADeposits,1);
    repmat({'fra'},nEONIAFRA,1);repmat({'swap'},nEONIASwaps,1)];
EONIAPeriod = [repmat(EONIADepositPeriod,nEONIADeposits,1);
    repmat(EONIAFRAPeriod,nEONIAFRA,1);repmat(EONIASwapPeriod,nEONIASwaps,1)];
EONIABasis = [repmat(EONIADepositBasis,nEONIADeposits,1);
    repmat(EONIAFRABasis,nEONIAFRA,1);repmat(EONIASwapBasis,nEONIASwaps,1)];
EONIAInstrumentData = [[repmat(Settle,[nEONIADeposits 1]);EONIAFRAStartDate;repmat(Settle,[nEONIASwaps 1])]
    [EONIADepositMat;EONIAFRAEndDate;EONIASwapMat] ...
    [EONIADepositRates;EONIAFRARates;EONIASwapRates]];
```

```
EONIACurve = IRDataCurve.bootstrap(CurveType,Settle,EONIAInstrumentTypes,...
    EONIAInstrumentData,'Compounding',CurveCompounding,'Basis',CurveBasis,...
    'InstrumentPeriod',EONIAPeriod,'InstrumentBasis',EONIABasis
EONIACurve =
    Type: zero
    Settle: 735466 (20-Aug-2013)
    Compounding: 1
    Basis: 3 (actual/365)
InterpMethod: linear
    Dates: [16x1 double]
    Data: [16x1 double]
```


## Construct a EURIBOR Forward Curve

The EURIBOR forward curve is built first using a single curve approach.

```
nEURIBORDeposits = length(EURIBORDepositMat);
nEURIBORFRA = length(EURIBORFRAEndDate);
nEURIBORSwaps = length(EURIBORSwapMat);
EURIBORInstrumentTypes = [repmat({'deposit'},nEURIBORDeposits,1);
    repmat({'futures'},nEURIBORFRA,1);repmat({'swap'},nEURIBORSwaps,1)];
EURIBORPeriod = [repmat(EURIBORDepositPeriod,nEURIBORDeposits,1);
    repmat(EURIBORFRAPeriod,nEURIBORFRA,1);repmat(EURIBORSwapPeriod,nEURIBORSwaps,1)];
EURIBORBasis = [repmat(EURIBORDepositBasis,nEURIBORDeposits,1);
    repmat(EURIBORFRABasis,nEURIBORFRA,1);repmat(EURIBORSwapBasis,nEURIBORSwaps,1)];
EURIBORInstrumentData = [repmat(Settle,size(EURIBORInstrumentTypes)) ...
    [EURIBORDepositMat;EURIBORFRAEndDate;EURIBORSwapMat] ...
    [EURIBORDepositRates;EURIBORFRARates;EURIBORSwapRates]];
EURIBORCurve_Single = IRDataCurve.bootstrap(CurveType,Settle,EURIBORInstrumentTypes,...
    EURIBORInstrumentData,'Compounding',CurveCompounding,'Basis',CurveBasis,...
    'InstrumentPeriod', EURIBORPeriod,'InstrumentBasis', EURIBORBasis)
```

EURIBORCurve_Single =

```
    Type: zero
    Settle: 735466 (20-Aug-2013)
    Compounding: 1
    Basis: 3 (actual/365)
InterpMethod: linear
    Dates: [18x1 double]
    Data: [18x1 double]
```


## Build the EURIBOR Curve with the EONIA Curve

Next, build a curve using the EONIA curve as a discounting curve. To do this, specify the EONIA curve as an optional input argument.

```
EURIBORCurve = IRDataCurve.bootstrap(CurveType,Settle,EURIBORInstrumentTypes,...
    EURIBORInstrumentData,'DiscountCurve',EONIACurve,'Compounding',...
    CurveCompounding,'Basis',CurveBasis,'InstrumentPeriod',EURIBORPeriod,...
    'InstrumentBasis',EURIBORBasis)
EURIBORCurve =
    Type: zero
    Settle: 735466 (20-Aug-2013)
    Compounding: 1
    Basis: 3 (actual/365)
    InterpMethod: linear
    Dates: [18x1 double]
    Data: [18x1 double]
```


## Plot the Results

Plot the results to compare the curves.

```
PlottingDates = (Settle+20:30:Settle+365*30)';
TimeToMaturity = yearfrac(Settle,PlottingDates);
figure
plot(TimeToMaturity,EONIACurve.getZeroRates(PlottingDates),'b')
hold on
plot(TimeToMaturity,EURIBORCurve_Single.getZeroRates(PlottingDates),'r')
plot(TimeToMaturity,EURIBORCurve.getZeroRates(PlottingDates),'g')
title('Comparison of Single Curve and Dual Curve Bootstrapping')
```

```
legend({'EONIA','EURIBOR','EURIBOR w/ EONIA Discounting'},'location','south
```



As expected, the difference between the two different EURIBOR curves is small but nontrivial.

## Bibliography

This example draws from the following papers and journal articles:
[1] Ametrano, F and Bianchetti, M.,Everything You Always Wanted to Know About Multiple Interest Rate Curve Bootstrapping but Were Afraid to Ask, (April 2, 2013), available at: http://ssrn.com/abstract=2219548.
[2] Bianchetti, M., Two Curves, One Price, Risk Magazine, pages 74-80, August 2010.
[3] Fujii, M , Shimada, Y, Takahashi, A.,A Note on Construction of Multiple Swap Curves with and without Collateral, (January 2, 2010), CARF Working Paper Series No. CARF-F-154, available at: http://ssrn.com/abstract=1440633.
[4] Mercurio, Fabio, Interest Rates and The Credit Crunch: New Formulas and Market Models, (February 5, 2009), Bloomberg Portfolio Research Paper No. 2010-01-FRONTIERS.
[5] Nashikkar, A., Understanding OIS Discounting, Barclays Capital Interest Rate Strategy, February 24, 2011.

See Also
IRDataCurve | bootstrap | floatbyzero | swapbyzero | getZeroRates

## Creating an IRFunctionCurve Object

In this section...<br>"Fitting IRFunctionCurve Object Using a Function Handle" on page 9-22 "Fitting IRFunctionCurve Object Using Nelson-Siegel Method" on page 9-23 "Fitting IRFunctionCurve Object Using Svensson Method" on page 9-25 "Fitting IRFunctionCurve Object Using Smoothing Spline Method" on page 9-27<br>"Using fitFunction to Create Custom Fitting Function" on page 9-30

## Fitting IRFunctionCurve Object Using a Function Handle

You can use the constructor IRFunctionCurve with a MATLAB function handle to define an interest-rate curve. For more information on defining a function handle, see the MATLAB Programming Fundamentals documentation.

## Example

This example uses a FunctionHandle argument with a value @(t) t.^2 to construct an interest-rate curve:

```
rr = IRFunctionCurve('Zero',today,@(t) t.^2)
rr =
    Properties:
        FunctionHandle: @(t)t.^2
            Type: 'Zero'
            Settle: 733600
        Compounding: 2
            Basis: 0
```


## Fitting IRFunctionCurve Object Using Nelson-Siegel Method

Use the method, fitNelsonSiegel, for the Nelson-Siegel model that fits the empirical form of the yield curve with a prespecified functional form of the spot rates which is a function of the time to maturity of the bonds.

The Nelson-Siegel model represents a dynamic three-factor model: level, slope, and curvature. However, the Nelson-Siegel factors are unobserved, or latent, which allows for measurement error, and the associated loadings have economic restrictions (forward rates are always positive, and the discount factor approaches zero as maturity increases). For more information, see "Zero-coupon yield curves: technical documentation," BIS Papers, Bank for International Settlements, Number 25, October, 2005.

## Example

This example uses IRFunctionCurve to model the default-free term structure of interest rates in the United Kingdom.

Load the data:
load ukdata20080430
Convert repo rates to be equivalent zero coupon bonds:

```
RepoCouponRate = repmat(0,size(RepoRates));
RepoPrice = bndprice(RepoRates, RepoCouponRate, RepoSettle, RepoMaturity);
```

Aggregate the data:

```
Settle = [RepoSettle;BondSettle];
Maturity = [RepoMaturity;BondMaturity];
CleanPrice = [RepoPrice;BondCleanPrice];
CouponRate = [RepoCouponRate;BondCouponRate];
Instruments = [Settle Maturity CleanPrice CouponRate];
InstrumentPeriod = [repmat (0,6,1);repmat(2,31,1)];
CurveSettle = datenum('30-Apr-2008');
```

The IRFunctionCurve object provides the capability to fit a Nelson-Siegel curve to observed market data with the fitNelsonSiegel method. The fitting
is done by calling the Optimization Toolbox function lsqnonlin. This method has required inputs of Type, Settle, and a matrix of instrument data.

NSModel = IRFunctionCurve.fitNelsonSiegel('Zero', CurveSettle,...
Instruments, 'Compounding', -1, 'InstrumentPeriod', InstrumentPeriod);
Plot the Nelson-Siegel interest-rate curve for forward rates:

```
PlottingDates = CurveSettle+20:30:CurveSettle+365*25;
TimeToMaturity = yearfrac(CurveSettle,PlottingDates);
NSForwardRates = NSModel.getForwardRates(PlottingDates);
plot(TimeToMaturity,NSForwardRates)
title('Nelson Siegel model of UK instantaneous nominal forward curve')
```



## Fitting IRFunctionCurve Object Using Svensson Method

Use the method, fitSvensson, for the Svensson model to improve the flexibility of the curves and the fit for a Nelson-Siegel model. In 1994, Svensson extended Nelson and Siegel's function by adding a further term that allows for a second "hump." The extra precision is achieved at the cost of adding two more parameters, $\beta 3$ and $\tau 2$, which have to be estimated.

## Example

In this example of using the fitSvensson method, an IRFitOptions structure, previously defined using the IRFitOptions constructor, is used. Thus, you must specify FitType, InitialGuess, UpperBound, LowerBound, and the OptOptions optimization parameters for lsqnonlin.

Load the data:
load ukdata20080430

Convert repo rates to be equivalent zero coupon bonds:

```
RepoCouponRate = repmat(0,size(RepoRates));
RepoPrice = bndprice(RepoRates, RepoCouponRate, RepoSettle, RepoMaturity);
```

Aggregate the data:
Settle = [RepoSettle; BondSettle];

```
Maturity = [RepoMaturity;BondMaturity];
```

CleanPrice = [RepoPrice;BondCleanPrice];
CouponRate = [RepoCouponRate;BondCouponRate];
Instruments = [Settle Maturity CleanPrice CouponRate];
InstrumentPeriod $=[\operatorname{repmat}(0,6,1) ; \operatorname{repmat}(2,31,1)]$;
CurveSettle = datenum('30-Apr-2008');

Define OptOptions for the IRFitOptions constructor:

```
OptOptions = optimoptions('lsqnonlin','MaxFunEvals',1000);
fIRFitOptions = IRFitOptions([5.82 -2.55 -. 87 0.45 3.9 0.44],...
'FitType','durationweightedprice','OptOptions',OptOptions,...
'LowerBound',[0 -Inf -Inf -Inf 0 0],'UpperBound',[Inf Inf Inf Inf Inf Inf]);
```

Fit the interest-rate curve using a Svensson model:

```
SvenssonModel = IRFunctionCurve.fitSvensson('Zero',CurveSettle,...
Instruments,'IRFitOptions',fIRFitOptions, 'Compounding',-1,...
'InstrumentPeriod',InstrumentPeriod);
Local minimum possible.
lsqnonlin stopped because the final change in the sum of squares relative to
its initial value is less than the default value of the function tolerance.
```

The status message, output from lsqnonlin, indicates that the optimization to find parameters for the Svensson equation terminated successfully.

Plot the Svensson interest-rate curve for forward rates:

```
PlottingDates = CurveSettle+20:30:CurveSettle+365*25;
TimeToMaturity = yearfrac(CurveSettle,PlottingDates);
SvenssonForwardRates = SvenssonModel.getForwardRates(PlottingDates);
plot(TimeToMaturity,SvenssonForwardRates)
title('Svensson model of UK instantaneous nominal forward curve')
```



## Fitting IRFunctionCurve Object Using Smoothing Spline Method

Use the method, fitSmoothingSpline, to model the term structure with a spline, specifically, the term structure represents the forward curve with a cubic spline.

Note You must have a license for Curve Fitting Toolbox software to use the fitSmoothingSpline method.

## Example

The IRFunctionCurve object is used to fit a smoothing spline representation of the forward curve with a penalty function. Required inputs are Type, Settle, the matrix of Instruments, and Lambdafun, a function handle containing the penalty function

Load the data:
load ukdata20080430

Convert repo rates to be equivalent zero coupon bonds:

```
RepoCouponRate = repmat(0,size(RepoRates));
RepoPrice = bndprice(RepoRates, RepoCouponRate, RepoSettle, RepoMaturity);
```

Aggregate the data:

```
Settle = [RepoSettle;BondSettle];
Maturity = [RepoMaturity;BondMaturity];
CleanPrice = [RepoPrice;BondCleanPrice];
CouponRate = [RepoCouponRate;BondCouponRate];
Instruments = [Settle Maturity CleanPrice CouponRate];
InstrumentPeriod = [repmat(0,6,1);repmat(2,31,1)];
CurveSettle = datenum('30-Apr-2008');
```

Choose parameters for Lambdafun:

```
L = 9.2;
S = -1;
mu = 1;
```

Define the Lambdafun penalty function:

```
lambdafun = @(t) exp(L - (L-S)*exp(-t/mu));
t = 0:.1:25;
y = lambdafun(t);
figure
semilogy(t,y);
title('Penalty Function for VRP Approach')
ylabel('Penalty')
xlabel('Time')
```



Use the fitSmoothinSpline method to fit the interest-rate curve and model the Lambdafun penalty function:

VRPModel = IRFunctionCurve.fitSmoothingSpline('Forward',CurveSettle,...
Instruments, lambdafun, 'Compounding',-1, 'InstrumentPeriod', InstrumentPeriod);

Plot the smoothing spline interest-rate curve for forward rates:

```
PlottingDates = CurveSettle+20:30:CurveSettle+365*25;
TimeToMaturity = yearfrac(CurveSettle,PlottingDates);
VRPForwardRates = VRPModel.getForwardRates(PlottingDates);
plot(TimeToMaturity,VRPForwardRates)
title('Smoothing Spline model of UK instantaneous nominal forward curve')
```



## Using fitFunction to Create Custom Fitting Function

When using an IRFunctionCurve object, you can create a custom fitting function with the fitFunction method. To use fitFunction, you must define a FunctionHandle. In addition, you must also use the constructor IRFitOptions to define IRFitOptionsObj to support an InitialGuess for the parameters of the curve function.

## Example

The following example demonstrates the use of fitFunction with a FunctionHandle and an IRFitOptionsObj:

```
Settle = repmat(datenum('30-Apr-2008'),[6 1]);
Maturity = [datenum('07-Mar-2009');datenum('07-Mar-2011');...
datenum('07-Mar-2013');datenum('07-Sep-2016');...
datenum('07-Mar-2025');datenum('07-Mar-2036')];
```

```
CleanPrice = [100.1;100.1;100.8;96.6;103.3;96.3];
CouponRate = [0.0400;0.0425;0.0450;0.0400;0.0500;0.0425];
Instruments = [Settle Maturity CleanPrice CouponRate];
CurveSettle = datenum('30-Apr-2008');
```

Define the FunctionHandle:

```
functionHandle = @(t,theta) polyval(theta,t);
```

Define the OptOptions for IRFitOptions:
OptOptions = optimoptions('lsqnonlin','display','iter');

## Define fitFunction:

```
CustomModel = IRFunctionCurve.fitFunction('Zero', CurveSettle, ...
functionHandle,Instruments, IRFitOptions([.05 .05 .05],'FitType','price',...
'OptOptions',OptOptions));
```

| Iteration | Func-count | $f(x)$ | Norm of step | First-order optimality | CG-iterations |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 4 | 38036.7 |  | $4.92 \mathrm{e}+04$ |  |
| 1 | 8 | 38036.7 | 10 | $4.92 \mathrm{e}+04$ | 0 |
| 2 | 12 | 38036.7 | 2.5 | $4.92 \mathrm{e}+04$ | 0 |
| 3 | 16 | 38036.7 | 0.625 | $4.92 \mathrm{e}+04$ | 0 |
| 4 | 20 | 38036.7 | 0.15625 | $4.92 \mathrm{e}+04$ | 0 |
| 5 | 24 | 30741.5 | 0.0390625 | $1.72 \mathrm{e}+05$ | 0 |
| 6 | 28 | 30741.5 | 0.078125 | $1.72 \mathrm{e}+05$ | 0 |
| 7 | 32 | 30741.5 | 0.0195312 | $1.72 \mathrm{e}+05$ | 0 |
| 8 | 36 | 28713.6 | 0.00488281 | $2.33 \mathrm{e}+05$ | 0 |
| 9 | 40 | 20323.3 | 0.00976562 | $9.47 \mathrm{e}+05$ | 0 |
| 10 | 44 | 20323.3 | 0.0195312 | $9.47 \mathrm{e}+05$ | 0 |
| 11 | 48 | 20323.3 | 0.00488281 | $9.47 \mathrm{e}+05$ | 0 |
| 12 | 52 | 20323.3 | 0.0012207 | $9.47 \mathrm{e}+05$ | 0 |
| 13 | 56 | 19698.8 | 0.000305176 | $1.08 \mathrm{e}+06$ | 0 |
| 14 | 60 | 17493 | 0.000610352 | $7 \mathrm{e}+06$ | 0 |
| 15 | 64 | 17493 | 0.0012207 | $7 \mathrm{e}+06$ | 0 |
| 16 | 68 | 17493 | 0.000305176 | $7 \mathrm{e}+06$ | 0 |
| 17 | 72 | 15455.1 | 7.62939e-05 | $2.25 \mathrm{e}+07$ | 0 |
| 18 | 76 | 15455.1 | 0.000177499 | $2.25 \mathrm{e}+07$ | 0 |
| 19 | 80 | 13317.1 | 3.8147e-05 | $3.18 \mathrm{e}+07$ | 0 |


| 20 | 84 | 12865.3 | $7.62939 \mathrm{e}-05$ | $7.83 \mathrm{e}+07$ | 0 |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 21 | 88 | 11779.8 | $7.62939 \mathrm{e}-05$ | $7.58 \mathrm{e}+06$ | 0 |
| 22 | 92 | 11747.6 | 0.000152588 | $1.45 \mathrm{e}+05$ | 0 |
| 23 | 96 | 11720.9 | 0.000305176 | $2.33 \mathrm{e}+05$ | 0 |
| 24 | 100 | 11667.2 | 0.000610352 | $1.48 \mathrm{e}+05$ | 0 |
| 25 | 104 | 11558.6 | 0.0012207 | $3.55 \mathrm{e}+05$ | 0 |
| 26 | 108 | 11335.5 | 0.00244141 | $1.57 \mathrm{e}+05$ | 0 |
| 27 | 112 | 10863.8 | 0.00488281 | $6.36 \mathrm{e}+05$ | 0 |
| 28 | 116 | 9797.14 | 0.00976562 | $2.53 \mathrm{e}+05$ | 0 |
| 29 | 120 | 6882.83 | 0.0195312 | $9.18 \mathrm{e}+05$ | 0 |
| 30 | 124 | 6882.83 | 0.0373993 | $9.18 \mathrm{e}+05$ | 0 |
| 31 | 128 | 3218.45 | 0.00934981 | $1.96 \mathrm{e}+06$ | 0 |
| 32 | 132 | 612.703 | 0.0186996 | $3.01 \mathrm{e}+06$ | 0 |
| 33 | 136 | 13.0998 | 0.0253882 | $3.05 \mathrm{e}+06$ | 0 |
| 34 | 140 | 0.0762922 | 0.00154002 | $5.05 \mathrm{e}+04$ | 0 |
| 35 | 144 | 0.0731652 | $3.61102 \mathrm{e}-06$ | 29.9 | 0.063 |

Local minimum possible.
lsqnonlin stopped because the final change in the sum of squares relative to its initial value is less than the default value of the function tolerance.

Plot the custom function that is defined using fitFunction:

```
Yields = bndyield(CleanPrice,CouponRate,Settle(1),Maturity);
scatter(Maturity,Yields);
PlottingPoints = min(Maturity):30:max(Maturity);
hold on;
plot(PlottingPoints,CustomModel.getParYields(PlottingPoints), 'r');
datetick
legend('Market Yields','Fitted Yield Curve')
title('Custom Function fit to Market Data')
```



## Fitting Interest Rate Curve Functions

This example shows how to use objects to model the term structure of interest rates (also referred to as the yield curve). This can be contrasted with modeling the term structure with vectors of dates and data and interpolating between the points (which can currently be done with the function PRBYZERO).

The term structure can refer to at least three different curves: the discount curve, zero curve, or forward curve.

The new object IRFunctionCurve allows the user to model an interest rate curve as a function.

This example will explore using IRFunctionCurve to model the default-free term structure of interest rates in the United Kingdom. Three different forms for the term structure will be implemented, which will be discussed in more detail later:

- Nelson-Siegel
- Svensson
- Smoothing Cubic Spline with a so-called Variable Roughness Penalty (VRP)


## Choosing the Data

The first question in modeling the yield curve is what data should be used. To model a default-free yield curve, default-free, option-free market instruments must be used. The most significant component of the data set will be UK Government Bonds (known as Gilts). Historical data was retrieved from the following site:
http://www.dmo.gov.uk
Repo data will be used to construct the short end of the curve. Repo data was retrieved from the following site:
http://www.bba.org.uk

Note also that the data must be specified as a matrix where the columns are Settle, Maturity, CleanPrice and CouponRate -- and that instruments must be bonds or synthetically converted to bonds.

Market data for a close date of April 30, 2008, has been downloaded and saved to the following data file, which can be loaded into MATLAB® with the following command

```
% Load the data
load ukdata20080430
```

```
% Convert repo rates to be equivalent zero coupon bonds
RepoCouponRate = repmat(0,size(RepoRates));
RepoPrice = bndprice(RepoRates, RepoCouponRate, RepoSettle, RepoMaturity);
% Aggregate the data
Settle = [RepoSettle;BondSettle];
Maturity = [RepoMaturity;BondMaturity];
CleanPrice = [RepoPrice;BondCleanPrice];
CouponRate = [RepoCouponRate;BondCouponRate];
Instruments = [Settle Maturity CleanPrice CouponRate];
InstrumentPeriod = [repmat(0,6,1);repmat(2,31,1)];
CurveSettle = datenum('30-Apr-2008');
```


## Fit Nelson-Siegel Model to Market Data

The Nelson-Siegel model proposes that the instantaneous forward curve can be modeled with the following:

This can be integrated to derive an equation for the zero curve (see [6] for more information on the equations and the derivation):

See [1] for more information.

The IRFunctionCurve object provides the capability to fit a Nelson Siegel curve to observed market data with the FITNELSONSIEGEL method. The fitting is done by calling the Optimization Toolbox ${ }^{\mathrm{TM}}$ function LSQNONLIN.

This method has required inputs: Curve Type, Curve Settle, and a matrix of instrument data.

Optional input arguments, specified in parameter value pairs, are:

- IRFitOptions structure: Provides capability to choose which quantity to be minimized (price, yield, or duration weighted price) and other optimization parameters (e.g.: upper and lower bounds for parameters)
- Curve Compounding and Basis (Day Count Convention)
- Additional instrument parameters, Period, Basis, FirstCouponDate, etc.

```
NSModel = IRFunctionCurve.fitNelsonSiegel('Zero',CurveSettle,...
    Instruments,'InstrumentPeriod',InstrumentPeriod);
```


## Fit Svensson Model

A very similar model to the Nelson-Siegel is the Svensson model, which adds two additional parameters to account for greater flexibility in the term structure. This model proposes that the forward rate can be modeled with the following form:

As above, this can be integrated to derive an equation for the zero curve:

See [2] for more information.
Fitting the parameters to this model proceeds in a similar fashion to the Nelson-Siegel model.

```
SvenssonModel = IRFunctionCurve.fitSvensson('Zero',CurveSettle,...
    Instruments,'InstrumentPeriod',InstrumentPeriod);
```


## Fit Smoothing Spline

The term structure can also be modeled with a spline -- specifically, one way to model the term structure is by representing the forward curve with a cubic spline. To ensure that the spline is sufficiently smooth, a penalty is imposed relating to the curvature (second derivative) of the spline:
where the first term is the difference between the observed price P and the predicted price, P_hat, (weighted by the bond's duration, D) summed over all bonds in our data set and the second term is the penalty term (where lambda is a penalty function and $f$ is the spline)

See [3], [4], [5] below.
There have been different proposals for the specification of the penalty function lambda. One approach, advocated by [4], and currently used by the UK Debt Management Office, is a penalty function of the following form:

The parameters L, S, and mu are typically estimated from historical data.
The IRFunctionCurve object can be used to fit a smoothing spline representation of the forward curve with a penalty function.

Required inputs, like for methods above, are a Curve Type, Curve Settle, Instrument matrix, and new for this method, a function handle containing the penalty function

Optional parameters are similar to FITNELSONSIEGEL and FITSVENSSON

```
% Parameters chosen to be roughly similar to [4] below.
L = 9.2;
S = -1;
mu = 1;
lambdafun = @(t) exp(L - (L-S)*exp(-t/mu)); % Construct penalty function
```

```
t = 0:.1:25; % Construct data to plot penalty function
y = lambdafun(t);
figure
semilogy(t,y);
title('Penalty Function for VRP Approach')
ylabel('Penalty')
xlabel('Time')
VRPModel = IRFunctionCurve.fitSmoothingSpline('Forward',CurveSettle,...
    Instruments,lambdafun, 'Compounding', -1,...
    ' InstrumentPeriod', InstrumentPeriod);
```


## Use Fitted Curves and Plot Results

Once a curve has been constructed, methods can be called to extract the Forward and Zero Rates and the Discount Factors. This curve can also be converted into a RateSpec structure, to be used with functions in the Financial Instruments Toolbox ${ }^{\text {TM }}$

```
PlottingDates = CurveSettle+20:30:CurveSettle+365*25;
TimeToMaturity = yearfrac(CurveSettle,PlottingDates);
NSForwardRates = NSModel.getForwardRates(PlottingDates);
SvenssonForwardRates = SvenssonModel.getForwardRates(PlottingDates);
VRPForwardRates = VRPModel.getForwardRates(PlottingDates);
figure
hold on
plot(TimeToMaturity,NSForwardRates,'r')
plot(TimeToMaturity,SvenssonForwardRates,'g')
plot(TimeToMaturity,VRPForwardRates,'b')
title('UK instantaneous nominal forward curve')
xlabel('Years ahead')
legend({'Nelson Siegel','Svensson','VRP'})
```


## Compare with this Link

This link provides a live look at the derived yield curve published by the UK http://www.bankofengland.co.uk

## Bibliography

This example is based on the following papers and journal articles:
[1] Nelson, C.R., Siegel, A.F., (1987), "Parsimonious modelling of yield curves", Journal of Business, 60, pp 473-89
[2] Svensson, L.E.O. (1994), "Estimating and interpreting forward interest rates: Sweden 1992-4", International Monetary Fund, IMF Working Paper, 1994/114
[3] Fisher, M., Nychka, D., Zervos, D. (1995), "Fitting the term structure of interest rates with smoothing splines", Board of Governors of the Federal Reserve System, Federal Reserve Board Working Paper 95-1
[4] Anderson, N., Sleath, J. (1999), "New estimates of the UK real and nominal yield curves", Bank of England Quarterly Bulletin, November, pp 384-92
[5] Waggoner, D. (1997), "Spline Methods for Extracting Interest Rate Curves from Coupon Bond Prices", Federal Reserve Board Working Paper 97-10
[6] "Zero-coupon yield curves: technical documentation", BIS Papers No. 25 October 2005
[7] Bolder, D.J., Gusba,S (2002), "Exponentials, Polynomials, and Fourier Series: More Yield Curve Modelling at the Bank of Canada," Working Papers 02-29, Bank of Canada
[8] Bolder, D.J., Streliski, D (1999), "Yield Curve Modelling at the Bank of Canada," Technical Reports 84, Bank of Canada

# Converting an IRDataCurve or IRFunctionCurve Object 

In this section...<br>"Introduction" on page 9-40<br>"Using the toRateSpec Method" on page 9-40<br>"Using Vector of Dates and Data Methods" on page 9-42

## Introduction

The IRDataCurve and IRFunctionCurve objects for interest-rate curves support conversion to:

- A RateSpec structure. The RateSpec generated from an IRDataCurve or IRFunctionCurve object, using the toRateSpec method, is identical to the RateSpec structure created with intenvset using Financial Instruments Toolbox software.
- A vector of dates and data from an IRDataCurve object acceptable to prbyzero, bkcall, bkput, tfutbyprice, and tfutbyyield or any function that requires a term structure of interest rates.


## Using the toRateSpec Method

To convert an IRDataCurve or IRFunctionCurve object to a RateSpec structure, you must first create an interest-rate curve object. Then, use the toRateSpec method for an IRDataCurve object or thetoRateSpec method for an IRFunctionCurve object.

## Example

Create a data vector from the following data:
http://www.ustreas.gov/offices/domestic-finance/debt-management/ interest-rate/yield.shtml:


scatter(Dates, Data)
datetick


Create an IRDataCurve interest-rate curve object:
rr = IRDataCurve('Zero',today,Dates,Data);

Convert to a RateSpec:
rr.toRateSpec(today+30:30:today+365)
ans =
FinObj: 'RateSpec'
Compounding: 2
Disc: [12x1 double]
Rates: [12x1 double]
EndTimes: [12x1 double]
StartTimes: [12x1 double]
EndDates: [12x1 double]
StartDates: 733569
ValuationDate: 733569
Basis: 0
EndMonthRule: 1

## Using Vector of Dates and Data Methods

You can use the getZeroRates method for an IRDataCurve object with a Dates property to create a vector of dates and data acceptable for prbyzero in Financial Toolbox software and bkcall, bkput, tfutbyprice, and tfutbyyield in Financial Instruments Toolbox software.

## Example

This is an example of using the IRDataCurve method getZeroRates with prbyzero:

```
Data = [2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100;
Dates = daysadd(today,[360 2*360 3*360 5*360 7*360 10*360 20*360 30*360],1);
irdc = IRDataCurve('Zero',today,Dates,Data,'InterpMethod','pchip');
Maturity = daysadd(today,8*360,1);
CouponRate = .055;
ZeroDates = daysadd(today,180:180:8*360,1);
ZeroRates = irdc.getZeroRates(ZeroDates);
BondPrice = prbyzero([Maturity CouponRate], today, ZeroRates, ZeroDates)
BondPrice =
    113.9221
```


## Numerix Workflows

- "Working with Simple Numerix Trades" on page 10-2
- "Working with Advanced Numerix Trades" on page 10-5
- "Use Numerix to Price Cash Deposits" on page 10-12
- "Use Numerix for Interest-Rate Risk Assessment" on page 10-15


## Working with Simple Numerix Trades

This example shows how to price a callable reverse floater using Numerix ${ }^{\circledR}$ CAIL.

Initialize Numerix environment.

```
import com.numerix.integration.*;
import com.numerix.integration.implementation.*;
n = numerix('i:\NumeriX_java_10_3_0\data')
n =
```

```
    Path: 'i:\NumeriX java 10 3 0\data
```

    Path: 'i:\NumeriX java 10 3 0\data
    RepositoryPath: 'i:\NumeriX_java_10_3_0\data\Repository'
RepositoryPath: 'i:\NumeriX_java_10_3_0\data\Repository'
Repository: [1x1 com.numerix.integration.implementation.FileSystemRepository]
Repository: [1x1 com.numerix.integration.implementation.FileSystemRepository]
Context: [1x1 com.numerix.integration.implementation.LocalCalculationContext]
Context: [1x1 com.numerix.integration.implementation.LocalCalculationContext]
LookupsPath: 'i:\NumeriX_java_10_3_0\data\Data\LookupRules'
LookupsPath: 'i:\NumeriX_java_10_3_0\data\Data\LookupRules'
MarketsPath: 'i:\NumeriX_java_10_3_0\data\Data\Markets'
MarketsPath: 'i:\NumeriX_java_10_3_0\data\Data\Markets'
FixingsPath: 'i:\NumeriX_java_10_3_0\data\Data\Fixings
FixingsPath: 'i:\NumeriX_java_10_3_0\data\Data\Fixings
TradesPath: 'i:\NumeriX_java_10_3_0\data\Data\Trades'
TradesPath: 'i:\NumeriX_java_10_3_0\data\Data\Trades'
Parameters: [1x1 com.numerix.integration.implementation.CalculationParameters]

```
    Parameters: [1x1 com.numerix.integration.implementation.CalculationParameters]
```

Create a market.

```
quotes = java.util.HashMap;
quotes.put('IR.USD-LIBOR-3M.SWAP-1Y.MID', 0.0066056);
quotes.put('IR.USD-LIBOR-3M.SWAP-10Y.MID', 0.022465005);
quotes.put('IR.USD-LIBOR-3M.SWAP-20Y.MID', 0.027544995);
market = Market('EOD_14-NOV-2011', DateExtensions.date('14-Nov-2011'), quotes.entrySet);
```

Define a trade instance for a callable reverse floater based on instrument template located in the Repository.

```
tradeDescriptor = 'TRADE.IR.CALLABLEREVERSEFLOATER';
tradeParameters = java.util.HashMap;
tradeParameters.put('Trade ID','1001');
tradeParameters.put('Quote Type', 'MID');
tradeParameters.put('Currency', 'USD');
tradeParameters.put('Notional', 1000000.0);
```

```
tradeParameters.put('Effective Date', DateExtensions.date('1-Dec-2011'));
tradeParameters.put('Termination Date', DateExtensions.date('1-Dec-2021'));
tradeParameters.put('IR Index', 'LIBOR');
tradeParameters.put('IR Index Tenor', '3M');
tradeParameters.put('Structured Freq', '3M');
tradeParameters.put('Structured Side', 'Receive');
tradeParameters.put('Structured Coupon Floor', 0.0);
tradeParameters.put('Structured Coupon UpBd', 0.08);
tradeParameters.put('StructuredCoupon Multiplier', 1.4);
tradeParameters.put('Structured Coupon Cap', 0.05);
tradeParameters.put('Structured Basis', 'ACT/360');
tradeParameters.put('Funding Freq', '3M');
tradeParameters.put('Funding Side', 'Pay');
tradeParameters.put('Funding Spread', 0.003);
tradeParameters.put('Funding Basis', 'ACT/360');
tradeParameters.put('Call Start Date', DateExtensions.date('1-Dec-2013'));
tradeParameters.put('Call End Date', DateExtensions.date('1-Dec-2020'));
tradeParameters.put('Option Side', 'Short');
tradeParameters.put('Option Type', 'Right to Terminate');
tradeParameters.put('Call Frequency', '3M');
tradeParameters.put('Model', 'IR.USD-LIBOR-3M.MID.DET');
tradeParameters.put('Method', 'BackwardAnalytic');
```

Create the trade instance.
trade = RepositoryExtensions.createTradeInstance(n.Repository, tradeDescriptor, tradeParameters)
Price the trade.
results $=$ CalculationContextExtensions.calculate(n.Context, trade, market, Request.getAll);
Parse the results for MATLAB and display.
$r=n . p a r s e R e s u l t s(r e s u l t s)$
disp([r.Name r.Category r.Currency r.Data])
$r=$

Category: $\{13 \times 1$ cell\}
Currency: $\{13 \times 1$ cell $\}$
Name: \{13x1 cell\}

```
Data: {13x1 cell}
```

| 'Reporting Currency' | 'Price ' | 1 ' | 'USD' |
| :---: | :---: | :---: | :---: |
| 'Structured Cashflow Log' | 'Cashflow' | '' | \{41x20 cell $\}$ |
| 'Structured Leg PV Accrued' | 'Price' | 'USD ' | 0] |
| 'PV' | 'Price ${ }^{\prime}$ | 'USD ' | [ 6.4133e+04] |
| 'Structured Leg PV Clean' | 'Price' | 'USD ' | [ 4.2637e+05] |
| 'Option PV' | 'Price' | 'USD' | [-1.3220e+05] |
| 'Funding Cashflow Log' | 'Cashflow' | ' 1 | \{41x20 cell $\}$ |
| 'Structured Leg PV' | 'Price ' | 'USD ' | [ 4.2637e+05] |
| 'Funding Leg PV' | 'Price ' | ' USD ' | [-2.3004e+05] |
| 'Funding Leg PV Accrued' | 'Price ${ }^{\text {' }}$ | 'USD ' | ] |
| 'Funding Leg PV Clean' | 'Price' | 'USD' | [-2.3004e+05] |
| 'Yield Risk Report' | 1 | '' | \{ $4 \times 30$ cell $\}$ |
| 'Messages' | '1 | '' | \{ $4 \times 1$ cell $\}$ |

## See Also numerix I parseResults

Related - "Working with Advanced Numerix Trades" on page 10-5
Examples

- "Use Numerix to Price Cash Deposits" on page 10-12
- "Use Numerix for Interest-Rate Risk Assessment" on page 10-15


## External

- http://www.numerix.com/cail


## Working with Advanced Numerix Trades

## This example shows how to price multiple trades from MATLAB using

 Numerix CAIL.Initialize Numerix environment.

```
import com.numerix.integration.*;
import com.numerix.integration.implementation.*;
n = numerix('i:\NumeriX_java_10_3_0\data')
n =
Path: 'i:\NumeriX_java_10_3_0\data'
RepositoryPath: 'i:\NumeriX_java_10_3_0\data\Repository'
Repository: [1x1 com.numerix.integration.implementation.FileSystemRepository]
Context: [1x1 com.numerix.integration.implementation.LocalCalculationContext]
LookupsPath: 'i:\NumeriX_java_10_3_0\data\Data\LookupRules'
MarketsPath: 'i:\NumeriX_java_10_3_0\data\Data \Markets'
FixingsPath: 'i:\NumeriX_java_10_3_0\data\Data\Fixings'
TradesPath: 'i:\NumeriX_java_10_3_0\data\Data\Trades'
Parameters: [1x1 com.numerix.integration.implementation.CalculationParameters]
```

Specify the hybrid model for multiple trades.

```
hySpec = HybridModelSpecification;
hySpec.addHW1F('IR-USD', 'USD', 'LIBOR', '3M', 'MeanReversion(0.5),DiagonalSwaption(ATM, 10Y)');
hySpec.addHW1F('IR-EUR', 'EUR', 'EURIBOR', '6M', 'MeanReversion(0.5),DiagonalSwaption(ATM, 10Y)');
hySpec.addFXBlack('FX-USDEUR', 'USD', 'EUR', 'LIBOR', '3M', 'EURIBOR', '6M', 'StrikeFXEuropean(ATM, 10Y)');
%5 Specify the factor correlations.
hyCorrelations = HybridModelCorrelationMatrix(hySpec);
hyCorrelations.add('IR-USD', 'IR-EUR', 0.5);
hyCorrelations.add('IR-USD', 'FX-USDEUR', 0.25);
hyCorrelations.add('IR-EUR', 'FX-USDEUR', 0.25);
```

\% Specify the model parameters.
hybridModelParameters = java.util.HashMap;
hybridModelParameters.put('Quote Type', 'MID');
hybridModelParameters.put('Payout Currency', 'USD');

```
hybridModelParameters.put('Specification', hySpec);
hybridModelParameters.put('Correlations', hyCorrelations);
```


## Specify exposure calculation parameters.

```
observationDates = CustomObservationSchedule;
observationDates.add(DateExtensions.date(2011, 12, 1));
for y = 2012:2013
    for m = 1:12
            observationDates.add(DateExtensions.date(y, m, 1));
    end
end
exposureParameters = java.util.HashMap;
exposureParameters.put('Model ID', 'HYBRID');
exposureParameters.put('Observation Dates', observationDates);
```


## Define the first trade instance.

```
tradeParameters1 = java.util.HashMap;
tradeParameters1.put('Trade ID', 'RVFL1001');
tradeParameters1.put('Quote Type', 'MID');
tradeParameters1.put('Currency', 'USD');
tradeParameters1.put('Notional', 1000000.0);
tradeParameters1.put('Effective Date', DateExtensions.date('1-Dec-2011'));
tradeParameters1.put('Termination Date', DateExtensions.date('1-Dec-2021'));
tradeParameters1.put('IR Index', 'LIBOR');
tradeParameters1.put('IR Index Tenor', '3M');
tradeParameters1.put('Structured Freq', '3M');
tradeParameters1.put('Structured Side', 'Receive');
tradeParameters1.put('Structured Coupon Floor', 0.0);
tradeParameters1.put('Structured Coupon UpBd', 0.08);
tradeParameters1.put('StructuredCoupon Multiplier', 1.4);
tradeParameters1.put('Structured Coupon Cap', 0.05);
tradeParameters1.put('Structured Basis', 'ACT/360');
tradeParameters1.put('Funding Freq', '3M');
tradeParameters1.put('Funding Side', 'Pay');
tradeParameters1.put('Funding Spread', 0.003);
tradeParameters1.put('Funding Basis', 'ACT/360');
tradeParameters1.put('Call Start Date', DateExtensions.date('1-Dec-2013'));
```

```
tradeParameters1.put('Call End Date', DateExtensions.date('1-Dec-2020'));
tradeParameters1.put('Option Side', 'Short');
tradeParameters1.put('Option Type', 'Right to Terminate');
tradeParameters1.put('Call Frequency', '3M');
tradeParameters1.put('Model', 'HYBRID');
tradeParameters1.put('Method', 'BackwardMC');
tradeInstance1 = RepositoryExtensions.createTradeInstance(n.Repository, 'TRADE.IR.CALLABLEREVERSEFLOATER',
```


## Define the second trade instance.

```
tradeParameters2 = java.util.HashMap;
tradeParameters2.put('Trade ID', 'CASHDEP1001');
tradeParameters2.put('Quote Type', 'MID');
tradeParameters2.put('Currency', 'USD');
tradeParameters2.put('Coupon Rate', 0.05);
tradeParameters2.put('Yield', 0.044);
tradeParameters2.put('Notional', 100.0);
tradeParameters2.put('Effective Date', DateExtensions.date('1-Apr-2012'));
tradeParameters2.put('Maturity', DateExtensions.date('1-Apr-2013'));
tradeParameters2.put('IR Index', 'LIBOR');
tradeParameters2.put('IR Index Tenor', '3M');
tradeParameters2.put('Model', 'HYBRID');
tradeParameters2.put('Method', 'BACKWARDMC');
tradeInstance2 = RepositoryExtensions.createTradeInstance(n.Repository, 'IR.CASHDEPOSIT', tradeParameters2);
```

Create the third trade instance.

```
tradeParameters3 = java.util.HashMap;
tradeParameters3.put('Trade ID', 'FXFWD1001');
tradeParameters3.put('Quote Type', 'MID');
tradeParameters3.put('Base Currency', 'USD');
tradeParameters3.put('Term Currency', 'EUR');
tradeParameters3.put('Delivery Date', DateExtensions.date('1-Jun-2012'));
tradeParameters3.put('Contract FX Forward Rate', 80.5);
tradeParameters3.put('Base Notional', 10000000.0);
tradeParameters3.put('Base IR Index', 'LIBOR');
tradeParameters3.put('Term IR Index', 'EURIBOR');
tradeParameters3.put('Base IR Index Tenor', '3m');
tradeParameters3.put('Term IR Index Tenor', '6m');
tradeParameters3.put('Calendar', 'NewYork Target');
```

```
tradeParameters3.put('Spot Lag', '2bd');
tradeParameters3.put('Model', 'HYBRID');
tradeParameters3.put('Method', 'BACKWARDMC');
tradeInstance3 = RepositoryExtensions.createTradeInstance(n.Repository, 'FX.FXFORWARD', tradeParameters3);
```

Set tradeInstances for all three trade instances.

```
tradeInstances = java.util.ArrayList;
tradeInstances.add(tradeInstance1);
tradeInstances.add(tradeInstance2);
tradeInstances.add(tradeInstance3);
n.Parameters.setInstances(tradeInstances);
```

Add a custom lookup so these trade instances reference the hybrid model.

```
n.Parameters.getLookups.add(0,ExactLookupRule('HYBRID','MODEL.HYBRID',hybridModelParameters.entrySet));
```

Add another custom lookup so that exposure report has parameters defined.
n. Parameters.getLookups.add(1, ExactLookupRule('RISK.REPORT.EXPOSURE', 'REPORT.EXPOSURE', exposureParameters.en

Perform the calculation.

```
results = n.Context.calculate(n.Parameters, Request.getExposure);
```

Parse the results for MATLAB and display.

```
r = n.parseResults(results)
disp([r.Trade(2) r.Market(2)])
disp([r.Results{2}.Name r.Results{2}.Category r.Results{2}.Currency r.Results{2}.Data])
disp([r.Results{2}.Name{1}])
disp([r.Results{2}.Data{1}])
r =
    Trade: {3x1 cell}
    Market: {3\times1 cell}
    Results: {3\times1 cell}
'CASHDEP1001' 'EOD'
```

| 'Exposure' | '' | '' | $\{21 \times 501$ | cell $\}$ |
| :--- | :--- | :--- | :--- | :--- |
| 'Exposure.Discount Factors' | '' | ' | $\{21 \times 501$ | cell $\}$ |

Exposure
Columns 1 through 3

| 'DATE' |  |  | 'VALUE 1' | 'VALUE 2' |
| :---: | :---: | :---: | :---: | :---: |
| 'Tue May | 01 13:00:00 | EDT 2012' | [104.198166609924] | [103.386222783828] |
| 'Fri Jun | 01 13:00:00 | EDT 2012' | [ 104.09953599675] | [102.117465067435] |
| 'Sun Jul | 01 13:00:00 | EDT 2012' | [105.524567506006] | [100.055731577867] |
| 'Wed Aug | 01 13:00:00 | EDT 2012' | [105.787455961524] | [100.318762976796] |
| 'Sat Sep | 01 13:00:00 | EDT 2012' | [104.417483614373] | [100.764337265155] |
| 'Mon Oct | 01 13:00:00 | EDT 2012' | [104.692275556824] | [100.980213613911] |
| ' Thu Nov | 01 13:00:00 | EDT 2012' | [ 104.443818312902] | [101.478508725115] |
| 'Sat Dec | 01 12:00:00 | EST 2012' | [104.736646932343] | [101.679769557039] |
| 'Tue Jan | 01 12:00:00 | EST 2013' | [104.577562970494] | [102.423339265735] |
| 'Fri Feb | 01 12:00:00 | EST 2013' | [ 104.28994278039] | [103.117326879887] |
| 'Fri Mar | 01 12:00:00 | EST 2013' | [ 104.70469459715] | [104.232180198939] |
| 'Mon Apr | 01 13:00:00 | EDT 2013' | [ 105.07334321718] | [ 105.05089338769] |
| 'Wed May | 01 13:00:00 | EDT 2013' | [ 0] | 0] |
| 'Sat Jun | 01 13:00:00 | EDT 2013' | [ 0] | ] |
| 'Mon Jul | 01 13:00:00 | EDT 2013' | [ 0] | [ 0] |
| 'Thu Aug | 01 13:00:00 | EDT 2013' | [ 0] | [ 0] |
| 'Sun Sep | 01 13:00:00 | EDT 2013' | [ 0] | [ 0] |
| 'Tue Oct | 01 13:00:00 | EDT 2013' | [ 0] | [ 0] |
| 'Fri Nov | 01 13:00:00 | EDT 2013' | [ 0] | [ 0] |
| 'Sun Dec | 01 12:00:00 | EST 2013' | [ 0] | [ 0] |

Columns 499 through 501

| 'VALUE 498' | 'VALUE 499' | 'VALUE 500' |
| :--- | :--- | :--- |
| $[$ 105.36273206453] | $[104.335982034187]$ | $[104.141595030057]$ |
| $[105.904822463264]$ | $[104.238089023172]$ | $[104.276676080686]$ |


| $[103.893060436208]$ | $[103.613968079212]$ | $[106.188617261199]$ |
| :--- | :--- | :--- |
| $[103.183889382889]$ | $[105.499763150412]$ | $[105.440275818983]$ |
| $[103.310404527817]$ | $[105.233622768447]$ | $[105.267337892552]$ |
| $[103.274239052394]$ | $[104.716952177783]$ | $[104.33099834332]$ |
| $[103.583983117053]$ | $[104.710250522521]$ | $[105.501004542869]$ |
| $[103.379982561438]$ | $[105.146939039653]$ | $[104.681616459661]$ |
| $[103.821169954095]$ | $[105.567274949306]$ | $[104.835971977691]$ |
| $[104.016530403399]$ | $[105.254054161819]$ | $[104.842156238753]$ |
| $[104.481475787501]$ | $[105.197179985119]$ | $[104.962752610848]$ |
| $[105.061984636083]$ | $[105.077227736476]$ | $[105.077766765965]$ |
| $[$ | $0]$ | $[$ |
| $[$ | $0]$ | $[$ |

Plot the results for the second trade instance, CASHDEP1001, with the corresponding Exposure Discount Factors.

```
figure('Tag','NumerixAdvancedRiskExample');
for ii=1:3
    % Get dates
    dates = cell2mat(r.Results{ii}.Data{expIndex}(2:end,1));
    dates = dates(:,4:end);
    dates = floor(datenum(dates));
    % Get exposures
    mtm = cell2mat(r.Results{ii}.Data{expIndex}(2:end,2:end))';
    exposures = max(0,mtm); % Exposure at contract level, no netting
    EE = mean(exposures); % Expected Exposure
    PFE = prctile(exposures,95); % Potential Future Exposure
    subplot(3,1,ii)
    plot(dates,EE,dates,PFE)
    title(r.Trade{ii})
    datetick
end
```


## See Also

## Related <br> Examples

## External Web Sites


numerix | parseResults

- "Working with Simple Numerix Trades" on page 10-2
- "Use Numerix to Price Cash Deposits" on page 10-12
- "Use Numerix for Interest-Rate Risk Assessment" on page 10-15
- http://www.numerix.com/cail


## Use Numerix to Price Cash Deposits

This example shows how to use the Numerix CAIL API to price a cash deposit from MATLAB. The trade parameters are read from the Cashdeposit1.csv in the Numerix Data Trades directory.

Initialize Numerix environment.

```
import com.numerix.integration.*;
import com.numerix.integration.implementation.*;
n = numerix('i:\NumeriX_java_10_3_0\data')
n =
```

```
            Path: 'i:\NumeriX_java_10_3_0\data
```

            Path: 'i:\NumeriX_java_10_3_0\data
    RepositoryPath: 'i:\NumeriX_java_10_3_0\data\Repository'
    RepositoryPath: 'i:\NumeriX_java_10_3_0\data\Repository'
        Repository: [1x1 com.numerix.integration.implementation.FileSystemRepository]
        Repository: [1x1 com.numerix.integration.implementation.FileSystemRepository]
            Context: [1x1 com.numerix.integration.implementation.LocalCalculationContext]
            Context: [1x1 com.numerix.integration.implementation.LocalCalculationContext]
        LookupsPath: 'i:\NumeriX_java_10_3_0\data\Data\LookupRules
        LookupsPath: 'i:\NumeriX_java_10_3_0\data\Data\LookupRules
        MarketsPath: 'i:\NumeriX_java_10_3_0\data\Data\Markets'
        MarketsPath: 'i:\NumeriX_java_10_3_0\data\Data\Markets'
        FixingsPath: 'i:\NumeriX_java_10_3_0\data\Data\Fixings
        FixingsPath: 'i:\NumeriX_java_10_3_0\data\Data\Fixings
        TradesPath: 'i:\NumeriX_java_10_3_0\data\Data\Trades'
        TradesPath: 'i:\NumeriX_java_10_3_0\data\Data\Trades'
        Parameters: [1x1 com.numerix.integration.implementation.CalculationParameters]
    ```
        Parameters: [1x1 com.numerix.integration.implementation.CalculationParameters]
```

Create a market.
market $=$ Market('EOD_16-APR-2012', DateExtensions.date('16-APR-2012'), []);
Read the Cashdeposit1.csv file from the Numerix Trades directory.
[~,~,tradeInfo] = xlsread([n.TradesPath '\Cashdeposit1.csv'])
tradeInfo $=$

| 'Template' | 'String' | 'TRADE.IR.CASHDEPOSIT' |
| :--- | :--- | :--- |
| 'Trade ID' | 'ID' | 'CASHDEP1001' |
| 'Quote Type' | 'String' | 'MID' |
| 'Effective Date' | 'Date' | '4/1/2012' |
| 'Maturity' | 'Date' | '4/1/2013' |
| 'Notional' | 'Double' | [ |


| 'Currency ' | 'Currency' | 'USD' |  |
| :--- | :--- | :--- | :--- |
| 'Coupon Rate' | 'Double' | [ | $0.0500]$ |
| 'Yield' | 'Double' | [ | $0.0440]$ |
| 'IR Index' | 'String' | 'Libor' |  |
| 'IR Index Tenor' | 'Tenor' | '3m' |  |

Define a trade instance from the imported CASHDEP1001 instrument.

```
tradeDescriptor = tradeInfo{1,3};
tradeParameters = java.util.HashMap;
numTradeInfoFields = size(tradeInfo,1);
for i = 2:numTradeInfoFields
    switch tradeInfo{i,2}
        case {'DATE','Date'}
            tradeParameters.put(tradeInfo{i,1},DateExtensions.date(datestr(tradeInfo{i,3},'dd-mmm-yyyy')));
        otherwise
            tradeParameters.put(tradeInfo{i,1},tradeInfo{i,3});
    end
end
```

Create the trade instance.
trade $=$ RepositoryExtensions.createTradeInstance(n.Repository, tradeDescriptor, tradeParameters);

Price the trade.

```
results = CalculationContextExtensions.calculate(n.Context, trade, market, Request.getAll);
```

Parse the results for MATLAB and display.

```
r = n.parseResults(results)
disp([r.Name r.Category r.Currency r.Data])
```

$r=$
Category: \{9x1 cell\}
Currency: \{9x1 cell\}
Name: \{9x1 cell\}
Data: \{9x1 cell\}
'Modified Duration' 'Price' '' [ 0.9349]

| 'Accrued Interest' | 'Price' | 'USD' | [ 0.2083$]$ |
| :--- | :--- | :--- | :--- |
| 'Reporting Currency' | 'Price' | '' | 'USD' |
| 'PV' | 'Price' | 'USD' | [ 100.7607] |
| 'Instrument' | 'Price' | '' | [1x85 char] |
| 'Clean Price' | 'Price' | 'USD' | [ 100.5524] |
| 'Convexity' | 'Price' | '' | [ |
| 'YTM' | 1.7481] |  |  |
| 'Messages' | 'Price' | '' |  |

## See Also

numerix | parseResults

## Related <br> Examples

- "Working with Simple Numerix Trades" on page 10-2
- "Working with Advanced Numerix Trades" on page 10-5
- "Use Numerix for Interest-Rate Risk Assessment" on page 10-15

External Web Sites

## Use Numerix for Interest-Rate Risk Assessment

This example shows how to use the Numerix CAIL API for interest-rate curve stripping for risk assessment.

Initialize Numerix environment.

```
import com.numerix.integration.*;
import com.numerix.integration.implementation.*;
n = numerix('i:\NumeriX_java_10_3_0\data')
n =
```

```
            Path: 'i:\NumeriX_java_10_3_0\data
```

            Path: 'i:\NumeriX_java_10_3_0\data
    RepositoryPath: 'i:\NumeriX_java_10_3_0\data\Repository'
    RepositoryPath: 'i:\NumeriX_java_10_3_0\data\Repository'
        Repository: [1x1 com.numerix.integration.implementation.FileSystemRepository]
        Repository: [1x1 com.numerix.integration.implementation.FileSystemRepository]
            Context: [1x1 com.numerix.integration.implementation.LocalCalculationContext]
            Context: [1x1 com.numerix.integration.implementation.LocalCalculationContext]
    LookupsPath: 'i:\NumeriX_java_10_3_0\data\Data\LookupRules
    LookupsPath: 'i:\NumeriX_java_10_3_0\data\Data\LookupRules
    MarketsPath: 'i:\NumeriX_java_10_3_0\data\Data\Markets
    MarketsPath: 'i:\NumeriX_java_10_3_0\data\Data\Markets
    FixingsPath: 'i:\NumeriX_java_10_3_0\data\Data\Fixings
    FixingsPath: 'i:\NumeriX_java_10_3_0\data\Data\Fixings
    TradesPath: 'i:\NumeriX java_10_3_0\data\Data\Trades'
    TradesPath: 'i:\NumeriX java_10_3_0\data\Data\Trades'
    Parameters: [1x1 com.numerix.integration.implementation.CalculationParameters]
    ```
    Parameters: [1x1 com.numerix.integration.implementation.CalculationParameters]
```

Specify the current market associated with the Numerix CAIL environment.

```
markets = get(n.Parameters,'Markets');
currentMarket = markets.get(0);
outInstance = RefObject(currentMarket);
```

Define the interest-rate curve key IR.USD-LIBOR-3M.MID.
n. Context.tryResolveId('IR.USD-LIBOR-3M.MID', outInstance);
currentInstance = outInstance.argvalue;
Set the instance and market.
n. Parameters.setMarkets(java.util.Arrays.asList(currentMarket));
n. Parameters.setInstances(java.util.Arrays.asList(currentInstance)) ;

Calculate the interest-rate curve stripping.

```
results = n.Context.calculate(n.Parameters,Request.getAll);
```


## The calculation returns the results from stripping the interest-rate curve for

 IR.USD-LIBOR-3M.MID. Parse the results for MATLAB and display.\% IR.USD-LIBOR-3M.MID.
$r=n . p a r s e R e s u l t s(r e s u l t s)$
disp([r.Instance r.Market])
disp([r.Results\{1\}.Name r.Results\{1\}.Category r.Results\{1\}.Currency r.Results\{1\}.Data])
disp([r.Results\{1\}.Name\{1\}])
disp([r.Results\{1\}.Data\{1\}])
$r=$

Instance: \{'IR.USD-LIBOR-3M.MID'\}
Market: \{'EOD'\}
Results: \{[1x1 struct]\}

IR.USD-LIBOR-3M.MID' 'EOD'

| 'Curve Info' | '' | '' | $\{30 \times 3$ cell $\}$ |
| :--- | :--- | :--- | :--- |
| 'Messages | '' | '' | $\{7 \times 1$ cell $\}$ |

Curve Info

| ' CASH | RATE | 16-APR-2012 | 17-APR-2012' |
| :---: | :---: | :---: | :---: |
| ' CASH | RATE | 16-APR-2012 | 18-APR-2012' |
| ' CASH | RATE | 16-APR-2012 | 23-APR-2012' |
| ' CASH | RATE | 16-APR-2012 | 30-APR-2012' |
| ' CASH | RATE | 16-APR-2012 | 16-MAY-2012' |
| ' CASH | RATE | 16-APR-2012 | 18-JUN-2012' |
| ' CASH | RATE | 16-APR-2012 | 16-JUL-2012' |
| ' CASH | RATE | 18-MAY-2012 | 12-AUG-2012' |
| ' CASH | RATE | 20-JUN-2012 | 20-SEP-2012' |
| ' CASH | RATE | 18-JUL-2012 | 18-OCT-2012' |
| ' CASH | RATE | 15-AUG-2012 | 15-NOV-2012' |
| ' CASH | RATE | 19-SEP-2012 | 19-DEC-2012' |
| ' CASH | RATE | 17-OCT-2012 | 17-JAN-2013' |



10-16


## See Also numerix I parseResults

Related - "Working with Simple Numerix Trades" on page 10-2
Examples

- "Working with Advanced Numerix Trades" on page 10-5
- "Use Numerix to Price Cash Deposits" on page 10-12

External

- http://www.numerix.com/cail Web Sites


## Class Reference

- "@IRBootstrapOptions" on page A-2
- "@IRCurve" on page A-4
- "@IRDataCurve" on page A-7
- "@IRFitOptions" on page A-10
- "@IRFunctionCurve" on page A-12


## @IRBootstrapOptions

Create specific options for bootstrapping an interest-rate curve object

```
In this section...
"Hierarchy" on page A-2
"Constructor" on page A-2
"Public Read-Only Properties" on page A-2
"Methods" on page A-3
```


## Hierarchy

Superclasses: None
Subclasses: None

## Constructor

IRBootstrapOptions

## Public Read-Only Properties

| Name | Description |
| :--- | :--- |
| ConvexityAdjustment | Controls the convexity adjustment to interest <br> rate futures. This can be specified as a <br> function handle that takes time to maturity <br> as an input and returns a value which <br> is ConvexityAdjustment. Alternatively, <br> you can define ConvexityAdjustment <br> as an N-by-1 vector of values, where N is <br> the number of interest rate futures. In <br> either case, the ConvexityAdjustment is <br> subtracted from the futures rate. |
|  | For more information on defining a function <br> handle, see the MATLAB Programming <br> Fundamentals documentation. |

## Methods

There are no methods.

## @IRCurve

Base abstract class for interest-rate curve objects

```
In this section...
"Hierarchy" on page A-4
"Description" on page A-4
"Constructor" on page A-4
"Public Read-Only Properties" on page A-4
"Methods" on page A-6
```


## Hierarchy

Superclasses: None
Subclasses: @IRDataCurve, @IRFunctionCurve

## Description

IRCurve is an abstract class; you cannot create instances of it directly. You can create IRDataCurve and IRFunctionCurve objects that are derived from this class.

## Constructor

@IRCurve is an abstract class. To construct an IRCurve object, use one of the subclass constructors, IRDataCurve or IRFunctionCurve.

## Public Read-Only Properties

| Name | Description |
| :--- | :--- |
| Type | Type of interest-rate curve: zero, forward, or discount. |
| Settle | Scalar for the Settle date of the curve. |


| Name | Description |
| :---: | :---: |
| Compounding | Scalar that sets the compounding frequency per year for the IRCurve object: <br> - $-1=$ Continuous compounding <br> - $1=$ Annual compounding <br> - $2=$ Semiannual compounding (default) <br> - $3=$ Compounding three times per year <br> - 4 = Quarterly compounding <br> - $6=$ Bimonthly compounding <br> - $12=$ Monthly compounding |
| Basis | Day-count basis of the interest-rate curve. A vector of integers. <br> - $0=$ actual/actual (default) <br> - $1=30 / 360$ (SIA) <br> - 2 = actual/360 <br> - $3=$ actual/365 <br> - $4=30 / 360$ (BMA) <br> - $5=30 / 360$ (ISDA) <br> - $6=30 / 360$ (European) <br> - 7 = actual/365 (Japanese) <br> - $8=$ actual/actual (ICMA) <br> - $9=$ actual/360 (ICMA) <br> - $10=$ actual/365 (ICMA) <br> - $11=30 / 360 \mathrm{E}$ (ICMA) <br> - $12=$ actual/actual (ISDA) <br> - 13 = BUS/252 |

## Methods

Classes that inherit from the IRCurve abstract class must implement the following methods.

| Method | Description |
| :--- | :--- |
| getForwardRates | Returns forward rates for input dates. |
| getZeroRates | Returns zero rates for input dates. |
| getDiscountFactors | Returns discount factors for input dates. |
| getParYields | Returns par yields for input dates. |
| toRateSpec | Converts to be a RateSpec object. This is <br> identical to the RateSpec structure produced <br> by the Financial Instruments Toolbox function <br> intenvset. |

## @IRDataCurve

Represent interest-rate curve object based on vector of dates and data

```
In this section...
"Hierarchy" on page A-7
"Description" on page A-7
"Constructor" on page A-7
"Public Read-Only Properties" on page A-8
"Methods" on page A-9
```


## Hierarchy

Superclasses: @IRCurve

Subclasses: None

## Description

IRDataCurve is a representation of an interest-rate curve object with dates and data. You can construct this object directly by specifying dates and corresponding interest rates or discount factors; alternatively, you can bootstrap the object from market data. After an interest-rate curve object is constructed, you can:

- Calculate forward and zero rates and determine par yields.
- Extract the discount factors.
- Convert to a RateSpec structure that is identical to the RateSpec structure produced by the Financial Instruments Toolbox function intenvset.


## Constructor

IRDataCurve

## Public Read-Only Properties

| Name | Description |
| :---: | :---: |
| Type | Type of interest-rate curve: zero, forward, or discount. |
| Settle | Scalar for the Settle date of the curve. |
| Compounding | Scalar that sets the compounding frequency per year for the IRCurve object: <br> - $-1=$ Continuous compounding <br> - 1 = Annual compounding <br> - $2=$ Semiannual compounding (default) <br> - 3 = Compounding three times per year <br> - 4 = Quarterly compounding <br> - 6 = Bimonthly compounding <br> - $12=$ Monthly compounding |
| Basis | Day-count basis of the financial curve. A vector of integers. <br> - $0=$ actual/actual (default) <br> - $1=30 / 360$ (SIA) <br> - 2 = actual/360 <br> - 3 = actual/365 <br> - $4=30 / 360$ (BMA) <br> - $5=30 / 360$ (ISDA) <br> - $6=30 / 360$ (European) <br> - $7=$ actual/365 (Japanese) <br> - $8=$ actual/actual (ICMA) <br> - $9=$ actual/360 (ICMA) <br> - 10 = actual/365 (ICMA) <br> - $11=30 / 360 \mathrm{E}$ (ICMA) |


| Name | Description |
| :--- | :--- |
|  | • $12=$ actual/actual (ISDA) |
|  | $\bullet 13=$ BUS/252 |
| Dates | Dates corresponding to rate data. |
| Data | Interest-rate data or discount factors for the curve object. |
| InterpMethod | Values are: |
|  | - 'linear ' — Linear interpolation (default). |
|  | - 'constant - Piecewise constant interpolation. |
|  | - 'pchip' — Piecewise cubic Hermite interpolation. |
|  | - 'spline ' - Cubic spline interpolation. |

## Methods

The following table contains links to methods with supporting reference pages, including examples.

| Method | Description |
| :--- | :--- |
| getForwardRates | Returns forward rates for input dates. |
| getZeroRates | Returns zero rates for input dates. |
| getDiscountFactors | Returns discount factors for input dates. |
| getParYields | Returns par yields for input dates. |
| toRateSpec | Converts to be a RateSpec object. This <br> structure is identical to the RateSpec |
| bootstrap | Bootstraps an interest rate curve from market <br> data. |

## @IRFitOptions

Object to specify fitting options for an IRFunctionCurve interest-rate curve object

## In this section...

"Hierarchy" on page A-10
"Description" on page A-10
"Constructor" on page A-10
"Public Read-Only Properties" on page A-11
"Methods" on page A-11

## Hierarchy

## Superclasses: None

Subclasses: None

## Description

The IRFitOptions object allows you to specify options relating to the fitting process for an IRFunctionCurve object. Input arguments are specified in parameter/value pairs. The IRFitOptions structure provides the capability to choose which quantity to be minimized and other optimization parameters.

## Constructor

[^2]
## Public Read-Only Properties

| Name | Description |
| :--- | :--- |
| FitType | Price, Yield, or DurationWeightedPrice <br> determines which is minimized in the curve fitting <br> process. DurationWeightedPrice is the default. |
| InitialGuess | Initial guess for the parameters of the curve <br> function. |
| UpperBound | Upper bound for the parameters of the curve <br> function. |
| LowerBound | Lower bound for the parameters of the curve <br> function. |
| OptOptions | Optimization structure based on the output from <br> the Optimization Toolbox function optimset or <br> optimoptions. This optimization structure is <br> evaluated by lsqnonlin. |
| A | Inequality constraint for parameters, ignored if <br> OptimFunction is set to lsqnonlin. |
| b | Inequality constraint for parameters, ignored if <br> OptimFunction is set to lsqnonlin. |
| OptimFunction | Optimization function used to fit function, either <br> lsqnonlin or fmincon. |

## Methods

There are no methods.

## @IRFunctionCurve

Represent an interest-rate curve object using a function

## In this section...

"Hierarchy" on page A-12
"Description" on page A-12
"Constructor" on page A-12
"Public Read-Only Properties" on page A-13
"Methods" on page A-14

## Hierarchy

Superclasses: @IRCurve
Subclasses: None

## Description

IRFunctionCurve is a representation of an interest-rate curve object. You can construct this object directly by specifying a function handle or a function can be fit to market data using methods of the object. After an interest-rate curve object is constructed; you can:

- Calculate forward and zero rates and determine par yields.
- Extract the discount factors.
- Convert to a RateSpec structure; this is identical to the RateSpec structure produced by the Financial Instruments Toolbox function intenvset.


## Constructor

IRFunctionCurve

## Public Read-Only Properties

| Name | Description |
| :--- | :--- |
| Type | Type of interest-rate curve: zero, forward, or <br> discount. |
| Settle | Scalar for the Settle date of the curve. |
| Compounding | Scalar that sets the compounding frequency per year <br> for the IRCurve object: |
|  | - $1=$ Continuous compounding |
|  | - $1=$ Annual compounding |
|  | • $2=$ Semiannual compounding (default) |
|  | - $3=$ Compounding three times per year |
|  | - $4=$ Quarterly compounding |
|  | - $6=$ Bimonthly compounding |
| - $12=$ Monthly compounding |  |


| Name | Description |
| :--- | :--- |
|  | • $11=30 / 360 \mathrm{E}$ (ICMA) |
|  | $\bullet 12=$ actual/actual (ISDA) |
|  | $\bullet 13=\mathrm{BUS} / 252$ |$\quad$| FunctionHandle | Function handle that defines the interest-rate curve. <br> For more information on defining a function handle, <br> see the MATLAB Programming Fundamentals <br> documentation. |
| :--- | :--- |
| Parameters | Fitted parameters for function. |

## Methods

The following table contains links to methods with supporting reference pages, including examples.

| Method | Description |
| :--- | :--- |
| getForwardRates | Returns forward rates for input dates. |
| getZeroRates | Returns zero rates for input dates. |
| getDiscountFactors | Returns discount factors for input dates. |
| getParYields | Returns par yields for input dates. |
| toRateSpec | Converts to be a RateSpec object. This <br> is identical to the RateSpec structure |
| fitSvensson | Fits a Svensson function to market data. |
| fitNelsonSiegel | Fits a Nelson-Siegel function to market <br> data. |
| fitSmoothingSpline | Fits a smoothing spline function to <br> market data. |
| fitFunction | Fits a custom function to market data. |

Functions - Alphabetical
List

## asianbycrr

Purpose Price Asian option from Cox-Ross-Rubinstein binomial tree

Syntax Price = asianbycrr(CRRTree, OptSpec, Strike, Settle, ExerciseDates, AmericanOpt, AvgType, AvgPrice, AvgDate)

## Arguments

CRRTree Stock tree structure created by crrtree.
OptSpec NINST-by-1 list of string values 'Call' or 'Put'.
Strike NINST-by-1 vector of strike price values. Each row is the schedule for one option.

Settle NINST-by-1 vector of Settle dates. The settle date for every Asian option is set to the valuation date of the stock tree. The Asian argument Settle is ignored.
ExerciseDates For a European option (American0pt = 0):
NINST-by- 1 vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.

For an American option (AmericanOpt = 1):
NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is NINST-by-1, the option can be exercised between the valuation date of the stock tree and the single listed exercise date.

|  | Americanopt | (Optional) If AmericanOpt $=0$, NaN, or is unspecified, the option is a European option. If AmericanOpt $=1$, the option is an American option. |
| :---: | :---: | :---: |
|  | AvgType | (Optional) String = 'arithmetic 'for arithmetic average (default) or 'geometric' for geometric average. |
|  | AvgPrice | (Optional) Scalar representing the average price of the underlying asset at Settle. This argument is used when AvgDate < Settle. Default is the current stock price. |
|  | AvgDate | (Optional) Scalar representing the date on which the averaging period begins. Default $=$ Settle. |
| Description | Price = asia ExerciseDate calculates the compute the va NaN. Fixed-stri Floating-strike | (CRRTree, OptSpec, Strike, Settle, ricanOpt, AvgType, AvgPrice, AvgDate) of fixed- and floating-strike Asian options. To a floating-strike Asian option, specify Strike as an options are also known as average price options. options are also known as average strike options. |
|  | Price is a NINS | vector of expected prices at time 0 . |
|  | Asian options these options | ced using Hull-White (1993). Consequently, for root node contains a unique price. |
| Examples | Price a Float Tree | ke Asian Option Using a CRR Binomial |

This example shows how to price a floating-strike Asian option using a CRR binomial tree using the file deriv.mat, which provides CRRTree. The CRRTree structure contains the stock specification and time information needed to price the option.

## asianbycrr

```
load deriv.mat;
OptSpec = 'put';
Strike = NaN;
Settle = '01-Jan-2003';
ExerciseDates = '01-Jan-2004';
Price = asianbycrr(CRRTree, OptSpec, Strike, Settle, ...
ExerciseDates)
```

Price =
1.2177

References Hull, J., and A. White, "Efficient Procedures for Valuing European and American Path-Dependent Options," Journal of Derivatives, Volume 1, pp. 21-31.

See Also crrtree | instasian

## Purpose

Price Asian option from Equal Probabilities binomial tree

Syntax<br>Price = asianbyeqp(EQPTree, OptSpec, Strike, Settle, ExerciseDates, AmericanOpt, AvgType, AvgPrice, AvgDate)

## Arguments

| EQPTree | Stock tree structure created by eqptree. |
| :--- | :--- |
| OptSpec | NINST-by-1 list of string values 'Call' or 'Put'. |
| Strike | NINST-by-1 vector of strike price values. Each <br> row is the schedule for one option. |
| Settle | NINST-by-1 vector of Settle dates. The settle <br> date for every Asian option is set to the valuation <br> date of the stock tree. The Asian argument <br> Settle is ignored. |
|  | For a European option (AmericanOpt = 0): |
|  | NINST-by-1 vector of exercise dates. Each row <br> is the schedule for one option. For a European <br> option, there is only one exercise date, the option <br> expiry date. |
| For an American option (AmericanOpt = 1): |  |
| NINST-by-2 vector of exercise date boundaries. <br> For each instrument, the option can be exercised <br> on any tree date between or including the pair <br> of dates on that row. If only one non-NaN date is <br> listed, or if ExerciseDates is NINST-by-1, the |  |
| option can be exercised between the valuation |  |
| date of the stock tree and the single listed |  |
| exercise date. |  |

## asianbyeqp

|  | AmericanOpt | (Optional) If AmericanOpt $=0, \mathrm{NaN}$, or is unspecified, the option is a European option. If AmericanOpt = 1, the option is an American option. |
| :---: | :---: | :---: |
|  | AvgType | (Optional) String = 'arithmetic' for arithmetic average (default) or 'geometric' for geometric average. |
|  | AvgPrice | (Optional) Scalar representing the average price of the underlying asset at Settle. This argument is used when AvgDate < Settle. Default is the current stock price. |
|  | AvgDate | (Optional) Scalar representing the date on which the averaging period begins. |
| Description | Price = asia ExerciseDate calculates the compute the valu NaN. Fixed-stri Floating-strike | (EQPTree, OptSpec, Strike, Settle, ricanOpt, AvgType, AvgPrice, AvgDate) f fixed- and floating-strike Asian options. To a floating-strike Asian option, specify Strike as an options are also known as average price options. options are also known as average strike options. |
|  | Price is a NIN | vector of expected prices at time 0. |
| Examples | Price a Floo | ke Asian Option Using an EQP Equity Tree |
|  | This example an EQP equity EQPTree. The time informati | how to price a floating-strike Asian option using y loading the file deriv.mat, which provides ree structure contains the stock specification and ded to price the option. |
|  | load deriv.m |  |
|  | OptSpec = 'pu |  |
|  | Strike = NaN |  |
|  | Settle = '01 | 003'; |

```
ExerciseDates = '01-Jan-2004';
Price = asianbyeqp(EQPTree, OptSpec, Strike, Settle, ...
ExerciseDates)
```

Price $=$
1.2724
References Hull, J., and A. White, "Efficient Procedures for Valuing European and American Path-Dependent Options," Journal of Derivatives, Volume 1, pp. 21-31.
See Also eqptree | instasian

## asianbyitt

Purpose Price Asian options using implied trinomial tree (ITT)

```
Syntax Price = asianbyitt(ITtTree, OptSpec, Strike, Settle,
ExerciseDates, AmericanOpt, AvgType, AvgPrice, AvgDate)
```


## Arguments

| ITTTree | Stock tree structure created by itttree. |
| :--- | :--- |
| OptSpec | NINST-by-1 list of string values 'call' or 'put '. |
| Strike | NINST-by-1 vector of strike price values. Each <br> row represents the schedule for one option. |
| Settle | NINST-by-1 vector of Settle dates. The settle <br> date for every Asian option is set to the valuation <br> date of the stock tree. The Asian argument <br> Settle is ignored. |
|  | For a European option (AmericanOpt = 0): |
|  | NINST-by-1 vector of exercise dates. Each row <br> is the schedule for one option. For a European <br> option, there is only one exercise date which is <br> the option expiry date. |
| For an American option (AmericanOpt = 1): |  |
| NINST-by-2 vector of exercise date boundaries. <br> For each instrument, the option can be exercised <br> on any tree date between or including the pair <br> of dates on that row. If only one non-NaN date is |  |
| listed, or if ExerciseDates is NINST-by-1, the |  |
| option can be exercised between the valuation |  |
| date of the stock tree and the single listed |  |
| exercise date. |  |


| AmericanOpt | (Optional) If AmericanOpt $=0$, NaN, or is <br> unspecified, the option is a European option. If <br> AmericanOpt $=1$, the option is an American <br> option. |
| :--- | :--- |
| AvgType | (Optional) String = 'arithmetic' for arithmetic |
| average (default) or 'geometric' for geometric |  |
| average. |  |
| AvgPrice | (Optional) Scalar representing the average <br> price of the underlying asset at Settle. This <br> argument is used when AvgDate $<$ Settle. <br> Default is the current stock price. |
| AvgDate | (Optional) Scalar representing the date on which <br> the averaging period begins. |

Description

Examples

Price = asianbyitt(ITTTree, OptSpec, Strike, Settle, ExerciseDates, AmericanOpt, AvgType, AvgPrice, AvgDate) calculates the value of fixed- and floating-strike Asian options. To compute the value of a floating-strike Asian option, specify Strike as NaN. Fixed-strike Asian options are also known as average price options. Floating-strike Asian options are also known as average strike options.

Price is a NINST-by-1 vector of expected prices at time 0.

Note The Settle date for every Asian option is set to the ValuationDate of the stock tree. The Asian argument, Settle, is ignored.

## Price a Floating-Strike Asian Option Using an ITT Equity Tree

This example shows how to price a floating-strike Asian option using an ITT equity tree by loading the file deriv.mat, which provides ITTTree. The ITTTree structure contains the stock specification and time information needed to price the option.

## asianbyitt

```
load deriv.mat;
OptSpec = 'put';
Strike = NaN;
Settle = '01-Jan-2006';
ExerciseDates = '01-Jan-2007';
Price = asianbyitt(ITTTree, OptSpec, Strike, Settle, ExerciseDates)
Price =
    1.0778
References Hull, J., and A. White, "Efficient Procedures for Valuing European and
    American Path-Dependent Options," Journal of Derivatives, Volume
    1, 1993, pp. 21-31.
```

See Also instasian | itttree

## Purpose

Syntax

## Description

Price European or American Asian option using Longstaff-Schwartz model

```
Price = asianbyls(RateSpec,StockSpec,OptSpec,Strike,Settle,
ExerciseDates)
Price = asianbyls( __ ,Name,Value)
[Price,Paths,Times,Z] = asianbyls(RateSpec,StockSpec,OptSpec,Strike,S
ExerciseDates)
[Price,Paths,Times,Z] = asianbyls( __ ,Name,Value)
```

Price = asianbyls(RateSpec,StockSpec,OptSpec,Strike,Settle, ExerciseDates) returns fixed- and floating-strike Asian option prices using the Longstaff-Schwartz model. asianbyls computes prices of European and American Asian options. For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium. To compute the value of a floating-strike Asian option, Strike should be specified as NaN. Fixed-strike Asian options are also known as average price options and floating-strike Asian options are also known as average strike options.

Price = asianbyls( __ ,Name, Value) returns fixed- and floating-strike Asian option prices using optional name-value pair arguments and the Longstaff-Schwartz model. asianbyls computes prices of European and American Asian options. For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium. To compute the value of a floating-strike Asian option, Strike should be specified as NaN. Fixed-strike Asian options are also known as average price options and floating-strike Asian options are also known as average strike options.
[Price,Paths,Times, Z] = asianbyls(RateSpec,StockSpec,OptSpec,Strike,Settle, ExerciseDates) returns fixed- and floating-strike Asian option Price, Paths, Times, and $Z$ values using the Longstaff-Schwartz model. asianbyls computes prices of European and American Asian options.

## asianbyls

## Input Arguments

For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium. To compute the value of a floating-strike Asian option, Strike should be specified as NaN. Fixed-strike Asian options are also known as average price options and floating-strike Asian options are also known as average strike options.
[Price, Paths,Times, Z] = asianbyls( __ , Name, Value) returns fixed and floating strike Asian option Price, Paths, Times, and $Z$ values using optional name-value pair arguments and the Longstaff-Schwartz model. asianbyls computes prices of European and American Asian options. For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium. To compute the value of a floating-strike Asian option, Strike should be specified as NaN. Fixed-strike Asian options are also known as average price options and floating-strike Asian options are also known as average strike options.

## RateSpec - Interest-rate term structure <br> structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from intenvset. For information on the interest-rate specification, see intenvset.

## Data Types

struct

## StockSpec - Stock specification for underlying asset <br> structure

Stock specification for underlying asset, specified using StockSpec obtained from stockspec. For information on the stock specification, see stockspec.
stockspec can handle other types of underlying assets. For example, stocks, stock indices, and commodities. If dividends are not specified in StockSpec, dividends are assumed to be 0 .

## Data Types <br> struct

## OptSpec - Definition of option

string with a value of 'call' or 'put'
Definition of option, specified as 'call' or 'put' using a string.
Data Types
char

## Strike - Option strike price value

nonnegative scalar integer
Option strike price value, specified with a nonnegative scalar integer. To compute the value of a floating-strike Asian option, Strike should be specified as NaN. Floating-strike Asian options are also known as average strike options.

Data Types
double

## Settle - Settlement or trade date

date string | nonnegative scalar integer
Settlement or trade date for the Asian option, specified as a nonnegative scalar integer or date string.

Data Types
char

## ExerciseDates - Option exercise dates

date string | nonnegative scalar integer
Option exercise dates, specified as a nonnegative scalar integer or date string:

- For a European option, use a 1-by-1 vector of dates. For a European option, there is only one ExerciseDates on the option expiry date.


## asianbyls

- For an American option, use a 1-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is a 1-by-1 vector of serial date numbers or cell array of strings, the option can be exercised between Settle and the single listed ExerciseDates.


## Data Types

char

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.
Example: Price =
asianbyls(RateSpec,StockSpec,OptSpec,Strike,Settle, ExerciseDates, 'NumTria

## 'AmericanOpt' - Option type

0 European (default) | scalar with value [0,1]
Option type, specified as NINST-by-1 positive integer scalar flags with values:

- 0 - European
- 1 - American


## Data Types

single | double

## 'AvgType' - Average types

arithmetic (default) | string with values of arithmetic or geometric
Average types, specified as arithmetic for arithmetic average, or geometric for geometric average.

## Data Types <br> double

## 'AvgPrice' - Average price of underlying asset at Settle scalar

Average price of underlying asset at Settle, specified as a scalar.

Note Use this argument when AvgDate < Settle.

Data Types
double

## 'AvgDate' - Date averaging period begins

scalar
Date averaging period begins, specified as a scalar.
Data Types
double

## 'NumTrials' - Simulation trials

## 1000 (default) | scalar

Simulation trials, specified as a scalar number of independent sample paths.

## Data Types

double

## 'NumPeriods' - Simulation periods per trial <br> 100 (default) | scalar

Simulation periods per trial, specified as a scalar number. NumPeriods is considered only when pricing European Asian options. For American Asian options NumPeriod is equal to the number of exercise days during the life of the option.

## Data Types

double

## asianbyls

## 'Z' - Dependent random variates

## scalar | nonnegative integer

Dependent random variates used to generate the Brownian motion vector (i.e., Wiener processes) that drive the simulation, specified as a NumPeriods-by-2-by-NumTrials 3-D time series array.

Data Types
single | double

## 'Antithetic' - Indicator for antithetic sampling

false (default) | logical flag with value of true or false
Logical flag to indicate antithetic sampling, specified with a value of true or false.

## Data Types

logical

## Output Arguments

## Price - Expected price of Asian option

scalar
Expected price of the Asian option, returned as a 1-by-1 scalar.

## Paths - Simulated paths of correlated state variables

vector
Simulated paths of correlated state variables, returned as a (NumPeriods +1 )-by-1-by-NumTrials 3-D time series array. Each row of Paths is the transpose of the state vector $X(t)$ at time $t$ for a given trial.

## Times - Observation times associated with simulated paths vector

Observation times associated with the simulated paths, returned as a (NumPeriods +1 )-by- 1 column vector of observation times associated with the simulated paths. Each element of Times is associated with the corresponding row of Paths.

## Z - Dependent random variates

## vector

Dependent random variates, returned, if $Z$ is specified as an optional input argument, the same value is returned. Otherwise, $Z$ contains the random variates generated internally.

## Examples Compute the Price of an Asian Option Using the Longstaff-Schwartz Model

Define the RateSpec.
Rates $=0.05$;
StartDate = 'Jan-1-2013';
EndDate = 'Jan-1-2014';
RateSpec = intenvset('ValuationDate', StartDate, 'StartDates', StartD
'EndDates', EndDate,'Compounding', -1, 'Rates', Rates)

## RateSpec =

FinObj: 'RateSpec'
Compounding: -1
Disc: 0.9512
Rates: 0.0500
EndTimes: 1
StartTimes: 0
EndDates: 735600
StartDates: 735235
ValuationDate: 735235
Basis: 0
EndMonthRule: 1

Define the StockSpec for the asset.

```
AssetPrice = 100;
Sigma = 0.2;
StockSpec = stockspec(Sigma, AssetPrice)
```


## asianbyls

## StockSpec =

| FinObj: | 'StockSpec' |
| ---: | :--- |
| Sigma: | 0.2000 |
| AssetPrice: | 100 |
| DividendType: | [] |
| DividendAmounts: | 0 |
| ExDividendDates: [] |  |

Define the Asian 'call' option.

```
Settle = 'Jan-1-2013';
ExerciseDates = 'Jan-1-2014';
Strike = 110;
OptSpec = 'call';
```

Compute the price for the European arithmetic average price for the Asian option using the Longstaff-Schwartz model.

NumTrials = 10000;
NumPeriods = 100;
AvgType = 'arithmetic';
Antithetic= true;
Price= asianbyls(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDa 'NumTrials', NumTrials, 'NumPeriods', NumPeriods,'Antithetic', Antithetic

## Price =

1.9693

See Also asiansensbyls | asianbycrr | intenvset | stockspec | asianbykv | asianbylevy

[^3]Purpose

Syntax

## Description

Calculate European or American Asian option prices and sensitivities using Longstaff-Schwartz model

```
PriceSens = asiansensbyls(RateSpec,StockSpec,OptSpec,StrikeSettle,
ExerciseDates)
PriceSens = asiansensbyls( ___, Name,Value)
[PriceSens,Path,Times,Z] = asiansensbyls(RateSpec,StockSpec,OptSpec,Strik
ExerciseDates)
[PriceSens,Path,Times,Z] = asiansensbyls(___,Name,Value)
```

PriceSens = asiansensbyls(RateSpec,StockSpec,OptSpec,StrikeSettle, ExerciseDates) returns Asian option prices and sensitivities for fixedand floating-strike Asian options using the Longstaff-Schwartz model. asiansensbyls supports European and American Asian options. For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium. To compute the value of a floating-strike Asian option, Strike should be specified as NaN. Fixed-strike Asian options are also known as average price options and floating-strike Asian options are also known as average strike options.

PriceSens = asiansensbyls( __ , Name, Value) returns Asian option prices and sensitivities for fixed- and floating-strike Asian options using optional name-value pair arguments and the Longstaff-Schwartz model. asiansensbyls supports European and American Asian options. For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium. To compute the value of a floating-strike Asian option, Strike should be specified as NaN. Fixed-strike Asian options are also known as average price options and floating-strike Asian options are also known as average strike options.

```
[PriceSens,Path,Times,Z] =
asiansensbyls(RateSpec,StockSpec,OptSpec,StrikeSettle,
ExerciseDates) returns Asian option prices and sensitivities (PriceSens, Path, Times, and Z) for fixed- and floating-strike Asian
```

options using the Longstaff-Schwartz model. asiansensbyls supports European and American Asian options. For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium. To compute the value of a floating-strike Asian option, Strike should be specified as NaN. Fixed-strike Asian options are also known as average price options and floating-strike Asian options are also known as average strike options.

> [PriceSens, Path,Times, Z] = asiansensbyls ( $\overline{\text { ens }}$, Name, Value) returns Asian option prices and sensitivities (PriceSens, Path, Times, and Z) for fixed- and floating-strike Asian options using optional name-value pair arguments and the Longstaff-Schwartz model. asiansensbyls supports European and American Asian options. For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium. To compute the value of a floating-strike Asian option, Strike should be specified as NaN. Fixed-strike Asian options are also known as average price options and floating-strike Asian options are also known as average strike options.

## Input Arguments

RateSpec - Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from intenvset. For information on the interest-rate specification, see intenvset.

## Data Types

struct

## StockSpec - Stock specification for underlying asset

structure
Stock specification for underlying asset, specified using StockSpec obtained from stockspec. For information on the stock specification, see stockspec.

## asiansensbyls

stockspec can handle other types of underlying assets. For example, stocks, stock indices, and commodities. If dividends are not specified in StockSpec, dividends are assumed to be 0 .

## Data Types <br> struct

## OptSpec - Definition of option

string with value 'call' or 'put
Definition of option, specified as 'call' or 'put' using a string.

## Data Types <br> char

## Strike - Option strike price value

nonnegative scalar integer
Option strike price value, specified with a nonnegative scalar integer. To compute the value of a floating-strike Asian option, Strike should be specified as NaN. Floating-strike Asian options are also known as average strike options.

## Data Types <br> double

## Settle - Settlement date or trade date

nonnegative scalar integer | date string
Settlement date or trade date for the Asian option, specified as a nonnegative scalar integer or date string.

## Data Types <br> char

## ExerciseDates - Option exercise dates

nonnegative integer | data string
Option exercise dates, specified as a nonnegative scalar integer or date string:

- For a European option, use a 1-by-1 vector of dates. For a European option, there is only one ExerciseDates on the option expiry date.
- For an American option, use a 1-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non- NaN date is listed, or if ExerciseDates is a 1-by-1 vector of serial date numbers or cell array of strings, the option can be exercised between Settle and the single listed ExerciseDates.


## Data Types <br> char

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: PriceSens =
asiansensbyls(RateSpec,StockSpec,OptSpec,Strike,Settle, ExerciseDates, NumPeriods, 'Antithetic',Antithetic, 'AvgType', AvgType, 'OutSpec ', \{'All'

## 'AmericanOpt' - Option type

0 European (default) | scalar with values [0,1]
Option type, specified as NINST-by-1 positive integer scalar flags with values:

- 0 - European
- 1 - American

Data Types
single | double

## 'AvgType' - Average types

arithmetic (default) | string with values of arithmetic or geometric

## asiansensbyls

Average types, specified as arithmetic for arithmetic average, or geometric for geometric average.

## Data Types

double

## 'AvgPrice' - Average price of underlying asset at Settle scalar

Average price of underlying asset at Settle, specified as a scalar.

Note Use this argument when AvgDate < Settle.

```
Data Types
double
```


## 'AvgDate' - Date averaging period begins

```
scalar
```

Date averaging period begins, specified as a scalar.

## Data Types

double

## 'NumTrials' - Simulation trials

1000 (default) | scalar
Simulation trials, specified as a scalar number of independent sample paths.

Data Types
double

## 'NumPeriods' - Simulation periods per trial

## 100 (default) | scalar

Simulation periods per trial, specified as a scalar number. NumPeriods is considered only when pricing European Asian options. For American

Asian options NumPeriod is equal to the number of exercise days during the life of the option.

## Data Types

double

## 'Z' - Dependent random variates

scalar | nonnegative integer
Dependent random variates used to generate the Brownian motion vector (i.e., Wiener processes) that drive the simulation, specified as a NumPeriods-by-2-by-NumTrials 3-D time series array.

## Data Types <br> single | double

## 'Antithetic' - Indicates antithetic sampling

false (default) | logical flag with value of true or false
Indicates antithetic sampling, specified with a value of true or false.

## Data Types <br> logical

## 'OutSpec' - Define outputs

\{'Price'\} (default)| string with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | cell array of strings with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

Define outputs specifying NOUT- by-1 or 1-by-NOUT cell array of strings with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta' and 'All'.

OutSpec $=$ \{'All'\} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec to include each sensitivity:

```
Example: OutSpec =
{'delta','gamma','vega','lambda','rho','theta','price'}
```


## asiansensbyls

|  | Data Types char \| cell |
| :---: | :---: |
| Output Arguments | PriceSens - Expected price and sensitivities of Asian option vector |
|  | Expected price and sensitivities of the Asian option, returned as an NINST-by-1 vector. |
|  | Path - Simulated paths of correlated state variables vector |
|  | Simulated paths of correlated state variables, returned as a (NumPeriods +1 )-by-2-by-NumTrials 3-D time series array. Each row of Paths is the transpose of the state vector $X(t)$ at time $t$ for a given trial. |
|  | Times - Observation times associated with simulated paths vector |
|  | Observation times associated with simulated paths, returned as a (NumPeriods +1 )-by- 1 column vector of observation times associated with the simulated paths. Each element of Times is associated with the corresponding row of Paths. |
|  | Z - Dependent random variates vector |
|  | Dependent random variates, returned, if $Z$ is specified as an optional input argument, the same value is returned. Otherwise, $Z$ contains the random variates generated internally. |
| Examples | Compute the Price and Sensitivities of an Asian Option Using the Longstaff-Schwartz Model |
|  | Define the RateSpec. |
|  | Rates = 0.05; |
|  | $\begin{aligned} & \text { StartDate = 'Jan-1-2013'; } \\ & \text { EndDate = 'Jan-1-2014'; } \end{aligned}$ |

## Z - Dependent random variates

## vector

Dependent random variates, returned, if $Z$ is specified as an optional input argument, the same value is returned. Otherwise, $Z$ contains the random variates generated internally.

## Examples Compute the Price and Sensitivities of an Asian Option Using the Longstaff-Schwartz Model

Define the RateSpec.

```
EndDate = 'Jan-1-2014';
```

```
RateSpec = intenvset('ValuationDate', StartDate, 'StartDates', StartD'
'EndDates', EndDate,'Compounding', -1, 'Rates', Rates)
RateSpec =
    FinObj: 'RateSpec'
        Compounding: -1
            Disc: 0.9512
            Rates: 0.0500
            EndTimes: 1
        StartTimes: 0
            EndDates: 735600
            StartDates: 735235
ValuationDate: 735235
            Basis: 0
    EndMonthRule: 1
```

Define the StockSpec for the asset.
AssetPrice = 100;
Sigma = 0.2;
StockSpec = stockspec(Sigma, AssetPrice)

StockSpec =

```
                    FinObj: 'StockSpec'
                    Sigma: 0.2000
        AssetPrice: 100
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Define the Asian 'call' option.

## asiansensbyls

```
Settle = 'Jan-1-2013';
ExerciseDates = 'Jan-1-2014';
Strike = 110;
OptSpec = 'call';
```

Compute the price for the European arithmetic average price and sensitivities for the Asian option using the Longstaff-Schwartz model.

```
NumTrials = 10000;
```

NumPeriods = 100;
AvgType = 'arithmetic';
Antithetic= true;
OutSpec $=$ \{'Price', 'Delta', 'Gamma'\};
PriceSens = asiansensbyls(RateSpec, StockSpec, OptSpec, Strike, Settle,
'NumTrials', NumTrials, 'NumPeriods', NumPeriods,'Antithetic', Antithetic
AvgType, 'OutSpec', OutSpec)

## PriceSens =

### 1.9693

| See Also | asianbyls \| asianbycrr | stockspec | intenvset | asianbykv | <br> asianbylevy |
| :--- | :--- |
| Concepts | - "Asian Option" on page 3-35 |

## Purpose

Prices European geometric Asian options using Kemna-Vorst model

```
Syntax
```

Price = asianbykv(RateSpec,StockSpec,OptSpec,Strike,Settle, ExerciseDates)
Description

## Input Arguments

Price = asianbykv(RateSpec,StockSpec,OptSpec,Strike,Settle, ExerciseDates) returns prices of European geometric Asian options using the Kemna-Vorst model.

## RateSpec - Interest-rate term structure <br> structure

The annualized continuously compounded interest-rate term structure specified by the RateSpec obtained from intenvset. For information on the interest-rate specification, see intenvset.

## Data Types

struct

## StockSpec - Stock specification for underlying asset

## structure

Stock specification for underlying asset, specified using StockSpec obtained from stockspec. For information on the stock specification, see stockspec.
stockspec can handle other types of underlying assets. For example, stocks, stock indices and commodities. If dividends are not specified in StockSpec, dividends are assumed to be 0 .

## Data Types

struct

## OptSpec - Definition of option

string with value'call' or 'put ' | cell array of strings
Definition of option, specified as 'call' or 'put ' using a NINST-by-1 cell array of strings.

## asianbykv

```
Data Types
cell | char
```


## Strike - Option strike price values

```
nonnegative integer | vector of nonnegative integers
```

Option strike price values, specified with nonnegative integers using a NINST-by-1 vector.

Data Types
single | double

## Settle - Settlement dates or trade dates

nonnegative integer | vector of nonnegative integers | date string | cell array of date strings
Settlement dates or trade dates for the Asian option, specified as a date string or as nonnegative integers using a NINST-by-1 vector or cell array of dates.

Data Types
char | cell

## ExerciseDates - European option exercise dates

nonnegative integer | vector of nonnegative integers | date string | cell array of date strings

European option exercise dates, specified as nonnegative integers or date strings using a NINST-by-1 vector or cell array of exercise dates. For an European option, there is only one ExerciseDates on the option expiry date.

Data Types
char | cell

## Output Price - Expected prices of an Asian option <br> vector

Expected prices of the Asian option, returned as an NINST-by-1 vector.

## Examples

## Compute the Price of an Asian Option Using the Kemna-Vorst Model

Define the RateSpec.

```
StartDates = 'Jan-1-2013';
EndDates = 'Jan-1-2014';
Rates = 0.035;
Basis = 1;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', Start
'EndDates', EndDates,'Rates', Rates, 'Compounding', -1, 'Basis', Bas:
```

RateSpec =

```
                    FinObj: 'RateSpec
        Compounding: -1
            Disc: 0.9656
            Rates: 0.0350
            EndTimes: 1
        StartTimes: 0
            EndDates: 735600
        StartDates: 735235
    ValuationDate: 735235
            Basis: 1
    EndMonthRule: 1
```

Define the StockSpec for the asset.

```
AssetPrice = 100;
Sigma = 0.15;
DivType = 'continuous';
DivAmounts = 0.03;
StockSpec = stockspec(Sigma, AssetPrice, DivType, DivAmounts)
```


## asianbykv

StockSpec =

```
            FinObj: 'StockSpec'
            Sigma: 0.1500
        AssetPrice: 100
        DividendType: {'continuous'}
DividendAmounts: 0.0300
ExDividendDates: []
```

Define the Asian 'call' and 'put' options.
Strike = 102;
OptSpec = \{'put'; 'call'\};
Settle = 'Jan-1-2013';
Maturity = 'Apr-1-2013';
Compute the European geometric Average Price for the Asian option using the Kemna-Vorst model.

Price = asiansensbykv(RateSpec, StockSpec, OptSpec, Strike, Settle, Matur

Price =
2.8881
0.9210

See Also $\begin{gathered}\text { asiansensbykv | asianbycrr | intenvset | stockspec | asianbyls } \\ \text { | asianbylevy }\end{gathered}$
Concepts

- "Asian Option" on page 3-35


## Purpose

Syntax

Description

## Input <br> Arguments

Calculate prices and sensitivities of European geometric Asian options using Kemna-Vorst model

```
PriceSens = asiansensbykv(RateSpec,StockSpec,OptSpec,Strike,Settle,
ExerciseDates)
PriceSens = asiansensbykv( ___ ,Name,Value)
```

PriceSens = asiansensbykv(RateSpec,StockSpec,OptSpec, Strike, Settle, ExerciseDates) returns prices and sensitivities of European geometric Asian options using Kemna-Vorst model.

PriceSens = asiansensbykv( __ , Name, Value) returns prices and sensitivities of European geometric Asian options using optional Name-Value pairs for the Kemna-Vorst model.

## RateSpec - Interest-rate term structure

structure
The annualized continuously compounded interest-rate term structure specified by the RateSpec obtained from intenvset. For information on the interest-rate specification, see intenvset.

## Data Types

struct

## StockSpec - Stock specification for underlying asset structure

Stock specification for underlying asset, specified using StockSpec obtained from stockspec. For information on the stock specification, see stockspec.
stockspec can handle other types of underlying assets. For example, stocks, stock indices and commodities. If dividends are not specified in StockSpec, dividends are assumed to be 0 .

## asiansensbykv

```
Data Types
struct
```


## OptSpec - Definition of option

```
string with values 'call' or 'put' | cell array of strings
```

Definition of option, specified as 'call' or 'put ' using a NINST-by-1 cell array of strings.

Data Types
cell | char

## Strike - Option strike price values

nonnegative integer | vector of nonnegative integers
Option strike price values, specified with nonnegative integers using a NINST- by- 1 vector.

Data Types
single | double

## Settle - Settlement dates or trade dates

nonnegative integer | vector of nonnegative integers | date string | cell array of date strings

Settlement dates or trade dates for the Asian option, specified as nonnegative integers or date strings using a NINST-by-1 vector or cell array of dates.

Data Types
char | cell

## ExerciseDates - Option exercise dates

nonnegative integer | vector of nonnegative integers | date string | cell array of date strings

European option exercise dates, specified as nonnegative integers or date strings using a NINST-by-1 vector or cell array of exercise dates. For an European option, there is only one ExerciseDates on the option expiry date.

## Data Types <br> char | cell

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

Example: PriceSens = asiansensbykv(RateSpec,StockSpec,0ptSpec, Strike,Settle, ExerciseDates,

## 'OutSpec' - Define outputs

\{'Price'\} (default) | string with values: 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta' and 'All'. | cell array of strings with values: 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta' and 'All'.

Define outputs specifying NOUT- by-1 or 1-by-NOUT cell array of strings with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta' and 'All'.

OutSpec $=$ \{'All'\} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec as:

Example: OutSpec = \{'delta','gamma','vega','lambda','rho','theta','price'\}

## Data Types

char | cell

## Output Arguments

## PriceSens - Expected prices and sensitivities of the Asian option vector

Expected prices and sensitivities of the Asian option, returned as an 1 -by- 1 vector. If the OutSpec is not specified only price is returned.

## asiansensbykv

## Examples Compute the Price and Sensitivities of an Asian Option Using the Kemna-Vorst Model

Define the RateSpec.

```
StartDates = 'Jan-1-2013';
EndDates = 'Jan-1-2014';
Rates = 0.035;
Basis = 1;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDat\epsilon
'EndDates', EndDates,'Rates', Rates, 'Compounding', -1, 'Basis', Basis)
```

RateSpec =

```
                    FinObj: 'RateSpec
        Compounding: -1
            Disc: 0.9656
            Rates: 0.0350
            EndTimes: 1
        StartTimes: 0
            EndDates: 735600
            StartDates: 735235
ValuationDate: 735235
            Basis: 1
EndMonthRule: 1
```

Define the StockSpec for the asset.

```
AssetPrice = 100;
Sigma = 0.15;
DivType = 'continuous';
DivAmounts = 0.03;
StockSpec = stockspec(Sigma, AssetPrice, DivType, DivAmounts)
```


## StockSpec =

```
    FinObj: 'StockSpec'
    Sigma: 0.1500
        AssetPrice: 100
    DividendType: {'continuous'}
DividendAmounts: 0.0300
ExDividendDates: []
```

Define the Asian 'call' and 'put' options.
Strike = 102;
OptSpec = \{'put'; 'call'\};
Settle = 'Jan-1-2013';
ExerciseDates = 'Jan-1-2014';
Compute the European geometric Average Price and sensitivities for the Asian option using the Kemna-Vorst model.

```
OutSpec = {'Price', 'Delta', 'Gamma'};
PriceSens = asiansensbykv(RateSpec, StockSpec, OptSpec, Strike,...
Settle, ExerciseDates,'OutSpec', OutSpec)
```

PriceSens =
4.3871
2.5163

## See Also asianbykv | asianbycrr | stockspec | intenvset | asianbyls | asianbylevy <br> Concepts <br> - "Asian Option" on page 3-35

## asianbylevy

## Purpose Price of European arithmetic Asian options using Levy model

Syntax Price = asianbylevy (RateSpec,StockSpec,OptSpec,Strike, Settle, ExerciseDates)

## Description Price =

asianbylevy(RateSpec,StockSpec,OptSpec,Strike,Settle, ExerciseDates) returns European arithmetic average pricing for Asian options using the Levy model.

## Input Arguments

## RateSpec - Interest-rate term structure <br> structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from intenvset. For information on the interest-rate specification, see intenvset.

## Data Types <br> struct

## StockSpec - Stock specification for underlying asset structure

Stock specification for underlying asset, specified using StockSpec obtained from stockspec. For information on the stock specification, see stockspec.
stockspec can handle other types of underlying assets. For example, stocks, stock indices, and commodities. If dividends are not specified in StockSpec, dividends are assumed to be 0 .

## Data Types <br> struct

## OptSpec - Definition of option

string with values 'call' or 'put' | cell array of strings
Definition of option, specified as 'call' or 'put' using a NINST-by-1 cell array of strings.

Data Types<br>char | cell

## Strike - Option strike price values

nonnegative integer | vector of nonnegative integers
Option strike price values, specified with nonnegative integers as a NINST-by-1 vector.

Data Types
single | double

## Settle - Settlement dates or trade dates

```
nonnegative integer | vector of nonnegative integers | date string |
cell array of date strings
```

Settlement dates or trade dates for the Asian option, specified as nonnegative integers or date strings using a NINST-by-1 vector or cell array of dates.

## Data Types <br> char | cell

## ExerciseDates - Option exercise dates

nonnegative integer | vector of nonnegative integers | date string | cell array of date strings

Option exercise dates, specified as nonnegative integers or date strings using a NINST-by- 1 vector or cell array of exercise dates. For a European option, there is only one ExerciseDates on the option expiry date.

## Data Types <br> char | cell

## Output Price - Expected prices of Asian option <br> Arguments <br> vector

Expected prices of the Asian option, returned as a NINST-by-1 vector.

## Examples Compute the Price of an Asian Option Using the Levy Model

Define the RateSpec.

```
Rates = 0.07;
StartDates = 'Jan-1-2013';
EndDates = 'Jan-1-2014';
EndDates, 'Rates', Rates, 'Compounding', -1)
RateSpec =
```

```
            FinObj: 'RateSpec'
```

            FinObj: 'RateSpec'
        Compounding: -1
        Compounding: -1
            Disc: 0.9324
            Disc: 0.9324
            Rates: 0.0700
            Rates: 0.0700
            EndTimes: 1
            EndTimes: 1
        StartTimes: 0
        StartTimes: 0
            EndDates: 735600
            EndDates: 735600
            StartDates: 735235
            StartDates: 735235
    ValuationDate: 735235
ValuationDate: 735235
Basis: 0
Basis: 0
EndMonthRule: 1

```
EndMonthRule: 1
```

RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDate

Define the StockSpec for the asset.
AssetPrice = 6.8;
Sigma = 0.14;
DivType = 'continuous';
DivAmounts = 0.09;
StockSpec = stockspec(Sigma, AssetPrice, DivType, DivAmounts)

StockSpec =

```
            FinObj: 'StockSpec'
            Sigma: 0.1400
        AssetPrice: 6.8000
    DividendType: {'continuous'}
DividendAmounts: 0.0900
ExDividendDates: []
```

Define two options for 'call' and 'put'.

```
Settle = 'Jan-1-2013';
Maturity = 'July-1-2013';
Strike = 6.9;
OptSpec = {'call'; 'put'};
```

Compute the European arithmetic average price for the Asian option using the Levy model.

Price= asianbylevy(RateSpec, StockSpec, OptSpec, Strike, Settle, Matur

```
Price =
    0.0944
    0.2237
```

See Also asiansensbylevy | asianbycrr | intenvset | stockspec | asianbyls asianbykv
Concepts - "Asian Option" on page 3-35

## asiansensbylevy

| Purpose | Calculate prices and sensitivities of European arithmetic Asian options <br> using Levy model |
| :--- | :--- |
| Syntax | PriceSens = asiansensbylevy (RateSpec, StockSpec, OptSpec, StrikeSettle, <br> ExerciseDates) <br> PriceSens = asiansensbylevy (__ , Name, Value) |
| Description | PriceSens = <br> asiansensbylevy (RateSpec, StockSpec, OptSpec, StrikeSettle, |
|  | ExerciseDates) returns European average pricing and sensitivities for <br> arithmetic Asian options using the Levy model. |

PriceSens = asiansensbylevy( __ , Name, Value) returns European average pricing and sensitivities for arithmetic Asian options using the Levy model with optional name-value pair arguments.

## Input <br> Arguments

## RateSpec - Interest-rate term structure

structure
Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from intenvset. For information on the interest-rate specification, see intenvset.

Data Types
struct

## StockSpec - Stock specification for underlying asset

structure
Stock specification for underlying asset, specified using StockSpec obtained from stockspec. For information on the stock specification, see stockspec.
stockspec can handle other types of underlying assets. For example, stocks, stock indices, and commodities. If dividends are not specified in StockSpec, dividends are assumed to be 0 .

## Data Types <br> struct

## OptSpec - Definition of option

string with values 'call' or 'put' | cell array of strings
Definition of option, specified as 'call' or 'put' using a NINST-by-1 cell array of strings.

Data Types
char | cell

## Strike - Option strike price values

nonnegative integer | vector of nonnegative integers
Option strike price values, specified with nonnegative integers using a NINST-by- 1 vector.

Data Types
single | double

## Settle - Settlement dates or trade dates

nonnegative integer | vector of nonnegative integers | date string | cell array of date strings
Settlement dates or trade dates for the Asian option, specified as nonnegative integers or date strings using a NINST-by-1 vector or cell array of dates.

## Data Types

char | cell

## ExerciseDates - Option exercise dates

nonnegative integer | vector of nonnegative integers | date string | cell array of date strings
Option exercise dates, specified as nonnegative integers or date strings using a NINST-by- 1 vector or cell array of exercise dates. For a European option, there is only one ExerciseDates on the option expiry date.

## asiansensbylevy

## Data Types char | cell

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

Example: PriceSens = asiansensbylevy(RateSpec,StockSpec,OptSpec,Strike,Settle,ExerciseDates,'

## 'OutSpec' - Define outputs

\{'Price'\} (default) | string with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'. | cell array of strings with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

Define outputs specifying NOUT- by-1 or 1-by-NOUT cell array of strings with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta' and 'All'.

OutSpec $=$ \{'All' $\}$ specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec to include each sensitivity:

Example: OutSpec =
\{'delta','gamma','vega','lambda','rho','theta','price'\}

## Data Types <br> char | cell

## Output Arguments

## PriceSens - Expected prices and sensitivities of Asian option vector

Expected prices and sensitivities of the Asian option, returned as an 1 -by- 1 vector. If the OutSpec is not specified only the price is returned.

```
Examples Compute the Price and Sensitivities of an Asian Option Using
the Levy Model
Define the RateSpec.
Rates = 0.07;
StartDates = 'Jan-1-2013';
EndDates = 'Jan-1-2014';
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', Startl
EndDates, 'Rates', Rates, 'Compounding', -1)
RateSpec =
```

```
                    FinObj: 'RateSpec'
```

                    FinObj: 'RateSpec'
        Compounding: -1
        Compounding: -1
            Disc: 0.9324
            Disc: 0.9324
            Rates: 0.0700
            Rates: 0.0700
            EndTimes: 1
            EndTimes: 1
        StartTimes: 0
        StartTimes: 0
            EndDates: 735600
            EndDates: 735600
        StartDates: 735235
        StartDates: 735235
    ValuationDate: 735235
    ValuationDate: 735235
            Basis: 0
            Basis: 0
    EndMonthRule: 1
    ```
    EndMonthRule: 1
```

Define the StockSpec for the asset.

```
AssetPrice = 6.8;
Sigma = 0.14;
DivType = 'continuous';
DivAmounts = 0.09;
StockSpec = stockspec(Sigma, AssetPrice, DivType, DivAmounts)
```

StockSpec =

## asiansensbylevy

```
            FinObj: 'StockSpec'
            Sigma: 0.1400
        AssetPrice: 6.8000
        DividendType: {'continuous'}
DividendAmounts: 0.0900
ExDividendDates: []
```

Define two options for a 'call' and 'put'.

```
Settle = 'Jan-1-2013';
ExerciseDates = 'Jan-1-2014';
Strike = 6.9;
OptSpec = {'call'; 'put'};
```

Compute the European arithmetic average price and sensitivities for the Asian option using the Levy model.

```
OutSpec = {'Price', 'Delta', 'Gamma'};
PriceSens = asiansensbylevy(RateSpec, StockSpec, OptSpec, Strike,...
Settle, ExerciseDates,'OutSpec', OutSpec)
```

PriceSens =
0.1358
0.2921

| See Also | asianbykv \| asianbycrr | stockspec | intenvset | asianbyls | <br> asianbykv |
| :--- | :--- |
| Concepts | - "Asian Option" on page 3-35 |


| Purpose | Determine price of asset-or-nothing digital options using Black-Scholes <br> model |
| :--- | :--- |
| Syntax | Price = assetbybls(RateSpec, StockSpec, Settle, Maturity, <br> OptSpec, Strike) |

Arguments

## Description Price = assetbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike) computes asset-or-nothing option prices using the Black-Scholes option pricing model. <br> Price is a NINST-by- 1 vector of expected option prices. <br> Examples <br> Compute Asset-Or-Nothing Digital Option Prices Using the Black-Scholes Option Pricing Model

Consider two asset-or-nothing put options on a nondividend paying stock with a strike of 95 and 93 and expiring on January 30, 2009. On November 3, 2008 the stock is trading at 97.50 . Using this data, calculate the price of the asset-or-nothing put options if the risk-free rate is $4.5 \%$ and the volatility is $22 \%$. First, create the RateSpec.

```
Settle = 'Nov-3-2008';
```

```
Maturity = 'Jan-30-2009';
Rates = 0.045;
Compounding = -1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding)
RateSpec =
                    FinObj: 'RateSpec'
        Compounding: -1
            Disc: 0.9893
            Rates: 0.0450
            EndTimes: 0.2391
        StartTimes: 0
            EndDates: 733803
        StartDates: 733715
ValuationDate: 733715
            Basis: 0
EndMonthRule: 1
```

Define the StockSpec.

```
AssetPrice = 97.50;
```

Sigma = .22;
StockSpec = stockspec(Sigma, AssetPrice)

StockSpec =

```
                    FinObj: 'StockSpec'
                    Sigma: 0.2200
            AssetPrice: 97.5000
        DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Define the put options.
OptSpec = \{'put'\};
Strike = [95;93];
Calculate the price.
Paon = assetbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Str:

Paon =
33.7666
26.9662

See Also assetsensbybls | cashbybls | gapbybls | supersharebybls

## Purpose

Syntax

PriceSens = assetsensbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike) PriceSens = assetsensbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, OutSpec)

## Arguments

| RateSpec | The annualized, continuously compounded rate <br> term structure. For information on the interest <br> rate specification, see intenvset. |
| :--- | :--- |
| StockSpec | Stock specification. See stockspec. |
| Settle | NINST-by-1 vector of settlement or trade dates. |
| Maturity | NINST-by-1 vector of maturity dates. |
| OptSpec | NINST-by-1 cell array of strings 'call' or 'put'. |
| Strike | NINST-by-1 vector of strike price values. |
|  | (Optional) All optional inputs are specified as <br> matching parameter name/value pairs. The |
|  | parameter name is specified as a character string, <br> followed by the corresponding parameter value. |
|  | You can specify parameter name/value pairs <br> in any order. Names are case-insensitive and <br> partial string matches are allowed provided no <br> ambiguities exist. Valid parameter names are: |
|  | - NouT-by-1 or 1-by-NOUT cell array of strings |
| indicating the nature and order of the outputs |  |,

should be Price, Lambda, and Rho, in that order.

To invoke from a function: [Price, Lambda, Rho] = assetsensbybls(..., 'OutSpec', \{'Price', 'Lambda', 'Rho'\})

OutSpec = \{'All'\} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec as OutSpec = \{'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', 'Price'\};.

- Default is OutSpec $=\{$ 'Price' $\}$.


## Description

PriceSens = assetsensbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike) computes asset-or-nothing option prices using the Black-Scholes option pricing model.

PriceSens = assetsensbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, OutSpec) includes the parameter/value pairs defined for OutSpec, and computes asset-or-nothing option prices and sensitivities using the Black-Scholes option pricing model.

PriceSens is a NINST-by-1 vector of expected option prices and sensitivities.

## Examples Compute Asset-Or-Nothing Digitial Option Prices and Sensitivities Using the Black-Scholes Option Pricing Model

Consider two asset-or-nothing put options on a nondividend paying stock with a strike of 95 and 93 and expiring on January 30, 2009. On November 3, 2008 the stock is trading at 97.50 . Using this data, calculate the price and sensitivity of the asset-or-nothing put options if the risk-free rate is $4.5 \%$ and the volatility is $22 \%$. First, create the RateSpec.

```
Settle = 'Nov-3-2008';
Maturity = 'Jan-30-2009';
Rates = 0.045;
Compounding = -1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding)
RateSpec =
\begin{tabular}{rl} 
FinObj: & 'RateSpec' \\
Compounding: & -1 \\
Disc: & 0.9893 \\
Rates: & 0.0450 \\
EndTimes: & 0.2391 \\
StartTimes: & 0 \\
EndDates: 733803 \\
StartDates: 733715 \\
ValuationDate: 733715 \\
Basis: 0 \\
EndMonthRule: 1
\end{tabular}
```

Define the StockSpec.
AssetPrice = 97.50;
Sigma = .22;
StockSpec = stockspec(Sigma, AssetPrice)

StockSpec =

```
                    FinObj: 'StockSpec
                    Sigma: 0.2200
        AssetPrice: 97.5000
        DividendType: []
DividendAmounts: 0
```


## ExDividendDates: []

Define the put options.

```
OptSpec = {'put'};
Strike = [95;93];
```

Calculate the delta, price, and gamma.

```
OutSpec = { 'delta';'price';'gamma'};
[Delta, Price, Gamma] = assetsensbybls(RateSpec, StockSpec, Settle,..
Maturity, OptSpec, Strike, 'OutSpec', OutSpec)
```

Delta =
-3. 0833
-2.8337
Price =
33.7666
26.9662
Gamma =
0.0941
0.1439

## See Also

assetbybls

## barrierbycrr

Purpose Price barrier option from Cox-Ross-Rubinstein binomial tree

Syntax [Price, PriceTree] = barrierbycrr(CRRTree, OptSpec, Strike, Settle, ExerciseDates, AmericanOpt, BarrierSpec, Barrier, Rebate, Options)

## Arguments

| CRRTree | Stock tree structure created by crrtree. |
| :--- | :--- |
| OptSpec | NINST-by-1 list of string values 'Call' or 'Put '. |
| Strike | NINST-by-1 vector of strike price values. Each <br> row is the schedule for one option. |
| Settle | NINST-by-1 vector of Settle dates. The settle <br> date for every barrier option is set to the <br> valuation date of the stock tree. The barrier <br> argument Settle is ignored. |
| ForciseDates | For a European option (AmericanOpt = 0): |
| NINST-by-1 vector of exercise dates. Each row <br> is the schedule for one option. For a European <br> option, there is only one exercise date, the <br> option expiry date. |  |
| For an American option (AmericanOpt = 1): |  |
| NINST-by-2 vector of exercise date boundaries. |  |
| For each instrument, the option can be exercised <br> on any tree date between or including the pair <br> of dates on that row. If only one non-NaN date is <br> listed, or if ExerciseDates is NINST-by-1, the |  |
| option can be exercised between the valuation |  |
| date of the stock tree and the single listed |  |
| exercise date. |  |


| Americanopt | If AmericanOpt $=0, \mathrm{NaN}$, or is unspecified, the option is a European option. If AmericanOpt = 1, the option is an American option. |
| :---: | :---: |
| BarrierSpec | List of string values: |
|  | 'UI': Up Knock In |
|  | 'U0': Up Knock Out |
|  | ' DI ' : Down Knock In |
|  | ' DO ' : Down Knock Out |
| Barrier | Vector of barrier values. |
| Rebate | (Optional) NINST-by-1 matrix of rebate values. Default $=0$. For Knock-in options, the rebate is paid at expiry. For Knock-out options, the rebate is paid when the barrier is reached. |
| Options | (Optional) Derivatives pricing options structure created with derivset. |

See instbarrier for a description of barrier contract arguments.

## Description

## Examples Price a Barrier Option Using a CRR Binomial Tree

This example shows how to to price a barrier option using a CRR binomial tree by loading the file deriv.mat, which provides CRRTree.

## barrierbycrr

The CRRTree structure contains the stock specification and time information needed to price the option.

```
load deriv.mat;
OptSpec = 'Call';
Strike = 105;
Settle = '01-Jan-2003';
ExerciseDates = '01-Jan-2006';
AmericanOpt = 1;
BarrierSpec = 'UI';
Barrier = 102;
Price = barrierbycrr(CRRTree, OptSpec, Strike, Settle, ...
ExerciseDates, AmericanOpt, BarrierSpec, Barrier)
```

Price =
12.1272

| References $\quad$ | Derman, E., I. Kani, D. Ergener and I. Bardhan, "Enhanced Numerical |
| :--- | :--- |
|  | Methods for Options with Barriers," Financial Analysts Journal, |
|  | (Nov. - Dec. 1995), pp. 65-74. |

See Also crrtree | instbarrier

## barrierbyeqp

## Purpose

Price barrier option from Equal Probabilities binomial tree

Syntax<br>[Price, PriceTree] = barrierbyeqp(EQPTree, OptSpec, Strike, ExerciseDates, AmericanOpt, BarrierSpec, Barrier, Rebate, Options)

## Arguments

| EQPTree | Stock tree structure created by eqptree. |
| :---: | :---: |
| OptSpec | NINST-by-1 list of string values 'Call' or 'Put'. |
| Strike | NINST-by- 1 vector of strike price values. Each row is the schedule for one option. |
| Settle | NINST-by- 1 vector of Settle dates. The settle date for every barrier option is set to the valuation date of the stock tree. The barrier argument Settle is ignored. |
| ExerciseDates | For a European option (AmericanOpt $=0$ ): <br> NINST-by-1 vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date. |
|  | For an American option (AmericanOpt = 1): <br> NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is NINST-by-1, the option can be exercised between the valuation date of the stock tree and the single listed exercise date. |

## barrierbyeqp

| AmericanOpt | If AmericanOpt $=0$, NaN, or is unspecified, the <br> option is a European option. If AmericanOpt $=$ <br> 1, the option is an American option. |
| :--- | :--- |
| BarrierSpec | List of string values: <br> 'UI ' : Up Knock In |
|  | 'UO' : Up Knock Out |
| Barrier | 'DI ' : Down Knock In |
| Rebate | 'DO' : Down Knock Out |
|  | Vector of barrier values. <br> (Optional) NINST-by-1 matrix of rebate values. |
| Options | Default $=0$. For Knock-in options, the rebate <br> is paid at expiry. For Knock-out options, the <br> rebate is paid when the barrier is reached. <br> (Optional) Derivatives pricing options structure |
|  | created with derivset. |

See instbarrier for a description of barrier contract arguments.

## Description

[Price, PriceTree] = barrierbyeqp(EQPTree, OptSpec, Strike, ExerciseDates, AmericanOpt, BarrierSpec, Barrier, Rebate, Options) computes the price of barrier options using an equal probabilities binomial tree.

Price is a NINST-by- 1 vector of expected prices at time 0 .
PriceTree is a tree structure with a vector of instrument prices at each node.

## Examples Price a Barrier Option Using an EQP Equity Tree

This example shows how to to price a barrier option using an EQP equity tree by loading the file deriv.mat, which provides EQPTree.

## barrierbyeqp

The EQPTree structure contains the stock specification and time information needed to price the option.

```
load deriv.mat;
OptSpec = 'Call';
Strike = 105;
Settle = '01-Jan-2003';
ExerciseDates = '01-Jan-2006';
AmericanOpt = 1;
BarrierSpec = 'UI';
Barrier = 102;
Price = barrierbyeqp(EQPTree, OptSpec, Strike, Settle, ...
ExerciseDates, AmericanOpt, BarrierSpec, Barrier)
```

Price =
12.2632
References Derman, E., I. Kani, D. Ergener and I. Bardhan, "Enhanced Numerical Methods for Options with Barriers," Financial Analysts Journal, (Nov. - Dec. 1995), pp. 65-74.
See Also eqptree | instbarrier

## barrierbyitt

Purpose
Price barrier options using implied trinomial tree (ITT)

```
Syntax
[Price, PriceTree] = barrierbyitt(ITTTree, OptSpec, Strike,
ExerciseDates, AmericanOpt, BarrierSpec, Barrier, Rebate,
Options)
```


## Arguments

| ITTTree | Stock tree structure created by itttree. |
| :--- | :--- |
| OptSpec | NINST-by-1 list of string values 'Call' or 'Put' |
| Strike | European and American option, NINST-by-1 <br> vector of strike price values. Each row is the <br> schedule for one option. |
| Settle | NINST-by-1 vector of Settle dates. The settle <br> date for every barrier option is set to the <br> valuation date of the stock tree. The barrier <br> argument Settle is ignored. |
| For a European option (AmericanOpt = 0): |  |


| AmericanOpt | If AmericanOpt $=0$, NaN, or is unspecified, the <br> option is a European option. If AmericanOpt $=$ <br> 1, the option is an American option. |
| :--- | :--- |
| BarrierSpec | List of string values: <br> 'UI ' : Up Knock In |
|  | 'UO ' : Up Knock Out |
| Barrier | 'DI ' : Down Knock In |
| Rebate | 'DO ' D Down Knock Out |
|  | Vector of barrier values. <br> (Optional) NINST-by-1 matrix of rebate values. |
| Options | Default $=0$. For Knock In options, the rebate <br> is paid at expiry. For Knock Out options, the <br> rebate is paid when the barrier is reached. |
|  | (Optional) Derivatives pricing options structure <br> created with derivset. |

See instbarrier for a description of barrier contract arguments.

## Description

[Price, PriceTree] = barrierbyitt(ITTTree, OptSpec, Strike, ExerciseDates, AmericanOpt, BarrierSpec, Barrier, Rebate, Options) computes the price of barrier options using an implied trinomial tree.

Price is a NINST-by- 1 vector of expected prices at time 0 .
PriceTree is a tree structure with a vector of instrument prices at each node.

Note The Settle date for every barrier option is set to the ValuationDate of the stock tree. The barrier argument, Settle, is ignored.

## barrierbyitt

## Examples Price a Barrier Option Using an ITT Tree

This example shows how to to price a barrier option using an ITT tree by loading the file deriv.mat, which provides ITTTree. The ITTTree structure contains the stock specification and time information needed to price the option.

```
load deriv.mat;
OptSpec = 'Call';
Strike = 85;
Settle = '01-Jan-2006';
ExerciseDates = '31-Dec-2008';
AmericanOpt = 1;
BarrierSpec = 'UI';
Barrier = 115;
```

Price = barrierbyitt(ITTTree,OptSpec,Strike,Settle, ExerciseDates,Americar
BarrierSpec, Barrier)

## Price =

$$
2.4074
$$

## References Derman, E., I. Kani, D. Ergener, and I. Bardhan, "Enhanced Numerical Methods for Options with Barriers," Financial Analysts Journal, Nov.-Dec.,1995.

See Also<br>instbarrier | itttree

## basketbyiu

Purpose
Price European basket options using Nengjiu Ju approximation model

Price = basketbyju(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Maturity)

Description

## Input <br> Arguments

## Output Arguments

Price = basketbyju(RateSpec, BasketStockSpec, OptSpec, Strike,
Settle, Maturity) prices European basket options using the Nengjiu Ju approximation model.

## RateSpec

Annualized, continuously compounded rate term structure. For more information on the interest rate specification, see intenvset.

## BasketStockSpec

BasketStock specification. For information on the basket of stocks specification, see basketstockspec.

## OptSpec

String or 2-by-1 cell array of the strings 'call' or 'put'.

## Strike

Scalar for the option strike price.

## Settle

Scalar of the settlement or trade date specified as a string or serial date number.

## Maturity

Maturity date specified as a string or serial date number.

## Price

Price of the basket option.

## basketbyiu

Examples
Find a European call basket option of two stocks. Assume that the stocks are currently trading at $\$ 10$ and $\$ 11.50$ with annual volatilities of $20 \%$ and $25 \%$, respectively. The basket contains one unit of the first stock and one unit of the second stock. The correlation between the assets is $30 \%$. On January 1, 2009, an investor wants to buy a 1 -year call option with a strike price of $\$ 21.50$. The current annualized, continuously compounded interest rate is $5 \%$. Use this data to compute the price of the call basket option with the Ju approximation model.

```
Settle = 'Jan-1-2009';
Maturity = 'Jan-1-2010';
% Define RateSpec
Rate = 0.05;
Compounding = -1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', ...
Settle, 'EndDates', Maturity, 'Rates', Rate, 'Compounding', Compounding);
% Define the Correlation matrix. Correlation matrices are symmetric, and
% have ones along the main diagonal.
Corr = [1 0.30; 0.30 1];
% Define BasketStockSpec
AssetPrice = [10;11.50];
Volatility = [0.2;0.25];
Quantity = [1;1];
BasketStockSpec = basketstockspec(Volatility, AssetPrice, Quantity, Corr);
%Compute the price of the call basket option
OptSpec = {'call'};
Strike = 21.5;
PriceCorr30 = basketbyju(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Maturity)
```

This returns:
PriceCorr30 =
2.12214

## basketbyiu

Compute the price of the basket instrument for these two stocks with a correlation of $60 \%$. Then compare this cost to the total cost of buying two individual call options:

```
Corr = [1 0.60; 0.60 1];
% Define the new BasketStockSpec
BasketStockSpec = basketstockspec(Volatility, AssetPrice, Quantity, Corr);
%Compute the price of the call basket option with Correlation = -0.60
PriceCorr60 = basketbyju(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Maturity)
```


## This returns:

PriceCorr60 =
2.27566

The following table summarizes the sensitivity of the option to correlation changes. In general, the premium of the basket option decreases with lower correlation and increases with higher correlation.

| Correlation | -0.60 | -0.30 | 0 | 0.30 | 0.60 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Premium | 1.52830 | 1.76006 | 1.9527 | 2.1221 | 2.2756 |

Compute the cost of two vanilla 1-year call options using the Black-Scholes (BLS) model on the individual assets:

```
StockSpec = stockspec(Volatility, AssetPrice);
StrikeVanilla= [10;11.5];
PriceVanillaOption = optstockbybls(RateSpec, StockSpec, Settle, Maturity,...
OptSpec, StrikeVanilla)
```

This returns:

## basketbyiu

```
PriceVanillaOption =
    1.0451
    1.4186
Find the total cost of buying two individual call options:
sum(PriceVanillaOption)
This returns:
\(a n s=2.4637\)
```

The total cost of purchasing two individual call options is $\$ 2.4637$, compared to the maximum cost of the basket option of $\$ 2.27$ with a correlation of $60 \%$.

## References

Nengjiu Ju, "Pricing Asian and Basket Options Via Taylor Expansion", Journal of Computational Finance, Vol. 5, 2002.

## See Also <br> basketstockspec | basketsensbyju

- "Basket Option" on page 3-25

Purpose
Price basket options using Longstaff-Schwartz model

```
Syntax
Price = basketbyls(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, ExerciseDates)
Price = basketbyls(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, ExerciseDates, 'ParameterName', ParameterValue ...)
```

Description

## Input Arguments

Price = basketbyls(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, ExerciseDates) prices basket options using the Longstaff-Schwartz model.

Price = basketbyls(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, ExerciseDates, 'ParameterName', ParameterValue ...) accepts optional inputs as one or more comma-separated parameter/value pairs. 'ParameterName' is the name of the parameter inside single quotes. 'Parametervalue is the value corresponding to 'ParameterName'. Specify parameter-value pairs in any order. Names are case-insensitive and partial string matches are allowable, if no ambiguities exist.

## RateSpec

Annualized, continuously compounded rate term structure. For more information on the interest rate specification, see intenvset.

## BasketStockSpec

BasketStock specification. For information on the basket of stocks specification, see basketstockspec.

## OptSpec

String or 2-by-1 cell array of the strings 'call' or 'put'.

## Strike

The option strike price:

## basketbyls

- For a European or Bermuda option, Strike is a scalar (European) or 1-by-NSTRIKES (Bermuda) vector of strike price.
- For an American option, Strike is a scalar vector of the strike price.


## Settle

Scalar of the settlement or trade date specified as a string or serial date number.

## ExerciseDates

The exercise date for the option:

- For a European or Bermuda option, ExerciseDates is a 1-by-1 (European) or 1-by-NSTRIKES (Bermuda) vector of exercise dates. For a European option, there is only one ExerciseDate on the option expiry date.
- For an American option, ExerciseDates is a 1-by-2 vector of exercise date boundaries. The option exercises on any date between, or including, the pair of dates on that row. If there is only one non- NaN date, or if ExerciseDates is 1-by-1, the option exercises between the Settle date and the single listed ExerciseDate.


## Parameter-Value Pairs

## AmericanOpt

Parameter values are a scalar flag.

- 0 - European/Bermuda
- 1 - American

Default: 0

## NumPeriods

Parameter value is a scalar number of simulation periods per trial. NumPeriods is considered only when pricing European basket options.

For American and Bermuda basket options, NumPeriod equals the number of exercise days during the life of the option.

Default: 100

## NumTrials

Parameter value is a scalar number of independent sample paths (simulation trials).

Default: 1000

## Output

Arguments

## Examples

## Price

Price of the basket option.

## Prices Basket Options Using the Longstaff-Schwartz Model

Find an American call basket option of three stocks. The stocks are currently trading at $\$ 35, \$ 40$ and $\$ 45$ with annual volatilities of $12 \%$, $15 \%$ and $18 \%$, respectively. The basket contains $33.33 \%$ of each stock. Assume the correlation between all pair of assets is $50 \%$. On May 1, 2009, an investor wants to buy a three-year call option with a strike price of $\$ 42$. The current annualized continuously compounded interest rate is $5 \%$. Use this data to compute the price of the call basket option using the Longstaff-Schwartz model.

```
Settle = 'May-1-2009';
Maturity = 'May-1-2012';
% Define RateSpec
Rate = 0.05;
Compounding = -1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates',...
Settle, 'EndDates', Maturity, 'Rates', Rate, 'Compounding', Compoundir
% Define the Correlation matrix. Correlation matrices are symmetric,
% and have ones along the main diagonal.
```


## basketbyls

```
Corr =[[1 0.50 0.50; 0.50 1 0.50;0.50 0.50 1];
% Define BasketStockSpec
AssetPrice = [35;40;45];
Volatility = [0.12;0.15;0.18];
Quantity = [0.333;0.333;0.333];
BasketStockSpec = basketstockspec(Volatility, AssetPrice, Quantity, Corr)
% Compute the price of the call basket option
OptSpec = {'call'};
Strike = 42;
AmericanOpt = 1; % American option
Price = basketbyls(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Ma
'AmericanOpt',American0pt)
Price =
    5.4687
Increase the number of simulation trials to 2000 to give the following results:
NumTrial = 2000;
Price = basketbyls(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Ma 'AmericanOpt',AmericanOpt, 'NumTrials', NumTrial)
Price =
5.5501
\begin{tabular}{ll} 
References \(\quad\) & Longstaff, F.A., and E.S. Schwartz, "Valuing American Options \\
by Simulation: A Simple Least-Squares Approach", The Review of \\
& Financial Studies, Vol. 14, No. 1, Spring 2001, pp. 113-147.
\end{tabular}
```

| See Also | basketstockspec \| basketsensbyls |
| :--- | :--- |
| How To | - "Basket Option" on page 3-25 |

## basketsensbyju

| Purpose | Determine European basket options price and sensitivities using Nengjiu Ju approximation model |
| :---: | :---: |
| Syntax | ```PriceSens = basketsensbyju(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Maturity) PriceSens = basketsensbyju(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Maturity, 'ParameterName', ParameterValue ...)``` |
| Description | PriceSens = basketsensbyju(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Maturity) calculates prices and sensitivities for basket options using the Nengjiu Ju approximation model. |
|  | PriceSens = basketsensbyju(RateSpec, BasketStockSpec, OptSpec, Strike, <br> Settle, Maturity, 'ParameterName', ParameterValue <br> ...) accepts optional inputs as one or more comma-separated parameter/value pairs. 'ParameterName' is the name of the parameter inside single quotes. 'Parametervalue is the value corresponding to 'ParameterName'. Specify parameter-value pairs in any order. Names are case-insensitive and partial string matches are allowable, if no ambiguities exist. |
| Input | RateSpec |
| Arguments | Annualized, continuously compounded rate term structure. For more information on the interest rate specification, see intenvset. |
|  | BasketStockSpec |
|  | BasketStock specification. For information on the basket of stocks specification, see basketstockspec. |
|  | OptSpec |

String or 2-by-1 cell array of the strings 'call' or 'put'.

## Strike

Scalar of the option strike price.

## Settle

Scalar of the settlement or trade date specified as a string or serial date number.

## Maturity

Maturity date, specified as a string or serial date number.

## Parameter-Value Pairs

## OutSpec

Parameter value is an NOUT-by-1 or 1-by-NOUT cell array of strings indicating the nature and order of the outputs for the function. Possible values are 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'. For example, OutSpec = \{'Price', 'Lambda', 'Rho'\} specifies that the output is Price, Lambda, and Rho, in that order.

OutSpec = \{'All'\} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec as OutSpec = \{'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', 'Price'\};.

Default: OutSpec = \{'Price' $\}$

## UndIdx

Scalar of the indice of the underlying instrument to compute the sensitivity.

> Default: UndIdx = [ ]

Arguments

## PriceSens

Expected prices or sensitivities values for the basket option.

## basketsensbyju

## Examples Calculate Prices and Sensitivities for Basket Options Using the Nengiiu Ju Approximation Model

Find a European call basket option of five stocks. Assume that the basket contains:

- $5 \%$ of the first stock trading at $\$ 110$
- $15 \%$ of the second stock trading at $\$ 75$
- $20 \%$ of the third stock trading at $\$ 40$
- $25 \%$ of the fourth stock trading at $\$ 125$
- $35 \%$ of the fifth stock trading at $\$ 92$

These stocks have annual volatilities of $20 \%$ and the correlation between the assets is zero. On May 1, 2009, an investor wants to buy a 1-year call option with a strike price of $\$ 90$. The current annualized, continuol compounded interest is $5 \%$. Use this data to compute price and delta of the call basket option with the Ju approximation model.

```
Settle = 'May-1-2009';
Maturity = 'May-1-2010';
% Define RateSpec
Rate = 0.05;
Compounding = -1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', ...
% have ones along the main diagonal.
NumInst = 5;
InstIdx = ones(NumInst,1);
Corr = diag(ones(5,1), 0);
% Define BasketStockSpec
AssetPrice = [110; 75; 40; 125; 92];
Volatility = 0.2;
Quantity = [0.05; 0.15; 0.2; 0.25; 0.35];
```

Settle, 'EndDates', Maturity, 'Rates', Rate, 'Compounding', Compounding);
\% Define the Correlation matrix. Correlation matrices are symmetric, and

```
BasketStockSpec = basketstockspec(Volatility, AssetPrice, Quantity, C(
% Compute the price of the call basket option. Calculate also the del
% of the first stock.
OptSpec = {'call'};
Strike = 90;
OutSpec = {'Price','Delta'};
UndIdx = 1; % First element in the basket
[Price, Delta] = basketsensbyju(RateSpec, BasketStockSpec, OptSpec, S
Maturity, 'OutSpec', OutSpec,'UndIdx', UndIdx)
Price =
    5.1610
Delta =
    0.0297
```

Compute Delta with respect to the second asset:

```
UndIdx = 2; % Second element in the basket
OutSpec = {'Delta'};
Delta = basketsensbyju(RateSpec, BasketStockSpec, OptSpec, Strike, Se
'OutSpec',OutSpec,'UndIdx',UndIdx)
```

Delta =
0.0906

## References Nengjiu Ju, "Pricing Asian and Basket Options Via Taylor Expansion", Journal of Computational Finance, Vol. 5, 2002.

## basketsensbyju

See Also basketstockspec | basketbyju<br>How To . "Basket Option" on page 3-25

## basketsensbyls

| Purpose | Determine price and sensitivities for basket options using Longstaff-Schwartz model |
| :---: | :---: |
| Syntax | ```PriceSens = basketsensbyls(RateSpec, BasketStockSpec, OptSpec, Strike Settle, ExerciseDates) PriceSens = basketsensbyls(RateSpec, BasketStockSpec, OptSpec, Strike Settle, ExerciseDates, 'ParameterName', ParameterValue ...)``` |
| Description | PriceSens = basketsensbyls(RateSpec, BasketStockSpec, OptSpec, Strike, <br> Settle, ExerciseDates) prices basket options using the Longstaff-Schwartz model. <br> PriceSens = basketsensbyls(RateSpec, BasketStockSpec, OptSpec, Strike, <br> Settle, ExerciseDates, 'ParameterName', ParameterValue ...) accepts optional inputs as one or more comma-separated parameter/value pairs. 'ParameterName' is the name of the parameter inside single quotes. 'Parametervalue is the value corresponding to 'ParameterName'. Specify parameter-value pairs in any order. Names are case-insensitive and partial string matches are allowable, if no ambiguities exist. |
| Input Arguments | RateSpec |
|  | Annualized, continuously compounded rate term structure. For more information on the interest rate specification, see intenvset. |
|  | BasketStockSpec |
|  | BasketStock specification. For information on the basket of stocks specification, see basketstockspec. |
|  | OptSpec |

String or 2-by-1 cell array of the strings 'call' or 'put'.

## Strike

## basketsensbyls

The option strike price:

- For a European or Bermuda option, Strike is a scalar (European) or 1-by-NSTRIKES (Bermuda) vector of strike price.
- For an American option, Strike is a scalar vector of strike price.


## Settle

Scalar of settlement or trade date.

## ExerciseDates

The exercise date for the option:

- For a European or Bermuda option, ExerciseDates is a 1-by-1 (European) or 1-by-NSTRIKES (Bermuda) vector of exercise dates. For a European option, there is only one ExerciseDate on the option expiry date.
- For an American option, ExerciseDates is a 1-by-2 vector of exercise date boundaries. The option exercises on any date between or including the pair of dates on that row. If there is only one non- NaN date, or if ExerciseDates is 1-by-1, the option exercises between the Settle date and the single listed ExerciseDate.


## Parameter-Value Pairs

## AmericanOpt

Parameter values are a scalar flag.

- 0 - European/Bermuda
- 1 - American

Default: 0

## NumPeriods

## basketsensbyls

Parameter value is a scalar number of simulation periods. NumPeriods is considered only when pricing European basket options. For American and Bermuda basket options, NumPeriod equals the number of exercise days during the life of the option.

Default: 100

## NumTrials

Parameter value is a scalar number of independent sample paths (simulation trials).

Default: 1000

## OutSpec

Parameter value is an NOUT-by-1 or 1-by-NOUT cell array of strings indicating the nature and order of the outputs for the function. Possible values are 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'. For example, OutSpec = \{'Price', 'Lambda', 'Rho' \} specifies that the output is Price, Lambda, and Rho, in that order.

OutSpec $=$ \{'All' $\}$ specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec as OutSpec = \{'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', 'Price'\};.

Default: OutSpec = \{'Price' $\}$

## UndIdx

Scalar of the indice of the underlying instrument to compute the sensitivity.

Default: UndIdx = [ ]

## basketsensbyls

## Output <br> Arguments

## Examples

## PriceSens

Expected prices or sensitivities values.

## Calculate Price and Sensitivities for Basket Options Using the Longstaff-Schwartz Model

Find a European put basket option of two stocks. The basket contains $50 \%$ of each stock. The stocks are currently trading at $\$ 90$ and $\$ 75$, with annual volatilities of $15 \%$. Assume that the correlation between the assets is zero. On May 1, 2009, an investor wants to buy a one-year put option with a strike price of $\$ 80$. The current annualized, continuously compounded interest is $5 \%$. Use this data to compute price and delta of the put basket option with the Longstaff-Schwartz approximation model.

```
Settle = 'May-1-2009';
Maturity = 'May-1-2010';
% Define RateSpec
Rate = 0.05;
Compounding = -1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates',...
Settle, 'EndDates', Maturity, 'Rates', Rate, 'Compounding', Compounding);
% Define the Correlation matrix. Correlation matrices are symmetric,
% and have ones along the main diagonal.
NumInst = 2;
InstIdx = ones(NumInst,1);
Corr = diag(ones(NumInst,1), 0);
% Define BasketStockSpec
AssetPrice = [90; 75];
Volatility = 0.15;
Quantity = [0.50; 0.50];
BasketStockSpec = basketstockspec(Volatility, AssetPrice, Quantity, Corr)
% Compute the price of the put basket option. Calculate also the delta
```


## basketsensbyls

```
% of the first stock.
OptSpec = {'put'};
Strike = 80;
OutSpec = {'Price','Delta'};
UndIdx = 1; % First element in the basket
[PriceSens, Delta] = basketsensbyls(RateSpec, BasketStockSpec, OptSpe
Strike, Settle, Maturity,'OutSpec', OutSpec,'UndIdx', UndIdx)
PriceSens =
    0.9822
Delta =
    -0.0995
```

Compute the Price and Delta of the basket with a correlation of - $20 \%$ :

```
NewCorr = [1 -0.20; -0.20 1];
```

\% Define the new BasketStockSpec.
BasketStockSpec = basketstockspec(Volatility, AssetPrice, Quantity,
\% Compute the price and delta of the put basket option.
[PriceSens, Delta] = basketsensbyls(RateSpec, BasketStockSpec, OptSpe Strike, Settle, Maturity,'OutSpec', OutSpec,'UndIdx', UndIdx)

PriceSens =
0.7814

## basketsensbyls

```
Delta =
\(-0.0961\)
```

References Longstaff, F.A., and E.S. Schwartz, "Valuing American Options by Simulation: A Simple Least-Squares Approach", The Review of Financial Studies,Vol. 14, No. 1, Spring 2001, pp. 113-147.

See Also basketstockspec | basketbyls
How To . "Basket Option" on page 3-25

## Purpose

Specify basket stock structure using Longstaff-Schwartz model
Syntax
BasketStockSpec = basketstockspec(Sigma, AssetPrice, Quantity, Correlation)
BasketStockSpec = basketstockspec(Sigma, AssetPrice, Quantity, Correlation, 'ParameterName',ParameterValue ...)

## Description

## Input <br> Arguments

BasketStockSpec = basketstockspec(Sigma, AssetPrice, Quantity, Correlation) creates a basket stock structure.

BasketStockSpec = basketstockspec(Sigma, AssetPrice, Quantity, Correlation, 'ParameterName',ParameterValue ...) accepts optional inputs as one or more comma-separated parameter/value pairs. 'ParameterName' is the name of the parameter inside single quotes. 'Parametervalue is the value corresponding to 'ParameterName'. Specify parameter-value pairs in any order. Names are case-insensitive and partial string matches are allowable, if no ambiguities exist.

## Sigma

NINST-by-1 vector of decimal annual price volatility of the underlying security.

## AssetPrice

NINST-by-1 vector of underlying asset price values at time 0 .

## Quantity

NINST-by- 1 vector of quantities of the instruments contained in the basket.

## Correlation

NINST-by-NINST matrix of correlation values.

## basketstockspec

## Parameter-Value Pairs

## DividendAmounts

NINST-by-1 cell array specifying the dividend amounts for basket instruments. Each element of the cell array is a 1 -by-NDIV row vector of cash dividends or a scalar representing a continuous annualized dividend yield for the corresponding instrument.

## DividendType

NINST-by-1 cell array of strings specifying each stock's dividend type. Dividend type must be either cash for actual dollar dividends or continuous for continuous dividend yield. .

## ExDividendDates

NINST-by- 1 cell array specifying the ex-dividend dates for the basket instruments. Each row is a 1 -by-NDIV matrix of ex-dividend dates for cash type. For rows that correspond to basket instruments with continuous dividend type, the cell is empty. If none of the basket instruments pay continuous dividends, do not specify ExDividendDates.

## Output <br> Arguments

## Examples

## BasketStockSpec

Structure encapsulating the properties of a basket stock structure.

## Create a Basket Stock Structure for Three Stocks

Find a basket option of three stocks. The stocks are currently trading at $\$ 56, \$ 92$ and $\$ 125$ with annual volatilities of $20 \%, 12 \%$ and $15 \%$, respectively. The basket option contains $25 \%$ of the first stock, $40 \%$ of the second stock, and $35 \%$ of the third. The first stock provides a continuous dividend of $1 \%$, while the other two provide no dividends. The correlation between the first and second asset is $30 \%$, between the second and third asset $11 \%$, and between the first and third asset $16 \%$. Use this data to create the BasketStockSpec structure:

## basketstockspec

```
AssetPrice = [56;92;125];
Sigma = [0.20;0.12;0.15];
% Create the Correlation matrix. Correlation matrices are symmetric ar
% have ones along the main diagonal.
NumInst = 3;
Corr = zeros(NumInst,1);
Corr(1,2) = . 30;
Corr(2,3) = .11;
Corr(1,3) = .16;
Corr = triu(Corr,1) + tril(Corr',-1) + diag(ones(NumInst,1), 0);
% Define dividends
DivType = cell(NumInst,1);
DivType{1}='continuous';
DivAmounts = cell(NumInst,1);
DivAmounts{1} = 0.01;
Quantity = [0.25; 0.40; 0.35];
BasketStockSpec = basketstockspec(Sigma, AssetPrice, Quantity, Corr,
'DividendType', DivType, 'DividendAmounts', DivAmounts)
BasketStockSpec =
                    FinObj: 'BasketStockSpec'
                    Sigma: [3x1 double]
                    AssetPrice: [3x1 double]
                    Quantity: [3x1 double]
            Correlation: [3x3 double]
            DividendType: {3x1 cell}
        DividendAmounts: {3x1 cell}
    ExDividendDates: {3x1 cell}
```

Examine the BasketStockSpec structure.

## basketstockspec

BasketStockSpec.Correlation
ans =

| 1.0000 | 0.3000 | 0.1600 |
| :--- | :--- | :--- |
| 0.3000 | 1.0000 | 0.1100 |
| 0.1600 | 0.1100 | 1.0000 |

## Create a Basket Stock Structure for Two Stocks

Find a basket option of two stocks. The stocks are currently trading at $\$ 60$ and $\$ 55$ with volatilities of $30 \%$ per annum. The basket option contains $50 \%$ of each stock. The first stock provides a cash dividend of $\$ 0.25$ on May 1, 2009 and September 1, 2009. The second stock provides a continuous dividend of $3 \%$. The correlation between the assets is $40 \%$. Use this data to create the structure BasketStockSpec:

```
AssetPrice = [60;55];
Sigma = [0.30;0.30];
% Create the Correlation matrix. Correlation matrices are symmetric and
% have ones along the main diagonal.
Correlation = [1 0.40;0.40 1];
% Define dividends
NumInst = 2;
DivType = cell(NumInst,1);
DivType{1}='cash';
DivType{2}='continuous';
DivAmounts = cell(NumInst,1);
DivAmounts{1} = [0.25 0.25];
DivAmounts{2} = 0.03;
ExDates = cell(NumInst,1);
```

```
ExDates{1} = {'May-1-2009' 'Sept-1-2009'};
Quantity = [0.5; 0.50];
BasketStockSpec = basketstockspec(Sigma, AssetPrice, Quantity, Correla
'DividendType', DivType, 'DividendAmounts', DivAmounts, 'ExDividendDa'
BasketStockSpec =
            FinObj: 'BasketStockSpec'
                    Sigma: [2x1 double]
            AssetPrice: [2x1 double]
                    Quantity: [2x1 double]
            Correlation: [2x2 double]
            DividendType: {2x1 cell}
                DividendAmounts: {2x1 cell}
                ExDividendDates: {2x1 cell}
```

Examine the BasketStockSpec structure.
BasketStockSpec.DividendType

```
ans =
    'cash'
    'continuous'
```

See Also basketbyls | basketbyju | basketsensbyju | basketsensbyls | stockspec | intenvset
How To - "Basket Option" on page 3-25

## bdtprice

# Purpose <br> Instrument prices from Black-Derman-Toy interest-rate tree 

```
Syntax
[Price, PriceTree] = bdtprice(BDTTree, InstSet, Options)
```


## Arguments

## Description

[Price, PriceTree] = bdtprice(BDTTree, InstSet, Options) computes arbitrage-free prices for instruments using an interest-rate tree created with bdttree. All instruments contained in a financial instrument variable, InstSet, are priced.

Price is a number of instruments (NINST)-by- 1 vector of prices for each instrument at time 0 . The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, NaN is returned.

PriceTree is a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node.

PriceTree.PTree contains the clean prices.
PriceTree.AITree contains the accrued interest.
PriceTree.tObs contains the observation times.
bdtprice handles instrument types: 'Bond', 'CashFlow', 'OptBond', 'OptEmBond', 'OptFloat', 'OptEmFloat', 'Fixed', 'Float', 'Cap',
'Floor', 'RangeFloat', 'Swap'. See instadd to construct defined types.
Related single-type pricing functions are:

- bondbybdt: Price a bond from a BDT tree.
- capbybdt: Price a cap from a BDT tree.
- cfbybdt: Price an arbitrary set of cash flows from a BDT tree.
- fixedbybdt: Price a fixed-rate note from a BDT tree.
- floatbybdt: Price a floating-rate note from a BDT tree.
- floorbybdt: Price a floor from a BDT tree.
- optbndbybdt: Price a bond option from a BDT tree.
- optfloatbybdt: Price a floating-rate note with an option from a BDT tree.
- optemfloatbybdt: Price a floating-rate note with an embedded option from a BDT tree.
- optembndbybdt: Price a bond with embedded option by a BDT tree.
- rangefloatbybdt: Price range floating note using a BDT tree.
- swapbybdt: Price a swap from a BDT tree.
- swaptionbybdt: Price a swaption from a BDT tree.


## Examples Price a Cap and Bond Instruments Contained in an Instrument Set

Load the BDT tree and instruments from the data file deriv.mat to price the cap and bond instruments contained in the instrument set.

```
load deriv.mat;
BDTSubSet = instselect(BDTInstSet,'Type', {'Bond', 'Cap'});
instdisp(BDTSubSet)
```


## bdtprice

| Index | Type | CouponRate | Settle | Maturity | Period | d Basis | s EndMonthRule | IssueDate | FirstCouponDate |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Bond | 0.1 | 01-Jan-2000 | 01-Jan-2003 | 1 | NaN | NaN | NaN | NaN |
| 2 | Bond | 0.1 | 01-Jan-2000 | 01-Jan-2004 | 2 | NaN | NaN | NaN | NaN |
| Index | Type | Strike Set | tle | Maturity C | CapReset B | Basis P | Principal Name | Quantit |  |
| 3 | cap | 0.15 01-J | Jan-2000 | 01-Jan-2004 1 | 1 N | NaN N | $\mathrm{NaN} 15 \%$ | Cap 30 |  |

Price the bond and cap.
[Price, PriceTree] = bdtprice(BDTTree, BDTSubSet)

```
Warning: Not all cash flows are aligned with the tree. Result will be approximated.
> In checktree at 289
    In bdtprice at }8
Price =
    95.5030
    93.9079
        1.4375
PriceTree =
\begin{tabular}{|c|c|c|c|c|}
\hline PTree: \(\{[3 \times 1\) double] & [3x2 double] & [3x3 double] & [3x4 double] & [3×4 double]\} \\
\hline AITree: \(\{[3 \times 1\) double] & [3x2 double] & [3x3 double] & [3x4 double] & [3x4 double]\} \\
\hline
\end{tabular}
```

You can use the treeviewer function to see the prices of these three instruments along the price tree.


First 10\% Bond (Maturity 2003)


Second 10\% Bond (Maturity 2004)


15\% Cap

## Price Multi-Stepped Coupon Bonds

Price the following multi-stepped coupon bonds using the following data:

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
```


## bdtprice

```
% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
% Create a portfolio of stepped coupon bonds with different maturities
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011';'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2011' .042;'01-Jan-2012' .05; '01-Jan-2013' .06;
% Display the instrument portfolio
ISet = instbond(CouponRate, Settle, Maturity, 1);
instdisp(ISet)
\begin{tabular}{lllllll} 
Index & Type CouponRate & Settle & Maturity & Period Basis EndMontr \\
1 & Bond [Cell] & 01-Jan-2010 & \(01-\) Jan-2011 & 1 & 0 & 1 \\
2 & Bond [Cell] & \(01-\) Jan-2010 & \(01-\) Jan-2012 & 1 & 0 & 1 \\
3 & Bond [Cell] & 01 -Jan-2010 & 01-Jan-2013 & 1 & 0 & 1 \\
4 & Bond [Cell] & 01-Jan-2010 & 01-Jan-2014 & 1 & 0 & 1
\end{tabular}
```

Build a BDTTree to price the stepped coupon bonds. Assume the volatility to be $10 \%$

```
Sigma = 0.1;
```

BDTTimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length(Enc
BDTT = bdttree(BDTVolSpec, RS, BDTTimeSpec);
\% Compute the price of the stepped coupon bonds
PBDT = bdtprice(BDTT, ISet)

PBDT =

## bdtprice

## Price a Portfolio of Stepped Callable Bonds and Stepped Vanilla Bonds

Price a portfolio of stepped callable bonds and stepped vanilla bonds using the following data: The data for the interest rate term structure is as follows:

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
%Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDat
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
% Create an instrument portfolio of 3 stepped callable bonds and thre
% stepped vanilla bonds
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2011' .042;'01-Jan-2012' .05; '01-Jan-2013' .0@
OptSpec='call';
Strike=100;
ExerciseDates='01-Jan-2011'; %Callable in one year
% Bonds with embedded option
ISet = instoptembnd(CouponRate, Settle, Maturity, OptSpec, Strike,...
ExerciseDates, 'Period', 1);
% Vanilla bonds
ISet = instbond(ISet, CouponRate, Settle, Maturity, 1);
```


## bdtprice

```
% Display the instrument portfolio
instdisp(ISet)
```

| Index | Type | CouponRate | Settle | Maturity | OptSpec Strike |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | OptEmBond | [Cell] | $01-$ Jan-2010 | $01-$ Jan-2012 | call | 100 |
| 2 | OptEmBond | [Cell] | $01-$ Jan-2010 | $01-$ Jan-2013 | call | 100 |
| 3 | OptEmBond | [Cell] | $01-$ Jan-2010 | $01-$ Jan-2014 | call | 100 |


| Index | Type CouponRate | Settle | Maturity | Period | Basis EndMonth |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 4 | Bond [Cell] | 01-Jan-2010 | 01-Jan-2012 | 1 | 0 | 1 |
| 5 | Bond [Cell] | 01-Jan-2010 | 01-Jan-2013 | 1 | 0 | 1 |
| 6 | Bond [Cell] | 01-Jan-2010 | 01-Jan-2014 | 1 | 0 | 1 |

Build a BDTTree and price the instruments. Build the tree Assume the volatility to be $10 \%$

```
Sigma = 0.1;
BDTTimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length(Eno
BDTT = bdttree(BDTVolSpec, RS, BDTTimeSpec);
%The first three rows corresponds to the price of the stepped callable bc
%last three rows corresponds to the price of the stepped vanilla bonds.
PBDT = bdtprice(BDTT, ISet)
PBDT =
```

    100.4799
    100.3228
    100.0840
    100.7368
    100.9266
    101.0115
    
## Price a Portfolio with Range Notes and a Floating Rate Note

Compute the price of a portfolio with range notes and a floating rate note using the following data: The data for the interest rate term structure is as follows:

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
Compounding = 1;
% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDat
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
% Create an instrument portfolio with two range notes and a floating
% note with the following data:
Spread = 200;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';
% First Range Note:
RateSched(1).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(1).Rates = [0.045 0.055; 0.0525 0.0675; 0.06 0.08];
% Second Range Note:
RateSched(2).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(2).Rates = [0.048 0.059; 0.055 0.068 ; 0.07 0.09];
% Create InstSet
InstSet = instadd('RangeFloat', Spread, Settle, Maturity, RateSched);
% Add a floating-rate note
InstSet = instadd(InstSet, 'Float', Spread, Settle, Maturity);
% Display the portfolio instrument
```


## bdtprice

```
instdisp(InstSet)
```

| Index | Type | Spread | Settle | Maturity | RateSched FloatRese |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | RangeFloat 200 | $01-$ Jan-2011 | $01-$ Jan-2014 | [Struct] | 1 |
| 2 | RangeFloat 200 | $01-$ Jan-2011 | $01-$ Jan-2014 | [Struct] | 1 |

Build a BDTTree and price the instruments. Build the tree Assume the volatility to be $10 \%$.

```
Sigma = 0.1;
BDTTS = bdttimespec(ValuationDate, EndDates, Compounding);
BDTVS = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length(EndDates
BDTT = bdttree(BDTVS, RS, BDTTS);
% Price the portfolio
Price = bdtprice(BDTT, InstSet)
```

Price =
100.2841
98.0757
105.5147

## See Also

bdtsens | bdttree | instadd | intenvprice | intenvsens

## Purpose

Syntax

## Arguments

## Description

Instrument prices and sensitivities from Black-Derman-Toy interest-rate tree
[Delta, Gamma, Vega, Price] = bdtsens(BDTTree, InstSet, Options)

BDTTree
Interest-rate tree structure created by bdttree.
InstSet Variable containing a collection of NINST instruments. Instruments are categorized by type. Each type can have different data fields. The stored data field is a row vector or string for each instrument.

Options (Optional) Derivatives pricing options structure created with derivset.
[Delta, Gamma, Vega, Price] = bdtsens(BDTTree, InstSet, Options) computes instrument sensitivities and prices for instruments using an interest-rate tree created with the bdttree function. NINST instruments from a financial instrument variable, InstSet, are priced. bdtsens handles instrument types: 'Bond', 'CashFlow', 'OptBond', 'OptEmBond', 'OptEmBond', 'OptFloat', 'OptEmFloat', 'Fixed', 'Float', 'Cap', 'Floor', 'RangeFloat', 'Swap'. See instadd for information on instrument types.

Delta is an NINST-by- 1 vector of deltas, representing the rate of change of instrument prices with respect to changes in the interest rate. Delta is computed by finite differences in calls to bdttree. See bdttree for information on the observed yield curve.

Gamma is an NINST-by- 1 vector of gammas, representing the rate of change of instrument deltas with respect to the changes in the interest rate. Gamma is computed by finite differences in calls to bdttree.

Vega is an NINST-by-1 vector of vegas, representing the rate of change of instrument prices with respect to the changes in the volatility
$\sigma(t, T)$. Vega is computed by finite differences in calls to bdttree. See bdtvolspec for information on the volatility process.

Note All sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, divide by the respective instrument price.

Price is an NINST-by- 1 vector of prices of each instrument. The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, NaN is returned.
Delta and Gamma are calculated based on yield shifts of 100 basis points. Vega is calculated based on a $1 \%$ shift in the volatility process.

## Examples Compute Instrument Sensitivities and Prices for Cap and Bond instruments

Load the tree and instruments from the deriv.mat data file.

```
load deriv.mat;
BDTSubSet = instselect(BDTInstSet,'Type', {'Bond', 'Cap'});
instdisp(BDTSubSet)
\begin{tabular}{lllllll} 
Index & Type CouponRate Settle & Maturity & Period Basis EndMontr \\
1 & Bond 0.1 & 01-Jan-2000 & 01-Jan-2003 & 1 & NaN & NaN \\
2 & Bond 0.1 & 01-Jan-2000 & 01-Jan-2004 & 2 & NaN & NaN
\end{tabular}
\begin{tabular}{llllll} 
Index & Type Strike Settle & Maturity & CapReset Basis Principal \\
3 & Cap 0.15 & \(01-J a n-2000\) & \(01-J a n-2004\) & 1 & NaN NaN
\end{tabular}
```

Compute Delta and Gamma for the cap and bond instruments contained in the instrument set.

```
[Delta, Gamma] = bdtsens(BDTTree, BDTSubSet)
```

Warning: Not all cash flows are aligned with the tree. Result will be approximated.

Delta =
$-232.6681$

- 281.0517 63.8102

Gamma =
$1.0 \mathrm{e}+03$ *
0.8037
1.1819
1.8535

See Also
bdtprice | bdttree | bdtvolspec | instadd

## bdttimespec

```
Purpose Specify time structure for Black-Derman-Toy interest-rate tree
Syntax TimeSpec = bdttimespec(ValuationDate, Maturity, Compounding)
```


## Arguments

| ValuationDate | Scalar date marking the pricing date and first observation in the tree. Specify as serial date number or date string. |
| :---: | :---: |
| Maturity | Number of levels (depth) of the tree. A number of levels (NLEVELS)-by- 1 vector of dates marking the cash flow dates of the tree. Cash flows with these maturities fall on tree nodes. Maturity should be in increasing order. |
| Compounding | (Optional) Scalar value representing the rate at which the input zero rates were compounded when annualized. Default = 1. This argument determines the formula for the discount factors: |
|  | Compounding $=1,2,3,4,6,12$ |
|  | Disc $=(1+Z / F)^{\wedge}(-T)$, where $F$ is the compounding frequency, $Z$ is the zero rate, and T is the time in periodic units; for example, $\mathrm{T}=$ $F$ is 1 year. |
|  | Compounding $=365$ |
|  | Disc $=(1+Z / F)^{\wedge}(-T)$, where $F$ is the number of days in the basis year and $T$ is a number of days elapsed computed by basis. |
|  | Compounding $=-1$ |
|  | Disc $=\exp \left(-T^{*} \mathrm{Z}\right)$, where T is time in years. |

## bdttimespec

## Description

TimeSpec = bdttimespec(ValuationDate, Maturity, Compounding) sets the number of levels and node times for a BDT tree and determines the mapping between dates and time for rate quoting.

TimeSpec is a structure specifying the time layout for bdttree. The state observation dates are [ValuationDate; Maturity(1:end-1)]. Because a forward rate is stored at the last observation, the tree can value cash flows out to Maturity.

## Examples Specify a Five-Period Tree with Annual Nodes

This example shows how to specify a five-period tree with annual nodes and use annual compounding to report rates.

```
Compounding = 1;
ValuationDate = '01-01-2000';
Maturity = ['01-01-2001'; '01-01-2002'; '01-01-2003';
'01-01-2004'; '01-01-2005'];
TimeSpec = bdttimespec(ValuationDate, Maturity, Compounding)
```

TimeSpec =
FinObj: 'BDTTimeSpec'
ValuationDate: 730486
Maturity: [5x1 double] Compounding: 1

Basis: 0
EndMonthRule: 1

See Also bdttree | bdtvolspec

# Purpose Construct Black-Derman-Toy interest-rate tree 

```
Syntax BDTTree = bdttree(VolSpec, RateSpec, TimeSpec)
```


## Arguments

## Description BDTTree $=$ bdttree (VolSpec, RateSpec, TimeSpec) creates a structure containing time and interest-rate information on a recombining tree.

## Examples

Using the data provided, create a BDT volatility specification (VolSpec), rate specification (RateSpec), and tree time layout specification (TimeSpec). Then use these specifications to create a BDT tree with bdttree.

```
Compounding = 1;
ValuationDate = '01-01-2000';
StartDate = ValuationDate;
EndDates = ['01-01-2001'; '01-01-2002'; '01-01-2003';
'01-01-2004'; '01-01-2005'];
Rates = [.1; .11; .12; .125; .13];
Volatility = [.2; .19; .18; .17; .16];
RateSpec = intenvset('Compounding', Compounding,...
```

```
    'ValuationDate', ValuationDate,...
    'StartDates', StartDate,...
    'EndDates', EndDates,...
    'Rates', Rates);
BDTTimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Volatility);
BDTTree = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec)
BDTTree =
    FinObj: 'BDTFwdTree
    VolSpec: [1\times1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1\times1 struct]
        tObs: [[0 1 1 2 3 4
        dObs: [730486 730852 731217 731582 731947]
    TFwd: {[5x1 double] [4\times1 double] [3\times1 double] [2x1 double] [4]}
    CFlowT: {[5\times1 double] [4\times1 double] [3\times1 double] [2x1 double] [5]}
    FwdTree: {[1.1000] [1.0979 1.1432] [1.0976 1.1377 1.1942] [1.0872 1.1183 1.1606 1.2179]
```

Use treeviewer to observe the tree you have created.
treeviewer(BDTTree)

## bdttree



See Also
bdtprice | bdttimespec | bdtvolspec | intenvset

## Purpose Specify Black-Derman-Toy interest-rate volatility process

```
Syntax
Volspec = bdtvolspec(ValuationDate, VolDates, VolCurve,
    InterpMethod)
```


## Arguments

| ValuationDate | Scalar value representing the observation date <br> of the investment horizon. |
| :--- | :--- |
| VolDates | Number of points (NPOINTS) -by-1 vector of yield <br> volatility end dates. |
| VolCurve | NPOINTS-by-1 vector of yield volatility values in <br> decimal form. |
| InterpMethod | (Optional) Interpolation method. Default is <br> 'linear'. See interp1 for more information. |

## Description Volspec = bdtvolspec(ValuationDate, VolDates, VolCurve, InterpMethod) creates a structure specifying the volatility for bdttree. <br> Examples Create a BDT Volatility Specification

This example shows how to create a BDT volatility specification (VolSpec) using the following data.

```
ValuationDate = '01-01-2000';
EndDates = ['01-01-2001'; '01-01-2002'; '01-01-2003';
'01-01-2004'; '01-01-2005'];
Volatility = [.2; .19; .18; .17; .16];
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Volatility)
```


## bdtvolspec

FinObj: 'BDTVolSpec'<br>ValuationDate: 730486<br>VolDates: [5x1 double]<br>VolCurve: [5x1 double]<br>VolInterpMethod: 'linear'

See Also
bdttree | interp1

## Purpose

Instrument prices from Black-Karasinski interest-rate tree

## Syntax

Arguments

## Description

[Price, PriceTree] = bkprice(BKTree, InstSet, Options) computes arbitrage-free prices for instruments using an interest-rate tree created with bktree. All instruments contained in a financial instrument variable, InstSet, are priced.

Price is a number of instruments (NINST)-by-1 vector of prices for each instrument. The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, NaN is returned.
PriceTree is a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node.

PriceTree. PTree contains the clean prices.
PriceTree.AITree contains the accrued interest.
PriceTree.tObs contains the observation times.
bkprice handles instrument types: 'Bond', 'CashFlow', 'OptBond', 'OptEmBond', 'OptEmBond', 'OptFloat', 'OptEmFloat', 'Fixed',

## bkprice

'Float', 'Cap', 'Floor', 'RangeFloat', 'Swap'. See instadd to construct defined types.

Related single-type pricing functions are:

- bondbybk: Price a bond from a Black-Karasinski tree.
- capbybk: Price a cap from a Black-Karasinski tree.
- cfbybk: Price an arbitrary set of cash flows from a Black-Karasinski tree.
- fixedbybk: Price a fixed-rate note from a Black-Karasinski tree.
- floatbybk: Price a floating-rate note from a Black-Karasinski tree.
- floorbybk: Price a floor from a Black-Karasinski tree.
- optbndbybk: Price a bond option from a Black-Karasinski tree.
- optembndbybk: Price a bond with embedded option by a Black-Karasinski tree.
- optfloatbybdt: Price a floating-rate note with an option from a Black-Karasinski tree.
- optemfloatbybdt: Price a floating-rate note with an embedded option from a Black-Karasinski tree.
- rangefloatbybk: Price range floating note from a Black-Karasinski tree.
- swapbybk: Price a swap from a Black-Karasinski tree.
- swaptionbybk: Price a swaption from a Black-Karasinski tree.


## Examples Load the BK tree and instruments from the data file deriv.mat. Price

 the cap and bond instruments contained in the instrument set.```
load deriv.mat;
BKSubSet = instselect(BKInstSet,'Type', {'Bond', 'Cap'});
instdisp(BKSubSet)
```

```
%Table of instrument portfolio partially displayed:
Index Type CouponRate Settle Maturity Period ... Name ...
1 Bond 0.03 01-Jan-2004 01-Jan-2007 1 ... 3% bond
2 Bond 0.03 01-Jan-2004 01-Jan-2008 2 ... 3% bond
Index Type Strike Settle Maturity CapReset ... Name ...
3 Cap 0.04 01-Jan-2004 01-Jan-2008 1 ... 4% Cap
[Price, PriceTree] = bkprice(BKTree, BKSubSet);
Price =
    98.1096
    95.6734
    2.2706
```

You can use treeviewer to see the prices of these three instruments along the price tree.

```
treeviewer(PriceTree, BKSubSet)
```


## bkprice



First 3\% Bond (Maturity 2007)


Second 3\% Bond (Maturity 2008)


4\% Cap

Price the following multi-stepped coupon bonds using the following data:

```
% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
% Create a portfolio of stepped coupon bonds with different maturities
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011';'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2011' .042;'01-Jan-2012' .05; '01-Jan-2013' .06; '01-Jan-2014' .07}};
ISet = instbond(CouponRate, Settle, Maturity, 1);
instdisp(ISet)
%Table of instrument portfolio partially displayed:
\begin{tabular}{llllllll} 
Index Type CouponRate Settle & Maturity & Period & Basis & EndMonthRule & ... Face \\
1 Bond [Cell] & \(01-J a n-2010\) & \(01-J a n-2011\) & 1 & 0 & 1 & \(\ldots\) & 100 \\
2 Bond [Cell] & \(01-J a n-2010\) & \(01-J a n-2012\) & 1 & 0 & 1 & \(\ldots\) & 100 \\
3 Bond [Cell] & \(01-J a n-2010\) & \(01-J a n-2013\) & 1 & 0 & 1 & \(\ldots\) & 100 \\
4 Bond [Cell] & 01 -Jan-2010 & 01 -Jan-2014 & 1 & 0 & 1 & \(\ldots\). & 100
\end{tabular}
% Build the tree with the following data
VolDates = ['1-Jan-2011'; '1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'];
VolCurve = 0.01;
AlphaDates = '01-01-2014';
AlphaCurve = 0.1;
BKVolSpec = bkvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
BKTimeSpec = bktimespec(RS.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVolSpec, RS, BKTimeSpec);
```


## bkprice

```
% Compute the price of the stepped coupon bonds
PBK = bkprice(BKT, ISet)
PBK =
    100.6763
    100.7368
    100.9266
    101.0115
```

Price a portfolio of stepped callable bonds and stepped vanilla bonds using the following data:

```
% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
%Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
% Create an instrument portfolio of 3 stepped callable bonds and three
% stepped vanilla bonds
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2011' .042;'01-Jan-2012' .05; '01-Jan-2013' .06; '01-Jan-2014' .07}};
OptSpec='call';
Strike=100;
ExerciseDates='01-Jan-2011'; %Callable in one year
% Bonds with embedded option
```

```
ISet = instoptembnd(CouponRate, Settle, Maturity, OptSpec, Strike,...
ExerciseDates, 'Period', 1);
% Vanilla bonds
ISet = instbond(ISet, CouponRate, Settle, Maturity, 1);
% Display the instrument portfolio
instdisp(ISet)
%Table of instrument portfolio partially displayed:
\begin{tabular}{llclllllll} 
Index & Type CouponRate & Settle & Maturity & OptSpec & Strike ExerciseDates & \(\ldots\) & AmericanOpt \\
1 & OptEmBond [Cell] & \(01-J a n-2010\) & \(01-J a n-2012\) & call & 100 & \(01-J a n-2011\) & \(\ldots\) & 0 \\
2 & OptEmBond [Cell] & \(01-J a n-2010\) & \(01-J a n-2013\) & call & 100 & \(01-J a n-2011\) & \(\ldots\) & 0 \\
3 & OptEmBond [Cell] & \(01-\) Jan-2010 & \(01-J a n-2014\) & call & 100 & \(01-J a n-2011\) & \(\ldots\) & 0
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline Index & Type & CouponRate & Settle & Maturity & Period & Basis & EndMonthRule & Face \\
\hline 4 & Bond & [Cell] & 01-Jan-2010 & 01-Jan-2012 & 1 & 0 & 1 & 100 \\
\hline 5 & Bond & [Cell] & 01-Jan-2010 & 01-Jan-2013 & 1 & 0 & 1 & 100 \\
\hline 6 & Bond & Cell & 01-Jan-2010 & 01-Jan-2014 & 1 & 0 & 1 & 100 \\
\hline
\end{tabular}
% Build the tree with the following data
VolDates = ['1-Jan-2011'; '1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'];
VolCurve = 0.01;
AlphaDates = '01-01-2014';
AlphaCurve = 0.1;
```

BKVolSpec = bkvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
BKTimeSpec = bktimespec(RS.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVolSpec, RS, BKTimeSpec);
\% The first three rows corresponds to the price of the stepped callable bonds
\% and the last three rows corresponds to the price of the stepped vanilla bonds.
PBK = bkprice(BKT, ISet)

## bkprice

```
PBK =
    100.6735
    100.6763
    100.6763
    100.7368
    100.9266
    101.0115
```

Compute the price of a portfolio using the following data:

```
% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
Compounding = 1;
% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates',...
StartDates, 'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
% Create an instrument portfolio with two range notes and a floating rate
% note with the following data:
Spread = 200;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';
% First Range Note:
RateSched(1).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(1).Rates = [0.045 0.055; 0.0525 0.0675; 0.06 0.08];
% Second Range Note:
RateSched(2).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
```

```
RateSched(2).Rates = [0.048 0.059; 0.055 0.068; 0.07 0.09];
```

```
% Create InstSet
InstSet = instadd('RangeFloat', Spread, Settle, Maturity, RateSched);
% Add a floating-rate note
InstSet = instadd(InstSet, 'Float', Spread, Settle, Maturity);
% Display the portfolio instrument
instdisp(InstSet)
```

| Index | Type | Spread | Settle | Maturity | RateSched | FloatReset | Basis | Principal EndMonthRule |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| Index | Type Spread Settle | Maturity |  | FloatReset | Basis | Principal EndMonthRule |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 3 | Float 200 | $01-$ Jan-2011 | $01-$ Jan-2014 | 1 | 0 | 100 | 1 |

\% The data to build the tree is as follows:
VolDates = ['1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014';'1-Jan-2015'];
VolCurve = 0.01;
AlphaDates = '01-01-2015';
AlphaCurve $=0.1$;
BKVS = bkvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
BKTS = bktimespec(RS.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVS, RS, BKTS);
\% Price the portfolio
Price = bkprice(BKT, InstSet)
Price =
105.5147

## bkprice

101.4740
105.5147

See Also bksens | bktree | instadd | intenvprice | intenvsens

## Purpose

## Arguments

## Description

Syntax $\begin{aligned} & \text { [Delta, Gamma, Vega, Price] }=\text { bksens(BKTree, InstSet, } \\ & \text { Options) }\end{aligned}$
Instrument prices and sensitivities from Black-Karasinski interest-rate tree

BKTree Interest-rate tree structure created by bktree.
InstSet Variable containing a collection of NINST instruments. Instruments are categorized by type. Each type can have different data fields. The stored data field is a row vector or string for each instrument.

Options (Optional) Derivatives pricing options structure created with derivset.
[Delta, Gamma, Vega, Price] = bksens(BKTree, InstSet, Options) computes instrument sensitivities and prices for instruments using an interest-rate tree created with the bktree function. NINST instruments from a financial instrument variable, InstSet, are priced. bksens handles instrument types: 'Bond', 'CashFlow', 'OptBond', 'OptEmBond', 'OptEmBond', 'OptFloat', 'OptEmFloat', 'Fixed', 'Float', 'Cap', 'Floor', 'RangeFloat', 'Swap'. See instadd for information on instrument types.

Delta is an NINST-by- 1 vector of deltas, representing the rate of change of instrument prices with respect to changes in the interest rate. Delta is computed by finite differences in calls to bktree. See bktree for information on the observed yield curve.

Gamma is an NINST-by- 1 vector of gammas, representing the rate of change of instrument deltas with respect to the changes in the interest rate. Gamma is computed by finite differences in calls to bktree.

## bksens

Vega is an NINST-by-1 vector of vegas, representing the rate of change of instrument prices with respect to the changes in the volatility $\sigma(t, T)$. Vega is computed by finite differences in calls to bktree. See bkvolspec for information on the volatility process.

Note All sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, divide by the respective instrument price.

Price is an NINST-by-1 vector of prices of each instrument. The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, NaN is returned.

Delta and Gamma are calculated based on yield shifts of 100 basis points. Vega is calculated based on a $1 \%$ shift in the volatility process.

## Examples <br> Compute Instrument Sensitivities and Prices for Cap and Bond Instruments

Load the tree and instruments from the deriv.mat data file.

```
load deriv.mat;
BKSubSet = instselect(BKInstSet,'Type', {'Bond', 'Cap'});
instdisp(BKSubSet)
```

| Index | Type CouponRate | Settle | Maturity | Period Basis EndMontr |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | Bond 0.03 | 01-Jan-2004 | $01-$ Jan-2007 | 1 | 0 | 1 |
| 2 | Bond 0.03 | 01-Jan-2004 | 01-Jan-2008 | 1 | 0 | 1 |


| Index | Type Strike Settle | Maturity | CapReset Basis Principal |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 3 | Cap 0.04 | 01-Jan-2004 | 01-Jan-2008 | 1 | 0 | 100 |

Compute Delta and Gamma for the cap and bond instruments contained in the instrument set.

```
[Delta, Gamma] = bksens(BKTree, BKSubSet)
Delta =
    -285.7151
    -365.7048
    189.5319
Gamma =
    1.0e+03 *
    0.8456
    1.4345
    6.9999
```

See Also
bkprice | bktree | bkvolspec | instadd

## bktimespec

Purpose Specify time structure for Black-Karasinski tree<br>Syntax $\quad$ TimeSpec $=$ bktimespec(ValuationDate, Maturity, Compounding)

## Arguments

| ValuationDate | Scalar date marking the pricing date and first observation in the tree. Specify as a serial date number or date string. |
| :---: | :---: |
| Maturity | Number of levels (depth) of the tree. A number of levels (NLEVELS)-by- 1 vector of dates marking the cash flow dates of the tree. Cash flows with these maturities fall on tree nodes. Maturity should be in increasing order. |
| Compounding | (Optional) Scalar value representing the rate at which the input zero rates were compounded when annualized. Default $=-1$ (continuous compounding). This argument determines the formula for the discount factors: |
|  | Compounding $=1,2,3,4,6,12=\mathrm{F}$ |
|  | Disc $=(1+Z / F)^{\wedge}(-T)$, where $F$ is the compounding frequency, $Z$ is the zero rate, and T is the time in periodic units; for example, $\mathrm{T}=$ $F$ is 1 year. |
|  | Compounding $=365$ |
|  | Disc $=(1+Z / F)^{\wedge}(-T)$, where $F$ is the number of days in the basis year and $T$ is a number of days elapsed computed by basis. |

Compounding $=-1$
Disc $=\exp (-T * Z)$, where $T$ is time in years.

## Description

TimeSpec = bktimespec(ValuationDate, Maturity, Compounding) sets the number of levels and node times for an BK tree and determines the mapping between dates and time for rate quoting.

TimeSpec is a structure specifying the time layout for bktree. The state observation dates are [Settle; Maturity (1:end-1)]. Because a forward rate is stored at the last observation, the tree can value cash flows out to Maturity.

## Examples Specify a Four-Period Tree with Annual Nodes

This example shows how to specify a four-period tree with annual nodes using annual compounding to report rates.

```
ValuationDate = 'Jan-1-2004';
Maturity = ['12-31-2004'; '12-31-2005'; '12-31-2006';
'12-31-2007'];
Compounding = 1;
TimeSpec = bktimespec(ValuationDate, Maturity, Compounding)
```

TimeSpec =
FinObj: 'BKTimeSpec'
ValuationDate: 731947
Maturity: [4×1 double]
Compounding: 1
Basis: 0
EndMonthRule: 1

## See Also

## bktree

Purpose Construct Black-Karasinski interest-rate tree

Description

Input
Arguments

```
Syntax BKTree \(=\) bktree (VolSpec, RateSpec, TimeSpec)
BKTree = bktree(VolSpec, RateSpec, TimeSpec, Name, Value)
BKTree = bktree(VolSpec, RateSpec, TimeSpec)
BKTree = bktree(VolSpec, RateSpec, TimeSpec, Name,Value)
```

BKTree = bktree(VolSpec, RateSpec, TimeSpec) creates a structure containing time and interest-rate information on a recombining tree.

BKTree = bktree(VolSpec, RateSpec, TimeSpec, Name,Value) creates a structure containing time and interest-rate information on a recombining tree with additional options specified by one or more Name, Value pair arguments.

## VolSpec

Volatility process specification. See bkvolspec for information on the volatility process.

## RateSpec

Interest-rate specification for the initial rate curve. See intenvset for information on declaring an interest-rate variable.

## TimeSpec

Tree time layout specification. Defines the observation dates of the BK tree and the compounding rule for date to time mapping and price-yield formulas. See bktimespec for information on the tree structure.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

## 'Method'

String specifying the Hull-White method upon which the tree-node connectivity algorithm is based. Possible values are HW1996 and HW2000.

Note bktree supports two tree-node connectivity algorithms. HW1996 is based on the original paper published in the Journal of Derivatives, and HW2000 is the general version of the algorithm, as specified in the paper published in August 2000.

## Default: HW1996

## Output

Arguments

Examples

## BKTree

Structure containing time and interest rate information of a trinomial recombining tree.

Using the data provided, create a BK volatility specification (VolSpec), rate specification (RateSpec), and tree time layout specification (TimeSpec). Then use these specifications to create a BK tree using bktree.

```
Compounding = -1;
ValuationDate = '01-01-2004';
StartDate = ValuationDate;
VolDates = ['12-31-2004'; '12-31-2005'; '12-31-2006';
'12-31-2007'];
VolCurve = 0.01;
AlphaDates = '01-01-2008';
AlphaCurve = 0.1;
Rates = [0.0275; 0.0312; 0.0363; 0.0415];
BKVolSpec = bkvolspec(ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
RateSpec = intenvset('Compounding', Compounding,...
    'ValuationDate', ValuationDate,...
```


## bktree

```
    'StartDates', ValuationDate,...
    'EndDates', VolDates,...
    Rates', Rates);
BKTimeSpec = bktimespec(ValuationDate, VolDates, Compounding);
BKTree = bktree(BKVolSpec, RateSpec, BKTimeSpec)
BKTree =
    FinObj: 'BKFwdTree'
    VolSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
            tObs: [0 0.9973 1.9973 2.9973]
            dObs: [731947 732312 732677 733042]
    CFlowT: {[4\times1 double] [3x1 double] [2x1 double] [3.9973]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}
    Connect: {[2] [2 3 4] [2 2 3 4 4]}
    FwdTree: {1x4 cell}
```

Use treeviewer to observe the tree you have created.

```
treeviewer(BKTree)
```



Using the data provided, create a Hull-White volatility specification (VolSpec), rate specification (RateSpec), and tree time layout specification (TimeSpec). Then use these specifications to create a Hull-White tree using hwtree.

```
Compounding = -1;
ValuationDate = '01-01-2004';
StartDate = ValuationDate;
VolDates = ['12-31-2004'; '12-31-2005'; '12-31-2006';
'12-31-2007'];
VolCurve = 0.01;
AlphaDates = '01-01-2008';
AlphaCurve = 0.1;
Rates = [0.0275; 0.0312; 0.0363; 0.0415];
HWVolSpec = hwvolspec(ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
```


## bktree

```
RateSpec = intenvset('Compounding', Compounding,...
    'ValuationDate', ValuationDate,...
    'StartDates', ValuationDate,...
    'EndDates', VolDates,...
    'Rates', Rates);
HWTimeSpec = hwtimespec(ValuationDate, VolDates, Compounding);
HWTree = hwtree(HWVolSpec, RateSpec, HWTimeSpec)
HWTree =
        FinObj: 'HWFwdTree'
        VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
            tObs: [0 0.9973 1.9973 2.9973]
            dObs: [731947 732312 732677 733042]
        CFlowT: {[4\times1 double] [3x1 double] [2x1 double] [3.9973]}
        Probs: {[3x1 double] [3x3 double] [3x5 double]}
    Connect: {[2] [2 3 4] [2 2 3 4 4]}
    FwdTree: {1x4 cell}
```

Use treeviewer to observe the tree you have created.
treeviewer(HWTree)


## References

Hull, J., and A. White, "Using Hull-White Interest Rate Trees", Journal of Derivatives, 1996.

Hull, J., and A. White, "The General Hull-White Model and Super Calibration", August 2000.

## See Also

| bkprice | bktimespec | bkvolspec | intenvset

Tutorials

## bkvolspec

## Purpose Specify Black-Karasinski interest-rate volatility process

Syntax<br>Volspec = bkvolspec(ValuationDate, VolDates, VolCurve, AlphaDates, AlphaCurve, InterpMethod)

## Arguments

| ValuationDate | Scalar value representing the observation date <br> of the investment horizon. |
| :--- | :--- |
| VolDates | Number of points (NPOINTS)-by-1 vector of yield <br> volatility end dates. |
| VolCurve | NPOINTS-by-1 vector of annualized yield <br> volatility values in decimal form. Volatility is <br> the standard deviation of proportional changes <br> in the rate. |
| AlphaDates | NPOINTS-by-1 vector of mean reversion end <br> dates. |
| AlphaCurve | NPOINTS-by-1 vector of positive mean reversion <br> values in decimal form. |
| InterpMethod | (Optional) Interpolation method. Default is <br> 'linear'. See interp1 for more information. |

Description Volspec = bkvolspec(ValuationDate, VolDates, VolCurve, AlphaDates, AlphaCurve, InterpMethod) creates a structure specifying the volatility for bktree.

## Examples Create a Black-Karasinski Volatility Specification

This example shows how to create a Black-Karasinski volatility specification (VolSpec) using the following data.

```
ValuationDate = '01-01-2004';
StartDate = ValuationDate;
```

```
VolDates = ['12-31-2004'; '12-31-2005'; '12-31-2006';
'12-31-2007'];
VolCurve = 0.01;
AlphaDates = '01-01-2008';
AlphaCurve = 0.1;
BKVolSpec = bkvolspec(ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve)
BKVolSpec =
```

FinObj: 'BKVolSpec'<br>ValuationDate: 731947<br>VolDates: [4×1 double]<br>VolCurve: [4x1 double]<br>AlphaCurve: 0.1000<br>AlphaDates: 733408<br>VolInterpMethod: 'linear'

See Also bktree | interp1

## bondbybdt

Purpose
Price bond from Black-Derman-Toy interest-rate tree

```
Syntax
[Price, PriceTree] = bondbybdt(BDTTree, CouponRate, Settle,
Maturity)
[Price, PriceTree] = bondbybdt(BDTTree, CouponRate, Settle,
Maturity, Period, Basis, EndMonthRule, IssueDate,
FirstCouponDate, LastCouponDate, StartDate, Face,
Options)
[Price, PriceTree] = bondbybdt(BDTTree, CouponRate, Settle,
Maturity,Name,Value)
```

Input Arguments

| BDTTree | Interest-rate tree structure created by <br> bdttree. |
| :--- | :--- |
| CouponRate | Decimal annual rate. CouponRate is a <br> NINST-by-1 vector or NINST-by-1 cell array of <br> decimal annual rates, or decimal annual rate <br> schedules. For the latter case of a variable <br> coupon schedule, each element of the cell <br> array is a NumDates-by-2 cell array, where the <br> first column is dates and the second column <br> is its associated rate. The date indicates the |
| last day that the coupon rate is valid. |  |
| Settle | Settlement date. A vector of serial date <br> numbers or date strings. Settle must be <br> earlier than Maturity. |
| Maturity | Maturity date. A vector of serial date <br> numbers or date strings. |

The Settle date for every bond is set to the ValuationDate of the BDT tree. The bond argument Settle is ignored.

## Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

## Period

Coupons per year of the bond. A vector of integers. Values are 1, 2, $3,4,6$, and 12.

## Default: 2

## Basis

Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- $10=$ actual/365 (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- $12=\mathrm{actual} / 365$ (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.

## bondbybdt

Default: 0 (actual/actual)

## EndMonthRule

End-of-month rule. A NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Default: 1

## IssueDate

Date when a bond was issued.

## FirstCouponDateDate

Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

## LastCouponDateDate

Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

## StartDate

Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify StartDate, the effective start date is the Settle date.

## Face

Face or par value. Face is a NINST-by- 1 vector or NINST-by- 1 cell array of face values, or face value schedules. For the latter case, each element of the cell array is a NumDates-by- 2 cell array, where the first column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid.

Default: 100

## Options

Derivatives pricing options structure created with derivset.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

## AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. NINST-by-1 of logicals.

Default: false

## BusinessDayConvention

Require payment dates to be business dates. NINST-by-1 cell array with possible choices of business day convention:

- actual


## bondbybdt

- follow
- modifiedfollow
- previous
- modifiedprevious

Default: actual

## Holidays

Holidays used for business day convention. NHOLIDAYS-by-1 of MATLAB date numbers.

Default: If no dates are specified, holidays.m is used.

## Description

[Price, PriceTree] = bondbybdt(BDTTree, CouponRate, Settle, Maturity) computes the price of a bond from a BDT interest-rate tree.
[Price, PriceTree] = bondbybdt(BDTTree, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face, Options) computes the price of a bond from a BDT interest-rate tree using optional input arguments.
[Price, PriceTree] = bondbybdt(BDTTree, CouponRate, Settle, Maturity, Name, Value) computes the price of a bond from a BDT interest-rate tree with additional options specified by one or more Name, Value pair arguments.

Price is a number of instruments (NINST)-by-1 matrix of expected prices at time 0.

PriceTree is a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within PriceTree:

- PriceTree. PTree contains the clean prices.
- PriceTree.AITree contains the accrued interest.
- PriceTree.tObs contains the observation times.
bondbybdt computes prices of vanilla bonds, stepped coupon bonds, and amortizing bonds with no market purchase option and no call provisions.


## Definitions

## Vanilla Bond

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time. The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment.

## Stepped Coupon Bond

A step-up and step-down bond is a debt security with a predetermined coupon structure over time. With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond.

## Bond with an Amortization Schedule

An amortized bond is treated as an asset, with the discount amount being amortized to interest expense over the life of the bond.

## Examples Price a Bond Using a BDT Tree

Price a $10 \%$ bond using a BDT interest-rate tree.
Load deriv.mat, which provides BDTTree. The BDTTree structure contains the time and interest-rate information needed to price the bond.

```
load deriv.mat;
```

Define the bond using the required arguments. Other arguments use defaults.

CouponRate = 0.10;
Settle = '01-Jan-2000';

## bondbybdt

```
Maturity = '01-Jan-2003';
Period = 1;
```

Use bondbybdt to compute the price of the bond.

```
Price = bondbybdt(BDTTree, CouponRate, Settle, Maturity, Period)
```

Price =
95.5030

## Price a Stepped Coupon Bond

Price single stepped coupon bonds using market data.
Define the interest-rate term structure.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
```

Create the RateSpec.
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, EndDates, 'Rates', Rates, 'Compounding', Compounding)

RS =

## FinObj: 'RateSpec'

 Compounding: 1Disc: [4x1 double]
Rates: [4x1 double]
EndTimes: [4x1 double]

```
    StartTimes: [4x1 double]
    EndDates: [4\times1 double]
    StartDates: 734139
ValuationDate: 734139
    Basis: 0
EndMonthRule: 1
```

Create the stepped bond instrument.

```
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011';'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2012' .0425;'01-Jan-2014' .0750}};
Period = 1;
```

Build the BDT tree and assume the volatility to be $10 \%$ using the following market data:

```
Sigma = 0.1;
BDTTimeSpec = bdttimespec(ValuationDate, EndDates);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length
BDTT = bdttree(BDTVolSpec, RS, BDTTimeSpec)
BDTT =
    FinObj: 'BDTFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tObs: [0 1 2 3]
        dObs: [734139 734504 734869 735235]
        TFwd: {[4x1 double] [3x1 double] [2x1 double] [3]}
        CFlowT: {[4\times1 double] [3\times1 double] [2x1 double] [4]}
    FwdTree: {1x4 cell}
```


## bondbybdt

Compute the price of the stepped coupon bonds.

```
PBDT= bondbybdt(BDTT, CouponRate, Settle,Maturity , Period)
```

PBDT $=$
100.7246
100.0945
101.5900
102.0820

## Price Two Bonds with Amortization Schedules

Price two bonds with amortization schedules using the Face input argument to define the schedule.

Define the interest-rate term structure.

```
Rates = 0.035;
ValuationDate = '1-Nov-2011';
StartDates = ValuationDate;
EndDates = '1-Nov-2017';
Compounding = 1;
```

Create the RateSpec.
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDa
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
Create the bond instrument. The bonds have a coupon rate of $4 \%$ and $3.85 \%$, a period of one year, and mature on 1-Nov-2017.

```
CouponRate = [0.04; 0.0385];
Settle ='1-Nov-2011';
Maturity = '1-Nov-2017';
Period = 1;
```

Define the amortizing schedule.

```
Face = {{'1-Nov-2015' 100;'1-Nov-2016' 85;'1-Nov-2017' 70};
{'1-Nov-2015' 100;'1-Nov-2016' 90;'1-Nov-2017' 80}};
```

Build the BDT tree and assume the volatility to be $10 \%$.

```
MatDates = {'1-Nov-2012'; '1-Nov-2013';'1-Nov-2014';'1-Nov-2015';'1-N
BDTTimeSpec = bdttimespec(ValuationDate, MatDates);
Volatility = 0.1;
BDTVolSpec = bdtvolspec(ValuationDate, MatDates, Volatility*ones(1,ler
BDTT = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec);
Compute the price of the amortizing bonds.
```

```
Price = bondbybdt(BDTT, CouponRate, Settle, Maturity, 'Period',Period
```

Price = bondbybdt(BDTT, CouponRate, Settle, Maturity, 'Period',Period
'Face', Face)

```
'Face', Face)
```

Price =
102.4791
101.7786

## blackvolbyrebonato

| Purpose | Compute Black volatility for LIBOR Market Model using Rebonato formula |
| :---: | :---: |
| Syntax | ```outVol = blackvolbyrebonato(ZeroCurve,VolFunc,CorrMat, ExerciseDate,Maturity) outVol = blackvolbyrebonato( __ ,Name,Value)``` |
| Description | outVol = blackvolbyrebonato(ZeroCurve, VolFunc, CorrMat, ExerciseDate, Maturity) computes the Black volatility for a swaption using a LIBOR Market Model. |
|  | outVol = blackvolbyrebonato( $\qquad$ , Name, Value) computes the Black volatility for a swaption using a LIBOR Market Model with optional name-value pair arguments. |
| Input Arguments | ZeroCurve - Zero-curve for LiborMarketModel model structure |
|  | Zero-curve for the LiborMarketModel, specified using IRDataCurve or RateSpec. |
|  | Data Types struct |
|  | VolFunc - Function handle for volatility cell array of function handles |
|  | Function handle for volatility, specified by a NumRates-by- 1 cell array of function handles. Each function handle must take time as an input and return a scalar volatility |
|  | Data Types cell \| function_handle |
|  | CorrMat - Correlation matrix vector |

Correlation matrix, specified by NumRates-by-NumRates.

## blackvolbyrebonato

Data Types
single | double

## ExerciseDate - Swaption exercise date

nonnegative integer | vector of nonnegative integers | string of dates
Swaption exercise dates, specified by a NumSwaptions-by-1 vector of serial date numbers or date strings.

Data Types
single | double | cell

## Maturity - Swaption maturity date

nonnegative integer | vector of nonnegative integers | string of dates
Swaption maturity dates, specified using a NumSwaptions-by-1 vector of dates or date strings.

Data Types
single | double | cell

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

Example: Vol =
blackvolbyrebonato(irdc,VolFunc,CorrMat, ExerciseDate, Maturity, 'Period

## 'Period' - Compounding frequency of curve and reset of swaptions

2 (default) | positive integer from the set [1,2,3,4,6,12] | vector of positive integers from the set [1, $2,3,4,6,12]$

Compounding frequency of curve and reset of swaptions, specified as a positive integers for the values $1,2,4,6,12$ in a NumSwaptions-by-1 vector.

## blackvolbyrebonato

Data Types<br>single | double

## Output Arguments

## Algorithms

The Rebonato approximation formula relates the Black volatility for a European swaption, given a set of volatility functions and a correlation matrix

$$
\left(v_{\alpha, \beta}^{L F M}\right)^{2}=\sum_{i, j=\alpha+1}^{\beta} \frac{w_{i}(0) w_{j}(0) F_{i}(0) F_{j}(0) \rho_{i, j}}{S_{\alpha, \beta}(0)^{2}} \int_{0}^{T_{\alpha}} \sigma_{i}(t) \sigma_{j}(t) d t
$$

where:

$$
w_{i}(t)=\frac{\tau_{i} P\left(t, T_{i}\right)}{\sum_{k=\alpha+1}^{\beta} \tau_{\kappa} P\left(t, t_{\kappa}\right)}
$$

## Examples Price Swaption for LIBOR Market Model Using the Rebonato Formula

Define the input maturity and tenor for a LIBOR Market Model (LMM) specified by the cell array of volatility function handles, and a correlation matrix for the LMM.

```
Settle = datenum('11-Aug-2004');
% Zero Curve
CurveTimes = (1:10)';
CurveDates = daysadd(Settle,360*CurveTimes,1);
ZeroRates = [0.03 0.033 0.036 0.038 0.04 0.042 0.043 0.044 0.045 0.046]';
```


## blackvolbyrebonato

```
% Construct an IRCurve
irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);
LMMVolFunc = @(a,t) (a(1)*t + a(2)).*exp(-a(3)*t) + a(4);
LMMVolParams = [. 3 -. 02 .7 .14];
numRates = length(ZeroRates);
VolFunc(1:numRates-1) = {@(t) LMMVolFunc(LMMVolParams,t)};
Beta = .08;
CorrFunc = @(i,j,Beta) exp(-Beta*abs(i-j));
CorrMat = CorrFunc(meshgrid(1:numRates-1)',meshgrid(1:numRates - 1),Beta
ExerciseDate = datenum('11-Aug-2009');
Maturity = daysadd(ExerciseDate,360*[3;4],1);
Vol = blackvolbyrebonato(irdc,VolFunc,CorrMat,ExerciseDate,Maturity,'
Vol =
    0.2113
    0.1990
```


## References

[1] Brigo, D. and F. Mercurio, Interest Rate Models - Theory and Practice, Springer Finance, 2006.

See Also LiborMarketModel
Related $\quad$ "Price Swaptions With Interest-Rate Models Using Simulation" on
Examples page $2-105$

## blackvolbysabr

Purpose Calculate implied Black volatility using SABR model

```
Syntax outVol = blackvolbysabr(Alpha,Beta,Rho,Nu,Settle,
```

ExerciseDate,ForwardValue,Strike)

```
ExerciseDate,ForwardValue,Strike)
outVol = blackvolbysabr(___,Name,Value)
```

```
outVol = blackvolbysabr(___,Name,Value)
```

```

Description

Input Arguments
outVol = blackvolbysabr(Alpha, Beta,Rho,Nu,Settle, ExerciseDate, ForwardValue, Strike) calculates the implied Black volatility using the SABR stochastic volatility model.
outVol = blackvolbysabr( __ , Name, Value) calculates the implied Black volatility using the SABR stochastic volatility model with optional name-value pair arguments.

\section*{Alpha - Current SABR volatility scalar}

Current SABR volatility, specified as a scalar.
Data Types
double

\section*{Beta-SABR constant elasticity of variance (CEV) exponent scalar}

SABR CEV exponent, specified as a scalar.
Data Types
double

\section*{Rho - Correlation between forward value and volatility} scalar

Correlation between forward value and volatility, specified as a scalar.
Data Types
double
Nu - Volatility of volatility

\section*{scalar}

Volatility of volatility, specified as a scalar.

\section*{Data Types}
double

\section*{Settle - Settlement date}
scalar for serial nonnegative date number | scalar for date string
Settlement date, specified as a scalar using a serial nonnegative date number or date string.

\section*{Data Types}
double | char

\section*{ExerciseDate - Option exercise date}
scalar for serial nonnegative date number | scalar for date string
Option exercise date, specified as a scalar using a serial nonnegative date number or date string.
```

Data Types
double | char

```

\section*{ForwardValue - Current forward value of underlying asset \\ scalar | vector}

Current forward value of the underlying asset, specified as a scalar or vector of size NumVols-by-1.

Data Types
double

\section*{Strike - Option strike price values}
scalar | vector
Option strike price values, specified as a scalar value or a vector of size NumVols-by-1.

Data Types
double

\section*{blackvolbysabr}

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

Example: outVol =
blackvolbysabr(Alpha, Beta, Rho,Nu,Settle, ExerciseDate, ForwardValue, Strike,

\section*{'Basis' - Day-count basis of instrument}

0 (actual/actual) (default) | positive integers of the set [1...13]
Day-count basis of the instrument specified as a positive integer of the set [1...13]. The Basis value represents the basis used when annualizing the input forward-rate tree:
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- \(3=\) actual \(/ 365\)
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- \(9=\) actual/360 (ISMA)
- \(10=\) actual/365 (ISMA)
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

\section*{Data Types}
double

\section*{'Model' - Version of SABR model}
'Hagan2002' (default) | value 'Obloj2008'
Version of SABR model, specified with either value:
- 'Hagan2002' — Original version by Hagan et al. (2002)
- 'Obloj2008' - Version by Obloj (2008)

\section*{Data Types}
char

\section*{Output Arguments}

\section*{Algorithms}

\section*{outVol - Implied Black volatility computed by SABR model \\ scalar | vector}

Implied Black volatility computed by SABR model, returned as a scalar or vector of size NumVols-by-1.

The SABR stochastic volatility model treats the underlying forward \(\hat{F}\) and volatility \(\hat{\alpha}\) as separate random processes, which are related with correlation \(\rho\) :
\[
\begin{aligned}
& d \hat{F}=\hat{\alpha} \hat{F}^{\beta} d W_{1} \\
& d \hat{\alpha}=v \hat{\alpha} d W_{2} \\
& d W_{1} d W_{2}=\rho d t \\
& \hat{F}(0)=F \\
& \hat{\alpha}(0)=\alpha
\end{aligned}
\]
where
- \(\hat{F}\) is the underlying forward (a variable).
- \(F\) is the current underlying forward (a constant).

\section*{blackvolbysabr}
- \(\hat{\alpha}\) is the SABR volatility (a variable).
- \(\alpha\) is the current SABR volatility (a constant).
- \(\beta\) is the SABR constant elasticity of variance (CEV) exponent.
- \(v\) is the volatility of volatility.
- \(d W_{1}\) is Brownian motion.
- \(d W_{2}\) is Brownian motion.
- \(\rho\) is the correlation between forward value and volatility.

In contrast, Black's lognormal model assumes a constant volatility, \(\sigma_{B}\).
\[
d \hat{F}=\sigma_{B} \hat{F} d W
\]

Hagan et al. (2002) derived the following closed-form approximation of implied Black lognormal volatility ( \(\sigma_{B}\) ) for the SABR model
\[
\begin{aligned}
& \sigma_{B}(F, K)=\frac{\alpha\left\{1+\left[\frac{(1-\beta)^{2}}{24} \frac{\alpha^{2}}{(F K)^{1-\beta}}+\frac{1}{4} \frac{\rho \beta v \alpha}{(F K)^{(1-\beta) / 2}}+\frac{2-3 \rho^{2}}{24} v^{2}\right] T+\ldots\right\}}{(F K)^{(1-\beta) / 2}\left\{1+\frac{(1-\beta)^{2}}{24} \log ^{2}(F / K)+\frac{(1-\beta)^{4}}{1920} \log ^{4}(F / K)+\ldots\right\}}\left(\frac{z}{x(z)}\right) \\
& z=\frac{v}{\alpha}(F K)^{(1-\beta) / 2} \log (F / K) \\
& x(z)=\log \left\{\frac{\sqrt{1-2 \rho z+z^{2}}+z-\rho}{1-\rho}\right\}
\end{aligned}
\]
where
- \(F\) is the current forward value of the underlying.
- \(\alpha\) is the current SABR volatility.
- \(K\) is the strike value.
- \(T\) is the time to option maturity.

Obloj (2008) advocated the following closed-form approximation of implied Black lognormal volatility for the SABR model (for \(\beta<1\) )
\[
\begin{aligned}
& \sigma_{B}(F, K)=\frac{v \log (F / K)}{x(z)}\left\{1+\left[\frac{(1-\beta)^{2}}{24} \frac{\alpha^{2}}{(F K)^{1-\beta}}+\frac{1}{4} \frac{\rho \beta v \alpha}{(F K)^{(1-\beta) / 2}}+\frac{2-3 \rho^{2}}{24} v^{2}\right] T+\ldots\right\} \\
& z=\frac{v}{\alpha} \frac{F^{(1-\beta)}-K^{(1-\beta)}}{1-\beta} \\
& x(z)=\log \left\{\frac{\sqrt{1-2 \rho z+z^{2}}+z-\rho}{1-\rho}\right\}
\end{aligned}
\]

These expressions can be simplified in special situations, such as the at-the-money \((F=K)\) and stochastic lognormal \((\beta=1)\) cases \([1,2]\).

\section*{Examples}

\section*{Compute the Implied Black Volatility Using the SABR Model}

Define the model parameters and option data.
```

ForwardRate = 0.0357;
Strike = 0.03;
Alpha = 0.036;
Beta = 0.5;
Rho = -0.25;
Nu = 0.35;
Settle = datenum('15-Sep-2013');
ExerciseDate = datenum('15-Sep-2015');

```

Compute the Black volatility using the SABR model.
ComputedVols = blackvolbysabr(Alpha, Beta, Rho, Nu, Settle, ... ExerciseDate, ForwardRate, Strike)

\section*{blackvolbysabr}

\section*{ComputedVols = \\ 0.2122}

\section*{References}
[1] Hagan, P. S., D. Kumar, A.S. Lesniewski, and D.E. Woodward Managing smile risk, Wilmott Magazine, September, pp. 84-108, 2002.
[2] Obloj, J. Fine-tune your smile: Correction to Hagan et. al. Wilmott Magazine, 2008.

\section*{See Also swaptionbyblk}
\(\begin{array}{ll}\text { Related } & \text { - "Calibrate the SABR Model" on page 2-20 } \\ \text { Examples } & \text { "Price a Swaption Using the SABR Model" on page 2-26 }\end{array}\)
\begin{tabular}{|c|c|}
\hline Purpose & Price bond from Black-Karasinski interest-rate tree \\
\hline Syntax & ```
[Price, PriceTree] = bondbybk(BKTree, CouponRate,
Settle, Maturity)
[Price, PriceTree] = bondbybk(BKTree, CouponRate,
Settle, Maturity, Period, Basis, EndMonthRule, IssueDate,
FirstCouponDate, LastCouponDate, StartDate, Face,
Options)
[Price, PriceTree] = bondbybk(BKTree, CouponRate,
Settle, Maturity, Name,Value)
``` \\
\hline \multirow[t]{3}{*}{\begin{tabular}{l}
Input \\
Arguments
\end{tabular}} & BKTree \(\quad\)\begin{tabular}{l} 
Forward rate tree structure created by \\
bktree.
\end{tabular} \\
\hline & \begin{tabular}{l}
CouponRate \\
Decimal annual rate. CouponRate is a NINST-by- 1 vector or NINST-by- 1 cell array of decimal annual rates, or decimal annual rate schedules. For the latter case of a variable coupon schedule, each element of the cell array is a NumDates-by-2 cell array, where the first column is dates and the second column is its associated rate. The date indicates the last day that the coupon rate is valid.
\end{tabular} \\
\hline & \begin{tabular}{ll} 
Settle & \begin{tabular}{l} 
Settlement date. A vector of serial date \\
numbers or date strings. Settle must be \\
earlier than Maturity. The Settle date for \\
every bond is set to the ValuationDate of
\end{tabular} \\
the BK Tree. The bond argument, Settle, is \\
Maturity & \begin{tabular}{l} 
igabuedity date. A vector of serial date numbers \\
or date strings.
\end{tabular}
\end{tabular} \\
\hline
\end{tabular}

\section*{bondbybk}

\section*{Ordered Input or Name-Value Pair Arguments}

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

\section*{Period}

Coupons per year of the bond. A vector of integers. Values are 1, 2, \(3,4,6\), and 12.

\section*{Default: 2}

\section*{Basis}

Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual \(/ 360\)
- \(3=\) actual \(/ 365\)
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- \(9=\) actual/360 (ISMA)
- \(10=\mathrm{actual} / 365\) (ISMA)
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.

Default: 0 (actual/actual)

\section*{EndMonthRule}

End-of-month rule. A NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.
- \(0=\) Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Default: 1

\section*{IssueDate}

Date when a bond was issued.

\section*{FirstCouponDateDate}

Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

\section*{LastCouponDateDate}

Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

\section*{StartDate}

Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify StartDate, the effective start date is the Settle date.

\section*{Face}

Face or par value. Face is a NINST-by- 1 vector or NINST-by- 1 cell array of face values, or face value schedules. For the latter case, each element of the cell array is a NumDates-by- 2 cell array, where the first column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid.

Default: 100

\section*{Options}

Derivatives pricing options structure created with derivset.

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

\section*{AdjustCashFlowsBasis}

Adjust the cash flows based on the actual period day count. NINST-by-1 of logicals.

Default: false

\section*{BusinessDayConvention}

Require payment dates to be business dates. NINST-by-1 cell array with possible choices of business day convention:
- actual
- follow
- modifiedfollow
- previous
- modifiedprevious

Default: actual

\section*{Holidays}

Holidays used for business day convention. NHOLIDAYS-by-1 of MATLAB date numbers.

Default: If no dates are specified, holidays.m is used.

\section*{Description}
[Price, PriceTree] = bondbybk(BKTree, CouponRate, Settle, Maturity) computes the price of a bond from a Black-Karasinski interest-rate tree.
```

[Price, PriceTree] = bondbybk(BKTree, CouponRate,
Settle, Maturity, Period, Basis, EndMonthRule, IssueDate,
FirstCouponDate, LastCouponDate, StartDate, Face,
Options) computes the price of a bond from a Black-Karasinski
interest-rate tree using optional input arguments.

```
[Price, PriceTree] = bondbybk(BKTree, CouponRate, Settle, Maturity, Name, Value) computes the price of a bond from a Black-Karasinski interest-rate tree with additional options specified by one or more Name, Value pair arguments.

Price is a number of instruments (NINST)-by-1 matrix of expected prices at time 0.

PriceTree is a structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within PriceTree:
- PriceTree. PTree contains the clean prices.

\section*{bondbybk}
- PriceTree.AITree contains the accrued interest.
- PriceTree.tObs contains the observation times.
bondbybk computes prices of vanilla bonds, stepped coupon bonds, and amortizing bonds with no market purchase option and no call provisions.

\section*{Definitions}

\section*{Examples}

\section*{Vanilla Bond}

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time. The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment.

\section*{Stepped Coupon Bond}

A step-up and step-down bond is a debt security with a predetermined coupon structure over time. With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond.

\section*{Bond with an Amortization Schedule}

An amortized bond is treated as an asset, with the discount amount being amortized to interest expense over the life of the bond.

\section*{Price a Bond Using a BK Tree}

Price a \(4 \%\) bond using a Black-Karasinski interest-rate tree.
Load deriv.mat, which provides BKTree. The BKTree structure contains the time and interest-rate information needed to price the bond.
load deriv.mat;
Define the bond using the required arguments. Other arguments use defaults.

CouponRate = 0.04;
```

Settle = '01-Jan-2004';
Maturity = '31-Dec-2008';

```

Use bondbybk to compute the price of the bond.
```

Price = bondbybk(BKTree, CouponRate, Settle, Maturity)

```

Warning: Not all cash flows are aligned with the tree. Result will be approximated.

Price =
98.0300

\section*{Price a Stepped Coupon Bond}

Price single stepped coupon bonds using market data.
Define the interest-rate term structure.
```

Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012';...
'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
Create the RateSpec.
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDate
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)

```
RS =

\section*{FinObj: 'RateSpec'}

Compounding: 1

\section*{bondbybk}
```

            Disc: [4x1 double]
            Rates: [4x1 double]
            EndTimes: [4x1 double]
            StartTimes: [4x1 double]
            EndDates: [4x1 double]
    StartDates: 734139
    ValuationDate: 734139
Basis: 0
EndMonthRule: 1

```

Create the stepped bond instrument.
```

Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011';'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2012' .0425;'01-Jan-2014' .0750}};
Period = 1;

```

Build the BK tree using the following market data:
```

VolDates = ['1-Jan-2011'; '1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'];
VolCurve = 0.01;
AlphaDates = '01-01-2014';
AlphaCurve = 0.1;
BKVolSpec = bkvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
BKTimeSpec = bktimespec(RS.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVolSpec, RS, BKTimeSpec);

```

Compute the price of the stepped coupon bonds.
```

PBK= bondbybk(BKT, CouponRate, Settle,Maturity , Period)

```
PBK =
100.7246
100.0945
101.5900
102.0820

\section*{Price a Bond with an Amortization Schedule}

Price a bond with an amortization schedule using the Face input argument to define the schedule.

Define the interest-rate term structure.
```

Rates = 0.065;
ValuationDate = '1-Jan-2011';
StartDates = ValuationDate;
EndDates= '1-Jan-2017';
Compounding = 1;

```

Create the RateSpec.
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', Sta
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
RateSpec =
                    FinObj: 'RateSpec'
        Compounding: 1
            Disc: 0.6853
            Rates: 0.0650
            EndTimes: 6
            StartTimes: 0
            EndDates: 736696
            StartDates: 734504
        ValuationDate: 734504
            Basis: 0

\section*{bondbybk}

\section*{EndMonthRule: 1}

Create the bond instrument. The bond has a coupon rate of \(7 \%\), a period of one year, and matures on 1-Jan-2017.
```

CouponRate = 0.07;
Settle ='1-Jan-2011';
Maturity = '1-Jan-2017';
Period = 1;
Face = {{'1-Jan-2015' 100;'1-Jan-2016' 90;'1-Jan-2017' 80}};

```

Build the BK tree with the following market data:
```

VolDates = ['1-Jan-2012'; '1-Jan-2013';...
'1-Jan-2014';'1-Jan-2015';'1-Jan-2016';'1-Jan-2017'];
VolCurve = 0.01;
AlphaDates = '01-01-2017';
AlphaCurve = 0.1;
BKVolSpec = bkvolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
BKTimeSpec = bktimespec(RateSpec.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVolSpec, RateSpec, BKTimeSpec);

```

Compute the price of the amortizing bond.
Price = bondbybk(BKT, CouponRate, Settle, Maturity, 'Period', Period,...
'Face', Face)

Price =
102.3155

Compare the results with price of a vanilla bond.
```

PriceVanilla = bondbybk(BKT, CouponRate, Settle, Maturity, Period)
PriceVanilla =

```
    102.4205
See Also bkprice | bktree | cfamounts | hwprice | hwtree | instbond

\section*{bondbyhim}

Purpose
Price bond from Heath-Jarrow-Morton interest-rate tree
```

Syntax
[Price, PriceTree] = bondbyhjm(HJMTree, CouponRate, Settle,
Maturity)
[Price, PriceTree] = bondbyhjm(HJMTree, CouponRate, Settle,
Maturity, Period, Basis, EndMonthRule, IssueDate,
FirstCouponDate, LastCouponDate, StartDate, Face,
Options)
[Price, PriceTree] = bondbyhjm(HJMTree, CouponRate,
Settle, Maturity, Name,Value)

```

\section*{Input} Arguments
\begin{tabular}{ll} 
HJMTree & \begin{tabular}{l} 
Forward rate tree structure created by \\
hjmtree.
\end{tabular} \\
CouponRate & \begin{tabular}{l} 
Decimal annual rate. CouponRate is a \\
NINST-by-1 vector or NINST-by-1 cell array of \\
decimal annual rates, or decimal annual rate \\
schedules. For the latter case of a variable \\
coupon schedule, each element of the cell \\
array is a NumDates-by-2 cell array, where the \\
first column is dates and the second column \\
is its associated rate. The date indicates the \\
last day that the coupon rate is valid.
\end{tabular} \\
Settle & \begin{tabular}{l} 
Settlement date. A vector of serial date \\
numbers or date strings. Settle must be \\
earlier than Maturity.
\end{tabular} \\
Maturity & \begin{tabular}{l} 
Maturity date. A vector of serial date \\
numbers or date strings.
\end{tabular}
\end{tabular}

The Settle date for every bond is set to the ValuationDate of the HJM tree. The bond argument Settle is ignored.

\section*{Ordered Input or Name-Value Pair Arguments}

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

\section*{Period}

Coupons per year of the bond. A vector of integers. Values are 1, 2, \(3,4,6\), and 12.

\section*{Default: 2}

\section*{Basis}

Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- 3 = actual/365
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- \(10=\) actual/365 (ISMA)
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- \(12=\) actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.

Default: 0 (actual/actual)

\section*{EndMonthRule}

End-of-month rule. A NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.
- \(0=\) Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Default: 1

\section*{IssueDate}

Date when a bond was issued.

\section*{FirstCouponDateDate}

Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

\section*{LastCouponDateDate}

Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

\section*{StartDate}

Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify StartDate, the effective start date is the Settle date.

\section*{Face}

Face or par value. Face is a NINST-by- 1 vector or NINST-by- 1 cell array of face values, or face value schedules. For the latter case, each element of the cell array is a NumDates-by- 2 cell array, where the first column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid.

Default: 100

\section*{Options}

Derivatives pricing options structure created with derivset.

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

\section*{AdjustCashFlowsBasis}

Adjust the cash flows based on the actual period day count. NINST-by-1 of logicals.

Default: false

\section*{BusinessDayConvention}

Require payment dates to be business dates. NINST-by-1 cell array with possible choices of business day convention:
```

- actual

```

\section*{bondbyhim}
- follow
- modifiedfollow
- previous
- modifiedprevious

Default: actual

\section*{Holidays}

Holidays used for business day convention. NHOLIDAYS-by-1 of MATLAB date numbers.

Default: If no dates are specified, holidays.m is used.

\section*{Description}
[Price, PriceTree] = bondbyhjm(HJMTree, CouponRate, Settle, Maturity) computes the price of a bond from an HJM forward-rate tree.
```

[Price, PriceTree] = bondbyhjm(HJMTree, CouponRate, Settle,
Maturity, Period, Basis, EndMonthRule, IssueDate,
FirstCouponDate, LastCouponDate, StartDate, Face,
Options) computes the price of a bond from an HJM forward-rate tree
with optional input arguments.
[Price, PriceTree] = bondbyhjm(HJMTree, CouponRate, Settle, Maturity, Name, Value) computes the price of a bond from an HJM forward-rate tree with additional options specified by one or more Name, Value pair arguments.
Price is a number of instruments (NINST)-by-1 matrix of expected prices at time 0.
PriceTree is a structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within PriceTree:

```
- PriceTree. PBush contains the clean prices.
- PriceTree.AIBush contains the accrued interest.
- PriceTree.tObs contains the observation times.
bondbyhjm computes prices of vanilla bonds, stepped coupon bonds, and amortizing bonds with no market purchase option and no call provisions.

\section*{Definitions}

\section*{Vanilla Bond}

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time. The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment.

\section*{Stepped Coupon Bond}

A step-up and step-down bond is a debt security with a predetermined coupon structure over time. With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond.

\section*{Bond with an Amortization Schedule}

An amortized bond is treated as an asset, with the discount amount being amortized to interest expense over the life of the bond.

\section*{Examples Price a Bond Using an HJM Tree}

Price a \(4 \%\) bond using an HJM interest-rate tree.
Load deriv.mat, which provides HJMTree. The HJMTree structure contains the time and interest-rate information needed to price the bond.
```

load deriv.mat;

```

Define the bond using the required arguments. Other arguments use defaults.
```

CouponRate = 0.04;
Settle = '01-Jan-2000';

```

\section*{bondbyhim}

Maturity = '01-Jan-2004';

Use bondbyhjm to compute the price of the bond.
Price = bondbyhjm(HJMTree, CouponRate, Settle, Maturity)

Warning: Not all cash flows are aligned with the tree. Result will be approximated.

Price =
97.5280

\section*{Price a Stepped Coupon Bond}

Price single stepped coupon bonds using market data.
Define the interest-rate term structure.
```

Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;

```

Create the RateSpec.
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, 'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);

Create the stepped bond instrument.
```

Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011';'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2012' .0425;'01-Jan-2014' .0750}};
Period = 1;

```

Build the HJM tree using the following market data:
```

Volatility = [.2; .19; .18; .17];
CurveTerm = [ 1; 2; 3; 4];
HJMTimeSpec = hjmtimespec(ValuationDate, EndDates);
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec,RS,HJMTimeSpec);

```

Compute the price of the stepped coupon bonds.
PHJM \(=\) bondbyhjm(HJMT, CouponRate, Settle, Maturity , Period)

\section*{PHJM =}
100.7246
100.0945
101.5900
102.0820

\section*{Price a Bond with an Amortization Schedule}

Price a bond with an amortization schedule using the Face input argument to define the schedule.

Define the interest-rate term structure.
```

Rates = 0.065;
ValuationDate = '1-Jan-2011';
StartDates = ValuationDate;
EndDates= '1-Jan-2017';
Compounding = 1;

```

Create the RateSpec.
```

RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', Star
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)

```

\section*{bondbyhim}
```

RateSpec =
FinObj: 'RateSpec'
Compounding: 1
Disc: 0.6853
Rates: 0.0650
EndTimes: 6
StartTimes: 0
EndDates: 736696
StartDates: 734504
ValuationDate: 734504
Basis: 0
EndMonthRule: 1

```

Create the bond instrument. The bond has a coupon rate of \(7 \%\), a period of one year, and matures on 1-Jan-2017.
```

CouponRate = 0.07;
Settle ='1-Jan-2011';
Maturity = '1-Jan-2017';
Period = 1;
Face = {{'1-Jan-2015' 100;'1-Jan-2016' 90;'1-Jan-2017' 80}};

```

Build the HJM tree using the following market data:
```

Volatility = [.2; .19; .18; .17];
CurveTerm = [ 1; 2; 3; 4];
MaTree = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015';...
'Jan-1-2016'; 'Jan-1-2017'};
HJMTimeSpec = hjmtimespec(ValuationDate, MaTree);
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec,RateSpec,HJMTimeSpec);

```

Compute the price of the amortizing bond.
```

Price = bondbyhjm(HJMT, CouponRate, Settle, Maturity, 'Period',...
Period, 'Face' , Face)
Price =
102.3155
Compare the results with price of a vanilla bond.
PriceVanilla = bondbyhjm(HJMT, CouponRate, Settle, Maturity, Period)
PriceVanilla =

```
102.4205

\section*{See Also}
hjmtree | cfamounts | hjmprice | instbond

\section*{bondbyhw}

Purpose
Price bond from Hull-White interest-rate tree
```

Syntax
[Price, PriceTree] = bondbyhw(HWTree, CouponRate,
Settle, Maturity)
[Price, PriceTree] = bondbyhw(HWTree, CouponRate,
Settle, Maturity, Period, Basis, EndMonthRule, IssueDate,
FirstCouponDate, LastCouponDate, StartDate, Face,
Options)
[Price, PriceTree] = bondbyhw(HWTree, CouponRate,
Settle, Maturity, Name,Value)

```

Input Arguments
\begin{tabular}{ll} 
HWTree & \begin{tabular}{l} 
Forward-rate tree structure created by \\
hwtree.
\end{tabular} \\
CouponRate & \begin{tabular}{l} 
Decimal annual rate. CouponRate is a \\
NINST-by-1 vector or NINST-by-1 cell array of \\
decimal annual rates, or decimal annual rate \\
schedules. For the latter case of a variable \\
coupon schedule, each element of the cell \\
array is a NumDates-by-2 cell array, where the \\
first column is dates and the second column \\
is its associated rate. The date indicates the
\end{tabular} \\
last day that the coupon rate is valid. \\
Settle & \begin{tabular}{l} 
Settlement date. A vector of serial date \\
numbers or date strings. Settle must be \\
earlier than Maturity.
\end{tabular} \\
Maturity & \begin{tabular}{l} 
Maturity date. A vector of serial date \\
numbers or date strings.
\end{tabular}
\end{tabular}

The Settle date for every bond is set to the ValuationDate of the HW tree. The bond argument Settle is ignored.

\section*{Ordered Input or Name-Value Pair Arguments}

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

\section*{Period}

Coupons per year of the bond. A vector of integers. Values are 1, 2, \(3,4,6\), and 12.

\section*{Default: 2}

\section*{Basis}

Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- 3 = actual/365
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- \(10=\) actual/365 (ISMA)
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- \(12=\mathrm{actual} / 365\) (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.

Default: 0 (actual/actual)

\section*{EndMonthRule}

End-of-month rule. NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.
- \(0=\) Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Default: 1

\section*{IssueDate}

Date when a bond was issued.

\section*{FirstCouponDateDate}

Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

\section*{LastCouponDateDate}

Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

\section*{StartDate}

Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify StartDate, the effective start date is the Settle date.

\section*{Face}

Face or par value. Face is a NINST-by- 1 vector or NINST-by- 1 cell array of face values, or face value schedules. For the latter case, each element of the cell array is a NumDates-by- 2 cell array, where the first column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid.

Default: 100

\section*{Options}

Derivatives pricing options structure created with derivset.

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

\section*{AdjustCashFlowsBasis}

Adjust the cash flows based on the actual period day count. NINST-by-1 of logicals.

Default: false

\section*{BusinessDayConvention}

Require payment dates to be business dates. NINST-by-1 cell array with possible choices of business day convention:
- actual

\section*{bondbyhw}
- follow
- modifiedfollow
- previous
- modifiedprevious

Default: actual

\section*{Holidays}

Holidays used for business day convention. NHOLIDAYS-by-1 of MATLAB date numbers.

Default: If no dates are specified, holidays.m is used.

\section*{Description}
[Price, PriceTree] = bondbyhw(HWTree, CouponRate, Settle, Maturity) computes the price of a bond from a Hull-White interest-rate tree.
```

[Price, PriceTree] = bondbyhw(HWTree, CouponRate,
Settle, Maturity, Period, Basis, EndMonthRule, IssueDate,
FirstCouponDate, LastCouponDate, StartDate, Face,
Options) computes the price of a bond from a Hull-White interest-rate tree with optional input arguments.

```
[Price, PriceTree] = bondbyhw(HWTree, CouponRate, Settle, Maturity, Name, Value) computes the price of a bond from a Hull-White interest-rate tree with additional options specified by one or more Name, Value pair arguments.

Price is a number of instruments (NINST)-by- 1 matrix of expected prices at time 0.

PriceTree is a structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within PriceTree:
- PriceTree. PTree contains the clean prices.
- PriceTree.AITree contains the accrued interest.
- PriceTree.tObs contains the observation times.
bondbyhw computes prices of vanilla bonds, stepped coupon bonds, and amortizing bonds with no market purchase option and no call provisions.

\section*{Definitions Vanilla Bond}

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time. The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment.

\section*{Stepped Coupon Bond}

A step-up and step-down bond is a debt security with a predetermined coupon structure over time. With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond.

\section*{Bond with an Amortization Schedule}

An amortized bond is treated as an asset, with the discount amount being amortized to interest expense over the life of the bond.

\section*{Examples Price a Bond Using the HW Tree}

Price a \(4 \%\) bond using a Hull-White interest-rate tree.
Load deriv.mat, which provides HWTree. The HWTree structure contains the time and interest-rate information needed to price the bond.
load deriv.mat;
Define the bond using the required arguments. Other arguments use defaults.

CouponRate = 0.04;

\section*{bondbyhw}
```

Settle = '01-Jan-2004';
Maturity = '31-Dec-2008';

```

Use bondbyhw to compute the price of the bond.
Price = bondbyhw(HWTree, CouponRate, Settle, Maturity)
Warning: Not all cash flows are aligned with the tree. Result will be approximated.

Price =
98.0483

\section*{Price a Stepped Coupon Bond}

Price single stepped coupon bonds using market data.
Define the interest-rate term structure.
```

Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;

```

Create the RateSpec.
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,. 'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)

RS =
```

            FinObj: 'RateSpec'
            Compounding: 1
            Disc: [4x1 double]
    ```
```

            Rates: [4x1 double]
            EndTimes: [4x1 double]
    StartTimes: [4x1 double]
    EndDates: [4\times1 double]
    StartDates: 734139
    ValuationDate: 734139
Basis: 0
EndMonthRule: 1

```

Create the stepped bond instrument.
```

Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011';'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2012' .0425;'01-Jan-2014' .0750}};
Period = 1;

```

Build the HW tree using the following market data:
```

VolDates = ['1-Jan-2011'; '1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'];
VolCurve = 0.01;
AlphaDates = '01-01-2014';
AlphaCurve = 0.1;
HWVolSpec = hwvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RS.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVolSpec, RS, HWTimeSpec);

```

Compute the price of the stepped coupon bonds.
```

PHW= bondbyhw(HWT, CouponRate, Settle,Maturity , Period)

```
PHW =
    100.7246

\section*{bondbyhw}
100.0945
101.5900
102.0820

\section*{Price Two Bonds with Amortization Schedules}

Price two bonds with amortization schedules using the Face input argument to define the schedules.

Define the interest rate term structure.
```

Rates = 0.035;
ValuationDate = '1-Nov-2011';
StartDates = ValuationDate;
EndDates = '1-Nov-2017';
Compounding = 1;

```

Create the RateSpec.
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDa
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);

Create the bond instrument. The bonds have a coupon rate of \(4 \%\) and \(3.85 \%\), a period of one year, and mature on 1-Nov-2017.
```

CouponRate = [0.04; 0.0385];
Settle ='1-Nov-2011';
Maturity = '1-Nov-2017';
Period = 1;

```

Define the amortizing schedule.
```

Face = {{'1-Nov-2015' 100;'1-Nov-2016' 85;'1-Nov-2017' 70};
{'1-Nov-2015' 100;'1-Nov-2016' 90;'1-Nov-2017' 80}};

```

Build the HW tree and assume the volatility to be \(10 \%\).
```

VolDates = ['1-Nov-2012'; '1-Nov-2013';'1-Nov-2014';'1-Nov-2015';'1-Nc
VolCurve = 0.1;
AlphaDates = '01-01-2017';
AlphaCurve = 0.1;
HWVolSpec = hwvolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RateSpec.ValuationDate, VolDates, Compounding)
HWT = hwtree(HWVolSpec, RateSpec, HWTimeSpec);
Compute the price of the amortizing bonds.

```
```

Price = bondbyhw(HWT, CouponRate, Settle, Maturity, 'Period',Period,.

```
Price = bondbyhw(HWT, CouponRate, Settle, Maturity, 'Period',Period,.
'Face', Face)
'Face', Face)
Price =
    102.4791
    101.7786
```

See Also
bkprice | bktree | cfamounts | hwprice | hwtree | instbond

## bondbyzero

## Purpose <br> Price bond from set of zero curves

| Syntax | [Price, DirtyPrice, CFlowAmounts, CFlowDates] $=$ |
| :--- | :--- |
| bondbyzero(RateSpec, CouponRate, Settle, Maturity) |  |
|  | [Price, DirtyPrice, CFlowAmounts, CFlowDates] = |
|  | bondbyzero(RateSpec, CouponRate, Settle, Maturity, |
|  | Period, Basis, EndMonthRule, IssueDate, |
|  | FirstCouponDate, LastCouponDate, StartDate, Face) |
|  | [Price, DirtyPrice, CFlowAmounts, CFlowDates] $=$ |
|  | bondbyzero(RateSpec, CouponRate, Settle, |
|  | Maturity, Name, Value) |

## Description

[Price, DirtyPrice, CFlowAmounts, CFlowDates] = bondbyzero(RateSpec, CouponRate, Settle, Maturity) returns a NINST-by-NUMCURVES matrix of clean bond prices. Each column arises from one of the zero curves.
[Price, DirtyPrice, CFlowAmounts, CFlowDates] = bondbyzero(RateSpec, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face) returns a NINST-by-NUMCURVES matrix of clean bond prices. Each column arises from one of the zero curves.
[Price, DirtyPrice, CFlowAmounts, CFlowDates] = bondbyzero(RateSpec, CouponRate, Settle, Maturity, Name, Value) returns a NINST-by-NUMCURVES matrix of clean bond prices (each column arises from one of the zero curves) with additional options specified by one or more Name, Value pair arguments.
bondbyzero computes prices of vanilla bonds, stepped coupon bonds, and amortizing bonds with no market purchase option and no call provisions.

## Input Arguments

## RateSpec

Structure containing the properties of an interest-rate structure. See intenvset for information on creating RateSpec.

## CouponRate

Decimal annual rate. CouponRate is a NINST-by-1 vector or NINST-by-1 cell array of decimal annual rates, or decimal annual rate schedules. For the latter case of a variable coupon schedule, each element of the cell array is a NumDates-by-2 cell array, where the first column is dates and the second column is its associated rate. The date indicates the last day that the coupon rate is valid.

## Settle

Settlement date. Settle must be either a scalar or NINST-by-1 vector of serial date numbers or date strings of the same value which represent the settlement date for each bond. Settle must be earlier than Maturity.

## Maturity

Maturity date. NINST-by-1 vector of dates representing the maturity date for each swap.

## Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

## Period

Coupons per year of the bond. A vector of integers. Values are 1, 2, $3,4,6$, and 12.

Default: 2

## Basis

## bondbyzero

Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual $/ 365$
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- $9=$ actual/360 (ISMA)
- $10=\mathrm{actual} / 365$ (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

## EndMonthRule

End-of-month rule. A NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Default: 1

## IssueDate

Date when a bond was issued.

## FirstCouponDateDate

Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

## LastCouponDateDate

Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

## StartDate

Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify StartDate, the effective start date is the Settle date.

## Face

Face or par value. Face is a NINST-by- 1 vector or NINST-by- 1 cell array of face values, or face value schedules. For the latter case, each element of the cell array is a NumDates-by-2 cell array, where the first column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid.

Default: 100

## bondbyzero

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

## AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. NINST-by-1 of logicals.

Default: false

## BusinessDayConvention

Require payment dates to be business dates. NINST-by-1 cell array with possible choices of business day convention:

- actual
- follow
- modifiedfollow
- previous
- modifiedprevious

Default: actual

## Holidays

Holidays used for business day convention. NHOLIDAYS-by-1 of MATLAB date numbers.

Default: If no dates are specified, holidays.m is used.

## Output Arguments

## Price

NINST-by-NUMCURVES matrix of clean bond prices. Each column arises from one of the zero curves.

## DirtyPrice

NINST-by-NUMCURVES matrix of dirty bond price (clean + accrued interest). Each column arises from one of the zero curves.

## CFlowAmounts

NINST-by-NUMCFS matrix of cash flows for each bond

## CFlowDates

NUMCFS-by-1 matrix of payment dates for each bond

## Definitions Vanilla Bond

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time. The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment.

## Stepped Coupon Bond

A step-up and step-down bond is a debt security with a predetermined coupon structure over time. With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond.

## Bond with an Amortization Schedule

An amortized bond is treated as an asset, with the discount amount being amortized to interest expense over the life of the bond.

## bondbyzero

## Examples Price a Vanilla Bond

Price a $4 \%$ bond using a zero curve.
Load deriv.mat, which provides ZeroRateSpec, the interest-rate term structure, needed to price the bond.

```
load deriv.mat;
CouponRate = 0.04;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2004';
Price = bondbyzero(ZeroRateSpec, CouponRate, Settle, Maturity)
```

Price =
97.5334

## Price a Stepped Coupon Bond

Price single stepped coupon bonds using market data.
Define data for the interest-rate term structure.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
Create the RateSpec.
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,.
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```

```
            FinObj: 'RateSpec'
    Compounding: 1
            Disc: [4\times1 double]
            Rates: [4x1 double]
            EndTimes: [4x1 double]
    StartTimes: [4x1 double]
            EndDates: [4x1 double]
    StartDates: 734139
ValuationDate: 734139
            Basis: 0
EndMonthRule: 1
```

Create the stepped bond instrument.

```
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011';'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2012' .0425;'01-Jan-2014' .0750}};
Period = 1;
```

Compute the price of the stepped coupon bonds.

```
PZero= bondbyzero(RS, CouponRate, Settle, Maturity ,Period)
```

PZero =
100.7246
100.0945
101.5900
102.0820

## bondbyzero

## Price a Bond with an Amortizing Schedule

Price a bond with an amortizing schedule using the Face input argument to define the schedule.

Define data for the interest-rate term structure.
Rates = 0.065;
ValuationDate = '1-Jan-2011';
StartDates = ValuationDate;
EndDates= '1-Jan-2017';
Compounding = 1;
Create the RateSpec.
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDa
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)

RateSpec =

```
                    FinObj: 'RateSpec'
        Compounding: 1
            Disc: 0.6853
            Rates: 0.0650
        EndTimes: 6
        StartTimes: 0
        EndDates: 736696
        StartDates: 734504
    ValuationDate: 734504
            Basis: 0
    EndMonthRule: 1
```

Create and price the amortizing bond instrument. The bond has a coupon rate of $7 \%$, a period of one year, and matures on 1-Jan-2017.

CouponRate = 0.07;

```
Settle ='1-Jan-2011';
Maturity = '1-Jan-2017';
Period = 1;
Face = {{'1-Jan-2015' 100;'1-Jan-2016' 90;'1-Jan-2017' 80}};
Price = bondbyzero(RateSpec, CouponRate, Settle, Maturity, 'Period',
Period, 'Face', Face)
Price =
    102.3155
```

Compare the results with price of a vanilla bond.
PriceVanilla = bondbyzero(RateSpec, CouponRate, Settle, Maturity,Peric

## PriceVanilla =

102.4205

See Also | swapbyzero | cfamounts | cfbyzero | fixedbyzero | floatbyzero

## bushpath

Purpose Extract entries from node of bushy tree
Syntax Values = bushpath (Tree, BranchList)
Arguments

$$
\begin{array}{ll}
\text { Tree } & \text { Bushy tree. } \\
\text { BranchList } & \begin{array}{l}
\text { Number of paths (NUMPATHS) by path length } \\
\text { (PATHLENGTH) matrix containing the sequence of } \\
\\
\\
\text { branchings. }
\end{array}
\end{array}
$$

Description

Values = bushpath(Tree, BranchList) extracts entries of a node
of a bushy tree. The node path is described by the sequence of
branchings taken, starting at the root. The top branch is number 1 , the
second-to-top is 2 , and so on. Set the branch sequence to zero to obtain
the entries at the root node.

Values is a number of values (NUMVALS)-by-NUMPATHS matrix containing
the retrieved entries of a bushy tree.

## Examples Create an HJM tree by loading the example file.

load deriv.mat;
Then
FwdRates = bushpath(HJMTree.FwdTree, [llll)
returns the rates at the tree nodes located by taking the up branch, then the down branch, and finally the up branch again.

FwdRates =
1.0356
1.0364
1.0526

You can visualize this with the treeviewer function.
treeviewer (HJMTree)


## See Also

bushshape | mkbush

## bushshape

Purpose Retrieve shape of bushy tree

Syntax [NumLevels, NumChild, NumPos, NumStates, Trim] = bushshape(Tree)

Arguments
Tree Bushy tree.

## Description

[NumLevels, NumChild, NumPos, NumStates, Trim] = bushshape(Tree) returns information on a bushy tree's shape.

NumLevels is the number of time levels of the tree.
NumChild is a 1-by-number of levels (NUMLEVELS) vector with the number of branches (children) of the nodes in each level.

NumPos is a 1 -by-NUMLEVELS vector containing the length of the state vectors in each level.

NumStates is a 1 -by-NUMLEVELS vector containing the number of state vectors in each level.

Trim is 1 if NumPos decreases by 1 when moving from one time level to the next. Otherwise, it is 0 .

Examples Create an HJM tree by loading the example file.
load deriv.mat;
With treeviewer you can see the general shape of the HJM interest-rate tree.

With this tree
[NumLevels, NumChild, NumPos, NumStates, Trim] =... bushshape(HJMTree.FwdTree)
returns
NumLevels =
NumLevels =
4
4
NumChild =
NumChild =
2 2 2 0
2 2 2 0
NumPos
NumPos
NumStates =
NumStates =
1 2 4 4 8
1 2 4 4 8

## bushshape

Trim =
1

You can recreate this tree using the mkbush function.
Tree $=$ mkbush(NumLevels, NumChild(1), NumPos(1), Trim); Tree $=$ mkbush(NumLevels, NumChild, NumPos);

See Also
bushpath | mkbush

## Purpose

Price cap instrument from Black-Derman-Toy interest-rate tree

## Syntax

Arguments

BDTTree Interest-rate tree structure created by bdttree.
Strike Number of instruments (NINST)-by-1 vector of rates at which the cap is exercised.
Settle Settlement dates. NINST-by-1 vector of dates representing the settlement dates of the cap.

Maturity NINST-by-1 vector of dates representing the maturity dates of the cap.

Reset (Optional) NINST-by-1 vector representing the frequency of payments per year. Default $=1$.
Basis (Optional) Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- $10=\mathrm{actual} / 365$ (ICMA)


## capbybdt

- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Principal (Optional) The notional principal amount. Default $=100$.

Options (Optional) Derivatives pricing options structure created with derivset.

## Description

[Price, PriceTree] = capbybdt(BDTTree, Strike, Settle, Maturity, Reset, Basis, Principal, Options) computes the price of a cap instrument from a BDT interest-rate tree.

Price is the expected price of the cap at time 0.
PriceTree is the tree structure with values of the cap at each node.
The Settle date for every cap is set to the ValuationDate of the BDT tree. The cap argument Settle is ignored.

## Examples Price a 3\% Cap Instrument Using a BDT Interest-Rate Tree

Load the file deriv.mat, which provides BDTTree. The BDTTree structure contains the time and interest-rate information needed to price the cap instrument.
load deriv.mat;
Set the required values. Other arguments will use defaults.
Strike = 0.03;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2004';
Use capbybdt to compute the price of the cap instrument.

```
Price = capbybdt(BDTTree, Strike, Settle, Maturity)
```

Price =
28.4001

## Price a 10\% Cap Instrument Using a BDT Interest-Rate Tree

Set the required arguments for the three specifications required to create a BDT tree.

```
Compounding = 1;
ValuationDate = '01-01-2000';
StartDate = ValuationDate;
EndDates = ['01-01-2001'; '01-01-2002'; '01-01-2003';
'01-01-2004'; '01-01-2005'];
Rates = [.1; .11; .12; .125; .13];
Volatility = [.2; .19; .18; .17; .16];
```

Create the specifications.

```
RateSpec = intenvset('Compounding', Compounding,...
'ValuationDate', ValuationDate,...
'StartDates', StartDate,...
'EndDates', EndDates,...
'Rates', Rates);
BDTTimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Volatility);
Create the BDT tree from the specifications.
```

```
BDTTree = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec)
```

```
    FinObj: 'BDTFwdTree'
    VolSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
    tObs: [00 1 2 3 4 4]
    dObs: [730486 730852 731217 731582 731947]
    TFwd: {[5x1 double] [4x1 double] [3x1 double] [2x1 double]
    CFlowT: {[5x1 double] [4x1 double] [3x1 double] [2x1 double]
FwdTree: {1x5 cell}
```

Set the cap arguments. Remaining arguments will use defaults.

```
CapStrike = 0.10;
Settlement = ValuationDate;
Maturity = '01-01-2002';
CapReset = 1;
```

Use capbybdt to find the price of the cap instrument.
Price= capbybdt(BDTTree, CapStrike, Settlement, Maturity,... CapReset)

Price =
1.7169

See Also
bdttree | cfbybdt | floorbybdt | swapbybdt

## Purpose

Price cap instrument from Black-Karasinski interest-rate tree

Syntax<br>[Price, PriceTree] = capbybk(BKTree, Strike, Settle, Maturity, Reset, Basis, Principal, Options)

## Arguments

| BKTree | Interest-rate tree structure created by bktree. |
| :---: | :---: |
| Strike | Number of instruments (NINST)-by- 1 vector of rates at which the cap is exercised. |
| Settle | Settlement dates. NINST-by- 1 vector of dates representing the settlement dates of the cap. |
| Maturity | NINST-by- 1 vector of dates representing the maturity dates of the cap. |
| Reset | (Optional) NINST-by-1 vector representing the frequency of payments per year. Default $=1$. |
| Basis | (Optional) Day-count basis of the instrument. A |
|  | - $0=$ actual/actual (default) |
|  | - $1=30 / 360$ (SIA) |
|  | - $2=$ actual/360 |
|  | - 3 = actual/365 |
|  | - $4=30 / 360$ (BMA) |
|  | - $5=30 / 360$ (ISDA) |
|  | - $6=30 / 360$ (European) |
|  | - $7=$ actual/365 (Japanese) |
|  | - $8=$ actual/actual (ICMA) |
|  | - 9 = actual/360 (ICMA) |
|  | - $10=$ actual/365 (ICMA) |

## capbybk

- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Principal (Optional) The notional principal amount. Default $=100$.

Options (Optional) Derivatives pricing options structure created with derivset.

```
Description [Price, PriceTree] = capbybk(BKTree, Strike, Settle,
Maturity,
Reset, Basis, Principal, Options) computes the price of a cap
instrument from a Black-Karasinski interest-rate tree.
```

Price is the expected price of the cap at time 0 .
PriceTree is the tree structure with values of the cap at each node.
The Settle date for every cap is set to the ValuationDate of the BK tree. The cap argument Settle is ignored.

## Examples Price a 3\% Cap Instrument Using a Black-Karasinski Interest-Rate Tree

Load the file deriv.mat, which provides BKTree. The BKTree structure contains the time and interest-rate information needed to price the cap instrument.
load deriv.mat;
Set the required values. Other arguments will use defaults.

```
Strike = 0.03;
Settle = '01-Jan-2005';
Maturity = '01-Jan-2009';
```

Use capbybk to compute the price of the cap instrument.
Price = capbybk(BKTree, Strike, Settle, Maturity)
Warning: Caps are valued at Tree ValuationDate rather than Settle Warning: Not all cash flows are aligned with the tree. Result will be approximated.
Warning: Not all cash flows are aligned with the tree. Result will be approximated.

Price =
6.8337

See Also cfbybk | floorbybk | bktree | swapbybk

Purpose
Price caps using Black option pricing model

## Syntax

[CapPrice, Caplets] = capbyblk(RateSpec, Strike, Settle, Maturity, Volatility)
[CapPrice, Caplets] = capbyblk(RateSpec, Strike, Settle, Maturity, Volatility, 'Name1', Value1...)

## Arguments

| RateSpec | The annualized, continuously compounded rate term <br> structure. For more information, see intenvset. |
| :--- | :--- |
| Strike | NINST-by-1 vector of rates at which the cap is <br> exercised, as a decimal number. |
| Settle | Scalar representing the settle date of the cap. |
| Maturity | Scalar representing the maturity date of the cap. <br> Volatility <br> NINST-by-1 vector of volatilities. |
| Reset | (Optional) NINST-by-1 vector representing the <br> frequency of payments per year. Default is 1. |
| Principal | (Optional) NINST-by-1 vector representing the <br> notional principal amount. Default is 100. |
|  | NINST-by-1 vector representing the basis used when |
|  | annualizing the input forward rate. |

- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
ValuationDate (Optional) Scalar representing the observation date of the investment horizons. The default is the Settle date.

ProjectionCur(Optional) The rate curve to be used in generating the future forward rates. This structure must be created using intenvset. Use this optional input if the forward curve is different from the discount curve. If ProjectionCurve is not specified, then RateSpec is used both for discounting cash flows and projecting future forward rates.

Note All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. You can specify parameter name/value pairs in any order. Names are case-insensitive and partial string matches are allowed provided no ambiguities exist.

## Description

[CapPrice, Caplets] = capbyblk(RateSpec, Strike, Settle, Maturity, Volatility)
[CapPrice, Caplets] = capbyblk(RateSpec, Strike, Settle, Maturity, Volatility, 'Name1', Value1...)

Use capbyblk to price caps using the Black option pricing model.
The outputs are:

- CapPrice - NINST-by-1 expected prices of the cap.
- Caplets - NINST-by-NCF array of caplets, padded with NaNs.


## Examples

## Price a Cap Using the Black Option Pricing Model

Consider an investor who gets into a contract that caps the interest rate on a $\$ 100,000$ loan at $8 \%$ quarterly compounded for 3 months, starting on January 1, 2009. Assuming that on January 1, 2008 the zero rate is $6.9394 \%$ continuously compounded and the volatility is $20 \%$, use this data to compute the cap price. First, calculate the RateSpec:

```
ValuationDate = 'Jan-01-2008';
EndDates ='April-01-2010';
Rates = 0.069394;
Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, ...
'StartDates', ValuationDate,'EndDates', EndDates, ...
'Rates', Rates,'Compounding', Compounding,'Basis', Basis)
```


## RateSpec =

## FinObj: 'RateSpec'

Compounding: -1
Disc: 0.8554
Rates: 0.0694
EndTimes: 2.2500
StartTimes: 0
EndDates: 734229

```
StartDates: 733408 ValuationDate: 733408
Basis: 1
EndMonthRule: 1
```

Compute the price of the cap.

```
Settle = 'Jan-01-2009'; % cap starts in a year
Maturity = 'April-01-2009';
Volatility = 0.20;
CapRate = 0.08;
CapReset = 4;
Principal=100000;
CapPrice = capbyblk(RateSpec, CapRate, Settle, Maturity, Volatility,
'Reset',CapReset,'ValuationDate',ValuationDate,'Principal', Principal,
'Basis', Basis)
```

CapPrice =
51.6125

## Price a Cap Using a Different Curve to Generate the Future Forward Rates

Define the OIS and Libor rates.

```
Settle = datenum('15-Mar-2013');
CurveDates = daysadd(Settle,360*[1/12 2/12 3/12 6/12 1 2 3 4 5 7 10],
OISRates = [.0018 . 0019 .0021 . 0023 .0031 .006 . 011 . 017 . 021 . 026
LiborRates = [.0045 .0047 . 005 .0055 .0075 .0109 .0162 .0216 .0262
```

Create an associated RateSpec for the OIS and Libor curves.

OISCurve = intenvset('Rates',OISRates,'StartDate', Settle,'EndDates', Curve LiborCurve = intenvset('Rates', LiborRates,'StartDate',Settle,'EndDates', 0

Define the Cap instruments.

```
Maturity = {'15-Mar-2018';'15-Mar-2020'};
Strike = [0.04;0.05];
BlackVol = 0.2;
```

Price the cap instruments using the term structure OISCurve both for discounting the cash flows and generating future forward rates.

```
[Price, Caplets] = capbyblk(OISCurve, Strike, Settle, Maturity, BlackVol)
```

Price =
0.7472
0.9890

## Caplets =

| 0 | 0.0000 | 0.0033 | 0.2996 | 0.4443 | NaN | NaN |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | 0.0000 | 0.0003 | 0.1134 | 0.2112 | 0.2292 | 0.4349 |

Price the cap instruments using the term structure LiborCurve to generate future forward rates. The term structure OISCurve is used for discounting the cash flows.
[PriceLC, CapletsLC] = capbyblk(OISCurve, Strike, Settle, Maturity, Black

PriceLC =
1.3293

### 1.6329

```
CapletsLC =
    0 0.0000 0.0337 0.4250 0.8706 NaN
    0.0000
        0.0052
        0.1767
        0.4849
        0.3663
\(0 \quad 0.0000\)
0.0052
0.1767
0.4849
0.3663
```

0.599

## See Also

floorbyblk | intenvset

Purpose Price cap instrument from Heath-Jarrow-Morton interest-rate tree

```
Syntax
[Price, PriceTree] = capbyhjm(HJMTree, Strike,
Settle, Maturity, Reset, Basis, Principal, Options)
```


## Arguments

| HJMTree | Forward-rate tree structure created by hjmtree. |
| :---: | :---: |
| Strike | Number of instruments (NINST)-by-1 vector of rates a which the cap is exercised. |
| Settle | Settlement dates. NINST-by- 1 vector of dates representing the settlement dates of the cap. |
| Maturity | NINST-by- 1 vector of dates representing the maturity dates of the cap. |
| Reset | (Optional) NINST-by-1 vector representing the frequency of payments per year. Default $=1$. |
| Basis | (Optional) Day-count basis of the instrument. A |
|  | vector of integers. |
|  | - $0=$ actual/actual (default) |
|  | - $1=30 / 360$ (SIA) |
|  | - $2=$ actual/360 |
|  | - 3 = actual/365 |
|  | - $4=30 / 360$ (BMA) |
|  | - $5=30 / 360$ (ISDA) |
|  | - $6=30 / 360$ (European) |
|  | - $7=$ actual/365 (Japanese) |
|  | - $8=$ actual/actual (ICMA) |
|  | - 9 = actual/360 (ICMA) |
|  | - $10=\mathrm{actual} / 365$ (ICMA) |

- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Principal (Optional) The notional principal amount. Default $=100$.

Options (Optional) Derivatives pricing options structure created with derivset.

## Description

[Price, PriceTree] = capbyhjm(HJMTree, Strike, Settle, Maturity, Reset, Basis, Principal, Options) computes the price of a cap instrument from an HJM tree.

Price is the expected price of the cap at time 0.
PriceTree is the tree structure with values of the cap at each node.
The Settle date for every cap is set to the ValuationDate of the HJM tree. The cap argument Settle is ignored.

## Examples Price a 3\% Cap Instrument Using an HJM Forward-Rate Tree

Load the file deriv.mat, which provides HJMTree. The HJMTree structure contains the time and forward-rate information needed to price the cap instrument.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
Strike = 0.03;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2004';
```

Use capbyhjm to compute the price of the cap instrument.

## capbyhim

```
Price = capbyhjm(HJMTree, Strike, Settle, Maturity)
Price =
    6.2831
```

See Also cfbyhjm | floorbyhjm | hjmtree | swapbyhjm

## Purpose

Price cap instrument from Hull-White interest-rate tree

Syntax<br>[Price, PriceTree] = capbyhw(HWTree, Strike, Settle, Maturity, Reset, Basis, Principal, Options)

## Arguments

| HWTree | Interest-rate tree structure created by hwtree. |
| :--- | :--- |
| Strike | Number of instruments (NINST)-by-1 vector of rates <br> at which the cap is exercised. |
| Settle | Settlement dates. NINST-by-1 vector of dates <br> representing the settlement dates of the cap. |
| Maturity | NINST-by-1 vector of dates representing the maturity <br> dates of the cap. |
| Reset | (Optional) NINST-by-1 vector representing the <br> frequency of payments per year. Default =1. <br> (Optional) Day-count basis of the instrument. A <br> vector of integers. |

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Principal (Optional) The notional principal amount. Default $=100$.

Options (Optional) Derivatives pricing options structure created with derivset.

## Description

[Price, PriceTree] = capbyhw(HWTree, Strike, Settle, Maturity, Reset, Basis, Principal, Options) computes the price of a cap instrument from a Hull-White interest-rate tree.

Price is the expected price of the cap at time 0.
PriceTree is the tree structure with values of the cap at each node.
The Settle date for every cap is set to the ValuationDate of the HW tree. The cap argument Settle is ignored.

## Examples <br> Price a 3\% Cap Instrument Using a Hull-White Interest-Rate Tree

Load the file deriv.mat, which provides HWTree. The HWTree structure contains the time and interest-rate information needed to price the cap instrument.
load deriv.mat;
Set the required values. Other arguments will use defaults.

```
Strike = 0.03;
Settle = '01-Jan-2005';
Maturity = '01-Jan-2009';
```

Use capbyhw to compute the price of the cap instrument.
Price = capbyhw(HWTree, Strike, Settle, Maturity)

Warning: Caps are valued at Tree ValuationDate rather than Settle
Warning: Not all cash flows are aligned with the tree. Result will be approximated.
Warning: Not all cash flows are aligned with the tree. Result will be approximated.

Price =
7.0707

See Also cfbyhw | floorbyhw | hwtree | swapbyhw

## capbylg2f

Purpose Price cap using Linear Gaussian two-factor model
Syntax CapPrice = capbylg2f(ZeroCurve, a, b, sigma, eta, rho, Strike, Maturity) CapPrice = capbylg2f( __, Name,Value)

Description

Input Arguments

CapPrice = capbylg2f(ZeroCurve, $a, b$, sigma, eta, rho, Strike, Maturity) returns cap price for a two-factor additive Gaussian interest-rate model.

CapPrice = capbylg2f(__ , Name, Value) returns cap price for a two-factor additive Gaussian interest-rate model using optional name-value pairs.

## ZeroCurve - Zero-curve for Linear Gaussian two-factor model

 structureZero-curve for the Linear Gaussian two-factor model, specified using IRDataCurve or RateSpec.

## Data Types

struct

## a-Mean reversion for first factor for Linear Gaussian two-factor model

scalar
Mean reversion for first factor for the Linear Gaussian two-factor model, specified as a scalar.

## Data Types <br> single | double

## b-Mean reversion for second factor for Linear Gaussian two-factor model

scalar

Mean reversion for second factor for the Linear Gaussian two-factor model, specified as a scalar.

Data Types
single | double

## sigma-Volatility for first factor for Linear Gaussian two-factor model

scalar
Volatility for first factor for the Linear Gaussian two-factor model, specified as a scalar.

Data Types
single | double
eta - Volatility for second factor for Linear Gaussian two-factor model
scalar
Volatility for second factor for the Linear Gaussian two-factor model, specified as a scalar.

Data Types
single | double

## rho - Scalar correlation of the factors

scalar
Scalar correlation of the factors, specified as a scalar.
Data Types
single | double

## Strike - Cap strike price

nonnegative integer | vector of nonnegative integers
Cap strike price, specified as a nonnegative integer using a NumCaps-by-1 vector.

Data Types<br>single | double

## Maturity - Cap maturity date

nonnegative integer | vector of nonnegative integers | string of dates
Cap maturity date, specified using a NumCaps-by-1 vector of serial date numbers or date strings.

```
Data Types
single | double | cell
```


## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

Example: Price =
capbylg2f(irdc,a,b,sigma, eta,rho,Strike, CapMaturity, 'Reset', 1, 'Notional'

## 'Reset' - Frequency of cap payments per year

2 (default) | positive integer from the set[1,2,3,4,6,12] | vector of positive integers from the set [1, $2,3,4,6,12$ ]

Frequency of cap payments per year, specified as positive integers for the values $1,2,4,6,12$ ] in a NumCaps-by- 1 vector.

## Data Types <br> single | double

## 'Notional' - Notional value of cap

100 (default) | nonnegative integer | vector of nonnegative integers
Notional value of cap, specified as a nonnegative integer using a NumCaps-by- 1 vector of notional amounts.

Data Types<br>single | double

## Output Arguments

## Algorithms

## Examples

## CapPrice - Cap price

scalar | vector
Expected prices of cap, returned as a scalar or an NumCaps-by-1 vector.

The following defines the two-factor additive Gaussian interest rate model, given the ZeroCurve, $a, b$, sigma, eta, and rho parameters:

$$
\begin{aligned}
& r(t)=x(t)+y(t)+\phi(t) \\
& d x(t)=-a(x)(t) d t+\sigma\left(d W_{1}(t), x(0)=0\right. \\
& d y(t)=-b(y)(t) d t+\eta\left(d W_{2}(t), y(0)=0\right.
\end{aligned}
$$

where $d W_{1}(t) d W_{2}(t)=\rho d t$ is a two-dimensional Brownian motion with correlation $\rho$ and $\varphi$ is a function chosen to match the initial zero curve.

## Price a Cap Using a Linear Gaussian Two-Factor Model

Define the ZeroCurve, a, b, sigma, eta, and rho parameters to price the cap.

```
Settle = datenum('15-Dec-2007');
ZeroTimes = [3/12 6/12 1 5 7 10 20 30]';
ZeroRates = [0.033 0.034 0.035 0.040 0.042 0.044 0.048 0.0475]';
CurveDates = daysadd(Settle,360*ZeroTimes);
irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);
a = .07;
b = .5;
sigma = .01;
eta = .006;
rho = -.7;
CapMaturity = daysadd(Settle,360*[1:5 7 10 15 20 25 30],1);
```

```
Strike = [l0.035 0.037 0.038 0.039 0.040 0.042 0.044 0.046 0.047 0.047 0.0
Price = capbylg2f(irdc,a,b,sigma,eta,rho,Strike,CapMaturity)
Price =
    0.0316
    0.3225
    0.7761
    1.3240
    1.9394
    3.1247
    4.8451
    7.3752
    9.8582
    11.4673
    12.7850
```


## References

[1] Brigo, D. and F. Mercurio, Interest Rate Models - Theory and Practice, Springer Finance, 2006.

See Also floorbylg2f | swaptionbylg2f | LinearGaussian2F
$\begin{array}{ll}\text { Related } & \text { - "Price Swaptions With Interest-Rate Models Using Simulation" on } \\ \text { Examples page } 2-105\end{array}$

\author{
Purpose Determine price of cash-or-nothing digital options using Black-Scholes model <br> ```
Syntax <br> Price = cashbybls(RateSpec, StockSpec, Settle, Maturity, <br> OptSpec, Strike, Payoff)

```
}

Arguments

\section*{Description}

\section*{Examples}

Price = cashbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, Payoff) computes cash-or-nothing option prices using the Black-Scholes option pricing model.

Price is a NINST-by-1 vector of expected option prices.

\section*{Compute Cash-or-Nothing Option Prices Using the Black-Scholes Option Pricing Model}

Consider a European call and put cash-or-nothing options on a futures contract with and exercise strike price of \(\$ 90\), a fixed payoff of \(\$ 10\) that expires on October 1, 2008. Assume that on January 1, 2008, the contract trades at \(\$ 110\), and has a volatility of \(25 \%\) per annum and the risk-free rate is \(4.5 \%\) per annum. Using this data, calculate the price of
the call and put cash-or-nothing options on the futures contract. First, create the RateSpec:
```

Settle = 'Jan-1-2008';
Maturity = 'Oct-1-2008';
Rates = 0.045;
Compounding = -1;
Basis = 1;
RateSpec =
FinObj: 'RateSpec'
Compounding: -1
Disc: 0.9668
Rates: 0.0450
EndTimes: 0.7500
StartTimes: 0
EndDates: 733682
StartDates: 733408
ValuationDate: 733408
Basis: 1
EndMonthRule: 1

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Basis'

Define the StockSpec.
```

AssetPrice = 110;
Sigma = .25;
DivType = 'Continuous';
DivAmount = Rates;
StockSpec = stockspec(Sigma, AssetPrice, DivType, DivAmount)

```
StockSpec =
```

            FinObj: 'StockSpec'
            Sigma: 0.2500
        AssetPrice: 110
        DividendType: {'continuous'}
    DividendAmounts: 0.0450
ExDividendDates: []

```

Define the call and put options.
```

OptSpec = {'call'; 'put'};
Strike = 90;
Payoff = 10;

```

Calculate the prices.
```

Pcon = cashbybls(RateSpec, StockSpec, Settle,...
Maturity, OptSpec, Strike, Payoff)

```
Pcon =
    7.6716
    1.9965

See Also assetbybls | cashsensbybls | gapbybls | supersharebybls

\section*{Purpose}

Syntax
PriceSens = cashsensbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, Payoff) PriceSens = cashsensbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, Payoff, OutSpec)

\section*{Arguments}
\begin{tabular}{|c|c|}
\hline RateSpec & The annualized, continuously compounded rate term structure. For information on the interest rate specification, see intenvset. \\
\hline StockSpec & Stock specification. See stockspec. \\
\hline Settle & NINST-by-1 vector of settlement or trade dates. \\
\hline Maturity & NINST-by- 1 vector of maturity dates. \\
\hline OptSpec & NINST-by-1 cell array of strings 'call' or 'put'. \\
\hline Strike & NINST-by-1 vector of strike price values. \\
\hline Payoff & NINST-by-1 vector of payoff values or the amount to be paid at expiration. \\
\hline OutSpec & \begin{tabular}{l}
(Optional) All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. You can specify parameter name/value pairs in any order. Names are case-insensitive and partial string matches are allowed provided no ambiguities exist. Valid parameter names are: \\
- NOUT-by-1 or 1-by-NOUT cell array of strings indicating the nature and order of the outputs for the function. Possible values are 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', or 'All'.
\end{tabular} \\
\hline
\end{tabular}

> For example, OutSpec \(=\{\) 'Price';
> 'Lambda'; 'Rho'\} specifies that the output should be Price, Lambda, and Rho, in that order.
> To invoke from a function: [Price, Lambda, Rho] = cashsensbybls(..., 'OutSpec', \(\{' P r i c e ', ~ ' L a m b d a ', ~ ' R h o '\}) ~\)
> OutSpec = \{'All'\} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec as OutSpec = \{'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', 'Price'\};.
- Default is OutSpec = \{'Price' \(\}\).

\section*{Description}

PriceSens = cashsensbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, Payoff) computes cash-or-nothing option prices using the Black-Scholes option pricing model.

> PriceSens = cashsensbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, Payoff, OutSpec) includes an OutSpec argument defined as parameter/value pairs, and computes cash-or-nothing option prices and sensitivities using the Black-Scholes option pricing model.

PriceSens is a NINST-by-1 vector of expected option prices and sensitivities.

\section*{Examples Compute Cash-or-Nothing Option Prices and Sensitivities Using the Black-Scholes Option Pricing Model}

Consider a European call and put cash-or-nothing options on a futures contract with an exercise price of \(\$ 90\), and a fixed payoff of \(\$ 10\) that expires on January 1, 2009. Assume that on October 1, 2008 the
contract trades at \(\$ 110\), and has a volatility of \(25 \%\) per annum and the risk-free rate is \(4.5 \%\) per annum. Using this data, calculate the price and sensitivity of the call and put cash-or-nothing options on the futures contract. First, create the RateSpec:
```

Settle = 'Jan-1-2008';
Maturity = 'Oct-1-2008';
Rates = 0.045;
Compounding = -1;
Basis = 1;
RateSpec =
FinObj: 'RateSpec'
Compounding: -1
Disc: 0.9668
Rates: 0.0450
EndTimes: 0.7500
StartTimes: 0
EndDates: 733682
StartDates: 733408
ValuationDate: 733408
Basis: 1
EndMonthRule: 1

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Basis'

Define the StockSpec.
```

AssetPrice = 110;

```
Sigma = .25;
DivType = 'Continuous';
DivAmount = Rates;
StockSpec = stockspec(Sigma, AssetPrice, DivType, DivAmount)

\section*{StockSpec =}
```

            FinObj: 'StockSpec'
            Sigma: 0.2500
        AssetPrice: 110
        DividendType: {'continuous'}
    DividendAmounts: 0.0450
ExDividendDates: []

```

Define the call and put options.
```

OptSpec = {'call'; 'put'};
Strike = 90;
Payoff = 10;

```

Compute the gamma, theta, and price.
OutSpec = \{ 'gamma';'theta';'price'\};
[Gamma, Theta, Price] = cashsensbybls(RateSpec, StockSpec,... Settle, Maturity, OptSpec, Strike, Payoff, 'OutSpec', OutSpec)

Gamma =
- 0.0050
0.0050

Theta =
-2.2489
1.8139

Price =

See Also cashbybls

\section*{Purpose}

Price cash flows from Black-Derman-Toy interest-rate tree

\author{
Syntax \\ [Price, PriceTree] = cfbybdt(BDTTree, CFlowAmounts, CFlowDates, Settle, Basis, Options)
}

\section*{Arguments}

CFIowAmounts Number of instruments (NINST) by maximum number of cash flows (MOSTCFS) matrix of cash flow amounts. Each row is a list of cash flow values for one instrument. If an instrument has fewer than MOSTCFS cash flows, the end of the row is padded with NaNs.

CFlowDates NINST-by-MOSTCFS matrix of cash flow dates. Each entry contains the serial date number of the corresponding cash flow in CFlowAmounts.
Settle Settlement date. A vector of serial date numbers or date strings. The Settle date for every cash flow is set to the ValuationDate of the BDT tree. The cash flow argument, Settle, is ignored.

Basis
(Optional) Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- \(3=\) actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)

\section*{cfbybdt}
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- \(12=\) actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Options
(Optional) Derivatives pricing options structure created with derivset.
[Price, PriceTree] = cfbybdt(BDTTree, CFlowAmounts, CFlowDates, Settle, Basis, Options) prices cash flows from a BDT interest-rate tree.
Price is an NINST-by-1 vector of expected prices at time 0.
PriceTree is a tree structure with a vector of instrument prices at each node.

\section*{Examples}

Price a portfolio containing two cash flow instruments paying interest annually over the four-year period from January 1, 2000 to January 1, 2004.

Load the file deriv.mat, which provides BDTTree. The BDTTree structure contains the time and interest-rate information needed to price the instruments.
load deriv.mat;
The valuation date (settle date) specified in BDTTree is January 1, 2000 (date number 730486).
```

BDTTree.RateSpec.ValuationDate
ans =

```
730486

Provide values for the other required arguments.
CFlowAmounts =[5 NaN 5.5 105; 506 105]; CFlowDates = [730852, NaN, 731582, 731947; 730852, 731217, 731582, 731947];

Use this information to compute the prices of the two cash flow instruments.
[Price, PriceTree] = cfbybdt(BDTTree, CFlowAmounts, ... CFlowDates, BDTTree.RateSpec.ValuationDate)

Price =
74.0112
74.3671

PriceTree =
FinObj: 'BDTPriceTree'
tObs: [0 1.002 .003 .004 .00\(]\)
PTree: \{1x5 cell\}
You can visualize the prices of the two cash flow instruments with the treeviewer function.
treeviewer(PriceTree)


See Also
bdttree | bdtprice | cfamounts | instcf

\section*{Purpose}

Price cash flows from Black-Karasinski interest-rate tree

\author{
Syntax \\ [Price, PriceTree] = cfbybk(BKTree, CFlowAmounts, CFlowDates, Settle, Basis, Options)
}

BKTree Forward-rate tree structure created by bktree.
CFlowAmounts Number of instruments (NINST) by maximum number of cash flows (MOSTCFS) matrix of cash flow amounts. Each row is a list of cash flow values for one instrument. If an instrument has fewer than MOSTCFS cash flows, the end of the row is padded with NaNs.

CFlowDates NINST-by-MOSTCFS matrix of cash flow dates. Each entry contains the serial date number of the corresponding cash flow in CFlowAmounts.
Settle Settlement date. A vector of serial date numbers or date strings. The Settle date for every cash flow is set to the ValuationDate of the BK tree. The cash flow argument, Settle, is ignored.

Basis
(Optional) Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- \(3=\) actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)

\section*{cfbybk}
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- \(10=\mathrm{actual} / 365\) (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- \(12=\) actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Options
(Optional) Derivatives pricing options structure created with derivset.

Description
[Price, PriceTree] = cfbybk(BKTree, CFlowAmounts, CFlowDates,
Settle, Basis, Options) prices cash flows from a Black-Karasinski interest-rate tree.

Price is an NINST-by- 1 vector of expected prices at time 0.
PriceTree is a tree structure with a vector of instrument prices at each node.

\section*{Examples}

Price a portfolio containing two cash flow instruments paying interest annually over the four-year period from January 1, 2005 to January 1, 2009.

Load the file deriv.mat, which provides BKTree. The BKTree structure contains the time and interest-rate information needed to price the instruments.
load deriv.mat;
The valuation date (settle date) specified in BKTree is January 1, 2004 (date number 731947).
```

BKTree.RateSpec.ValuationDate
ans =

```
731947

Provide values for the other required arguments.
CFlowAmounts \(=[5 \mathrm{NaN} 5.5\) 105; 506 105];
CFlowDates \(=[732678, \mathrm{NaN}, 733408,733774\);
    732678, 733034, 733408, 734774];

Use this information to compute the prices of the two cash flow instruments.
```

[Price, PriceTree] = cfbybk(BKTree, CFlowAmounts, CFlowDates,...
BKTree.RateSpec.ValuationDate)
Price =
93.3600
81.6218
PriceTree =
FinObj: 'BKPriceTree'
tObs: [0 0 1 2 3 4]
PTree: {[2x1 double] [2x3 double] [2x5 double] [2\times5
double] [2x5 double]}
Connect: {[2] [2 2 3 4] [2 2 2 3 4 4 4]}
Probs: {[3x1 double] [3x3 double] [3x5 double]}

```

You can visualize the prices of the two cash flow instruments with the treeviewer function.
treeviewer(PriceTree)


See Also
bktree | bkprice | cfamounts | instcf

\section*{Purpose}

Price cash flows from Heath-Jarrow-Morton interest-rate tree

\author{
Syntax \\ [Price, PriceTree] = cfbyhjm(HJMTree, CFlowAmounts, CFlowDates, Settle, Basis, Options)
}

\section*{Arguments}

\section*{cfbyhim}
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- \(12=\) actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Options
(Optional) Derivatives pricing options structure created with derivset.

Description

\section*{Examples}
[Price, PriceTree] = cfbyhjm(HJMTree, CFlowAmounts, CFlowDates, Settle, Basis, Options) prices cash flows from an HJM interest-rate tree.

Price is an NINST-by- 1 vector of expected prices at time 0.
PriceTree is a tree structure with a vector of instrument prices at each node.

Price a portfolio containing two cash flow instruments paying interest annually over the four-year period from January 1, 2000 to January 1, 2004.

Load the file deriv.mat, which provides HJMTree. The HJMTree structure contains the time and forward-rate information needed to price the instruments.
load deriv.mat;
The valuation date (settle date) specified in HJMTree is January 1, 2000 (date number 730486).
```

HWTree.RateSpec.ValuationDate
ans =

```
730486

Provide values for the other required arguments.
```

CFlowAmounts =[5 NaN 5.5 105; 5 0 6 105];
CFlowDates = [730852, NaN, 731582, 731947;
730852, 731217, 731582, 731947];

```

Use this information to compute the prices of the two cash flow instruments.
[Price, PriceTree] = cfbyhjm(HJMTree, CFlowAmounts,... CFlowDates, HJMTree.RateSpec.ValuationDate)

Price =
96.7805
97.2188

PriceTree =
FinObj: 'HJMPriceTree'
tObs: [0 1.002 .003 .004 .00\(]\) PBush: \{1x5 cell\}

You can visualize the prices of the two cash flow instruments with the treeviewer function.
treeviewer(PriceTree)


\section*{See Also}
cfamounts | hjmprice | hjmtree | instcf

\section*{Purpose}

Price cash flows from Hull-White interest-rate tree

\author{
Syntax \\ [Price, PriceTree] = cfbyhw(HWTree, CFlowAmounts, CFlowDates, Settle, Basis, Options)
}

HWTree Forward-rate tree structure created by hwtree.
CFlowAmounts Number of instruments (NINST) by maximum number of cash flows (MOSTCFS) matrix of cash flow amounts. Each row is a list of cash flow values for one instrument. If an instrument has fewer than MOSTCFS cash flows, the end of the row is padded with NaNs.

CFlowDates NINST-by-MOSTCFS matrix of cash flow dates. Each entry contains the serial date number of the corresponding cash flow in CFlowAmounts.
Settle Settlement date. A vector of serial date numbers or date strings. The Settle date for every cash flow is set to the ValuationDate of the HW tree. The cash flow argument, Settle, is ignored.

Basis
(Optional) Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- \(3=\) actual \(/ 365\)
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)

\section*{cfbyhw}
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.

Options
(Optional) Derivatives pricing options structure created with derivset.

Description
[Price, PriceTree] = cfbyhw(HWTree, CFlowAmounts, CFlowDates, Settle, Basis, Options) prices cash flows from a Hull-White interest-rate tree.

Price is an NINST-by-1 vector of expected prices at time 0.
PriceTree is a tree structure with a vector of instrument prices at each node.

\section*{Examples}

Price a portfolio containing two cash flow instruments paying interest annually over the four-year period from January 1, 2005 to January 1, 2009.

Load the file deriv.mat, which provides HWTree. The HWTree structure contains the time and interest-rate information needed to price the instruments.
load deriv.mat;
The valuation date (settle date) specified in HWTree is January 1, 2004 (date number 731947).
```

HWTree.RateSpec.ValuationDate
ans =

```
731947

Provide values for the other required arguments.
```

CFlowAmounts =[5 NaN 5.5 105; 5 0 6 105];
CFlowDates = [732678, NaN, 733408, 733774;
732678, 733034, 733408, 734774];

```

Use this information to compute the prices of the two cash flow instruments.
```

[Price, PriceTree] = cfbyhw(HWTree, CFlowAmounts, CFlowDates,...
HWTree.RateSpec.ValuationDate)
Price =
93.3789
81.7651
PriceTree =
FinObj: 'HWPriceTree'
tObs: [00 1 1 2 3 4]
PTree: {[2x1 double] [2x3 double] [2x5 double] [2x5
double] [2x5 double]}
Connect: {[2] [2 3 4] [2 2 3 4 4]}
Probs: {[3x1 double] [3x3 double] [3x5 double]}

```

You can visualize the prices of the two cash flow instruments with the treeviewer function.
```

treeviewer(PriceTree)

```


See Also
cfamounts | hwtree | hwprice | instcf

\section*{Purpose}

Price cash flows from set of zero curves
Syntax \(\quad \begin{aligned} & \text { Price }=\text { cfbyzero(RateSpec, CFlowAmounts, CFlowDates, Settle, } \\ & \text { Basis) }\end{aligned}\)

\section*{Arguments}
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- \(10=\mathrm{actual} / 365\) (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- \(12=\) actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.

Description Price \(=\) cfbyzero(RateSpec, CFlowAmounts, CFlowDates, Settle,
Basis) computes Price, an NINST-by-NUMCURVES matrix of cash flows prices. Each column arises from one of the zero curves.

\section*{Examples Compute the Price and Sensitivity From the Interest-Rate Term Structure}

This example shows how to price a portfolio containing two cash flow instruments paying interest annually over the four-year period from January 1, 2000 to January 1, 2004. Load the file deriv.mat, which provides ZeroRateSpec. The ZeroRateSpec structure contains the interest-rate information needed to price the instruments.
```

load deriv.mat
CFlowAmounts =[5 NaN 5.5 105;5 0 6 105];
CFlowDates = [730852, NaN, 731582,731947;

```

730852, 731217, 731582, 731947];
Settle = 730486; Price = cfbyzero(ZeroRateSpec, CFlowAmounts, CFlowDates, Settle)

Price =
96.7804
97.2187

See Also bondbyzero | fixedbyzero | floatbyzero | swapbyzero

Purpose
Price European simple chooser options using Black-Scholes model
Syntax Price = chooserbybls(RateSpec, StockSpec, Settle, Maturity, Strike)

\section*{Arguments}

\section*{Description}

RateSpec The annualized continuously compounded rate term structure. For information on the interest rate specification, see intenvset.

StockSpec Stock specification. See stockspec.
Settle NINST-by-1 vector of settlement or trade dates.
Maturity NINST-by-1 vector of maturity dates.
Strike NINST-by-1 vector of strike price values.
ChooseDate NINST-by-1 vector of chooser dates.

Price = chooserbybls(RateSpec, StockSpec, Settle, Maturity, Strike) computes the price for European simple chooser options using the Black-Scholes model.

Price is a NINST-by- 1 vector of expected prices.

Note Only dividends of type continuous can be considered for choosers.

\section*{Examples Price European Simple Chooser Options Using the Black-Scholes Model}

Consider a European chooser option with an exercise price of \(\$ 60\) on June 1, 2007. The option expires on December 2, 2007. Assume the underlying stock provides a continuous dividend yield of \(5 \%\) per annum,
is trading at \(\$ 50\), and has a volatility of \(20 \%\) per annum. The annualized continuously compounded risk-free rate is \(10 \%\) per annum. Assume that the choice must be made on August 31, 2007. Using this data:
```

AssetPrice = 50;
Strike = 60;
Settlement = 'Jun-1-2007';
Maturity = 'Dec-2-2007';
ChooseDate = 'Aug-31-2007';
RiskFreeRate = 0.1;
Sigma = 0.20;
Yield = 0.05

```
Yield =
    0.0500

Define the RateSpec and StockSpec.
```

RateSpec = intenvset('Compounding', -1, 'Rates', RiskFreeRate, 'Start[
Settlement, 'EndDates', Maturity);
StockSpec = stockspec(Sigma, AssetPrice,'continuous',Yield);

```

Price the chooser option.
Price = chooserbybls(RateSpec, StockSpec, Settlement, Maturity,... Strike, ChooseDate)

\section*{Price =}
8.9308

References Rubinstein, Mark, "Options for the Undecided," Risk 4, 1991.

\author{
See Also \\ blsprice | intenvset
}

Purpose
Create financial structure or return financial structure class name
Syntax
Obj = classfin(ClassName)
Obj = classfin(Struct, ClassName)
ClassName = classfin(Obj)

\section*{Arguments}

\section*{Description \(\quad 0 b j=\) classfin(ClassName) and Obj = classfin(Struct, ClassName) create a financial structure of class ClassName. \\ ClassName = classfin(Obj) returns a string containing a financial structure's class name.}

\section*{Examples Create a Financial Structure}

This example shows how to create a financial structure HJMTimeSpec and complete its fields. (Typically, the function hjmtimespec is used to create HJMTimeSpec structures).
```

TimeSpec = classfin('HJMTimeSpec');
TimeSpec.ValuationDate = datenum('Dec-10-1999');
TimeSpec.Maturity = datenum('Dec-10-2002');
TimeSpec.Compounding = 2;
TimeSpec.Basis = 0;
TimeSpec.EndMonthRule = 1;
TimeSpec

```

TimeSpec =
```

    FinObj: 'HJMTimeSpec
    ValuationDate: 730464
Maturity: 731560
Compounding: 2
Basis: 0
EndMonthRule: 1

```

\section*{Convert an Existing MATLAB Structure into a Financial Structure}

This example shows how to convert an existing MATLAB structure into a financial structure.

TSpec.ValuationDate = datenum('Dec-10-1999');
TSpec.Maturity = datenum('Dec-10-2002');
TSpec.Compounding \(=2\);
TSpec.Basis = 0;
TSpec.EndMonthRule = 0;
TimeSpec = classfin(TSpec, 'HJMTimeSpec')

TimeSpec =
ValuationDate: 730464
Maturity: 731560
Compounding: 2
Basis: 0
EndMonthRule: 0
FinObj: 'HJMTimeSpec'

\section*{Return a String Containing a Financial Structure's Class Name}
```

structure's class name.
load deriv.mat
ClassName $=$ classfin(HJMTree)
ClassName =
HJMFwdTree

```

This example shows how to obtain a string containing a financial

\section*{See Also \\ isafin}

Purpose Price compound option from Cox-Ross-Rubinstein binomial tree
```

Syntax [Price, PriceTree] = compoundbycrr(CRRTree, UOptSpec, UStrike,
USettle, UExerciseDates, UAmericanOpt, COptSpec,
CStrike, CSettle, CExerciseDates, CAmericanOpt)

```

\section*{Arguments}
\begin{tabular}{|c|c|}
\hline CRRTree & Stock tree structure created by crrtree. \\
\hline UOptSpec & String = 'Call' or 'Put'. \\
\hline UStrike & 1-by-1 vector of strike price values. \\
\hline USettle & 1-by-1 vector of Settle dates. \\
\hline \multirow[t]{4}{*}{UExerciseDates} & For a European option (UAmericanOpt \(=0\) ): \\
\hline & 1-by-1 vector of exercise dates. For a European option, there is only one exercise date, the option expiry date. \\
\hline & For an American option (UAmericanOpt = 1): \\
\hline & 1-by-2 vector of exercise date boundaries. The option can be exercised on any tree date. If only one non-NaN date is listed, or if ExerciseDates is 1 -by-1, the option can be exercised between the valuation date of the stock tree and the single listed exercise date. \\
\hline UAmericanOpt & If UAmericanOpt \(=0, N a N\), or is unspecified, the option is a European option. If UAmericanOpt \(=1\), the option is an American option. \\
\hline COptSpec & NINST-by-1 list of string values 'Call' or 'Put' of the compound option. \\
\hline CStrike & NINST-by- 1 vector of strike price values. Each row is the schedule for one option. \\
\hline
\end{tabular}
\begin{tabular}{ll} 
CSettle & \begin{tabular}{l} 
1-by-1 vector containing the settlement or trade \\
date.
\end{tabular} \\
CExerciseDates & \begin{tabular}{l} 
For a European option (CAmericanOpt \(=0\) ): \\
NINST-by-1 vector of exercise dates. Each row \\
is the schedule for one option. For a European \\
option, there is only one exercise date, the \\
option expiry date.
\end{tabular} \\
& For an American option (CAmericanOpt = 1): \\
& NINST-by-2 vector of exercise date boundaries. \\
For each instrument, the option can be \\
exercised on any tree date between or including \\
the pair of dates on that row. If only one \\
non-NaN date is listed, or if ExerciseDates is \\
NINST-by-1, the option can be exercised between \\
the valuation date of the stock tree and the \\
single listed exercise date.
\end{tabular}

\section*{Description}
[Price, PriceTree] = compoundbycrr(CRRTree, UOptSpec, UStrike,
USettle, UExerciseDates, UAmericanOpt, COptSpec, CStrike, CSettle, CExerciseDates, CAmericanOpt) calculates the value of a compound option.

Price is a NINST-by- 1 vector of expected prices at time 0 .
PriceTree is a tree structure with a vector of instrument prices at each node.
```

Examples Price a Compound Option Using a CRR Binomial Tree
This example shows how to price a compound option using a CRR
binomial tree by loading the file deriv.mat, which provides CRRTree.
The CRRTree structure contains the stock specification and time
information needed to price the option.
load deriv.mat
UOptSpec = 'Call';
UStrike = 130;
USettle = '01-Jan-2003';
UExerciseDates = '01-Jan-2006';
UAmericanOpt = 1;
COptSpec = 'Put';
CStrike = 5;
CSettle = '01-Jan-2003';
CExerciseDates = '01-Jan-2005';
Price = compoundbycrr(CRRTree, UOptSpec, UStrike, USettle, ...
UExerciseDates, UAmericanOpt, COptSpec, CStrike, CSettle, ...
CExerciseDates)
Price =
2.8482
References Rubinstein, Mark, "Double Trouble,"Risk 5, 1991, p. }73
See Also crrtree | instcompound

```

\section*{Purpose}

Price compound option from Equal Probabilities binomial tree
```

Syntax
[Price, PriceTree] = compoundbyeqp(EQPTree, UOptSpec, UStrike,
USettle, UExerciseDates, UAmericanOpt, COptSpec,
CStrike, CSettle, CExerciseDates, CAmericanOpt)

```

\section*{Arguments}
\begin{tabular}{ll} 
EQPTree & Stock tree structure created by eqptree. \\
UOptSpec & String = 'Call' or 'Put '. \\
UStrike & \begin{tabular}{l} 
1-by-1 vector of strike price values.
\end{tabular} \\
USettle & \begin{tabular}{l} 
1-by-1 vector of Settle dates.
\end{tabular} \\
UExerciseDates & \begin{tabular}{l} 
For a European option (UAmericanOpt = 0): \\
1-by-1 vector of exercise dates. For a European \\
option, there is only one exercise date, the \\
option expiry date.
\end{tabular} \\
& \begin{tabular}{l} 
For an American option (UAmericanOpt = 1): \\
1-by-2 vector of exercise date boundaries. The \\
option can be exercised on any tree date. If only \\
one non-NaN date is listed, or if ExerciseDates \\
is 1-by-1, the option can be exercised between \\
the valuation date of the stock tree and the \\
single listed exercise date.
\end{tabular} \\
UAmericanOpt & \begin{tabular}{l} 
If UAmericanOpt = 0, NaN, or is unspecified, the \\
option is a European option. If UAmericanOpt \\
\(=\)
\end{tabular} \\
1, the option is an American option.
\end{tabular}
\(\left.\begin{array}{ll}\text { CSettle } & \begin{array}{l}\text { 1-by-1 vector containing the settlement or trade } \\ \text { date. }\end{array} \\ \text { CExerciseDates } & \begin{array}{l}\text { For a European option (CAmericanOpt }=0 \text { ): } \\ \text { NINST-by-1 vector of exercise dates. Each row } \\ \text { is the schedule for one option. For a European } \\ \text { option, there is only one exercise date, the } \\ \text { option expiry date. }\end{array} \\ & \text { For an American option (CAmericanOpt = 1): } \\ & \begin{array}{l}\text { NINST-by-2 vector of exercise date boundaries. }\end{array} \\ & \begin{array}{l}\text { For each instrument, the option can be } \\ \text { exercised on any tree date between or including }\end{array} \\ \text { the pair of dates on that row. If only one }\end{array}\right\}\)

\section*{Description}
[Price, PriceTree] = compoundbyeqp(EQPTree, UOptSpec, UStrike,
USettle, UExerciseDates, UAmericanOpt, COptSpec, CStrike, CSettle, CExerciseDates, CAmericanOpt) calculates the value of a compound option.

Price is a NINST-by- 1 vector of expected prices at time 0 .
PriceTree is a tree structure with a vector of instrument prices at each node.
Examples Price a Compound Option Using an EQP Equity TreeThis example shows how to price a compound option using a EQPequity tree by loading the file deriv.mat, which provides EQPTree. TheEQPTree structure contains the stock specification and time informationneeded to price the option.
```

load deriv.mat
UOptSpec = 'Call';
UStrike = 130;
USettle = '01-Jan-2003';
UExerciseDates = '01-Jan-2006';
UAmericanOpt = 1;
COptSpec = 'Put';
CStrike = 5;
CSettle = '01-Jan-2003';
CExerciseDates = '01-Jan-2005';

```
Price = compoundbyeqp(EQPTree, UOptSpec, UStrike, USettle, ...
UExerciseDates, UAmericanOpt, COptSpec, CStrike, CSettle, ...
CExerciseDates)
Price =
    3.3931
References Rubinstein, Mark, "Double Trouble," Risk 5, 1991, p. 73
See Also eqptree | instcompound

Purpose Price compound options using implied trinomial tree (ITT)
```

Syntax [Price, PriceTree] = compoundbyitt(ITTTree, UOptSpec, UStrike,
USettle, UExerciseDates, UAmericanOpt, COptSpec,
CStrike, CSettle, CExerciseDates, CAmericanOpt)

```

\section*{Arguments}
\begin{tabular}{ll} 
ITTTree & Stock tree structure created by itttree. \\
UOptSpec & String = 'call' or 'put'. \\
UStrike & \begin{tabular}{l} 
1-by-1 vector of strike price values. \\
USettle \\
UExerciseDates
\end{tabular} \\
\begin{tabular}{l} 
1-by-1 vector of Settle dates. \\
For a European option (UAmericanOpt = 0): \\
1-by-1 vector of exercise dates. For a European \\
option, there is only one exercise date, the option \\
expiry date.
\end{tabular} \\
& \begin{tabular}{l} 
For an American option (UAmericanOpt = 1):
\end{tabular} \\
\begin{tabular}{l} 
1-by-2 vector of exercise date boundaries. The \\
option can be exercised on any tree date. If only \\
one non-NaN date is listed, or if ExerciseDates is \\
1-by-1, the option can be exercised between the \\
valuation date of the stock tree and the single \\
listed exercise date.
\end{tabular} \\
UAmericanOpt & \begin{tabular}{l} 
If UAmericanOpt = 0, NaN, or is unspecified, the \\
option is a European option. If UAmericanOpt = \\
1, the option is an American option.
\end{tabular} \\
COptSpec & \begin{tabular}{l} 
NINST-by-1 list of string values 'Call' or 'Put ' \\
of the compound option.
\end{tabular} \\
CStrike & \begin{tabular}{l} 
NINST-by-1 vector of strike price values. Each \\
row is the schedule for one option.
\end{tabular}
\end{tabular}
\begin{tabular}{ll} 
CSettle & \begin{tabular}{l} 
1-by-1 vector containing the settlement or trade \\
date.
\end{tabular} \\
CExerciseDates \\
For a European option (CAmericanOpt = 0): \\
NINST-by-1 vector of exercise dates. Each row \\
is the schedule for one option. For a European \\
option, there is only one exercise date, the option \\
expiry date. \\
For an American option (CAmericanOpt = 1): \\
& \begin{tabular}{l} 
NINST-by-2 vector of exercise date boundaries. \\
For each instrument, the option can be exercised \\
on any tree date between or including the pair \\
of dates on that row. If only one non-NaN date \\
is listed, or if ExerciseDates is NINST-by-1, the \\
option can be exercised between the valuation \\
date of the stock tree and the single listed \\
exercise date.
\end{tabular} \\
CAmericanOpt & \begin{tabular}{l} 
(Optional) If CAmericanOpt \(=0\), NaN, or is
\end{tabular} \\
unspecified, the option is a European option. If \\
CAmericanOpt = 1, the option is an American \\
option.
\end{tabular}

\section*{Description}
[Price, PriceTree] = compoundbyitt(ITTTree, UOptSpec, UStrike, USettle, UExerciseDates, UAmericanOpt, COptSpec, CStrike, CSettle, CExerciseDates, CAmericanOpt) calculates the value of a compound option by an ITT trinomial tree.

Price is a NINST-by- 1 vector of expected prices at time 0 .
PriceTree is a tree structure with a vector of instrument prices at each node.

Note The Settle date is set to the ValuationDate of the stock tree.
```

Examples Price a Compound Option Using an ITT Tree
This example shows how to price a compound option using a ITT tree by loading the file deriv.mat, which provides ITTTree. The ITTTree structure contains the stock specification and time information needed to price the option.

```
```

load deriv.mat

```
load deriv.mat
UOptSpec = 'Call';
UOptSpec = 'Call';
UStrike = 99;
UStrike = 99;
USettle = '01-Jan-2006';
USettle = '01-Jan-2006';
UExerciseDates = '01-Jan-2010';
UExerciseDates = '01-Jan-2010';
UAmericanOpt = 1;
UAmericanOpt = 1;
COptSpec = 'Put';
COptSpec = 'Put';
CStrike = 5;
CStrike = 5;
CSettle = '01-Jan-2006';
CSettle = '01-Jan-2006';
CExerciseDates = '01-Jan-2010';
CExerciseDates = '01-Jan-2010';
Price = compoundbyitt(ITTTree, UOptSpec, UStrike, USettle, ...
Price = compoundbyitt(ITTTree, UOptSpec, UStrike, USettle, ...
UExerciseDates, UAmericanOpt, COptSpec, CStrike, CSettle, ...
UExerciseDates, UAmericanOpt, COptSpec, CStrike, CSettle, ...
CExerciseDates)
CExerciseDates)
Price =
    2.7271
    References Rubinstein, Mark, "Double Trouble," Risk 5, 1991.
    See Also instcompound | itttree
```


## Purpose

Instrument prices from Cox-Ross-Rubinstein tree
Syntax [Price, PriceTree] = crrprice(CRRTree, InstSet, Options)
Arguments

| CRRTree | Stock price tree structure created by crrtree. |
| :--- | :--- |
| InstSet | Variable containing a collection of NINST instruments. <br> Instruments are categorized by type; each type can <br> have different data fields. The stored data field <br> is a row vector or string for each instrument. For <br> more information about how to create the InstSet <br> structure, see instadd. |
| Options | (Optional) Derivatives pricing options structure <br> created with derivset. |

## Description

[Price, PriceTree] = crrprice(CRRTree, InstSet, Options) computes stock option prices using a CRR binomial tree created with crrtree.

Price is a number of instruments (NINST)-by-1 vector of prices for each instrument. The prices are computed by backward dynamic programming on the stock tree. If an instrument cannot be priced, NaN is returned.

PriceTree is a MATLAB structure of trees containing vectors of instrument prices and a vector of observation times for each node.

PriceTree.PTree contains the prices.
PriceTree.tObs contains the observation times.
PriceTree.dObs contains the observation dates.
crrprice handles instrument types: 'Asian', 'Barrier', 'Compound', 'Lookback', 'OptStock'. See instadd to construct defined types.

Related single-type pricing functions are:

## crrprice

- asianbycrr: Price an Asian option from a CRR tree.
- barrierbycrr: Price a barrier option from a CRR tree.
- compoundbycrr: Price a compound option from a CRR tree.
- lookbackbycrr: Price a lookback option from a CRR tree.
- optstockbycrr: Price an American, Bermuda, or European option from a CRR tree.


## Examples

Load the CRR tree and instruments from the data file deriv.mat. Price the barrier and lookback options contained in the instrument set.

```
load deriv.mat;
CRRSubSet = instselect(CRRInstSet,'Type', ...
{'Barrier', 'Lookback'});
instdisp(CRRSubSet)
%Table of instrument portfolio partially displayed:
Index Type OptSpec Strike Settle ExerciseDates AmericanOpt BarrierSpec ...
1 Barrier call 105 01-Jan-2003 01-Jan-2006 1 ui ...
Index Type OptSpec Strike Settle ExerciseDates AmericanOpt Name Quantity
L Lookback call 115 01-Jan-2003 01-Jan-2006 0 Lookback1 7
3 Lookback call 115 01-Jan-2003 01-Jan-2007 0 Lookback2 9
[Price, PriceTree] = crrprice(CRRTree, CRRSubSet)
Price =
    12.1272
    7.6015
    11.7772
PriceTree =
```

```
FinObj: 'BinPriceTree'
    PTree: {1x5 cell}
    tObs: [[0 1 1 2 3 4}
    dObs: [731582 731947 732313 732678 733043]
```

You can use treeviewer to see the prices of these three instruments along the price tree.
treeviewer(PriceTree, CRRSubSet)


Barrierl


Lookback 1


Lookback2

## crrprice

See Also crrsens | crrtree | instadd

## Purpose

Instrument prices and sensitivities from Cox-Ross-Rubinstein tree

Syntax<br>[Delta, Gamma, Vega, Price] = crrsens(CRRTree, InstSet, Options)

## Arguments

## Description

CRRTree Interest-rate tree structure created by crrtree.
InstSet Variable containing a collection of NINST instruments. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or string for each instrument.

Options (Optional) Derivatives pricing options structure created with derivset.
[Delta, Gamma, Vega, Price] = crrsens(CRRTree, InstSet, Options) computes dollar sensitivities and prices for instruments using a binomial tree created with crrtree. NINST instruments from a financial instrument variable, InstSet, are priced. crrsens handles instrument types: 'Asian', 'Barrier', 'Compound', 'Lookback', 'OptStock'. See instadd for information on instrument types.

Delta is an NINST-by- 1 vector of deltas, representing the rate of change of instrument prices with respect to changes in the stock price. Delta is computed by finite differences in calls to crrtree. See crrtree for information on the stock tree.

Gamma is an NINST-by- 1 vector of gammas, representing the rate of change of instrument deltas with respect to the changes in the stock price. Gamma is computed by finite differences in calls to crrtree.

Vega is an NINST-by-1 vector of vegas, representing the rate of change of instrument prices with respect to the changes in the volatility of the stock. Vega is computed by finite differences in calls to crrtree.

Note All sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, divide by the respective instrument price.

## Examples

## Compute Sensitivities for Barrier and Lookback Instruments Using a crrtree

Load the CRR tree and instruments from the data file deriv.mat. Compute the Delta and Gamma sensitivities of the barrier and lookback options contained in the instrument set.

```
load deriv.mat;
CRRSubSet = instselect(CRRInstSet,'Type', ...
{'Barrier', 'Lookback'});
instdisp(CRRSubSet)
\begin{tabular}{lllllll} 
Index & Type & OptSpec Strike Settle & ExerciseDates & AmericanOpt Ba \\
1 & Barrier call & 105 & \(01-J a n-2003\) & \(01-J a n-2006\) & 1
\end{tabular}
Index Type OptSpec Strike Settle ExerciseDates AmericanOpt
2 Lookback call 115 01-Jan-2003 01-Jan-2006 0
3 Lookback call 115 01-Jan-2003 01-Jan-2007 0
```

Obtain the Delta and Gamma for the barrier and lookback options contained in the instrument set.

```
[Delta, Gamma] = crrsens(CRRTree, CRRSubSet)
```

Delta =
0.6885
0.6049
0.8187

```
Gamma =
    0.0310
    -0.0000
        0
```

See Also crrprice | crrtree

## crrtimespec

| Purpose | Specify time structure for Cox-Ross-Rubinstein tree |
| :--- | :--- |
| Syntax | TimeSpec $=$ crrtimespec (ValuationDate, Maturity, NumPeriods) |

## Arguments

## Description

| ValuationDate | Scalar date indicating the pricing date and first <br> observation in the tree. A serial date number or <br> date string. |
| :--- | :--- |
| Maturity | Scalar date indicating depth of the tree. |
| NumPeriods | Scalar determining number of time steps in the <br> tree. |

## Examples

TimeSpec = crrtimespec(ValuationDate, Maturity, NumPeriods) sets the number of levels and node times for a CRR binomial tree.

TimeSpec is a structure specifying the time layout for a CRR binomial tree.

## Set the Number of Levels and Node Times for a CRR Binomial Tree

This example shows how to specify a four-period CRR tree with time steps of 1 year.

```
ValuationDate = '1-July-2002';
Maturity = '1-July-2006';
TimeSpec = crrtimespec(ValuationDate, Maturity, 4)
```

TimeSpec =
FinObj: 'BinTimeSpec'
ValuationDate: 731398
Maturity: 732859

```
    NumPeriods: 4
        Basis: 0
EndMonthRule: 1
        tObs: [0 1 1 2 3 4]
        dObs: [731398 731763 732128 732493 732859]
```

See Also crrtree | stockspec

```
Purpose Construct Cox-Ross-Rubinstein stock tree
Syntax CRRTree \(=\) crrtree(StockSpec, RateSpec, TimeSpec)
```


## Arguments

## Description

CRRTree = crrtree(StockSpec, RateSpec, TimeSpec) creates a structure specifying the time layout for a CRR binomial tree.

Examples Using the data provided, create a stock specification (StockSpec), rate specification (RateSpec), and tree time layout specification (TimeSpec). Then use these specifications to create a CRR tree with crrtree.

```
Sigma = 0.20;
AssetPrice = 50;
DividendType = 'cash';
DividendAmounts = [0.50; 0.50; 0.50; 0.50];
ExDividendDates = {'03-Jan-2003'; '01-Apr-2003'; '05-July-2003';
```

```
'01-Oct-2003'}
StockSpec = stockspec(Sigma, AssetPrice, DividendType, ...
DividendAmounts, ExDividendDates)
StockSpec =
                    FinObj: 'StockSpec'
                        Sigma: 0.2000
            AssetPrice: 50
        DividendType: 'cash'
    DividendAmounts: [4x1 double]
    ExDividendDates: [4x1 double]
RateSpec = intenvset('Rates', 0.05, 'StartDates',...
'01-Jan-2003', 'EndDates', '31-Dec-2003')
RateSpec =
                    FinObj: 'RateSpec'
        Compounding: 2
            Disc: 0.9519
            Rates: 0.0500
            EndTimes: 1.9945
        StartTimes: 0
            EndDates: 731946
            StartDates: 731582
        ValuationDate: 731582
        Basis: 0
    EndMonthRule: 1
ValuationDate = '1-Jan-2003';
Maturity = '31-Dec-2003';
TimeSpec = crrtimespec(ValuationDate, Maturity, 4)
TimeSpec =
```

```
        FinObj: 'BinTimeSpec'
    ValuationDate: 731582
        Maturity: 731946
        NumPeriods: 4
            Basis: 0
        EndMonthRule: 1
        tObs: [0 0.2493 0.4986 0.7479 0.9972]
        dObs: [731582 731673 731764 731855 731946]
CRRTree = crrtree(StockSpec, RateSpec, TimeSpec)
CRRTree =
    FinObj: 'BinStockTree'
    Method: 'CRR'
StockSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
            tObs: [0 0.2493 0.4986 0.7479 0.9972]
            dObs: [731582 731672 731763 731856 731946]
            STree: {1x5 cell}
    UpProbs: [0.5370 0.5370 0.5370 0.5370]
```

Use treeviewer to observe the tree you have created.


See Also
crrtimespec | intenvset | stockspec

Purpose Convert inverse-discount tree to interest-rate tree
Syntax RateTree = cvtree(Tree)
Arguments
Tree Heath-Jarrow-Morton, Black-Derman-Toy, Hull-White, or Black-Karasinski tree structure using inverse-discount notation for forward rates.

Description RateTree $=$ cvtree (Tree) converts a tree structure using inverse-discount notation to a tree structure using rate notation for forward rates.

Examples Convert a Hull-White tree using inverse-discount notation to a Hull-White tree displaying interest-rate notation.

```
load deriv.mat;
HWTree
HWTree =
    FinObj: 'HWFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tObs: [00 1 2 3
        dObs: [l731947 732313 732678 733043]
    CFlowT: {[4x1 double] [3\times1 double] [2x1 double] [4]}
        Probs: {[3x1 double] [3x3 double] [3x5 double]}
    Connect: {[2] [2 3 4] [2 2 3 4 4]}
    FwdTree: {1x4 cell}
```

HWTree. FwdTree\{1\}

```
HWTree.FwdTree{2}
ans =
    1.0528 1.0356 1.0186
```

Use treeviewer to display the path of interest rates expressed in inverse-discount notation.
treeviewer(HWTree)


Use cvtree to convert the inverse-discount notation to interest-rate notation.

RTree $=$ cvtree (HWTree)
RTree =

```
    FinObj: 'HwRateTree'
    VolSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
    tObs: [0 1 1 2 3]
    dObs: [731947 732313 732678 733043]
    CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}
Connect: {[2] [2 3 4] [2 2 3 4 4]}
RateTree: {1x4 cell}
RTree.RateTree{1}
ans =
    0.0275
RTree.RateTree{2}
ans =
    0.0514 0.0349 0.0185
```

Now use treeviewer to display the converted tree, showing the path of interest rates expressed as forward rates.


## See Also <br> disc2rate | rate2disc

## date2time

Purpose Time and frequency from dates

```
Syntax [Times, F] = date2time(Settle, Dates, Compounding, Basis,
EndMonthRule)
```


## Arguments

Settle Settlement date. A vector of serial date numbers or date strings.

Dates Vector of dates corresponding to the compounding value.

Compounding (Optional) Scalar value representing the rate at which the input zero rates were compounded when annualized. This argument determines the formula for the discount factors:

Compounding $=1,2,3,4,6,12($ Default $=2$.
Disc $=(1+Z / F)^{\wedge}(-T)$, where $F$ is the compounding frequency, $Z$ is the zero rate, and T is the time in periodic units; for example, $\mathrm{T}=$ $F$ is 1 year.

Compounding $=365$
Disc $=(1+Z / F)^{\wedge}(-T)$, where $F$ is the number of days in the basis year and $T$ is a number of days elapsed computed by basis.

Compounding $=-1$
Disc $=\exp (-T * Z)$, where $T$ is time in years.

Basis
(Optional) Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual $/ 365$
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual $/ 365$ (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.

EndMonthRule (Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. $1=$ set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.

## date2time

## Description

[Times, F] = date2time(Settle, Dates, Compounding, Basis, EndMonthRule) computes time factors appropriate to compounded rate quotes beyond the settlement date.

Times is a vector of time factors.
$F$ is a scalar of related compounding frequencies.

Note To obtain accurate results from this function, the Basis and Dates arguments must be consistent. If the Dates argument contains months that have 31 days, Basis must be one of the values that allow months to contain more than 30 days; for example, Basis $=0,3$, or 7 .
date2time is the inverse of time2date.

## See Also

cftimes | disc2rate | rate2disc | time2date

| Purpose | Display date entries |
| :--- | :--- |
| Syntax | datedisp (NumMat, DateForm) <br> CharMat $=$ datedisp(NumMat, DateForm) |

Arguments

Description

Examples $\quad$ NumMat $=[$ 730730, 0.03, 1200, 730100; 730731, 0.05, 1000, NaN]

NumMat $=$
$1.0 \mathrm{e}+05$ *

| 7.3073 | 0.0000 | 0.0120 | 7.3010 |
| ---: | ---: | ---: | ---: |
| 7.3073 | 0.0000 | 0.0100 | NaN |

datedisp(NumMat)
01-Sep-2000 0.03 1200 11-Dec-1998

Tips

See Also

This function is identical to the datedisp function in Financial Toolbox software.

datenum | datestr

| Purpose | Get derivatives pricing options |
| :--- | :--- |
| Syntax | Value $=\operatorname{derivget}($ Options, 'Parameter') |
| Arguments |  |

Options Existing options specification structure, probably created from previous call to derivset.
Parameter Must be 'Diagnostics', 'Warnings', 'ConstRate', or 'BarrierMethod'. It is sufficient to type only the leading characters that uniquely identify the parameter. Case is ignored for parameter names.

## Description

Value = derivget(Options, 'Parameter') extracts the value of the named parameter from the derivative options structure Options. Parameter values can be 'off' or 'on', except for 'BarrierMethod', which can be 'unenhanced' or 'interp'. Specifying 'unenhanced' uses no correction calculation. Specifying 'interp' uses an enhanced valuation interpolating between nodes on barrier boundaries.

## Examples

Example 1. Create an Options structure with the value of Diagnostics set to 'on'.

```
Options = derivset('Diagnostics','on')
```

Use derivget to extract the value of Diagnostics from the Options structure.

Value = derivget(Options, 'Diagnostics')
Value =
on
Example 2. Use derivget to extract the value of ConstRate.

## derivget

```
Value = derivget(Options, 'ConstRate')
Value =
```

on

Because the value of 'ConstRate' was not previously set with derivset, the answer represents the default setting for 'ConstRate'.

Example 3. Find the value of 'BarrierMethod' in this structure.
derivget(Options ,'BarrierMethod')
ans =
unenhanced

See Also barrierbycrr | barrierbyeqp | derivset

## Purpose

Set or modify derivatives pricing options
Syntax

```
Options = derivset(Options, 'Parameter1', Value1,
    ... 'Parameter4', Value4)
Options = derivset(OldOptions, NewOptions)
Options = derivset
derivset
```


## Arguments

Options (Optional) Existing options specification structure, probably created from a previous call to derivset.

Parametern The parameter must be 'Diagnostics', 'Warnings', 'ConstRate', or 'BarrierMethod'. Parameters can be entered in any order.
Valuen (BDT, BK, HJM, or HW pricing only) The parameter values for the following three options can be 'on' or 'off':

- 'Diagnostics' 'on' generates diagnostic information. The default is 'Diagnostics' 'off'.
- 'Warnings' 'on' (default) displays a warning message when executing a pricing function.
- 'ConstRate' 'on' (default) assumes a constant rate between tree nodes.

For pricing barrier options, the 'BarrierMethod' pricing option can be 'unenhanced' (default) or 'interp'. Specifying 'unenhanced' uses no correction calculation. Specifying 'interp' uses an enhanced valuation interpolating between nodes on barrier boundaries.

OldOptions Existing options specification structure.
NewOptions New options specification structure.

## Description

## Examples

Options = derivset(Options, 'Parameter1', Value1, ... 'Parameter4', Value4) creates a derivatives pricing options structure Options in which the named parameters have the specified values. Any unspecified value is set to the default value for that parameter when Options is passed to the pricing function. It is sufficient to type only the leading characters that uniquely identify the parameter name. Case is also ignored for parameter names.

If the optional input argument Options is specified, derivset modifies an existing pricing options structure by changing the named parameters to the specified values.

Note For parameter values, correct case and the complete string are required; if an invalid string is provided, the default is used.

Options = derivset(OldOptions, NewOptions) combines an existing options structure OldOptions with a new options structure NewOptions. Any parameters in NewOptions with nonempty values overwrite the corresponding old parameters in OldOptions.

Options = derivset creates an options structure Options whose fields are set to the default values.
derivset with no input or output arguments displays all parameter names and information about their possible values.

Options = derivset('Diagnostics','on')
enables the display of additional diagnostic information that appears when executing pricing functions.

```
Options = derivset(Options, 'ConstRate', 'off')
```

changes the ConstRate parameter in the existing Options structure so that the assumption of constant rates between tree nodes no longer applies.

With no input or output arguments derivset displays all parameter names and information about their possible values.
derivset

| Diagnostics: | on \| \{off \} |  |
| :---: | :---: | :---: |
| Warnings: | \{on\} \| off |  |
| ConstRate: | \{on\} \| off |  |
| BarrierMethod: | \{unenhanced\} | interp |

See Also
barrierbycrr | barrierbyeqp | derivget

Purpose Interest rates from cash flow discounting factors

## Syntax <br> Usage 1: Interval points are input as times in periodic units.

```
Rates = disc2rate(Compounding, Disc, EndTimes)
Rates = disc2rate(Compounding, Disc, EndTimes, StartTimes)
```

Usage 2: ValuationDate is passed and interval points are input as dates.
[Rates, EndTimes, StartTimes] = disc2rate(Compounding, Disc, EndDates, StartDates, ValuationDate)
[Rates, EndTimes, StartTimes] = disc2rate(Compounding, Disc, EndDates, StartDates, ValuationDate, Basis, EndMonthRule)

## Arguments

| Compounding | Scalar value representing the rate at which <br> the input zero rates were compounded when <br> annualized. This argument determines the <br> formula for the discount factors: <br> Compounding $=1,2,3,4,6,12$ |
| :--- | :--- |
| Disc $=(1+Z / F)^{\wedge}(-T)$, where $F$ is the |  |
| compounding frequency, $Z$ is the zero rate, and |  |
|  | T is the time in periodic units; for example, $T=$ |
|  | Fis 1 year. |
|  | Compounding $=365$ |$\quad$| Disc $=(1+Z / F)^{\wedge}(-T)$, where $F$ is the number |
| :--- |
| of days in the basis year and T is a number of |
| days elapsed computed by basis. |

unit bond prices over investment intervals from StartTimes, when the cash flow is valued, to EndTimes, when the cash flow is received.

EndTimes

StartTimes

EndDates

StartDates (Optional) NPOINTS-by-1 vector or scalar of serial dates starting the interval to discount over. Default = ValuationDate. StartDates must be earlier than EndDates.
ValuationDate Scalar value in serial date number form representing the observation date of the investment horizons entered in StartDates and EndDates. Required in Usage 2. Omitted or passed as an empty matrix to invoke Usage 1.

Basis
(Optional) Day-count basis of the instrument when using Usage 2. A vector of integers.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
EndMonthRule (Optional) End-of-month rule when using Usage 2. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. $0=$ ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. $1=$ set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.

## Description

Usage 1: Rates = disc2rate(Compounding, Disc, EndTimes) or Rates = disc2rate(Compounding, Disc, EndTimes, StartTimes) where interval points are input as times in periodic units.

Usage 2: [Rates, EndTimes, StartTimes] = disc2rate(Compounding, Disc, EndDates, StartDates, ValuationDate) or [Rates, EndTimes, StartTimes] = disc2rate(Compounding, Disc, EndDates, StartDates, ValuationDate, Basis, EndMonthRule) where ValuationDate is passed and interval points are input as dates.
disc2rate computes the yields over a series of NPOINTS time intervals given the cash flow discounts over those intervals. NCURVES different rate curves can be translated at once if they have the same time structure. The time intervals can represent a zero or a forward curve.

Rates is an NPOINTS-by-NCURVES column vector of yields in decimal form over the NPOINTS time intervals.

Specify the investment intervals with either input times (Usage 1) or input dates (Usage 2). Entering ValuationDate invokes the date interpretation; omitting ValuationDate invokes the default time interpretations.

## For Usage 1:

- StartTimes is an NPOINTS-by-1 column vector of times starting the interval to discount over, measured in periodic units.
- EndTimes is an NPOINTS-by-1 column vector of times ending the interval to discount over, measured in periodic units.


## For Usage 2:

- StartDates is an NPOINTS-by-1 column vector of serial dates starting the interval to discount over, measured in days.
- EndDates is an NPOINTS-by- 1 column vector of serial date ending the interval to discount over, measured in days.

If Compounding = 365 (daily), StartTimes and EndTimes are measured in days for Usage 2. Otherwise, for Usage 1, the arguments contain
values, T, computed from SIA semiannual time factors, Tsemi, by the formula $T=T s e m i / 2 * F$, where $F$ is the compounding frequency.

## See Also <br> rate2disc | ratetimes

## Purpose

Instrument prices from Equal Probabilities binomial tree

## Syntax

[Price, PriceTree] = eqpprice(EQPTree, InstSet, Options)

## Arguments

## Description

[Price, PriceTree] = eqpprice(EQPTree, InstSet, Options) computes stock option prices using an EQP binomial tree created with eqptree.

Price is a number of instruments (NINST)-by-1 vector of prices for each instrument. The prices are computed by backward dynamic programming on the stock tree. If an instrument cannot be priced, NaN is returned.

PriceTree is a MATLAB structure of trees containing vectors of instrument prices and a vector of observation times for each node.

PriceTree.PTree contains the prices.
PriceTree.tObs contains the observation times.
PriceTree.dObs contains the observation dates.
eqpprice handles instrument types: 'Asian', 'Barrier', 'Compound', 'Lookback', 'OptStock'. See instadd to construct defined types.

Related single-type pricing functions are:

- asianbyeqp: Price an Asian option from an EQP tree.
- barrierbyeqp: Price a barrier option from an EQP tree.
- compoundbyeqp: Price a compound option from an EQP tree.
- lookbackbyeqp: Price a lookback option from an EQP tree.
- optstockbyeqp: Price an American, Bermuda, or European option from an EQP tree.

Examples
Load the EQP tree and instruments from the data file deriv.mat. Price the put options contained in the instrument set.

```
load deriv.mat;
EQPSubSet = instselect(EQPInstSet, 'FieldName', 'OptSpec', ...
'Data', 'put')
instdisp(EQPSubSet)
%Table of instrument portfolio partially displayed:
Index Type OptSpec Strike Settle ExerciseDates AmericanOpt Name...
1 OptStock put 105 01-Jan-2003 01-Jan-2006 0 Put 105...
Index Type OptSpec Strike Settle ExerciseDates AmericanOpt AvgType...
2 Asian put 110 01-Jan-2003 01-Jan-2006 0 arithmetic...
3 Asian put 110 01-Jan-2003 01-Jan-2007 0 arithmetic...
[Price, PriceTree] = eqpprice(EQPTree, EQPSubSet)
Price =
    2.6375
    4.7444
    3.9178
PriceTree =
    FinObj: 'BinPriceTree'
```

```
PTree: {1x5 cell}
    tObs: [00 1 2 3 4]
    dObs: [731582 731947 732313 732678 733043]
```

You can use treeviewer to see the prices of these three instruments along the price tree.

```
treeviewer(PriceTree, EQPSubSet)
```



Putl


Asianll


Asian2
See Also
eqpsens | eqptimespec | eqptree

Purpose

Syntax

Arguments

| EQPTree | Interest-rate tree structure created by eqptree. |
| :--- | :--- |
| InstSet | Variable containing a collection of NINST instruments. <br> Instruments are categorized by type; each type can <br> have different data fields. The stored data field is a <br> row vector or string for each instrument. |
| Options | (Optional) Derivatives pricing options structure <br> created with derivset. |

## Description

Instrument prices and sensitivities from Equal Probabilities binomial tree
[Delta, Gamma, Vega, Price] = eqpsens(EQPTree, InstSet, Options)

Options (Optional) Derivatives pricing options structure created with derivset.
[Delta, Gamma, Vega, Price] = eqpsens(EQPTree, InstSet, Options) computes dollar sensitivities and prices for instruments using a binomial tree created with eqptree. NINST instruments from a financial instrument variable, InstSet, are priced. eqpsens handles instrument types: 'Asian', 'Barrier', 'Compound', 'Lookback', and 'OptStock'. See instadd for information on instrument types.
Delta is an NINST-by- 1 vector of deltas, representing the rate of change of instrument prices with respect to changes in the stock price. Delta is computed by finite differences in calls to eqptree. See eqptree for information on the stock tree.

Gamma is an NINST-by- 1 vector of gammas, representing the rate of change of instrument deltas with respect to the changes in the stock price. Gamma is computed by finite differences in calls to eqptree.

Vega is an NINST-by- 1 vector of vegas, representing the rate of change of instrument prices with respect to the changes in the volatility of the stock. Vega is computed by finite differences in calls to eqptree.

> Note All sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, divide by the respective instrument price.

## Examples

## Compute Sensitivities for Instruments Using an eqptree

Load the EQP tree and instruments from the data file deriv.mat.
Compute the Delta and Gamma sensitivities of the put options contained in the instrument set.

```
load deriv.mat;
EQPSubSet = instselect(EQPInstSet, 'FieldName', 'OptSpec', ...
'Data', 'put')
instdisp(EQPSubSet)
```

EQPSubSet =
FinObj:
IndexTable: $[1 \times 1$ struct $]$
Type: $\{5 \times 1$ cell $\}$
FieldName: $\{5 \times 1$ cell $\}$
FieldClass: $\{5 \times 1$ cell $\}$
FieldData: $\{5 \times 1$ cell $\}$

| Index | Type | OptSpec | Strike Settle | ExerciseDates | AmericanO |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | OptStock put | 105 | $01-$ Jan-2003 | $01-$ Jan-2006 | 0 |


| Index | Type 0 OtSpec | Strike Settle | ExerciseDates | AmericanOpt |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | Asian put | 110 | $01-$ Jan-2003 | $01-$ Jan-2006 | 0 |
| 3 | Asian put | 110 | $01-$ Jan-2003 | $01-$ Jan-2007 | 0 |

Obtain the Delta and Gamma for the put options contained in the instrument set.
[Delta, Gamma] = eqpsens(EQPTree, EQPSubSet)

Delta =
-0.2336
-0.5443
-0.4516

Gamma =
0.0218
0.0000
0.0000

## See Also

eqpprice | eqptree

## Purpose

Specify time structure for Equal Probabilities binomial tree

## Syntax

TimeSpec = eqptimespec(ValuationDate, Maturity, NumPeriods)

## Arguments

## Description

TimeSpec = eqptimespec(ValuationDate, Maturity, NumPeriods) sets the number of levels and node times for an equal probabilities tree.

TimeSpec is a structure specifying the time layout for an equal probabilities tree.

## Examples

## Set the Number of Levels and Node Times for an EQP Tree

This example shows how to set the number of levels and node times for an EQP tree by specifying a four-period tree with time steps of 1 year.

```
ValuationDate = '1-July-2002';
Maturity = '1-July-2006';
TimeSpec = eqptimespec(ValuationDate, Maturity, 4)
```

TimeSpec =
FinObj: 'BinTimeSpec'
ValuationDate: 731398
Maturity: 732859
NumPeriods: 4

## eqptimespec

```
Basis: 0
EndMonthRule: 1
tObs: [0 1234 4]
dObs: [731398 731763732128732493732859\(]\)
```

See Also eqptree \| stockspec

# Purpose Construct Equal Probabilities stock tree 

```
Syntax EQPTree = eqptree(StockSpec, RateSpec, TimeSpec)
```


## Arguments

## Description

## Examples

Using the data provided, create a stock specification (StockSpec), rate specification (RateSpec), and tree time layout specification (TimeSpec). Then use these specifications to create an EQP tree with eqptree.

```
Sigma = 0.20;
AssetPrice = 50;
```

```
DividendType = 'cash';
DividendAmounts = [0.50; 0.50; 0.50; 0.50];
ExDividendDates = {'03-Jan-2003'; '01-Apr-2003'; '05-July-2003';
'01-Oct-2003'}
StockSpec = stockspec(Sigma, AssetPrice, DividendType, ...
DividendAmounts, ExDividendDates)
StockSpec =
            FinObj: 'StockSpec'
                    Sigma: 0.2000
            AssetPrice: 50
        DividendType: 'cash'
    DividendAmounts: [4x1 double]
    ExDividendDates: [4x1 double]
RateSpec = intenvset('Rates', 0.05, 'StartDates',...
'01-Jan-2003', 'EndDates', '31-Dec-2003')
RateSpec =
                    FinObj: 'RateSpec'
        Compounding: 2
            Disc: 0.9519
            Rates: 0.0500
            EndTimes: 1.9945
        StartTimes: 0
            EndDates: 731946
            StartDates: 731582
    ValuationDate: 731582
            Basis: 0
        EndMonthRule: 1
ValuationDate = '1-Jan-2003';
Maturity = '31-Dec-2003';
TimeSpec = eqptimespec(ValuationDate, Maturity, 4)
```

```
TimeSpec =
    FinObj: 'BinTimeSpec'
    ValuationDate: 731582
        Maturity: 731946
        NumPeriods: 4
        Basis: O
        EndMonthRule: 1
        tObs: [0 0.2493 0.4986 0.7479 0.9972]
        dObs: [731582 731673 731764 731855 731946]
EQPTree = eqptree(StockSpec, RateSpec, TimeSpec)
EQPTree =
        FinObj: 'BinStockTree
        Method: 'EQP
    StockSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
            tObs: [0 0.2493 0.4986 0.7479 0.9972]
            dObs: [731582 731672 731763 731856 731946]
            STree: {1x5 cell}
        UpProbs: [0.5000 0.5000 0.5000 0.5000]
```

Use treeviewer to observe the tree you have created.


See Also
eqptimespec | intenvset | stockspec

## Purpose

Compute exposure profiles from credit exposures

## Syntax

Description

Input
Arguments

```
profilestructs = exposureprofiles(dates,exposures)
profilestructs = exposureprofiles(__ ,Name,Value)
```

profilestructs = exposureprofiles(dates, exposures) computes common counterparty credit exposures profiles from an array of exposures.
profilestructs = exposureprofiles( __ , Name, Value) computes common counterparty credit exposures profiles from an array of exposures using optional name-value pair arguments.

## dates - Simulation dates

vector of date numbers | cell array of date strings

Simulation dates, specified as vector of date numbers or a cell array of string dates in a known format. For more information for known date formats, see the function datenum.

```
Data Types
char | cell
```


## exposures - 3-D array of potential losses due to counterparty default <br> ```array```

3-D array of potential losses due to counterparty default on a set of instruments simulated over a series of simulation dates and across many scenarios, specified as a NumDates-by-NumCounterParties-by-NumScenarios "cube" of credit exposures. Each row represents a different simulation date, each column a different counterparty, and each "page" is a different scenario from a Monte-Carlo simulation.

```
Data Types
double
```


## exposureprofiles

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

Example: profilestructs = exposureprofiles(dates, exposures,'ProfileSpec ', 'PFE', 'PFEProabilityLevel'

## 'ProfileSpec' - Exposure profiles

All (generate all profiles) (default) | string with possible values EE, PFE, MPE, EffEE, EPE, EffePE, All \| cell array with possible values EE, PFE, MPE, EffEE, EPE, EffEPE

Exposure profiles, specified as a string or cell array with the following possible valuess:

- EE - Expected Exposure. The mean of the distribution of exposures at each date. A [NumDates-by-1] vector.
- PFE - Potential Future Exposure. A high percentile (default 95\%) of the distribution of possible exposures at each date. This is sometimes referred to as "Peak Exposure." A [NumDates-by-1] vector.
- MPFE - Maximum Potential Future Exposure. The maximum potential future exposure (PFE) over all dates
- EffEE - Effective Expected Exposure. The maximum expected exposure (at a specific date) that occurs at that date or any prior date. This is the expected exposure, but constrained to be nondecreasing over time. A [NumDates-by-1] vector.
- EPE - Expected Positive Exposure. The weighted average over time of expected exposures. A scalar.
- Effepe - Effective Expected Positive Exposure. The weighted average over time of the effective expected exposure (EffEE). A scalar.
- All - Generate all of the previous profiles.

Note Exposure profiles are computed on a per-counterparty basis.

## Data Types

char | cell

## 'PFEProbabilityLevel' - Level for potential future exposure (PFE) and maximum potential future exposure (MPFE)

.95 (the 95th percentile) (default) | scalar with value [0..1]

Level for potential future exposure (PFE) and maximum potential future exposure (MPFE), specified as a scalar with value [0..1].

## Data Types <br> double

## Output Arguments

## profilestructs - Structure of credit exposure profiles

array of structs holding credit exposure profiles for each counterparty
Structure of credit exposure profiles, returned as an array of structs holding credit exposure profiles for each counterparty, returned as a struct, with the fields of the struct as the (abbreviated) names of every exposure profiles. Profiles listed in the ProfileSpec (and their related profiles) are populated, while those not requested contain empty ([]). profilestructs contains the dates information as a vector of MATLAB date numbers requested in the ProfileSpec argument.

## Examples

## View Exposure Profiles of a Particular Counterparty

After computing the mark-to-market contract values for a portfolio of swaps over many scenarios, view the exposure profiles of a particular counterparty.

First, load data containing the mark-to-market contract values for a portfolio of swaps over many scenarios.

```
load ccr.mat
```


## exposureprofiles

Compute the exposure by counterparty.

```
[exposures, expcpty] = creditexposures(values,swaps.Counterparty,...
'NettingID',swaps.NettingID);
```

Compute the credit exposure profiles for all counterparties.

```
cpProfiles = exposureprofiles(simulationDates,exposures)
```

cpProfiles =
5x1 struct array with fields:

## Dates

EE
PFE
MPFE
Effee
EPE
Effepe

Visualize the exposure profiles for a particular counterparty.

```
cpIdx = find(expcpty == 4);
numDates = numel(simulationDates);
plot(simulationDates,cpProfiles(cpIdx).PFE,...
    simulationDates,cpProfiles(cpIdx).MPFE * ones(numDates,1),...
    simulationDates,cpProfiles(cpIdx).EE,...
    simulationDates,cpProfiles(cpIdx).EPE * ones(numDates,1),...
    simulationDates,cpProfiles(cpIdx).EffEE,...
    simulationDates,cpProfiles(cpIdx).EffEPE * ones(numDates,1));
legend({'PFE (95%)','Max PFE','Exp Exposure (EE)',...
    'Time-Avg EE (EPE)','Max past EE (EffEE)',...
    'Time-Avg EffEE (EffEPE)'})
datetick('x','mmmyy','keeplimits')
```



## exposureprofiles

## References

Basel II: International Convergence of Capital Measurement and Capital Standards: A Revised Framework - Comprehensive Version at http://www.bis.org/publ/bcbs128.htm, 2006.

See Also creditexposures | datenum

Purpose Price fixed-rate note from Black-Derman-Toy interest-rate tree

```
Syntax
[Price, PriceTree] = fixedbybdt(BDTTree, CouponRate, Settle,
Maturity)
[Price, PriceTree] = fixedbybdt(BDTTree, CouponRate, Settle,
Maturity, Reset, Basis, Principal, Options, EndMonthRule)
[Price, PriceTree] = fixedbybdt(BDTTree, CouponRate, Settle,
Maturity, Name,Value)
```

Input Arguments

| BDTTree | Interest-rate tree structure created by bdttree. |
| :--- | :--- |
| CouponRate | Decimal annual rate. |
| Settle | Settlement dates. Number of instruments <br> (NINST)-by-1 vector of dates representing the <br> settlement dates of the fixed-rate note. |
| Maturity | NINST-by-1 vector of dates representing the maturity <br> dates of the fixed-rate note. |

## Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

## Reset

NINST-by-1 vector representing the frequency of payments per year.
Default: 1

## Basis

Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual


## fixedbybdt

- $1=30 / 360$ (SIA)
- $2=$ actual $/ 360$
- 3 = actual $/ 365$
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- $9=$ actual/360 (ISMA)
- $10=\mathrm{actual} / 365$ (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

## Principal

NINST-by-1 of notional principal amounts or NINST-by-1 cell array where each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Default: 100

## Options

Derivatives pricing options structure created with derivset.

## EndMonthRule

End-of-month rule. A NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Default: 1

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

## AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. NINST-by-1 of logicals.

Default: False

## BusinessDayConvention

Require payment dates to be business dates. NINST-by- 1 cell array with possible choices of business day convention:

- actual
- follow
- modifiedfollow
- previous
- modifiedprevious


## fixedbybdt

## Default: actual

## Holidays

Holidays used for business day convention. NHOLIDAYS-by-1 of MATLAB date numbers.

Default: If no dates are specified, holidays.m is used.

## Description

[Price, PriceTree] = fixedbybdt(BDTTree, CouponRate, Settle, Maturity) computes the price of a fixed-rate note from a BDT interest-rate tree.

```
[Price, PriceTree] = fixedbybdt(BDTTree, CouponRate,
Settle,
Maturity, Reset, Basis, Principal, Options, EndMonthRule)
computes the price of a fixed-rate note from a BDT interest-rate tree using additional input arguments.
```

[Price, PriceTree] = fixedbybdt(BDTTree, CouponRate, Settle, Maturity, Name, Value) computes the price of a price of a fixed-rate note from a BDT interest-rate tree with additional options specified by one or more Name, Value pair arguments.

Price is an NINST-by-1 vector of expected prices of the fixed-rate note at time 0 .

PriceTree is a structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node.

PriceTree. PTree contains the clean prices.
PriceTree.AITree contains the accrued interest.
PriceTree.tObs contains the observation times.
The Settle date for every fixed-rate note is set to the ValuationDate of the BDT tree. The fixed-rate note argument Settle is ignored.

## Examples

## Price a 10\% Fixed-Rate Note Using a BDT Interest-Rate Tree

This example shows how to price a $10 \%$ fixed-rate note using a BDT interest-rate tree by loading the file deriv.mat, which provides BDTTree. The BDTTree structure contains the time and interest-rate information needed to price the note.
load deriv.mat
CouponRate = 0.10;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2004';
Reset = 1;
Price = fixedbybdt(BDTTree, CouponRate, Settle, Maturity, Reset)

Price =
92.9974

See Also $\begin{aligned} & \text { bdttree | bondbybdt | capbybdt | cfbybdt | floatbybdt | } \\ & \text { floorbybdt | swapbybdt }\end{aligned}$

## fixedbybk

Purpose Price fixed-rate note from Black-Karasinski interest-rate tree

```
Syntax
[Price, PriceTree] = fixedbybk(BKTree, CouponRate, Settle,
Maturity)
[Price, PriceTree] = fixedbybk(BKTree, CouponRate, Settle,
Maturity, Reset, Basis, Principal, Options, EndMonthRule)
[Price, PriceTree] = fixedbybk(BKTree, CouponRate, Settle,
Maturity, Name,Value)
```


## Input Arguments

| BKTree | Interest-rate tree structure created by bktree. |
| :--- | :--- |
| CouponRate | Decimal annual rate. |
| Settle | Settlement dates. NINST-by-1 vector of dates <br> representing the settlement dates of the fixed-rate <br> note. |
| Maturity | NINST-by-1 vector of dates representing the maturity <br> dates of the fixed-rate note. |

## Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

## Reset

NINST-by-1 vector representing the frequency of payments per year.

## Default: 1

## Basis

Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual $/ 360$
- $3=$ actual $/ 365$
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- $9=$ actual/360 (ISMA)
- $10=\mathrm{actual} / 365$ (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

## Principal

NINST-by-1 of notional principal amounts or NINST-by-1 cell array where each element is a NumDates-by- 2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Default: 100

## Options

Derivatives pricing options structure created with derivset.

## EndMonthRule

## fixedbybk

End-of-month rule. A NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Default: 1

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

## AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. NINST-by-1 of logicals.

Default: False

## BusinessDayConvention

Require payment dates to be business dates. NINST-by-1 cell array with possible choices of business day convention:

- actual
- follow
- modifiedfollow
- previous
- modifiedprevious

Default: actual

## Holidays

Holidays used for business day convention. NHOLIDAYS-by-1 of MATLAB date numbers.

Default: If no dates are specified, holidays.m is used.

## Description

[Price, PriceTree] = fixedbybk(BKTree, CouponRate, Settle, Maturity) computes the price of a fixed-rate note from a Black-Karasinski tree.
[Price, PriceTree] = fixedbybk(BKTree, CouponRate, Settle, Maturity, Reset, Basis, Principal, Options, EndMonthRule) computes the price of a fixed-rate note from a Black-Karasinski tree using optional input arguments.
[Price, PriceTree] = fixedbybk(BKTree, CouponRate, Settle, Maturity, Name, Value) computes the price of a price of a fixed-rate note from a Black-Karasinski interest-rate tree with additional options specified by one or more Name, Value pair arguments.
Price is an NINST-by-1 vector of expected prices of the fixed-rate note at time 0.

PriceTree is a structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node.

PriceTree. PTree contains the clean prices.
PriceTree.AITree contains the accrued interest.
PriceTree.tObs contains the observation times.
The Settle date for every fixed-rate note is set to the ValuationDate of the BK tree. The fixed-rate note argument Settle is ignored.

## fixedbybk

## Examples Price a 5\% Fixed-Rate Note Using a Black-Karasinski Interest-Rate Tree

Load the file deriv.mat, which provides BKTree. The BKTree structure contains the time and interest-rate information needed to price the note.
load deriv.mat;
Set the required values. Other arguments will use defaults.
CouponRate = 0.05;
Settle = '01-Jan-2005';
Maturity = '01-Jan-2006';
Use fixedbybk to compute the price of the note.
Price = fixedbybk(BKTree, CouponRate, Settle, Maturity)
Warning: Fixed rate notes are valued at Tree ValuationDate rather than Se
Price =
103.5126

See Also bktree | bondbybk | capbybk | cfbybk | floatbybk | floorbybk | swapbybk

Purpose Price fixed-rate note from Heath-Jarrow-Morton interest-rate tree

Syntax<br>Input Arguments

```
[Price, PriceTree] = fixedbyhjm(HJMTree, CouponRate, Settle, Maturity)
[Price, PriceTree] = fixedbyhjm(HJMTree, CouponRate, Settle, Maturity, Reset, Basis, Principal, Options, EndMonthRule) [Price, PriceTree] = fixedbyhjm(HJMTree, CouponRate, Settle, Maturity, Name, Value)
[Price, PriceTree] = fixedbyhjm(HJMTree, CouponRate, Settle,
Maturity)
[Price, PriceTree] = fixedbyhjm(HJMTree, CouponRate, Settle,
Maturity, Reset, Basis, Principal, Options, EndMonthRule)
[Price, PriceTree] = fixedbyhjm(HJMTree, CouponRate, Settle,
Maturity, Name,Value)
```

| HJMTree | Forward-rate tree structure created by hjmtree. |
| :--- | :--- |
| CouponRate | Decimal annual rate. <br> Settle |
| Settlement dates. Number of instruments <br> (NINST)-by-1 vector of dates representing the <br> settlement dates of the fixed-rate note. |  |
| Maturity | NINST-by-1 vector of dates representing the maturity <br> dates of the fixed-rate note. |

## Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

## Reset

NINST-by-1 vector representing the frequency of payments per year.

## Default: 1

## Basis

Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual


## fixedbyhim

- $1=30 / 360$ (SIA)
- $2=$ actual $/ 360$
- $3=$ actual $/ 365$
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- $9=$ actual/360 (ISMA)
- $10=\mathrm{actual} / 365$ (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

## Principal

NINST-by-1 of notional principal amounts or NINST-by-1 cell array where each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Default: 100

## Options

Derivatives pricing options structure created with derivset.

## EndMonthRule

End-of-month rule. A NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Default: 1

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

## AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. NINST-by-1 of logicals.

> Default: False

## BusinessDayConvention

Require payment dates to be business dates. NINST-by-1 cell array with possible choices of business day convention:

- actual
- follow
- modifiedfollow
- previous
- modifiedprevious


## fixedbyhim

## Default: actual

## Holidays

Holidays used for business day convention. NHOLIDAYS-by-1 of MATLAB date numbers.

Default: If no dates are specified, holidays.m is used.

## Description

[Price, PriceTree] = fixedbyhjm(HJMTree, CouponRate, Settle, Maturity) computes the price of a fixed-rate note from a HJM forward-rate tree.

```
[Price, PriceTree] = fixedbyhjm(HJMTree, CouponRate,
Settle,
Maturity, Reset, Basis, Principal, Options, EndMonthRule)
computes the price of a fixed-rate note from a HJM forward-rate tree
using optional input asrguments.
```

[Price, PriceTree] = fixedbyhjm(HJMTree, CouponRate,
Settle,
Maturity, Name, Value) computes the price of a price of a fixed-rate
note from a HJM forward-rate tree with additional options specified by
one or more Name, Value pair arguments.

Price is an NINST-by-1 vector of expected prices of the fixed-rate note at time 0.

PriceTree is a structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node.

PriceTree. PBush contains the clean prices.
PriceTree.AIBush contains the accrued interest.
PriceTree.tObs contains the observation times.
The Settle date for every fixed-rate note is set to the ValuationDate of the HJM tree. The fixed-rate note argument Settle is ignored.

## Examples

## Price a 4\% Fixed-Rate Note Using an HJM Forward-Rate Tree

This example shows how to price a $4 \%$ fixed-rate note using an HJM forward-rate tree by loading the file deriv.mat, which provides HJMTree. The HJMTree structure contains the time and forward-rate information needed to price the note.
load deriv.mat
CouponRate = 0.04;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2003';
Price = fixedbyhjm(HJMTree, CouponRate, Settle, Maturity)

Price =
98.7159

See Also bondbyhjm | capbyhjm | cfbyhjm | floatbyhjm | floorbyhjm | hjmtree | swapbyhjm

## fixedbyhw

Purpose
Price fixed-rate note from Hull-White interest-rate tree

```
Syntax
[Price, PriceTree] = fixedbyhw(HWTree, CouponRate, Settle,
Maturity)
[Price, PriceTree] = fixedbyhw(HWTree, CouponRate, Settle,
Maturity, Reset, Basis, Principal, Options, EndMonthRule)
[Price, PriceTree] = fixedbyhw(HWTree, CouponRate, Settle,
Maturity, Name,Value)
```


## Arguments

| HWTree | Interest-rate tree structure created by hwtree. |
| :--- | :--- |
| CouponRate | Decimal annual rate. <br> Settle |
| Settlement dates. NINST-by-1 vector of dates <br> representing the settlement dates of the fixed-rate <br> note. |  |
| Maturity | NINST-by-1 vector of dates representing the maturity <br> dates of the fixed-rate note. |

## Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

## Reset

NINST-by-1 vector representing the frequency of payments per year.

## Default: 1

## Basis

Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual $/ 360$
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- $9=$ actual/360 (ISMA)
- $10=\mathrm{actual} / 365$ (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

## Principal

NINST-by-1 of notional principal amounts or NINST-by-1 cell array where each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Default: 100

## Options

Derivatives pricing options structure created with derivset.

## EndMonthRule

## fixedbyhw

End-of-month rule. A NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Default: 1

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

## AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. NINST-by-1 of logicals.

Default: False

## BusinessDayConvention

Require payment dates to be business dates. NINST-by-1 cell array with possible choices of business day convention:

- actual
- follow
- modifiedfollow
- previous
- modifiedprevious


## fixedbyhw

## Default: actual

## Holidays

Holidays used for business day convention. NHOLIDAYS-by-1 of MATLAB date numbers.

Default: If no dates are specified, holidays.m is used.

## Description

[Price, PriceTree] = fixedbyhw(HWTree, CouponRate, Settle, Maturity) computes the price of a fixed-rate note from a Hull-White tree.
[Price, PriceTree] = fixedbyhw(HWTree, CouponRate, Settle, Maturity, Reset, Basis, Principal, Options, EndMonthRule) computes the price of a fixed-rate note from a Hull-White tree using optional input arguments.
[Price, PriceTree] = fixedbyhw(HWTree, CouponRate, Settle, Maturity, Name, Value) computes the price of a price of a fixed-rate note from a Hull-White tree with additional options specified by one or more Name, Value pair arguments.

Price is an NINST-by-1 vector of expected prices of the fixed-rate note at time 0.

PriceTree is a structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node.

PriceTree. PTree contains the clean prices.
PriceTree.AITree contains the accrued interest.
PriceTree.tObs contains the observation times.
The Settle date for every fixed-rate note is set to the ValuationDate of the HW tree. The fixed-rate note argument Settle is ignored.

## fixedbyhw

## Examples Price a 5\% Fixed-Rate Note Using a Hull-White Interest-Rate Tree

Load the file deriv.mat, which provides HWTree. The HWTree structure contains the time and interest-rate information needed to price the note.
load deriv.mat;
Set the required values. Other arguments will use defaults.
CouponRate = 0.05;
Settle = '01-Jan-2005';
Maturity = '01-Jan-2006';
Use fixedbyhw to compute the price of the note.
Price = fixedbyhw(HWTree, CouponRate, Settle, Maturity)
Warning: Fixed rate notes are valued at Tree ValuationDate rather than Se
Price =
103.5126

See Also bondbyhw | capbyhw | cfbyhw | floatbyhw | floorbyhw | hwtree | swapbyhw

## fixedbyzero

| Purpose | Price fixed-rate note from set of zero curves |
| :---: | :---: |
| Syntax | [Price, DirtyPrice, CFlowAmounts, CFlowDates] = fixedbyzero(RateSpec, CouponRate, Settle, Maturity) [Price, DirtyPrice, CFlowAmounts, CFlowDates] = fixedbyzero(RateSpec, CouponRate, Settle, Maturity, Reset, Basis, Principal, EndMonthRule) <br> [Price, DirtyPrice, CFlowAmounts, CFlowDates] = fixedbyzero(RateSpec, CouponRate, Settle, Maturity, Name, Value) |
| Description | [Price, DirtyPrice, CFlowAmounts, CFlowDates] = fixedbyzero(RateSpec, CouponRate, Settle, Maturity) computes the price of a fixed-rate note from a set of zero curves. All inputs are either scalars or NINST-by-1 vectors unless otherwise specified. Any date may be a serial date number or date string. An optional argument may be passed as an empty matrix []. |
|  | [Price, DirtyPrice, CFlowAmounts, CFlowDates] = fixedbyzero(RateSpec, CouponRate, Settle, Maturity, Reset, Basis, Principal, EndMonthRule) computes the price of a fixed-rate note from a set of zero curves using optional input arguments All inputs are either scalars or NINST-by-1vectors unless otherwise specified. Any date may be a serial date number or date string. An optional argument may be passed as an empty matrix []. |
|  | [Price, DirtyPrice, CFlowAmounts, CFlowDates] = fixedbyzero(RateSpec, CouponRate, Settle, Maturity, Name, Value) computes the price of a fixed-rate note from a set of zero curves with additional options specified by one or more Name, Value pair arguments. |
| Input Arguments | RateSpec |
|  | Structure containing the properties of an interest-rate structure. See intenvset for information on creating RateSpec. |
|  | CouponRate |

## fixedbyzero

Decimal annual rate.

## Settle

Settlement date. Settle must be either a scalar or NINST-by- 1 vector of serial date numbers or date strings of the same value which represent the settlement date for each bond. Settle must be earlier than Maturity.

## Maturity

Maturity date.

## Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

## Reset

NINST-by-1 vector representing the frequency of payments per year.
Default: 1

## Basis

Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)


## fixedbyzero

- $8=$ actual/actual (ISMA)
- $9=$ actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

## Principal

NINST-by-1 of notional principal amounts or NINST-by-1 cell array where each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Default: 100

## EndMonthRule

End-of-month rule. A NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Default: 1

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can

## fixedbyzero

specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, Valuen.

## AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. NINST-by-1 of logicals.

Default: False

## BusinessDayConvention

Require payment dates to be business dates. NINST-by-1 cell array with possible choices of business day convention:

- actual
- follow
- modifiedfollow
- previous
- modifiedprevious

Default: actual

## Holidays

Holidays used for business day convention. NHOLIDAYS-by-1 of MATLAB date numbers.

Default: If no dates are specified, holidays.m is used.

## Output <br> Arguments

## Price

A number of instruments (NINST) by number of curves (NUMCURVES) matrix of fixed-rate note prices. Each column arises from one of the zero curves.

## DirtyPrice

A NINST-by-NUMCURVES matrix of dirty bond price (clean + accrued interest). Each column arises from one of the zero curves.

## CFIowAmounts

A NINST-by-NUMCFS matrix of cash flows for each bond.

## CFlowDates

A NINST-by-NUMCFS matrix of payment dates for each bond.

## Examples

## Price a 4\% Fixed-Rate Note Using a Set of Zero Curves

This example shows how to price a $4 \%$ fixed-rate note using a set of zero curves by loading the file deriv.mat, which provides ZeroRateSpec, the interest-rate term structure needed to price the note.

```
load deriv.mat
CouponRate = 0.04;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2003';
Price = fixedbyzero(ZeroRateSpec, CouponRate, Settle, Maturity)
```

Price =
98.7159

See Also | bondbyzero | cfbyzero | floatbyzero | swapbyzero

## floatbybdt

Purpose Price floating-rate note from Black-Derman-Toy interest-rate tree
Syntax [Price, PriceTree] = floatbybdt(BDTTree, Spread, Settle, Maturity)

[Price, PriceTree] = floatbybdt(BDTTree, Spread,

Settle, Maturity, Reset, Basis, Principal, Options,

EndMonthRule)

[Price, PriceTree] = floatbybdt(BDTTree, Spread, Settle,

Maturity, Name, Value)

## Input

Arguments

| BDTTree | Interest-rate tree structure created by bdttree. |
| :--- | :--- |
| Spread | Number of instruments (NINST)-by-1 vector of number <br> of basis points over the reference rate. |
| Settle | Settlement dates. NINST-by-1 vector of dates <br> representing the settlement dates of the floating-rate <br> note. |

Note The Settle date for every floating-rate note is set to the ValuationDate of the BDT Tree. The floating-rate note argument Settle is ignored.
Maturity NINST-by-1 vector of dates representing the maturity dates of the floating-rate note.

## Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

## Reset

NINST-by-1 vector representing the frequency of payments per year.

Note Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for a FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates can not be uniquely determined because there will be more than one possible path for connecting the two payment dates.

## Default: 1

## Basis

Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- $9=$ actual/360 (ISMA)
- $10=\mathrm{actual} / 365$ (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)


## floatbybdt

- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

## Principal

NINST-by-1 vector of notional principal amounts or NINST-by-1 cell array. For the latter case, each element of the cell array is a NumDates-by- 2 matrix where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Default: 100

## Options

Derivatives pricing options structure created with derivset.

## EndMonthRule

End-of-month rule. NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Default: 1

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

## AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. NINST-by-1 of logicals.

Default: false

## BusinessDayConvention

Require payment dates to be business dates. NINST-by-1 cell array with possible choices of business day convention:

- actual
- follow
- modifiedfollow
- previous
- modifiedprevious

Default: actual

## Holidays

Holidays used for business day convention. NHOLIDAYS-by-1 of MATLAB date numbers.

Default: If no dates are specified, holidays.m is used.

## CapRate

NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by- 2 cell array, and the cell array first column is dates, and the second column is associated cap rates. The date indicates the last day that the cap rate is valid.

## FloorRate

## floatbybdt

## Description

## Examples

NINST-by-1 decimal annual rate or NINST-by- 1 cell array, where each element is a NumDates-by- 2 cell array, and the cell array first column is dates, and the second column is associated floor rates. The date indicates the last day that the floor rate is valid.
[Price, PriceTree] = floatbybdt(BDTTree, Spread, Settle, Maturity) computes the price of a floating-rate note from a BDT tree.
[Price, PriceTree] = floatbybdt(BDTTree, Spread, Settle, Maturity, Reset, Basis, Principal, Options, EndMonthRule) computes the price of a floating-rate note with optional inputs from a BDT tree.
[Price, PriceTree] = floatbybdt(BDTTree, Spread, Settle, Maturity, Name, Value) computes the price of a floating-rate note from a BDT tree with additional options specified by one or more Name, Value pair arguments..

Price is an NINST-by-1 vector of expected prices of the floating-rate note at time 0 .

PriceTree is a structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node.

PriceTree. PTree contains the clean prices.
PriceTree.AITree contains the accrued interest.
PriceTree.tObs contains the observation times.
The Settle date for every floating-rate note is set to the ValuationDate of the BDT tree. The floating-rate note argument Settle is ignored.

## Price a Floating-Rate Note Using a BDT Tree

Price a 20-basis point floating-rate note using a BDT interest-rate tree.
Load the file deriv.mat, which provides BDTTree. The BDTTree structure contains the time and interest-rate information needed to price the note.

## floatbybdt

## load deriv.mat;

Define the floating-rate note using the required arguments. Other arguments use defaults.

```
Spread = 20;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2003';
```

Use floatbybdt to compute the price of the note.

```
Price = floatbybdt(BDTTree, Spread, Settle, Maturity)
```

Price =
100.4865

## Price an Amortizing Floating-Rate Note

Price an amortizing floating-rate note using the Principal input argument to define the amortization schedule.

Create the RateSpec.

```
Rates = [0.03583; 0.042147; 0.047345; 0.052707; 0.054302];
ValuationDate = '15-Nov-2011';
StartDates = ValuationDate;
EndDates = {'15-Nov-2012';'15-Nov-2013';'15-Nov-2014' ;'15-Nov-2015';
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', Star
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```

RateSpec =
FinObj: 'RateSpec'

## floatbybdt

```
    Compounding: 1
            Disc: [5x1 double]
            Rates: [5x1 double]
        EndTimes: [5\times1 double]
        StartTimes: [5x1 double]
        EndDates: [5\times1 double]
        StartDates: 734822
ValuationDate: 734822
        Basis: 0
EndMonthRule: 1
```

Create the floating-rate instrument using the following data:

```
Settle ='15-Nov-2011';
Maturity = '15-Nov-2015';
Spread = 15;
```

Define the floating-rate note amortizing schedule.

```
Principal ={{'15-Nov-2012' 100;'15-Nov-2013' 70;'15-Nov-2014' 40;'15-Nov-
```

Build the BDT tree and assume volatility is $10 \%$.

```
MatDates = {'15-Nov-2012'; '15-Nov-2013';'15-Nov-2014';'15-Nov-2015';'15-
BDTTimeSpec = bdttimespec(ValuationDate, MatDates);
Volatility = 0.10;
BDTVolSpec = bdtvolspec(ValuationDate, MatDates, Volatility*ones(1,lengtr
BDTT = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec);
```

Compute the price of the amortizing floating-rate note.
Price = floatbybdt(BDTT, Spread, Settle, Maturity, 'Principal', Principal
Price =
100.3059

## Price a Collar with a Floating-Rate Note

Price a collar with a floating-rate note using the CapRate and FloorRate input argument to define the collar pricing.

Create the RateSpec.

```
Rates = [0.0287; 0.03024; 0.03345; 0.03861; 0.04033];
ValuationDate = '1-April-2012';
StartDates = ValuationDate;
EndDates = {'1-April-2013';'1-April-2014';'1-April-2015' ;...
'1-April-2016';'1-April-2017'};
Compounding = 1;
```

Create the RateSpec.

```
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', Star
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
```

Build the BDT tree and assume volatility is $5 \%$.

```
MatDates = {'1-April-2013'; '1-April-2014';'1-April-2015';'1-April-20
BDTTimeSpec = bdttimespec(ValuationDate, MatDates);
Volatility = 0.05;
BDTVolSpec = bdtvolspec(ValuationDate, MatDates, Volatility*ones(1,ler
BDTT = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec);
```

Create the floating rate note instrument.

```
Settle ='1-April-2012';
Maturity = '1-April-2016';
Spread = 10;
Principal = 100;
```

Compute the price of a collared floating-rate note.

## floatbybdt

```
CapStrike = {{'1-April-2013' 0.03; '1-April-2015' 0.055}};
FloorStrike = {{'1-April-2013' 0.025; '1-April-2015' 0.04}};
Price = floatbybdt(BDTT, Spread, Settle, Maturity, 'CapRate',...
CapStrike, 'FloorRate', FloorStrike)
```


## Price =

101.2414

See Also bdttree | bondbybdt | capbybdt | cfbybdt | fixedbybdt | floorbybdt | swapbybdt

## Purpose

Price floating-rate note from Black-Karasinski interest-rate tree
Syntax

## Input

Arguments

```
[Price, PriceTree] = floatbybk(BKTree, Spread,
```

[Price, PriceTree] = floatbybk(BKTree, Spread,
Settle, Maturity)
Settle, Maturity)
[Price, PriceTree] = floatbybk(BKTree, Spread,
[Price, PriceTree] = floatbybk(BKTree, Spread,
Settle, Maturity, Reset, Basis, Principal, Options,
Settle, Maturity, Reset, Basis, Principal, Options,
EndMonthRule)
EndMonthRule)
[Price, PriceTree] = floatbybk(BKTree, Spread, Settle,
[Price, PriceTree] = floatbybk(BKTree, Spread, Settle,
Maturity,Name, Value)

```
Maturity,Name, Value)
```

| BKTree | Interest-rate tree structure created by bktree. |
| :--- | :--- |
| Spread | Number of instruments (NINST)-by-1 vector of number <br> of basis points over the reference rate. |
| Settle | Settlement dates. NINST-by-1 vector of dates <br> representing the settlement dates of the floating-rate <br> note. |

Note The Settle date for every floating-rate note is set to the ValuationDate of the BK tree. The floating-rate note argument Settle is ignored.

Maturity NINST-by-1 vector of dates representing the maturity dates of the floating-rate note.

## Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

## Reset

## floatbybk

NINST-by-1 vector representing the frequency of payments per year.

Note Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for a FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates can not be uniquely determined because there will be more than one possible path for connecting the two payment dates.

## Default: 1

## Basis

Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- $9=$ actual/360 (ISMA)
- $10=\mathrm{actual} / 365$ (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

## Principal

NINST-by-1 vector of notional principal amounts or NINST-by-1 cell array. For the latter case, each element of the cell array is a NumDates-by-2 matrix where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Default: 100

## Options

Derivatives pricing options structure created with derivset.

## EndMonthRule

End-of-month rule. NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- $1=$ Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Default: 1

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

## floatbybk

## AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. NINST-by-1 of logicals.

Default: false

## BusinessDayConvention

Require payment dates to be business dates. NINST-by-1 cell array with possible choices of business day convention:

- actual
- follow
- modifiedfollow
- previous
- modifiedprevious

Default: actual

## Holidays

Holidays used for business day convention. NHOLIDAYS-by-1 of MATLAB date numbers.

Default: If no dates are specified, holidays.m is used.

## CapRate

NINST-by-1 decimal annual rate or NINST-by- 1 cell array, where each element is a NumDates-by- 2 cell array, and the cell array first column is dates, and the second column is associated cap rates. The date indicates the last day that the cap rate is valid.

## FloorRate

NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by- 2 cell array, and the cell array first column is dates, and the second column is associated floor rates. The date indicates the last day that the floor rate is valid.

## Description

## Examples

[Price, PriceTree] = floatbybk(BKTree, Spread, Settle, Maturity) computes the price of a floating-rate note from a Black-Karasinski tree.
[Price, PriceTree] = floatbybk(BKTree, Spread, Settle, Maturity, Reset, Basis, Principal, Options, EndMonthRule) computes the price of a floating-rate note with optional inputs from a Black-Karasinski tree.
[Price, PriceTree] = floatbybk(BKTree, Spread, Settle, Maturity, Name, Value) computes the price of a floating-rate note from a Black-Karasinski tree with additional options specified by one or more Name, Value pair arguments.

Price is an NINST-by-1 vector of expected prices of the floating-rate note at time 0 .

PriceTree is a structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node.

PriceTree. PTree contains the clean prices.
PriceTree.AITree contains the accrued interest.
PriceTree.tObs contains the observation times.
The Settle date for every floating-rate note is set to the ValuationDate of the BK tree. The floating-rate note argument Settle is ignored.

## Price a Floating-Rate Note Using a Black-Karasinski Tree

Price a 20 -basis point floating-rate note using a Black-Karasinski interest-rate tree.

Load the file deriv.mat, which provides BKTree. The BKTree structure contains the time and interest-rate information needed to price the note.

## floatbybk

```
load deriv.mat;
```

Define the floating-rate note using the required arguments. Other arguments use defaults.

Spread = 20;
Settle = '01-Jan-2005';
Maturity = '01-Jan-2006';
Use floatbybk to compute the price of the note.
Price = floatbybk(BKTree, Spread, Settle, Maturity)
Warning: Floating range notes are valued at Tree ValuationDate rather the Settle.

Price =
100.3825

## Price an Amortizing Floating-Rate Note

Price an amortizing floating-rate note using the Principal input argument to define the amortization schedule.

Create the RateSpec.

```
Rates = [0.03583; 0.042147; 0.047345; 0.052707; 0.054302];
ValuationDate = '15-Nov-2011';
StartDates = ValuationDate;
EndDates = {'15-Nov-2012';'15-Nov-2013';'15-Nov-2014' ;'15-Nov-2015';'15-
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDa
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```

```
            FinObj: 'RateSpec'
    Compounding: 1
            Disc: [5x1 double]
            Rates: [5x1 double]
        EndTimes: [5x1 double]
    StartTimes: [5x1 double]
        EndDates: [5x1 double]
    StartDates: 734822
ValuationDate: 734822
        Basis: 0
    EndMonthRule: 1
```

Create the floating-rate instrument using the following data:

```
Settle ='15-Nov-2011';
Maturity = '15-Nov-2015';
Spread = 15;
```

Define the floating-rate note amortizing schedule.

```
Principal ={{'15-Nov-2012' 100;'15-Nov-2013' 70;'15-Nov-2014' 40;'15-1
```

Build the BK tree and assume the volatility is $10 \%$.

```
VolDates = ['15-Nov-2012'; '15-Nov-2013';'15-Nov-2014';'15-Nov-2015';
VolCurve = 0.1;
AlphaDates = '15-Nov-2017';
AlphaCurve = 0.1;
BKVolSpec = bkvolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
BKTimeSpec = bktimespec(RateSpec.ValuationDate, VolDates, Compounding)
BKT = bktree(BKVolSpec, RateSpec, BKTimeSpec);
```

Compute the price of the amortizing floating-rate note.

## floatbybk

```
Price = floatbybk(BKT, Spread, Settle, Maturity, 'Principal', Principal)
Price =
    100.3059
```


## Price a Collar with a Floating-Rate Note

Price a collar with a floating-rate note using the CapRate and FloorRate input argument to define the collar pricing.

Price a portfolio of collared floating-rate notes using the following data:

```
Rates = [0.0287; 0.03024; 0.03345; 0.03861; 0.04033];
ValuationDate = '1-April-2012';
StartDates = ValuationDate;
EndDates = {'1-April-2013';'1-April-2014';'1-April-2015' ;...
'1-April-2016';'1-April-2017'};
Compounding = 1;
```

Create the RateSpec.

```
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDa
```

'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);

Build the BK tree and assume the volatility to be $5 \%$.

```
VolDates = ['1-April-2013';'1-April-2014';'1-April-2015';'1-April-2016';.
'1-April-2017';'1-April-2018'];
VolCurve = 0.05;
AlphaDates = '15-Nov-2018';
AlphaCurve = 0.1;
BKVolSpec = bkvolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
BKTimeSpec = bktimespec(RateSpec.ValuationDate, VolDates, Compounding);
```

```
BKT = bktree(BKVolSpec, RateSpec, BKTimeSpec);
```

Create the floating-rate note instrument.

```
Settle ='1-April-2012';
Maturity = '1-April-2016';
Spread = [15;10];
Principal = 100;
```

Compute the price of the two vanilla floaters.

```
Price = floatbybk(BKT, Spread, Settle, Maturity)
```

Price =
100.5519
100.3680

Compute the price of the collared floating-rate notes.

```
CapStrike = {{'1-April-2013' 0.045; '1-April-2014' 0.05;...
'1-April-2015' 0.06}; 0.06};
FloorStrike = {{'1-April-2013' 0.035; '1-April-2014' 0.04;...
'1-April-2015' 0.05}; 0.03};
PriceCollared = floatbybk(BKT, Spread, Settle, Maturity,...
'CapRate', CapStrike,'FloorRate', FloorStrike)
```


## PriceCollared =

102.8537
100.4910

## floatbybk

## Pricing a Floating-Rate Note When the Reset Dates Are Not Tree Level Dates

When using floatbybk to price floating-rate notes, there are cases where the dates specified in the BK tree Time Specs are not aligned with the cash flow dates.

Price floating-rate notes using the following data:

```
ValuationDate = '13-Sep-2013';
ForwardRatesVector = [ 0.0001; 0.0001; 0.0010; 0.0015];
EndDatesVector = ['13-Dec-2013'; '14-Mar-2014'; '13-Jun-2014'; '13-Sep-2014'];
```

Create the RateSpec.

```
RateSpec = intenvset('ValuationDate',ValuationDate,'StartDates',...
ValuationDate,'EndDates',EndDatesVector,'Rates',ForwardRatesVector,'Compounding', 1);
```

Build the BK tree.

```
Volcurve = 0.1;
Alpha = 0.01;
BKVolatilitySpec = bkvolspec(RateSpec.ValuationDate, ...
    EndDatesVector, Volcurve,...
    EndDatesVector, Alpha);
```

BKTimeSpec = bktimespec(RateSpec.ValuationDate, EndDatesVector, 1);
BKT = bktree(BKVolatilitySpec, RateSpec, BKTimeSpec);

Create the floating-rate note instrument using the following data;

```
Spread = 0;
Maturity = '13-Jun-2014';
reset = 4;
```

Compute the price of the floating-rate note.

```
Price = floatbybk(BKT, Spread, RateSpec.ValuationDate,...
```

```
Maturity, 'Reset', reset)
Warning: Not all cash flows are aligned with the tree. Result will be approximated.
> In floatengbytrintree at }21
    In floatbybk at 136
Error using floatengbytrintree (line 319)
Instrument '1 ' has cash flow dates that span across tree nodes.
Error in floatbybk (line 136)
[Price, PriceTree, CFTree] = floatengbytrintree(BKTree, Spread, Settle, Maturity, OArgs{:});
```

This error indicates that it is not possible to determine the applicable rate used to calculate the payoff at the reset dates, given that the applicable rate needed cannot be calculated (the information was lost due to the recombination of the tree nodes). Note, if the reset period for a FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there will be more than one possible path for connecting the two payment dates. The simplest solution is to place the tree levels at the cash flow dates of the instrument, which is done by specifying BKTimeSpec. It is also acceptable to have reset dates between tree levels, as long as there are reset dates on the tree levels too.

To recover from this error, build a tree that lines up with the instrument.

```
Basis = intenvget(RateSpec, 'Basis');
EOM = intenvget(RateSpec, 'EndMonthRule');
resetDates = cfdates(ValuationDate, Maturity,reset,Basis,EOM);
BKTimeSpec = bktimespec(RateSpec.ValuationDate,resetDates,reset);
BKT = bktree(BKVolatilitySpec, RateSpec, BKTimeSpec);
Price = floatbybk(BKT, Spread, RateSpec.ValuationDate, ...
    Maturity, 'Reset', reset)
Price =
    100.0004
```

See Also
bktree | bondbybk | capbybk | cfbybk | fixedbybk | floorbybk | swapbybk

## floatbyhim

## Purpose

Price floating-rate note from Heath-Jarrow-Morton interest-rate tree
Syntax

## Input

Arguments

```
[Price, PriceTree] = floatbybk(HJMTree, Spread,
```

[Price, PriceTree] = floatbybk(HJMTree, Spread,
Settle, Maturity)
Settle, Maturity)
[Price, PriceTree] = floatbybk(HJMTree, Spread,
[Price, PriceTree] = floatbybk(HJMTree, Spread,
Settle, Maturity, Reset, Basis, Principal, Options,
Settle, Maturity, Reset, Basis, Principal, Options,
EndMonthRule)
EndMonthRule)
[Price, PriceTree] = floatbybk(HJMTree, Spread, Settle,
[Price, PriceTree] = floatbybk(HJMTree, Spread, Settle,
Maturity,Name, Value)

```
Maturity,Name, Value)
```

| HJMTree | Forward-rate tree structure created by hjmtree. |
| :--- | :--- |
| Spread | Number of instruments (NINST)-by-1 vector of number <br> of basis points over the reference rate. |
| Settle | Settlement dates. NINST-by-1 vector of dates <br> representing the settlement dates of the floating-rate <br> note. |

Note The Settle date for every floating-rate note is set to the ValuationDate of the HJM tree. The floating-rate note argument Settle is ignored.

Maturity NINST-by-1 vector of dates representing the maturity dates of the floating-rate note.

## Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

## Reset

## floatbyhim

NINST-by-1 vector representing the frequency of payments per year.

## Default: 1

## Basis

Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- $9=$ actual/360 (ISMA)
- $10=\mathrm{actual} / 365$ (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

## Principal

NINST-by-1 vector of notional principal amounts or NINST-by-1 cell array. For the latter case, each element of the cell array is a NumDates-by- 2 matrix where the first column is dates and the second

## floatbyhim

column is associated principal amount. The date indicates the last day that the principal value is valid.

Default: 100

## Options

Derivatives pricing options structure created with derivset.

## EndMonthRule

End-of-month rule. NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Default: 1

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

## AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. NINST-by-1 of logicals.

Default: false

## BusinessDayConvention

Require payment dates to be business dates. NINST-by- 1 cell array with possible choices of business day convention:

## floatbyhim

- actual
- follow
- modifiedfollow
- previous
- modifiedprevious

Default: actual

## Holidays

Holidays used for business day convention. NHOLIDAYS-by-1 of MATLAB date numbers.

Default: If no dates are specified, holidays.m is used.

## CapRate

NINST-by-1 decimal annual rate or NINST-by- 1 cell array, where each element is a NumDates-by-2 cell array, and the cell array first column is dates, and the second column is associated cap rates. The date indicates the last day that the cap rate is valid.

## FloorRate

NINST-by-1 decimal annual rate or NINST-by- 1 cell array, where each element is a NumDates-by- 2 cell array, and the cell array first column is dates, and the second column is associated floor rates. The date indicates the last day that the floor rate is valid.

## Description

[Price, PriceTree] = floatbybk(HJMTree, Spread, Settle, Maturity) computes the price of a floating-rate note from an HJM tree.
[Price, PriceTree] = floatbybk(HJMTree, Spread,
Settle, Maturity, Reset, Basis, Principal, Options,

## floatbyhim

EndMonthRule) computes the price of a floating-rate note with optional inputs from an HJM tree.
[Price, PriceTree] = floatbybk(HJMTree, Spread, Settle, Maturity, Name, Value) computes the price of a floating-rate note from an HJM tree with additional options specified by one or more Name, Value pair arguments.

Price is an NINST-by-1 vector of expected prices of the floating-rate note at time 0 .

PriceTree is a structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node.

PriceTree. PBush contains the clean prices.
PriceTree.AIBush contains the accrued interest.
PriceTree.tObs contains the observation times.
The Settle date for every floating-rate note is set to the ValuationDate of the HJM tree. The floating-rate note argument Settle is ignored.

## Examples

## Price a Floating-Rate Note Using an HJM Tree

Price a 20-basis point floating-rate note using an HJM forward-rate tree.
Load the file deriv.mat, which provides HJMTree. The HJMTree structure contains the time and interest-rate information needed to price the note.
load deriv.mat;
Define the floating-rate note using the required arguments. Other arguments use defaults.

```
Spread = 20;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2003';
```

Use floatbyhjm to compute the price of the note.

## floatbyhim

```
Price = floatbyhjm(HJMTree, Spread, Settle, Maturity)
```

Price =
100.5529

## Price an Amortizing Floating-Rate Note

Price an amortizing floating-rate note using the Principal input argument to define the amortization schedule.

Create the RateSpec.

```
Rates = [0.03583; 0.042147; 0.047345; 0.052707; 0.054302];
ValuationDate = '15-Nov-2011';
StartDates = ValuationDate;
EndDates = {'15-Nov-2012';'15-Nov-2013';'15-Nov-2014' ;'15-Nov-2015';'15-
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDa
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```


## RateSpec =

```
                    FinObj: 'RateSpec'
    Compounding: 1
            Disc: [5\times1 double]
            Rates: [5x1 double]
            EndTimes: [5x1 double]
        StartTimes: [5x1 double]
            EndDates: [5x1 double]
        StartDates: 734822
    ValuationDate: 734822
            Basis: 0
EndMonthRule: 1
```


## floatbyhim

Create the floating-rate instrument using the following data:

```
Settle ='15-Nov-2011';
Maturity = '15-Nov-2015';
Spread = 15;
```

Define the floating-rate note amortizing schedule.

```
Principal ={{'15-Nov-2012' 100;'15-Nov-2013' 70;'15-Nov-2014' 40;'15-1
```

Build the HJM tree using the following data:

```
MatDates = {'15-Nov-2012'; '15-Nov-2013';'15-Nov-2014';'15-Nov-2015';
HJMTimeSpec = hjmtimespec(RateSpec.ValuationDate, MatDates);
Volatility = [.10; .08; .06; .04];
CurveTerm = [ 1; 2; 3; 4];
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec,RateSpec,HJMTimeSpec);
```

Compute the price of the amortizing floating-rate note.

```
Price = floatbyhjm(HJMT, Spread, Settle, Maturity, 'Principal', Princ:
```

Price =
100.3059

## Price a Collar with a Floating-Rate Note

Price a collar with a floating-rate note using the CapRate and FloorRate input argument to define the collar pricing.

Price a portfolio of collared floating-rate notes using the following data:

## floatbyhim

```
Rates = [0.0287; 0.03024; 0.03345; 0.03861; 0.04033];
ValuationDate = '1-April-2012';
StartDates = ValuationDate;
EndDates = {'1-April-2013';'1-April-2014';'1-April-2015' ;...
'1-April-2016';'1-April-2017'};
Compounding = 1;
```

Create the RateSpec.

```
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDa
```

'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);

Build the HJM tree with the following data:

```
MatDates = {'1-April-2013'; '1-April-2014';'1-April-2015';...
'1-April-2016';'1-April-2017';'1-April-2018'};
HJMTimeSpec = hjmtimespec(RateSpec.ValuationDate, MatDates);
Volatility = [.10; .08; .06; .04];
CurveTerm = [ 1; 2; 3; 4];
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec,RateSpec,HJMTimeSpec);
```

Create the floating-rate note instrument.

```
Settle ='1-April-2012';
Maturity = '1-April-2016';
Spread = 10;
Principal = 100;
```

Compute the price of two capped collared floating-rate notes.
CapStrike = [0.04;0.055];
PriceCapped = floatbyhjm(HJMT, Spread, Settle, Maturity,...
'CapRate', CapStrike)

## PriceCapped =

## floatbyhim

$$
98.9986
$$

100.2051

Compute the price of two collared floating-rate notes.
FloorStrike = [0.035;0.040];
PriceCollared = floatbyhjm(HJMT, Spread, Settle, Maturity,... 'CapRate', CapStrike, 'FloorRate', FloorStrike)

PriceCollared =
99.9246
102.2321

See Also
bondbyhjm | capbyhjm | cfbyhjm | fixedbyhjm | floorbyhjm | hjmtree | swapbyhjm

## floatbyhw

Purpose
Price floating-rate note from Hull-White interest-rate tree

```
Syntax
[Price, PriceTree] = floatbyhw(HWTree, Spread,
Settle, Maturity)
[Price, PriceTree] = floatbyhw(HWTree, Spread,
Settle, Maturity, Reset, Basis, Principal, Options,
EndMonthRule)
[Price, PriceTree] = floatbyhw(HWTree, Spread, Settle,
Maturity,Name,Value)
```


## Input

Arguments

| HWTree | Interest-rate tree structure created by hwtree. |
| :--- | :--- |
| Spread | Number of instruments (NINST)-by-1 vector of number <br> of basis points over the reference rate. |
| Settle | Settlement dates. NINST-by-1 vector of dates <br> representing the settlement dates of the floating-rate <br> note. |

Note The Settle date for every floating-rate note is set to the ValuationDate of the HW tree. The floating-rate note argument Settle is ignored.

Maturity NINST-by-1 vector of dates representing the maturity dates of the floating-rate note.

## Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

## Reset

## floatbyhw

NINST-by-1 vector representing the frequency of payments per year.

Note Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for a FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates can not be uniquely determined because there will be more than one possible path for connecting the two payment dates.

## Default: 1

## Basis

Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual $/ 360$
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- $9=$ actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)


## floatbyhw

- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

## Principal

NINST-by-1 vector of notional principal amounts or NINST-by-1 cell array. For the latter case, each element of the cell array is a NumDates-by- 2 matrix where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Default: 100

## Options

Derivatives pricing options structure created with derivset.

## EndMonthRule

End-of-month rule. NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Default: 1

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

## AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. NINST-by-1 of logicals.

Default: false

## BusinessDayConvention

Require payment dates to be business dates. NINST-by- 1 cell array with possible choices of business day convention:

- actual
- follow
- modifiedfollow
- previous
- modifiedprevious

Default: actual

## Holidays

Holidays used for business day convention. NHOLIDAYS-by-1 of MATLAB date numbers.

Default: If no dates are specified, holidays.m is used.

## CapRate

NINST-by-1 decimal annual rate or NINST-by- 1 cell array, where each element is a NumDates-by- 2 cell array, and the cell array first column is dates, and the second column is associated cap rates. The date indicates the last day that the cap rate is valid.

## FloorRate

## floatbyhw

## Description

## Examples

NINST-by-1 decimal annual rate or NINST-by- 1 cell array, where each element is a NumDates-by- 2 cell array, and the cell array first column is dates, and the second column is associated floor rates. The date indicates the last day that the floor rate is valid.
[Price, PriceTree] = floatbyhw(HWTree, Spread, Settle, Maturity) computes the price of a floating-rate note from a Hull-White tree.
[Price, PriceTree] = floatbyhw(HWTree, Spread, Settle, Maturity, Reset, Basis, Principal, Options, EndMonthRule) computes the price of a floating-rate note with optional inputs from a Hull-White tree.
[Price, PriceTree] = floatbyhw(HWTree, Spread, Settle, Maturity, Name, Value) computes the price of a floating-rate note from a Hull-White tree with additional options specified by one or more Name, Value pair arguments.

Price is an NINST-by-1 vector of expected prices of the floating-rate note at time 0 .

PriceTree is a structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node.

PriceTree. PTree contains the clean prices.
PriceTree.AITree contains the accrued interest.
PriceTree.tObs contains the observation times.
The Settle date for every floating-rate note is set to the ValuationDate of the HW tree. The floating-rate note argument Settle is ignored.

## Price a Floating-Rate Note Using a Hull-White Tree

Price a 20 -basis point floating-rate note using a Hull-White interest-rate tree.

Load the file deriv.mat, which provides HWTree. The HWTree structure contains the time and interest-rate information needed to price the note.

## floatbyhw

## load deriv.mat;

Define the floating-rate note using the required arguments. Other arguments use defaults.

```
Spread = 20;
Settle = '01-Jan-2005';
Maturity = '01-Jan-2006';
```

Use floatbyhw to compute the price of the note.
Price = floatbyhw(HWTree, Spread, Settle, Maturity)

Warning: Floating range notes are valued at Tree ValuationDate rather Settle.

Price =
100.3825

## Price an Amortizing Floating-Rate Note

Price an amortizing floating-rate note using the Principal input argument to define the amortization schedule.

Create the RateSpec.

```
Rates = [0.03583; 0.042147; 0.047345; 0.052707; 0.054302];
ValuationDate = '15-Nov-2011';
StartDates = ValuationDate;
EndDates = {'15-Nov-2012';'15-Nov-2013';'15-Nov-2014' ;'15-Nov-2015';
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', Star
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```

RateSpec =

## floatbyhw

```
    FinObj: 'RateSpec'
        Compounding: 1
            Disc: [5x1 double]
            Rates: [5x1 double]
            EndTimes: [5x1 double]
    StartTimes: [5x1 double]
    EndDates: [5x1 double]
    StartDates: 734822
ValuationDate: 734822
    Basis: 0
EndMonthRule: 1
```

Create the floating-rate instrument using the following data:

```
Settle ='15-Nov-2011';
Maturity = '15-Nov-2015';
Spread = 15;
```

Define the floating-rate note amortizing schedule.

```
Principal ={{'15-Nov-2012' 100;'15-Nov-2013' 70;'15-Nov-2014' 40;'15-Nov-
```

Build the HW tree and assume the volatility is $10 \%$.

```
VolDates = ['15-Nov-2012'; '15-Nov-2013';'15-Nov-2014';'15-Nov-2015';'15-
VolCurve = 0.1;
AlphaDates = '15-Nov-2017';
AlphaCurve = 0.1;
HWVolSpec = hwvolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RateSpec.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVolSpec, RateSpec, HWTimeSpec);
```

Compute the price of the amortizing floating-rate note.

## floatbyhw

```
Price = floatbyhw(HWT, Spread, Settle, Maturity, 'Principal', Princip
```

Price =
100.3059

## Price a Collar with a Floating-Rate Note

Price a collar with a floating-rate note using the CapRate and FloorRate input argument to define the collar pricing.

Price two collared floating-rate notes using the following data:

```
Rates = [0.0287; 0.03024; 0.03345; 0.03861; 0.04033];
ValuationDate = '1-April-2012';
StartDates = ValuationDate;
EndDates = {'1-April-2013';'1-April-2014';'1-April-2015' ;...
'1-April-2016';'1-April-2017'};
Compounding = 1;
```

Create the RateSpec.

```
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', Sta
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
```

Build the HW tree and assume the volatility to be $5 \%$.

```
VolDates = ['1-April-2013';'1-April-2014';'1-April-2015';...
'1-April-2016';'1-April-2017';'1-April-2018'];
VolCurve = 0.05;
AlphaDates = '15-Nov-2018';
AlphaCurve = 0.1;
HWVolSpec = hwvolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RateSpec.ValuationDate, VolDates, Compounding)
```


## floatbyhw

HWT = hwtree(HWVolSpec, RateSpec, HWTimeSpec);
Create the floating-rate note instrument.

```
Settle ='1-April-2012';
Maturity = '1-April-2016';
Spread = 10;
Principal = 100;
```

Compute the price of a vanilla floater.

```
Price = floatbyhw(HWT, Spread, Settle, Maturity)
```

Price =
100.3680

Compute the price of the collared floating-rate notes.

```
CapStrike = {{'1-April-2014' 0.045; '1-April-2015' 0.05;...
    '1-April-2016' 0.06}; 0.06};
FloorStrike = {{'1-April-2014' 0.035; '1-April-2015' 0.04;...
    '1-April-2016' 0.05}; 0.03};
PriceCollared = floatbyhw(HWT, Spread, Settle, Maturity,....
'CapRate', CapStrike,'FloorRate', FloorStrike)
PriceCollared =
```

    102.0317
    100.9189
    See Also
bondbyhw | capbyhw | cfbyhw | fixedbyhw | floorbyhw | hwtree | swapbyhw

## floatbyzero

| Purpose | Price floating-rate note from set of zero curves |
| :---: | :---: |
| Syntax | ```[Price,DirtyPrice,OutputCashFlows,CFlowDates] = floatbyzero(RateSpec,Spread,Settle,Maturity) [Price,DirtyPrice,OutputCashFlows,CFlowDates] = floatbyzero(RateSpec,Spread,Settle,Maturity, Reset,Basis,Principal,EndMonthRule) [Price,DirtyPrice,OutputCashFlows,CFlowDates] = floatbyzero(RateSpec,Spread,Settle,Maturity,Name,Value)``` |
| Description | [Price,DirtyPrice,OutputCashFlows,CFlowDates] = floatbyzero(RateSpec,Spread,Settle, Maturity) computes the price of a floating-rate note from a set of zero curves. <br> [Price,DirtyPrice,OutputCashFlows,CFlowDates] = floatbyzero(RateSpec, Spread, Settle, Maturity, Reset, Basis, Principal, EndMonthRule) computes the price of a floating-rate note from a set of zero curves using optional input arguments. <br> [Price,DirtyPrice,OutputCashFlows,CFlowDates] = floatbyzero(RateSpec,Spread,Settle, Maturity, Name, Value) computes the price of a floating-rate note from a set of zero curves with additional options specified by one or more Name, Value pair arguments. |
| Input Arguments | RateSpec <br> Structure containing the properties of an interest-rate structure. See intenvset for information on creating RateSpec. |

## Spread

Number of basis points over the reference rate.

## Settle

Settlement date. Settle must be either a scalar or NINST-by-1 vector of serial date numbers or date strings of the same value which represent
the settlement date for each bond. Settle must be earlier than Maturity.

## Maturity

Maturity date.

## Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

## Reset

NINST-by-1 vector representing the frequency of payments per year.

## Default: 1

## Basis

Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual $/ 365$
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- $9=$ actual/360 (ISMA)
- $10=$ actual/365 (ISMA)


## floatbyzero

- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

## Principal

NINST-by-1 vector of notional principal amounts or NINST-by-1 cell array. For the latter case, each element of the cell array is a NumDates-by- 2 matrix where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

## Default: 100

## EndMonthRule

End-of-month rule. NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Default: 1

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

## AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. NINST-by-1 of logicals.

Default: false

## BusinessDayConvention

Require payment dates to be business dates. NINST-by- 1 cell array with possible choices of business day convention:

- actual
- follow
- modifiedfollow
- previous
- modifiedprevious

Default: actual

## Holidays

Holidays used for business day convention. NHOLIDAYS-by-1 of MATLAB date numbers.

Default: If no dates are specified, holidays.m is used.

## LatestFloatingRate

Rate for the next floating payment set at the last reset date. NINST-by-1 of scalars.

Default: If no LatestFloatingRate is specified, the floating rate at the previous reset date is computed from the RateSpec.

## ProjectionCurve

## floatbyzero

The rate curve to be used in generating floating cash flows of the floater instruments. This structure must be created using intenvset.

Default: If no ProjectionCurve is specified, the RateSpec is used both for discounting and generating floating cash flows for the floater instrument.

## Output Arguments

## Examples

## Price

Number of instruments (NINST) by number of curves (NUMCURVES) matrix of floating-rate note prices. Each column arises from one of the zero curves.

## DirtyPrice

NINST-by-NUMCURVES matrix of dirty bond price (clean + accrued interest). Each column arises from one of the zero curves.

## OutputCashFlows

NINST-by-NUMCFS matrix of cash flows for each bond.

Note If there is more than one curve specified in the RateSpec input, then the first NCURVES rows correspond to the first bond, the second NCURVES rows correspond to the second bond, and so on.

## CFlowDates

NINST-by-NUMCFS matrix of payment dates for each bond.

## Price a Floating-Rate Note Using a Set of Zero Curves

Price a 20 -basis point floating-rate note using a set of zero curves.
Load deriv.mat, which provides ZeroRateSpec, the interest-rate term structure, needed to price the bond.

```
load deriv.mat;
```

Define the floating-rate note using the required arguments. Other arguments use defaults.

```
Spread = 20;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2003';
```

Use floatbyzero to compute the price of the note.

```
Price = floatbyzero(ZeroRateSpec, Spread, Settle, Maturity)
```

Price =
100.5529

## Price an Amortizing Floating-Rate Note

Price an amortizing floating-rate note using the Principal input argument to define the amortization schedule.

Create the RateSpec.

```
Rates = [0.03583; 0.042147; 0.047345; 0.052707; 0.054302];
ValuationDate = '15-Nov-2011';
StartDates = ValuationDate;
EndDates = {'15-Nov-2012';'15-Nov-2013';'15-Nov-2014' ;'15-Nov-2015';
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', Sta|
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```

RateSpec =
FinObj: 'RateSpec'
Compounding: 1

## floatbyzero

```
                    Disc: [5x1 double]
                    Rates: [5x1 double]
    EndTimes: [5x1 double]
    StartTimes: [5x1 double]
    EndDates: [5x1 double]
    StartDates: 734822
ValuationDate: 734822
    Basis: 0
EndMonthRule: 1
```

Create the floating-rate instrument using the following data:

```
Settle ='15-Nov-2011';
Maturity = '15-Nov-2015';
Spread = 15;
```

Define the floating-rate note amortizing schedule.

Principal $=\{\{' 15-$ Nov-2012' 100;'15-Nov-2013' 70;'15-Nov-2014' 40;'15-Nov-
Compute the price of the amortizing floating-rate note.
Price = floatbyzero(RateSpec, Spread, Settle, Maturity, 'Principal', Pri

Price =
100.3059

## Specify the Rate at the Instrument's Starting Date When it Cannot be Obtained From the RateSpec

If Settle is not on a reset date of a floating-rate note, floatbyzero attempts to obtain the latest floating rate before Settle from RateSpec or the LatestFloatingRate parameter. When the reset date for
this rate is out of the range of RateSpec (and LatestFloatingRate is not specified), floatbyzero fails to obtain the rate for that date and generates an error. This example shows how to use the LatestFloatingRate input parameter to avoid the error.

Create the error condition when a floating-rate instrument's StartDate cannot be determined from the RateSpec.

```
load deriv.mat;
```

```
Spread = 20;
Settle = '01-Jan-2000';
Maturity = '01-Dec-2003';
Price = floatbyzero(ZeroRateSpec, Spread, Settle, Maturity)
Error using floatbyzero (line 256)
The rate at the instrument starting date cannot be obtained from RateSpec.
    Its reset date (01-Dec-1999) is out of the range of dates contained in RateSpec.
This rate is required to calculate cash flows at the instrument starting date.
Consider specifying this rate with the 'LatestFloatingRate' input parameter.
```

Here, the reset date for the rate at Settle was 01-Dec-1999, which was earlier than the valuation date of ZeroRateSpec (01-Jan-2000). This error can be avoided by specifying the rate at the instrument's starting date using the LatestFloatingRate name-value pair argument.

Define LatestFloatingRate and calculate the floating-rate price.

```
Price = floatbyzero(ZeroRateSpec, Spread, Settle, Maturity, 'LatestFloatingRate', 0.03)
```

Price =
100.0285

## Price a Floating-Rate Note Using a Different Curve to Generate Floating Cash Flows

Define the OIS and Libor rates.

## floatbyzero

```
Settle = datenum('15-Mar-2013');
CurveDates = daysadd(Settle,360*[1/12 2/12 3/12 6/12 1 2 3 4 5 7 10],1);
OISRates = [.0018 .0019 .0021 .0023 .0031 .006 .011 .017 .021 . 026 .03]''
LiborRates = [.0045 .0047 . 005 . 0055 .0075 .011 .016 .022 .026 . 030 .0348
Plot the dual curves.
figure,plot(CurveDates,OISRates,'r');hold on;plot(CurveDates,LiborRates,' datetick
legend(\{'OIS Curve', 'Libor Curve'\})
```



Create an associated RateSpec for the OIS and Libor curves.
OISCurve = intenvset('Rates',OISRates,'StartDate',Settle,'EndDates',Cl LiborCurve = intenvset('Rates',LiborRates,'StartDate',Settle,'EndDates

Define the floating-rate note.

```
SwapMaturity = datenum('15-Mar-2018'); % Five year swap
FloatSpread = 0;
FixedRate = .025;
```


## floatbyzero

SwapRates = [FixedRate FloatSpread];

Compute the price for the floating-rate note. The LiborCurve term structure will be used to generate the floating cash flows of the floater instrument. The OISCurve term structure will be used for discounting the cash flows.
floatbyzero(OISCurve, 0, Settle, SwapMaturity,'ProjectionCurve',LiborCurv ans =
102.4214

Some instruments require using different interest-rate curves for generating the floating cash flows and discounting. This is when the ProjectionCurve parameter is useful. When you provide both RateSpec and ProjectionCurve, floatbyzero uses the RateSpec for the purpose of discounting and it uses the ProjectionCurve for generating the floating cash flows.

[^4]
## floorbybdt

## Purpose

Price floor instrument from Black-Derman-Toy interest-rate tree

Syntax<br>[Price, PriceTree] = floorbybdt(BDTTree, Strike, Settle, Maturity, Reset, Basis, Principal, Options)

## Arguments

BDTTree Interest-rate tree structure created by bdttree.
Strike Number of instruments (NINST)-by-1 vector of rates at which the floor is exercised.

Settle Settlement date. NINST-by-1 vector of dates representing the settlement dates of the floor. The Settle date for every floor is set to the ValuationDate of the BDT tree. The floor argument Settle is ignored.
Maturity NINST-by-1 vector of dates representing the maturity dates of the floor.

Reset (Optional) NINST-by-1 vector representing the frequency of payments per year. Default = 1 .
Basis (Optional) Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)


## floorbybdt

- $9=$ actual/360 (ICMA)
- $10=\mathrm{actual} / 365$ (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=\mathrm{actual} /$ actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Principal (Optional) The notional principal amount. Default $=100$.

Options (Optional) Derivatives pricing options structure created with derivset.

Description [Price, PriceTree] = floorbybdt(BDTTree, Strike, Settlement, Maturity, Reset, Basis, Principal, Options) computes the price of a floor instrument from a BDT interest-rate tree.
Price is an NINST-by- 1 vector of the expected prices of the floor at time 0.

PriceTree is the tree structure with values of the floor at each node.

## Examples

## Price a 10\% Floor Instrument Using a BDT Interest-Rate Tree

Load the file deriv.mat, which provides BDTTree. BDTTree contains the time and interest-rate information needed to price the floor instrument.

## load deriv.mat;

Set the required values. Other arguments will use defaults.

```
Strike = 0.10;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2004';
```


## floorbybdt

Use floorbybdt to compute the price of the floor instrument.

```
Price = floorbybdt(BDTTree, Strike, Settle, Maturity)
```


## Price =

0.2428

## Price a 10\% Floor Instrument Using a Newly Created BDT Interest-Rate Tree

First set the required arguments for the three needed specifications.

```
Compounding = 1;
ValuationDate = '01-01-2000';
StartDate = ValuationDate;
EndDates = ['01-01-2001'; '01-01-2002'; '01-01-2003';
'01-01-2004'; '01-01-2005'];
Rates = [.1; .11; .12; .125; .13];
Volatility = [.2; .19; .18; .17; .16];
```

Create the specifications.

```
RateSpec = intenvset('Compounding', Compounding,...
'ValuationDate', ValuationDate,...
'StartDates', StartDate,...
'EndDates', EndDates,...
'Rates', Rates);
BDTTimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Volatility);
```

Create the BDT tree from the specifications.

```
BDTTree = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec)
```


## floorbybdt

```
BDTTree =
    FinObj: 'BDTFwdTree'
    VolSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
        tObs: [00 1 2 2 3 4]
        dObs: [730486 730852 731217 731582 731947]
        TFwd: {[5x1 double] [4x1 double] [3x1 double] [2x1 double]
    CFlowT: {[5x1 double] [4x1 double] [3x1 double] [2x1 double]
FwdTree: {1x5 cell}
```

Set the floor arguments. Remaining arguments will use defaults.

```
FloorStrike = 0.10;
Settlement = ValuationDate;
Maturity = '01-01-2002';
FloorReset = 1;
```

Use floorbybdt to find the price of the floor instrument.
Price= floorbybdt(BDTTree, FloorStrike, Settlement, Maturity,... FloorReset)

Price =
0.0863

See Also bdttree | capbybdt | cfbybdt | swapbybdt

## floorbybk

## Purpose

Price floor instrument from Black-Karasinski interest-rate tree

Syntax<br>[Price, PriceTree] = floorbybk(BKTree, Strike, Settle, Maturity, Reset, Basis, Principal, Options)

## Arguments

BKTree Interest-rate tree structure created by bktree.

Strike Number of instruments (NINST)-by-1 vector of rates at which the floor is exercised.

Settle Settlement date. NINST-by-1 vector of dates representing the settlement dates of the floor. The Settle date for every floor is set to the ValuationDate of the BK tree. The floor argument Settle is ignored.

Maturity NINST-by-1 vector of dates representing the maturity dates of the floor.

Reset (Optional) NINST-by-1 vector representing the frequency of payments per year. Default $=1$.
Basis (Optional) Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual (default)
- 1 = 30/360 (SIA)
- 2 = actual/360
- $3=$ actual $/ 365$
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)


## floorbybk

- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Principal (Optional) The notional principal amount. Default $=100$.
Options (Optional) Derivatives pricing options structure created with derivset.

Description
[Price, PriceTree] = floorbybk(BKTree, Strike, Settlement, Maturity, Reset, Basis, Principal, Options) computes the price of a floor instrument from a Black-Karasinski tree.

Price is an NINST-by- 1 vector of the expected prices of the floor at time 0 .

PriceTree is the tree structure with values of the floor at each node.

## Examples

## Price a 3\% Floor Instrument Using a Black-Karasinski Interest-Rate Tree

Load the file deriv.mat, which provides BKTree. The BKTree structure contains the time and interest rate information needed to price the floor instrument.
load deriv.mat;
Set the required values. Other arguments will use defaults.
Strike = 0.03;
Settle = '01-Jan-2005';

## floorbybk

Maturity = '01-Jan-2009';
Use floorbybk to compute the price of the floor instrument.
Price $=$ floorbybk(BKTree, Strike, Settle, Maturity)

Warning: Floors are valued at Tree ValuationDate rather than Settle
Warning: Not all cash flows are aligned with the tree. Result will be approximated.
Warning: Not all cash flows are aligned with the tree. Result will be approximated.

Price $=$
0.2061

See Also
bktree | capbybk | cfbybk | swapbybk

## floorbyblk

Purpose
Price floors using Black option pricing model

```
Syntax [FloorPrice, Floorlets] = floorbyblk(RateSpec, Strike, Settle,
Maturity, Volatility)
[FloorPrice, Floorlets] = floorbyblk(RateSpec, Strike, Settle,
Maturity, Volatility, 'Name1', Value1...)
```


## Arguments

| RateSpec | The annualized, continuously compounded rate term <br> structure. For more information, see intenvset. |
| :--- | :--- |
| Strike | NINST-by-1 vector of rates at which the floor is <br> exercised, as a decimal number. |
| Settle | Scalar representing the settle date of the floor. |
| Maturity | Scalar representing the maturity date of the floor. <br> Volatility <br> NINST-by-1 vector of volatilities. |
| Reset | (Optional) NINST-by-1 vector representing the <br> frequency of payments per year. Default is 1. |
| Principal | (Optional) NINST-by-1 vector representing the <br> notional principal amount. Default is 100. |
|  | NINST-by-1 vector representing the basis used when |
| Basis | annualizing the input forward rate. |
|  | - $0=$ actual/actual (default) |
|  | - $1=30 / 360$ (SIA) |
|  | - $2=$ actual/360 |
|  | - $3=$ actual/365 |
|  | - $4=30 / 360$ (BMA) |
|  | - $5=30 / 360$ (ISDA) |
|  | - $6=30 / 360$ (European) |

## floorbyblk

- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
ValuationDate (Optional) Scalar representing the observation date of the investment horizons. The default is the Settle date.

ProjectionCur(Optional) The rate curve to be used in generating the future forward rates. This structure must be created using intenvset. Use this optional input if the forward curve is different from the discount curve. If ProjectionCurve is not specified, then RateSpec is used both for discounting cash flows and projecting future forward rates.

Note All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. You can specify parameter name/value pairs in any order. Names are case-insensitive and partial string matches are allowed provided no ambiguities exist.

## Description

[FloorPrice, Floorlets] = floorbyblk(RateSpec, Strike, Settle, Maturity, Volatility)

## floorbyblk

```
[FloorPrice, Floorlets] = floorbyblk(RateSpec, Strike,
Settle,
Maturity, Volatility, 'Name1', Value1...)
```

Usefloorbyblk to price floors using the Black option pricing model.
The outputs are:

- FloorPrice - NINST-by-1 expected prices of the floor.
- Floorlets - NINST-by-NCF array of floorlets, padded with NaNs.


## Examples

## Price a Floor Using the Black Option Pricing Model

This example shows how to price a floor using the Black option pricing model. Consider an investor who gets into a contract that floors the interest rate on a $\$ 100,000$ loan at $6 \%$ quarterly compounded for 3 months, starting on January 1, 2009. Assuming that on January 1, 2008 the zero rate is $6.9394 \%$ continuously compounded and the volatility is $20 \%$, use this data to compute the floor price.

```
ValuationDate = 'Jan-01-2008';
EndDates ='April-01-2010';
Rates = 0.069394;
Compounding = -1;
Basis = 1;
% calculate the RateSpec
RateSpec = intenvset('ValuationDate', ValuationDate, ...
'StartDates', ValuationDate,'EndDates', EndDates, ...
'Rates', Rates,'Compounding', Compounding,'Basis', Basis);
Settle = 'Jan-01-2009'; % floor starts in a year
Maturity = 'April-01-2009';
Volatility = 0.20;
FloorRate = 0.06;
FloorReset = 4;
Principal=100000;
```


## floorbyblk

```
FloorPrice = floorbyblk(RateSpec, FloorRate, Settle, Maturity, Volati
'Reset',FloorReset,'ValuationDate',ValuationDate,'Principal', Princip
'Basis', Basis)
```


## FloorPrice =

37.4864

## Price a Floor Using a Different Curve to Generate the Future Forward Rates

Define the OIS and Libor rates.

```
Settle = datenum('15-Mar-2013');
CurveDates = daysadd(Settle,360*[1/12 2/12 3/12 6/12 1 2 3 4 5 7 10],
OISRates = [.0018 .0019 .0021 .0023 .0031 .006 . 011 . 017 .021 . 026
LiborRates = [.0045 .0047 .005 .0055 .0075 .0109 .0162 . 0216 . 0262
```

Create an associated RateSpec for the OIS and Libor curves.
OISCurve = intenvset('Rates',OISRates,'StartDate',Settle,'EndDates', Cl LiborCurve = intenvset('Rates',LiborRates,'StartDate',Settle,'EndDates

Define the Floor instruments.

```
Maturity = {'15-Mar-2018';'15-Mar-2020'};
Strike = [.04;.05];
BlackVol = .2;
```

Price the floor instruments using the term structure OISCurve both for discounting the cash flows and generating future forward rates.
[Price, Floorlets] = floorbyblk(OISCurve, Strike, Settle, Maturity, B

## floorbyblk

Price =
9.9808
16.9057

Floorlets =

| 3.6783 | 3.0706 | 1.8275 | 0.7280 | 0.6764 | NaN | NaN |
| :--- | :--- | :--- | :--- | :--- | ---: | ---: |
| 4.6753 | 4.0587 | 2.7921 | 1.4763 | 1.3442 | 1.4130 | 1.1462 |

Price the floor instruments using the term structure LiborCurve to generate future forward rates. The term structure OISCurve is used for discounting the cash flows.
[PriceLC, FloorletsLC] = floorbyblk(OISCurve, Strike, Settle, Maturity,

```
PriceLC =
    8.0524
    14.3184
FloorletsLC =
\begin{tabular}{llllll}
3.2385 & 2.5338 & 1.2895 & 0.5889 & 0.4017 & NaN
\end{tabular}
\begin{tabular}{lllllll}
4.2355 & 3.5219 & 2.2286 & 1.2751 & 0.9169 & 1.1698 & 0.9706
\end{tabular}
```


## See Also <br> capbyblk | intenvset

## Purpose

Price floor instrument from Heath-Jarrow-Morton interest-rate tree

Syntax<br>[Price, PriceTree] = floorbyhjm(HJMTree, Strike, Settle, Maturity, Reset, Basis, Principal, Options)

## Arguments

| HJMTree | Forward-rate tree structure created by hjmtree. <br> Sumber of instruments (NINST)-by-1 vector of rates |
| :--- | :--- |
| Strike | Nat which the floor is exercised. |
| Settle | Settlement date. NINST-by-1 vector of dates <br> representing the settlement dates of the floor. <br> The Settle date for every floor is set to the <br> ValuationDate of the HJM tree. The floor argument <br> Settle is ignored. |
| Maturity | NINST-by-1 vector of dates representing the maturity <br> dates of the floor. |
| Reset | (Optional) NINST-by-1 vector representing the <br> frequency of payments per year. Default = 1. |
| Basis | (Optional) Day-count basis of the instrument. A |
|  | vector of integers. |
|  | - $0=$ actual/actual (default) |
|  | - $1=30 / 360$ (SIA) |
|  | - $2=$ actual/360 |
|  | - $3=$ actual/365 |
|  | - $4=30 / 360$ (BMA) |
|  | - $5=30 / 360$ (ISDA) |
|  | - $6=30 / 360$ (European) |
|  | - $7=$ actual/365 (Japanese) |
|  | - $8=$ actual/actual (ICMA) |

## floorbyhim

- $9=$ actual/360 (ICMA)
- $10=\mathrm{actual} / 365$ (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=\mathrm{actual} /$ actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Principal (Optional) The notional principal amount. Default $=100$.

Options (Optional) Derivatives pricing options structure created with derivset.

Description [Price, PriceTree] = floorbyhjm(HJMTree, Strike, Settlement, Maturity, Reset, Basis, Principal, Options) computes the price of a floor instrument from an HJM tree.
Price is an NINST-by-1 vector of the expected prices of the floor at time 0.

PriceTree is the tree structure with values of the floor at each node.

## Examples

## Price a 3\% Floor Instrument Using an HJM Forward-Rate Tree

This example shows how to price a $3 \%$ floor instrument using an HJM forward-rate tree by loading the file deriv.mat, which provides HJMTree. The HJMTree structure contains the time and forward-rate information needed to price the floor instrument.

```
load deriv.mat;
Strike = 0.03;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2004';
```

```
Price = floorbyhjm(HJMTree, Strike, Settle, Maturity)
Price =
    0.0486
```


## See Also capbyhjm | cfbyhjm | hjmtree \| swapbyhjm

## floorbyhw

Purpose

Arguments

```
Syntax
```

```
[Price, PriceTree] = floorbyhw(HWTree, Strike, Settle, Maturity,
```

[Price, PriceTree] = floorbyhw(HWTree, Strike, Settle, Maturity,
Reset, Basis, Principal, Options)

```
Reset, Basis, Principal, Options)
```

Price floor instrument from Hull-White interest-rate tree
HWTree Interest-rate tree structure created by hwtree.
Strike $\quad$ Number of instruments (NINST)-by-1 vector of rates at which the floor is exercised.
Settle Settlement date. NINST-by-1 vector of dates representing the settlement dates of the floor. The Settle date for every floor is set to the ValuationDate of the HW tree. The floor argument Settle is ignored.
Maturity NINST-by-1 vector of dates representing the maturity dates of the floor.
Reset (Optional) NINST-by-1 vector representing the frequency of payments per year. Default = 1 .
Basis (Optional) Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)


## floorbyhw

|  | - $9=$ actual/360 (ICMA) |
| :--- | :--- |
|  | - $10=\mathrm{actual} / 365$ (ICMA) |
|  | - $11=30 / 360 \mathrm{E}$ (ICMA) |
|  | - $12=$ actual/actual (ISDA) |
|  | - $13=\mathrm{BUS} / 252$ |
|  | For more information, see basis on page Glossary-1. |
| Principal | (Optional) The notional principal amount. Default |
| Options | $=100$. |
|  | (Optional) Derivatives pricing options structure <br> created with derivset. |

## Description [Price, PriceTree] = floorbyhw(HWTree, Strike, Settlement, Maturity, Reset, Basis, Principal, Options) computes the price of a floor instrument from an HW tree. <br> Price is an NINST-by-1 vector of the expected prices of the floor at time 0 . <br> PriceTree is the tree structure with values of the floor at each node. <br> Examples <br> Price a 3\% Floor Instrument Using a Hull-White Interest-Rate Tree

Load the file deriv.mat, which provides HWTree. The HWTree structure contains the time and interest rate information needed to price the floor instrument.
load deriv.mat;
Set the required values. Other arguments will use defaults.
Strike = 0.03;
Settle = '01-Jan-2005';

## floorbyhw

Maturity = '01-Jan-2009';

Use floorbyhw to compute the price of the floor instrument.
Price $=$ floorbyhw(HWTree, Strike, Settle, Maturity)

Warning: Floors are valued at Tree ValuationDate rather than Settle Warning: Not all cash flows are aligned with the tree. Result will be approximated.
Warning: Not all cash flows are aligned with the tree. Result will be approximated.

Price =
0.4616

See Also capbyhw | cfbyhw | hwtree | swapbyhw

Purpose
Price floor using Linear Gaussian two-factor model
Syntax FloorPrice = floorbylg2f(ZeroCurve, a, b, sigma, eta,rho,Strike, Maturity)
CapPrice = floorbylg2f( __ , Name, Value)

## Description

## Input Arguments

FloorPrice =
floorbylg2f(ZeroCurve, a, b, sigma, eta, rho, Strike, Maturity) returns the floor price for a two-factor additive Gaussian interest-rate model.

CapPrice = floorbylg2f( __ , Name, Value) returns the floor price for a two-factor additive Gaussian interest-rate model using optional name-value pairs.

## ZeroCurve - Zero curve for Linear Gaussian two-factor model structure

Zero curve for the Linear Gaussian two-factor model, specified using IRDataCurve or RateSpec.

Data Types
struct

## a-Mean reversion for first factor for Linear Gaussian two-factor model

scalar
Mean reversion for the first factor for the Linear Gaussian two-factor model, specified as a scalar.

## Data Types <br> single | double

## b-Mean reversion for second factor for Linear Gaussian two-factor model

scalar

## floorbylg2f

Mean reversion for the second factor for the Linear Gaussian two-factor model, specified as a scalar.

```
Data Types
single | double
```


## sigma - Volatility for first factor for Linear Gaussian two-factor model

scalar
Volatility for the first factor for the Linear Gaussian two-factor model, specified as a scalar.

```
Data Types
single | double
eta - Volatility for second factor for Linear Gaussian two-factor
model
```

scalar
Volatility for the second factor for the Linear Gaussian two-factor model, specified as a scalar.

Data Types
single | double

## rho - Scalar correlation of factors

scalar
Scalar correlation of the factors, specified as a scalar.

```
Data Types
single | double
```


## Strike - Floor strike price

nonnegative integer | vector of nonnegative integers
Floor strike price specified, as a nonnegative integer using a NumFloors-by- 1 vector of floor strike prices.

## floorbylg2f

```
Data Types
single | double
```


## Maturity - Floor maturity date

```
nonnegative integer | vector of nonnegative integers | string of dates
```

Floor maturity date, specified using a NumFloors-by-1 vector of serial date numbers or date strings.

Data Types
single | double | cell

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

Example: Price =
floorbylg2f(irdc, a, b, sigma, eta, rho,Strike, FloorMaturity, 'Reset', 1, 'No

## 'Reset' - Frequency of floor payments per year

2 (default) | positive integer from the set [1,2,3,4,6,12] | vector of positive integers from the set [1,2,3, 4, 6, 12]
Frequency of floor payments per year, specified as positive integers for the values [ $1,2,4,6,12$ ] in a NumFloors-by- 1 vector.

## Data Types

single | double

## 'Notional' - Notional value of floor

100 (default) | nonnegative integer | vector of nonnegative integers
Notional value of the floor, specified as a nonnegative integer using a NumFloors-by-1 vector of notional amounts.

Data Types<br>single | double

## floorbylg2f

## Output <br> Arguments

Algorithms

## Examples

## Price a Floor Using a Linear Gaussian Two-Factor Model

Define the ZeroCurve, a, b, sigma, eta, and rho parameters to compute the floor price.

```
Settle = datenum('15-Dec-2007');
ZeroTimes = [3/12 6/12 1 5 7 10 20 30]';
ZeroRates = [0.033 0.034 0.035 0.040 0.042 0.044 0.048 0.0475]';
CurveDates = daysadd(Settle,360*ZeroTimes,1);
irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);
a = .07;
b = .5;
sigma = .01;
eta = .006;
rho = -.7;
```

FloorMaturity $=$ daysadd(Settle,360*[1:5 7 10152025 30],1);

```
Strike = [llllol035 0.037 0.038 0.039 0.040 0.042 0.044 0.046 0.047 0.047
Price = floorbylg2f(irdc,a,b,sigma,eta,rho,Strike,FloorMaturity)
Price =
```

0
0.4041
0.8282
1.3103
1.8346
3.0636
4.9172
7.7614
9.7166
11.4163
12.7628

## References

[1] Brigo, D. and F. Mercurio, Interest Rate Models - Theory and Practice, Springer Finance, 2006.

See Also
capbylg2f | swaptionbylg2f | LinearGaussian2F
Related Examples

- "Price Swaptions With Interest-Rate Models Using Simulation" on page 2-105

Purpose
Determine price of gap digital options using Black-Scholes model

Arguments

## Description

## Examples

RateSpec The annualized, continuously compounded rate term structure. For information on the interest rate specification, see intenvset.
StockSpec Stock specification. See stockspec.
Settle NINST-by-1 vector of settlement or trade dates.
Maturity NINST-by-1 vector of maturity dates.
OptSpec NINST-by-1 cell array of strings 'call' or 'put'.
Strike NINST-by-1 vector of payoff strike price values.
StrikeThreshold NINST-by-1 vector of strike values that determine if the option pays off.

Price = gapbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, StrikeThreshold) computes gap option prices using the Black-Scholes option pricing model.

Price is a NINST-by-1 vector of expected option prices.

## Compute Gap Option Prices Using the Black-Scholes Option Pricing Model

This example shows how to compute gap option prices using the Black-Scholes option pricing model. Consider a gap call and put options on a nondividend paying stock with a strike of 57 and expiring on January 1, 2008. On July 1, 2008 the stock is trading at 50 . Using this data, compute the price of the option if the risk-free rate is $9 \%$, the strike threshold is 50 , and the volatility is $20 \%$.

```
Settle = 'Jan-1-2008';
Maturity = 'Jul-1-2008';
Compounding = -1;
Rates = 0.09;
% calculate the RateSpec
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,..
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Bas
% define the StockSpec
AssetPrice = 50;
Sigma = .2;
StockSpec = stockspec(Sigma, AssetPrice);
% define the call and put options
OptSpec = {'call'; 'put'};
Strike = 57;
StrikeThreshold = 50;
% calculate the price
Pgap = gapbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec,...
Strike, StrikeThreshold)
```

Pgap =
-0.0053
4.4866

## See Also

assetbybls | cashbybls | gapsensbybls | supersharebybls

## Purpose

Syntax

PriceSens = gapsensbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, StrikeThreshold) PriceSens = gapsensbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, StrikeThreshold, OutSpec)

## Arguments

> For example, OutSpec $=\{$ 'Price'; 'Lambda'; 'Rho'\} specifies that the output should be Price, Lambda, and Rho, in that order.
> To invoke from a function: [Price, Lambda, Rho] = gapsensbybls(..., 'OutSpec', $\{' P r i c e ', ~ ' L a m b d a ', ~ ' R h o '\}) ~$
> OutSpec $=\left\{\right.$ 'All' $\left.^{\prime}\right\}$ specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec as OutSpec = \{'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', 'Price'\};.

- Default is OutSpec = \{'Price'\}.


## Description

PriceSens = gapsensbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, StrikeThreshold) computes gap option prices using the Black-Scholes option pricing model.

```
PriceSens = gapsensbybls(RateSpec, StockSpec, Settle,
Maturity, OptSpec, Strike, StrikeThreshold, OutSpec) includes an OutSpec argument defined as parameter/value pairs, and computes gap option prices and sensitivities using the Black-Scholes option pricing model.
```

PriceSens is a NINST-by-1 vector of expected option prices and sensitivities.

## Examples

## Compute Gap Option Prices and Sensitivities Using the Black-Scholes Option Pricing Model

This example shows how to compute gap option prices and sensitivities using the Black-Scholes option pricing model. Consider a gap call and put options on a nondividend paying stock with a strike of 57 and expiring on January 1, 2008. On July 1, 2008 the stock is trading at 50.

Using this data, compute the price and sensitivity of the option if the risk-free rate is $9 \%$, the strike threshold is 50 , and the volatility is $20 \%$.

```
Settle = 'Jan-1-2008';
Maturity = 'Jul-1-2008';
Compounding = -1;
Rates = 0.09;
%create the RateSpec
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Basis'
% define the StockSpec
AssetPrice = 50;
Sigma = .2;
StockSpec = stockspec(Sigma, AssetPrice);
% define the call and put options
OptSpec = {'call'; 'put'};
Strike = 57;
StrikeThreshold = 50;
% compute the price
Pgap = gapbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec,...
Strike, StrikeThreshold)
% compute the gamma and delta
OutSpec = {'gamma'; 'delta'};
[Gamma ,Delta] = gapsensbybls(RateSpec, StockSpec, Settle, Maturity,...
OptSpec, Strike, StrikeThreshold, 'OutSpec', OutSpec)
```

Pgap =
-0.0053
4.4866

Gamma =
0.0724
0.0724

```
Delta =
    0.2852
    -0.7148
```

See Also
gapbybls

## hedgeopt

Purpose
Allocate optimal hedge for target costs or sensitivities

Syntax<br>[PortSens, PortCost, PortHolds] = hedgeopt(Sensitivities, Price, CurrentHolds, FixedInd, NumCosts, TargetCost, TargetSens, ConSet)

## Arguments

| Sensitivities | Number of instruments (NINST) by number of <br> sensitivities (NSENS) matrix of dollar sensitivities <br> of each instrument. Each row represents a <br> different instrument. Each column represents <br> a different sensitivity. |
| :--- | :--- |
| Price | NINST-by-1 vector of portfolio instrument unit <br> prices. |
| CurrentHolds | NINST-by-1 vector of contracts allocated to each <br> instrument. |
| FixedInd | (Optional) Number of fixed instruments <br> (NFIXED)-by-1 vector of indices of instruments <br> to hold fixed. For example, to hold the first and <br> third instruments of a 10 instrument portfolio |
| NumCosts | unchanged, set FixedInd $=$ [1 3]. Default $=[]$, <br> no instruments held fixed. <br> (Optional) Number of points generated along |
|  | the cost frontier when a vector of target costs <br> (TargetCost) is not specified. The default is |
| 10 equally spaced points between the point |  |
| of minimum cost and the point of minimum |  |
| exposure. When specifying TargetCost, enter |  |
| NumCosts as an empty matrix [ ]. |  |


| TargetCost | (Optional) Vector of target cost values along the <br> cost frontier. If TargetCost is empty, or not <br> entered, hedgeopt evaluates NumCosts equally <br> spaced target costs between the minimum cost <br> and minimum exposure. When specified, the <br> elements of TargetCost should be positive <br> numbers that represent the maximum amount of <br> money the owner is willing to spend to rebalance <br> the portfolio. |
| :--- | :--- |
| TargetSens | (Optional) 1-by-NSENS vector containing the <br> target sensitivity values of the portfolio. When |
| ConSet | specifying TargetSens, enter NumCosts and |
|  | (Optional) Number of constraints (NCONS) <br> by number of instruments (NINST) matrix <br> of additional conditions on the portfolio <br> reallocations. An eligible NINST-by-1 vector of <br> contract holdings, PortWts, satisfies all the <br> inequalities A*PortWts <= b, where |
| A = ConSet (:, 1: end-1) and b = |  |
| ConSet (: , end). |  |

## Notes

The user-specified constraints included in ConSet may be created with the functions pcalims or portcons. However, the portcons default PortHolds positivity constraints are typically inappropriate for hedging problems since short-selling is usually required.

NPOINTS, the number of rows in PortSens and PortHolds and the length of PortCost, is inferred from the inputs. When the target sensitivities, TargetSens, is entered, NPOINTS = 1; otherwise NPOINTS $=$ NumCosts, or is equal to the length of the TargetCost vector.

Not all problems are solvable (for example, the solution space may be infeasible or unbounded, or the solution may fail to converge). When a valid solution is not found, the corresponding rows of PortSens and PortHolds and the elements of PortCost are padded with NaNs as placeholders.

## Description

[PortSens, PortCost, PortHolds] = hedgeopt(Sensitivities, Price, CurrentHolds, FixedInd, NumCosts, TargetCost, TargetSens, ConSet) allocates an optimal hedge by one of two criteria:

- Minimize portfolio sensitivities (exposure) for a given set of target costs.
- Minimize the cost of hedging a portfolio given a set of target sensitivities.

Hedging involves the fundamental tradeoff between portfolio insurance and the cost of insurance coverage. This function lets investors modify portfolio allocations among instruments to achieve either of the criteria. The chosen criterion is inferred from the input argument list. The problem is cast as a constrained linear least-squares problem.

PortSens is a number of points (NPOINTS)-by-NSENS matrix of portfolio sensitivities. When a perfect hedge exists, PortSens is zeros. Otherwise, the best hedge possible is chosen.

PortCost is a 1-by-NPOINTS vector of total portfolio costs.
PortHolds is an NPOINTS-by-NINST matrix of contracts allocated to each instrument. These are the reallocated portfolios.

See Also hedgeslf | pcalims | portcons | portopt | lsqlin

## hedgeslf

Purpose
Self-financing hedge

Syntax<br>[PortSens, PortValue, PortHolds] = hedgeslf(Sensitivities, Price, CurrentHolds, FixedInd, ConSet)

## Arguments

| Sensitivities | Number of instruments (NINST) by number of sensitivities (NSENS) matrix of dollar sensitivities of each instrument. Each row represents a different instrument. Each column represents a different sensitivity. |
| :---: | :---: |
| Price | NINST-by-1 vector of instrument unit prices. |
| CurrentHolds | NINST-by- 1 vector of contracts allocated in each instrument. |
| FixedInd | (Optional) Empty or number of fixed instruments (NFIXED)-by- 1 vector of indices of instruments to hold fixed. The default is FixedInd $=1$; the holdings in the first instrument are held fixed. If NFIXED instruments will not be changed, enter all their locations in the portfolio in a vector. If no instruments are to be held fixed, enter FixedInd $=[]$. |
| ConSet | (Optional) Number of constraints (NCONS)-by-NINST matrix of additional conditions on the portfolio reallocations. An eligible NINST-by-1 vector of contract holdings, PortHolds, satisfies all the inequalities A*PortHolds <= b, where $\mathrm{A}=\operatorname{ConSet}(:, 1:$ end-1) and $\mathrm{b}=$ ConSet(:,end). |

## Description

[PortSens, PortValue, PortHolds] = hedgeslf(Sensitivities, Price, CurrentHolds, FixedInd, ConSet) allocates a self-financing hedge among a collection of instruments. hedgeslf finds the reallocation in a portfolio of financial instruments that hedges the portfolio against market moves and that is closest to being self-financing (maintaining constant portfolio value). By default the first instrument entered is hedged with the other instruments.

PortSens is a 1-by-NSENS vector of portfolio dollar sensitivities. When a perfect hedge exists, PortSens is zeros. Otherwise, the best possible hedge is chosen.

PortValue is the total portfolio value (scalar). When a perfectly self-financing hedge exists, PortValue is equal to dot (Price, CurrentWts) of the initial portfolio.

PortHolds is an NINST-by- 1 vector of contracts allocated to each instrument. This is the reallocated portfolio.

## Notes

- The constraints PortHolds(FixedInd) = CurrentHolds(FixedInd) are appended to any constraints passed in ConSet. Pass FixedInd = [] to specify all constraints through ConSet.
- The default constraints generated by portcons are inappropriate, since they require the sum of all holdings to be positive and equal to one.
- hedgeself first tries to find the allocations of the portfolio that make it closest to being self-financing, while reducing the sensitivities to 0 . If no solution is found, it finds the allocations that minimize the sensitivities. If the resulting portfolio is self-financing, PortValue is equal to the value of the original portfolio.

Examples Example 1. Perfect sensitivity cannot be reached.

```
Sens = [0.44 0.32; 1.0 0.0];
Price = [1.2; 1.0];
WO = [1; 1];
[PortSens, PortValue, PortHolds]= hedgeslf(Sens, Price, WO)
PortSens =
    0.0000
    0.3200
PortValue =
    0.7600
PortHolds =
    1.0000
    -0.4400
```

Example 2. Constraints are in conflict.

```
Sens = [0.44 0.32; 1.0 0.0];
Price = [1.2; 1.0];
WO = [1; 1];
ConSet = pcalims([2 2])
% O.K. if nothing fixed.
[PortSens, PortValue, PortHolds]= hedgeslf(Sens, Price, wo,...
[], ConSet)
PortSens =
    2.8800
    0.6400
```

```
PortValue =
    4.4000
PortHolds =
    2
    2
%WO(1) is not greater than 2.
[PortSens, PortValue, PortHolds] = hedgeslf(Sens, Price, W0,...
1, ConSet)
??? Error using ==> hedgeslf
Overly restrictive allocation constraints implied by ConSet and
by fixing the weight of instruments(s): 1
```

Example 3. Constraints are impossible to meet.

```
Sens = [0.44 0.32; 1.0 0.0];
Price = [1.2; 1.0];
WO = [1; 1];
ConSet = pcalims([2 2],[1 1]);
[PortSens, PortValue, PortHolds] = hedgeslf(Sens, Price, Wo,...
[],ConSet)
??? Error using ==> hedgeslf
Overly restrictive allocation constraints specified in ConSet
```


## See Also

hedgeopt | lsqlin | portcons

## himprice

Purpose Instrument prices from Heath-Jarrow-Morton interest-rate tree
Syntax Price = hjmprice(HJMTree, InstSet, Options)
Arguments

| HJMTree | Heath-Jarrow-Morton tree sampling a forward-rate <br> process. See hjmtree for information on creating <br> HJMTree. |
| :---: | :--- |
| InstSet | Variable containing a collection of instruments. <br> Instruments are categorized by type. Each type can <br> have different data fields. The stored data field is a <br> row vector or string for each instrument. |
| Options | (Optional) Derivatives pricing options structure <br> created with derivset. |

## Description

Price = hjmprice(HJMTree, InstSet, Options) computes arbitrage-free prices for instruments using an interest-rate tree created with hjmtree. A subset of NINST instruments from a financial instrument variable, InstSet, are priced.

Price is a NINST-by-1 vector of prices for each instrument. The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, NaN is returned.

PriceTree is a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node.

PriceTree. PBush contains the clean prices.
PriceTree.AIBush contains the accrued interest.
PriceTree.tObs contains the observation times.
hjmprice handles instrument types: 'Bond', 'CashFlow', 'OptBond', 'OptEmBond', 'OptEmBond', 'OptFloat', 'OptEmFloat', 'Fixed',
'Float', 'Cap', 'Floor', 'RangeFloat', 'Swap'. See instadd to construct defined types.
Related single-type pricing functions are:

- bondbyhjm: Price a bond from an HJM tree.
- capbyhjm: Price a cap from an HJM tree.
- cfbyhjm: Price an arbitrary set of cash flows from an HJM tree.
- fixedbyhjm: Price a fixed-rate note from an HJM tree.
- floatbyhjm: Price a floating-rate note from an HJM tree.
- floorbyhjm: Price a floor from an HJM tree.
- optbndbyhjm: Price a bond option from an HJM tree.
- optembndbyhjm: Price a bond with embedded option by an HJM tree.
- optfloatbybdt: Price a floating-rate note with an option from an HJM tree.
- optemfloatbybdt: Price a floating-rate note with an embedded option from an HJM tree.
- rangefloatbyhjm: Price range floating note using an HJM tree.
- swapbyhjm: Price a swap from an HJM tree.
- swaptionbyhjm: Price a swaption from an HJM tree.


## Examples

Load the HJM tree and instruments from the data file deriv.mat. Price the cap and bond instruments contained in the instrument set.

```
load deriv.mat;
HJMSubSet = instselect(HJMInstSet,'Type', {'Bond', 'Cap'});
instdisp(HJMSubSet)
```

\%Table of instrument portfolio partially displayed:

| Index | Type CouponRate Settle | Maturity | Period | Basis | ... Name | Quantity |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | Bond | 0.04 | 01-Jan-2000 | 01 -Jan-2003 | 1 | NaN | $\ldots$ | $4 \%$ |
| bond | 100 |  |  |  |  |  |  |  |

## himprice

```
2 Bond 0.04 01-Jan-2000 01-Jan-2004 \(2 \quad\) NaN ... 4\% bond 50
Index Type Strike Settle Maturity CapReset Basis ... Name Quantity
3 Cap 0.03 01-Jan-2000 01-Jan-2004 \(1 \quad\) NaN...\(\quad 3 \%\) Cap 30
```

```
[Price, PriceTree] = hjmprice(HJMTree, HJMSubSet)
```

[Price, PriceTree] = hjmprice(HJMTree, HJMSubSet)
Warning: Not all cash flows are aligned with the tree. Result will
be approximated.
Price =
98.7159
97.5280
6.2831
PriceTree =
FinObj: 'HJMPriceTree'

```

You can use treeviewer to see the prices of these three instruments along the price tree.
```

treeviewer(PriceTree, HJMSubSet)

```


First 4\% Bond (Maturity 2003)


Second 4\% Bond (Maturity 2004)


3\% Cap

Price the following multi-stepped coupon bonds using the following data:
```

% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);

```

\section*{himprice}
```

% Create a portfolio of stepped coupon bonds with different maturities
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011';'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2011' .042;'01-Jan-2012' .05; '01-Jan-2013' .06; '01-Jan-2014' .07}};
ISet = instbond(CouponRate, Settle, Maturity, 1);
instdisp(ISet)
%Table of instrument portfolio partially displayed:

```

```

% Build the tree with the following data
Volatility = [.2; .19; .18; .17];
CurveTerm = [ 1; 2; 3; 4];
HJMTimeSpec = hjmtimespec(ValuationDate, EndDates);
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec,RS,HJMTimeSpec);
% Compute the price of the stepped coupon bonds
PHJM = hjmprice(HJMT, ISet)
PHJM =
100.6763
100.7368
100.9266
101.0115

```

Price a portfolio of stepped callable bonds and stepped vanilla bonds using the following data:
```

% Price a portfolio of stepped callable bonds and stepped vanilla bonds
% using the following data.
% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
%Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
% Create an instrument portfolio of 3 stepped callable bonds and three
% stepped vanilla bonds
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2011' .042;'01-Jan-2012' .05; '01-Jan-2013' .06; '01-Jan-2014' .07}};
OptSpec='call';
Strike=100;
ExerciseDates='01-Jan-2011'; %Callable in one year
% Bonds with embedded option
ISet = instoptembnd(CouponRate, Settle, Maturity, OptSpec, Strike,...
ExerciseDates, 'Period', 1);
% Vanilla bonds
ISet = instbond(ISet, CouponRate, Settle, Maturity, 1);
% Display the instrument portfolio
instdisp(ISet)
%Table of instrument portfolio partially displayed:
Index Type CouponRate Settle Maturity OptSpec Strike ExerciseDates ... AmericanOpt
1 OptEmBond [Cell] 01-Jan-2010 01-Jan-2012 call 100 01-Jan-2011 ... 0

```

\section*{himprice}
\begin{tabular}{llllllll}
2 & OptEmBond [Cell] & \(01-\) Jan-2010 & 01 -Jan-2013 & call & 100 & \(01-\) Jan-2011. .0 \\
3 & OptEmBond [Cell] & \(01-J a n-2010\) & \(01-\) Jan-2014 & call & 100 & \(01-\) Jan-2011. .0
\end{tabular}
\begin{tabular}{lcccccc}
\multicolumn{3}{l}{ Index Type CouponRate Settle } & Maturity & Period Basis EndMonthRule & ... Face \\
4 & Bond [Cell] & 01 -Jan-2010 & 01 -Jan-2012 & 1 & 0 & 1
\end{tabular}
\% Build the tree with the following data
Volatility = [.2; .19; .18; .17];
CurveTerm = [ 1; 2; 3; 4];
HJMTimeSpec = hjmtimespec(ValuationDate, EndDates);
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec,RS,HJMTimeSpec);
\%The first three rows corresponds to the price of the stepped callable bonds \% and the last three rows corresponds to the price of the stepped vanilla bonds. PHJM = hjmprice(HJMT, ISet)

PHJM =
100.3682
100.1557
99.9232
100.7368
100.9266
101.0115

Compute the price of a portfolio using the following data:
```

% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
Compounding = 1;
% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates',...
StartDates, 'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
% Create an instrument portfolio with two range notes and a floating rate
% note with the following data:
Spread = 200;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';
% First Range Note:
RateSched(1).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(1).Rates = [0.045 0.055; 0.0525 0.0675; 0.06 0.08];
% Second Range Note:
RateSched(2).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(2).Rates = [0.048 0.059; 0.055 0.068; 0.07 0.09];
% Create InstSet
InstSet = instadd('RangeFloat', Spread, Settle, Maturity, RateSched);
% Add a floating-rate note
InstSet = instadd(InstSet, 'Float', Spread, Settle, Maturity);
% Display the portfolio instrument
instdisp(InstSet)

| Index | Type | Spread Settle | Maturity | RateSched FloatReset Basis Principal EndMonthRule |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

```

\section*{himprice}

2
RangeFloat 200 01-Jan-2011 01-Jan-2014 [Struct] 1
0100
1
\begin{tabular}{lllllll} 
Index Type Spread Settle & Maturity & FloatReset Basis Principal EndMonthRule \\
3 & Float 200 & \(01-\) Jan-2011 & \(01-\) Jan-2014 & 1 & 0 & 100
\end{tabular}
\% The data to build the tree is as follows:
Volatility = [.2; .19; .18; .17];
CurveTerm = [ 1; 2; 3; 4];
MaTree = \{'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'\};
HJMTS = hjmtimespec(ValuationDate, MaTree);
HJMVS = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVS, RS, HJMTS);
\% Price the portfolio
Price = hjmprice(HJMT, InstSet)

Price =
91.1555
90.6656
105.5147

See Also hjmsens | hjmtree | hjmvolspec | instadd | intenvprice | intenvsens

\section*{Purpose}

Syntax

\section*{Arguments}
\begin{tabular}{cl} 
HJMTree & \begin{tabular}{l} 
Heath-Jarrow-Morton tree sampling a forward-rate \\
process. See hjmtree for information on creating \\
HJMTree.
\end{tabular} \\
InstSet & \begin{tabular}{l} 
Variable containing a collection of instruments. \\
Instruments are categorized by type. Each type can \\
have different data fields. The stored data field is a \\
row vector or string for each instrument.
\end{tabular} \\
Options & \begin{tabular}{l} 
(Optional) Derivatives pricing options structure \\
created with derivset.
\end{tabular}
\end{tabular}

\section*{Description}

Instrument prices and sensitivities from Heath-Jarrow-Morton interest-rate tree
[Delta, Gamma, Vega, Price] = hjmsens(HJMTree, InstSet, Options)

InstSet Variable containing a collection of instruments. Instruments are categorized by type. Each type can have different data fields. The stored data field is a row vector or string for each instrument.
Options (Optional) Derivatives pricing options structure created with derivset.
[Delta, Gamma, Vega, Price] = hjmsens(HJMTree, InstSet, Options) computes instrument sensitivities and prices for instruments using an interest-rate tree created with hjmtree. NINST instruments from a financial instrument variable, InstSet, are priced. hjmsens handles instrument types: 'Bond', 'CashFlow', 'OptBond', 'OptEmBond', 'OptEmBond', 'OptFloat', 'OptEmFloat', 'Fixed', 'Float', 'Cap', 'Floor', 'RangeFloat', 'Swap'. See instadd for information on instrument types.

Delta is an NINST-by- 1 vector of deltas, representing the rate of change of instrument prices with respect to changes in the interest rate. Delta is computed by finite differences in calls to hjmtree. See hjmtree for information on the observed yield curve.

Gamma is an NINST-by- 1 vector of gammas, representing the rate of change of instrument deltas with respect to the changes in the interest rate. Gamma is computed by finite differences in calls to hjmtree.

Vega is an NINST-by-1 vector of vegas, representing the rate of change of instrument prices with respect to the changes in the volatility \(\sigma(t, T)\). Vega is computed by finite differences in calls to hjmtree. See hjmvolspec for information on the volatility process.

Note All sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, divide by the respective instrument price.

Price is an NINST-by- 1 vector of prices of each instrument. The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, NaN is returned.
Delta and Gamma are calculated based on yield shifts of 100 basis points. Vega is calculated based on a \(1 \%\) shift in the volatility process.

\section*{Examples}

\section*{Compute Instrument Sensitivities Using an HJM Interest-Rate Tree}

Load the tree and instruments from the deriv.mat data file. Compute Delta and Gamma for the cap and bond instruments contained in the instrument set.
```

load deriv.mat;
HJMSubSet = instselect(HJMInstSet,'Type', {'Bond', 'Cap'});
instdisp(HJMSubSet)

```
\begin{tabular}{lllllll} 
Index & Type CouponRate Settle & Maturity & Period Basis EndMontr \\
1 & Bond 0.04 & 01-Jan-2000 & 01-Jan-2003 & 1 & NaN & NaN \\
2 & Bond 0.04 & 01-Jan-2000 & 01-Jan-2004 & 2 & NaN & NaN
\end{tabular}
\begin{tabular}{llllll} 
Index & Type Strike Settle & Maturity & CapReset Basis Principal \\
3 & Cap 0.03 & 01 -Jan-2000 & 01-Jan-2004 & 1 & NaN NaN
\end{tabular}

Compute the Delta and Gamma for the cap and bond instruments.
```

[Delta, Gamma] = hjmsens(HJMTree, HJMSubSet)
Warning: Not all cash flows are aligned with the tree. Result will be
approximated.
Delta =
-272.6462
-347.4315
294.9700
Gamma =
1.0e+03 *
1.0299
1.6227
6.8526

```

See Also hjmprice | hjmtree | hjmvolspec | instadd

\section*{himtimespec}
\begin{tabular}{ll} 
Purpose & Specify time structure for Heath-Jarrow-Morton interest-rate tree \\
Syntax & TimeSpec \(=\) hjmtimespec (ValuationDate, Maturity, Compounding)
\end{tabular}

\section*{Arguments}
\begin{tabular}{ll} 
ValuationDate & \begin{tabular}{l} 
Scalar date marking the pricing date and first \\
observation in the tree. Specify as serial date \\
number or date string.
\end{tabular} \\
Maturity & \begin{tabular}{l} 
Number of levels (depth) of the tree. A number of \\
levels (NLEVELS)-by-1 vector of dates marking the \\
cash flow dates of the tree. Cash flows with these \\
maturities fall on tree nodes. Maturity should \\
be in increasing order.
\end{tabular} \\
Compounding & \begin{tabular}{l} 
(Optional) Scalar value representing the rate \\
at which the input zero rates were compounded \\
when annualized. Default \(=1\). This argument \\
determines the formula for the discount factors:
\end{tabular}
\end{tabular}

Compounding \(=1,2,3,4,6,12\)
Disc = (1 + Z/F)^(-T), where F is the compounding frequency, \(Z\) is the zero rate, and T is the time in periodic units; for example, \(\mathrm{T}=\) \(F\) is 1 year.

Compounding \(=365\)
Disc \(=(1+Z / F)^{\wedge}(-T)\), where \(F\) is the number of days in the basis year and \(T\) is a number of days elapsed computed by basis.
Compounding \(=-1\)
Disc \(=\exp (-T * Z)\), where \(T\) is time in years.

Description

\section*{Examples}

TimeSpec = hjmtimespec(ValuationDate, Maturity, Compounding) sets the number of levels and node times for an HJM tree and determines the mapping between dates and time for rate quoting.

TimeSpec is a structure specifying the time layout for hjmtree. The state observation dates are [Settle; Maturity (1:end-1)]. Because a forward rate is stored at the last observation, the tree can value cash flows out to Maturity.

\section*{Set the Number of Levels and Node Times for an HJM Tree}

This example shows how to specify an eight-period tree with semiannual nodes (every six months) and use exponential compounding to report rates.

Compounding = -1;
ValuationDate = '15-Jan-1999';
Maturity = datemnth(ValuationDate, 6*(1:8)');
TimeSpec \(=\) hjmtimespec(ValuationDate, Maturity, Compounding)

TimeSpec =
FinObj: 'HJMTimeSpec'
ValuationDate: 730135
Maturity: [8x1 double] Compounding: -1

Basis: 0
EndMonthRule: 1

See Also hjmtree | hjmvolspec

\section*{himtree}
\begin{tabular}{ll} 
Purpose & Construct Heath-Jarrow-Morton interest-rate tree \\
Syntax & HJMTree \(=\) hjmtree (VolSpec, RateSpec, TimeSpec)
\end{tabular}

Arguments

\section*{Description HJMTree \(=\) hjmtree (VolSpec, RateSpec, TimeSpec) creates a} structure containing time and forward-rate information on a bushy tree.
Examples \(\quad\)\begin{tabular}{l} 
Using the data provided, create an HJM volatility specification \\
(VolSpec), rate specification (RateSpec), and tree time layout \\
specification (TimeSpec). Then use these specifications to create an \\
\\
HJM tree using hjmtree.
\end{tabular}
```

HJMVolSpec = hjmvolspec('Stationary', Volatility , CurveTerm);
RateSpec = intenvset('Compounding', Compounding,...
'ValuationDate', ValuationDate,...
'StartDates', StartDate,...
'EndDates', EndDates,...
'Rates', Rates);
HJMTimeSpec = hjmtimespec(ValuationDate, EndDates, Compounding);
HJMTree = hjmtree(HJMVolSpec, RateSpec, HJMTimeSpec)
HJMTree =

```


Use treeviewer to observe the tree you have created.
treeviewer(HJMTree)

\section*{himtree}


See Also hjmprice | hjmtimespec | hjmvolspec | intenvset

\section*{Purpose}

Specify Heath-Jarrow-Morton interest-rate volatility process

\section*{Syntax}

Arguments
Volspec = hjmvolspec(varargin)
The arguments to hjmvolspec vary according to the type and number of volatility factors specified when calling the function. Factors are specified by pairs of names and parameter sets. Factor names can be 'Constant', 'Stationary', 'Exponential', 'Vasicek', or 'Proportional'. The parameter set is specific for each of these factor types:
- Constant volatility (Ho-Lee): VolSpec = hjmvolspec('Constant', Sigma_0)
- Stationary volatility: VolSpec = hjmvolspec('Stationary', CurveVol, CurveTerm)
- Exponential volatility: VolSpec = hjmvolspec('Exponential', Sigma_0, Lambda)
- Vasicek, Hull-White: VolSpec = hjmvolspec('Vasicek', Sigma_0, CurveDecay, CurveTerm)
- Nearly proportional stationary: VolSpec = hjmvolspec('Proportional', CurveProp, CurveTerm, MaxSpot)
You can specify more than one factor by concatenating names and parameter sets.

The following table defines the various arguments to hjmvolspec.
\begin{tabular}{ll}
\hline Argument & Description \\
\hline Sigma_0 & Scalar base volatility over a unit time. \\
Lambda & Scalar decay factor. \\
CurveVol & \begin{tabular}{l} 
Number of curves (NCURVES)-by1 vector of Vol values \\
at sample points.
\end{tabular}
\end{tabular}

\section*{himvolspec}

\section*{Argument Description \\ CurveDecay NCURVES-by-1 vector of Decay values at sample points. \\ CurveProp NCURVES-by-1 vector of Prop values at sample points. \\ CurveTerm NCURVES-by-1 vector of Term sample points.}

Note See the volatility specifications formulas below for a description of Vol, Decay, Prop, and Term.

\section*{Description}

Volspec = hjmvolspec(varargin) computes VolSpec, a structure that specifies the volatility model for hjmtree.
hjmvolspec specifies an HJM forward-rate volatility process. Each factor is specified with one of the functional forms.

\section*{Volatility Specification Formula}

Constant
Stationary

Exponential
Vasicek, Hull-White

Proportional
\(\sigma(t, T)=\) Sigma_0* \(\exp (-\operatorname{Decay}(T-t))\)
\(\sigma(t, T)=\) Sigma \(\_0\)
\(\sigma(t, T)=\operatorname{Vol}(\mathrm{T}-\mathrm{t})=\operatorname{Vol}(\mathrm{Term})\)
\(\sigma(t, T)=\) Sigma_0*exp(-Lambda*(T-t))
\(\sigma(t, T)=\operatorname{Prop}(T-\mathrm{t}) * \max (\operatorname{SpotRate}(\mathrm{t})\), MaxSpot)

\section*{himvolspec}

The volatility process is \(\sigma(t, T)\), where \(t\) is the observation time and \(T\) is the starting time of a forward rate. In a stationary process, the volatility term is \(T-t\). Multiple factors can be specified sequentially.

The time values \(T, t\), and Term are in coupon interval units specified by the Compounding input of hjmtimespec. For instance if Compounding = 2 , Term = 1 is a semiannual period (six months).

\section*{Examples Compute the VolSpec Structure to Specify a Proportional Volatility Model for HJMTree}

This example shows how to compute the VolSpec structure to specify the volatility model for hjmtree when volatility is single-factor proportional.
```

CurveProp = [0.11765; 0.08825; 0.06865];
CurveTerm = [1; 2; 3];
VolSpec = hjmvolspec('Proportional', CurveProp, CurveTerm, 1e6)

```

VolSpec =
FinObj: 'HJMVolSpec'
FactorModels: \{'Proportional'\}
FactorArgs: \{\{1x3 cell\}\}
SigmaShift: 0
NumFactors: 1
NumBranch: 2
PBranch: [0.5000 0.5000]
Fact2Branch: [-1 1]

\section*{Compute the VolSpec Structure to Specify an Exponential Volatility Model for HJMTree}

This example shows how to compute the VolSpec structure to specify the volatility model for hjmtree when volatility is two-factor exponential and constant.

\section*{himvolspec}
```

VolSpec = hjmvolspec('Exponential', 0.1, 1, 'Constant', 0.2)
VolSpec =
FinObj: 'HJMVolSpec'
FactorModels: {'Exponential' 'Constant'}
FactorArgs: {{1x2 cell} {1x1 cell}}
SigmaShift: 0
NumFactors: 2
NumBranch: 3
PBranch: [0.2500 0.2500 0.5000]
Fact2Branch: [2x3 double]

```

See Also hjmtimespec | hjmtree

\section*{HullWhite IF}

\section*{Superclasses}

Purpose

\section*{Create Hull-White one-factor model}

Description The Hull-White one-factor model is specified using the zero curve, alpha, and sigma parameters for the equation
\[
d r=[\theta(t)-a(t) r] d t+\sigma(t) d W
\]
where:
\(d r\) is the change in the short-term interest rate over a small interval.
\(r\) is the short-term interest rate.
\(\Theta(t)\) is a function of time determining the average direction in which \(r\) moves, chosen such that movements in \(r\) are consistent with today's zero coupon yield curve.
\(\alpha\) is the mean reversion rate.
\(d t\) is a small change in time.
\(\sigma\) is the annual standard deviation of the short rate.
\(W\) is the Brownian motion.
Construction OBJ = HullWhite1F(ZeroCurve, alpha, sigma) constructs an object for a Hull-White one-factor model.
For example:
```

Settle = datenum('15-Dec-2007');
CurveTimes = [1:5 7 10 20]';
ZeroRates = [.01 .018 .024 .029 .033 . 034 .035 .034]';
CurveDates = daysadd(Settle,360*CurveTimes,1);
irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);
alpha = .1;
sigma = .01;

```

\section*{HullWhite IF}

HW1F = HullWhite1F(irdc,alpha,sigma);

\section*{Properties \\ The following properties are from the HullWhite1F class.}

\section*{ZeroCurve}

ZeroCurve is specified using the output from IRDataCurve or RateSpec. This is the zero curve used to evolve the path of future interest rates.

Attributes:
SetAccess public
GetAccess public

\section*{Alpha}

Mean reversion specified either as a scalar or function handle which takes time as an input and returns a scalar mean reversion value.

\section*{Attributes:}
```

SetAccess
public
GetAccess public

```

\section*{Sigma}

Volatility specified either as a scalar or function handle which takes time as an input and returns a scalar mean volatility.

\section*{Attributes:}
\begin{tabular}{ll} 
SetAccess & public \\
GetAccess & public
\end{tabular}

\section*{HullWhite IF}

\section*{Methods}
simTermStructs
Simulate term structures for Hull-White one-factor model

\section*{Definitions}

\section*{Hull-White One-Factor Model}

The Hull-White model is a single-factor, no-arbitrage yield curve model in which the short-term rate of interest is the random factor or state variable. No-arbitrage means that the model parameters are consistent with the bond prices implied in the zero coupon yield curve.

\section*{Copy \\ Semantics \\ Value. To learn how value classes affect copy operations, see Copying Objects in the MATLAB documentation.}

\section*{Examples Construct a Hull-White One-Factor Model}

Construct a Hull-White one-factor model.
```

Settle = datenum('15-Dec-2007');
CurveTimes = [1:5 7 10 20]';
ZeroRates = [.01 .018 .024 .029 .033 .034 .035 .034]';
CurveDates = daysadd(Settle,360*CurveTimes,1);
irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);
alpha = .1;
sigma = .01;
HW1F = HullWhite1F(irdc,alpha,sigma)

```
HW1F =

HullWhite1F with properties:
ZeroCurve: [1x1 IRDataCurve]

\section*{HullWhite IF}
```

Alpha: @(t,V)inAlpha
Sigma: @(t,V)inSigma

```

References Hull, J. Options, Futures, and Other Derivatives, Prentice-Hall, 2011.

\section*{See Also}

LiborMarketModel | LinearGaussian2F | hwcalbycap | hwcalbyfloor | simTermStructs

\section*{Related Examples}
- "Price Swaptions With Interest-Rate Models Using Simulation" on page 2-105
- Class Attributes
- Property Attributes

\section*{HullWhite 1 F.simTermStructs}
\begin{tabular}{|c|c|}
\hline Purpose & Simulate term structures for Hull-White one-factor model \\
\hline Syntax & ```
[ZeroRates,ForwardRates] = simTermStructs(nPeriods)
[ZeroRates,ForwardRates] = simTermStructs(nPeriods,
Name,Value)
``` \\
\hline Description & \begin{tabular}{l}
[ZeroRates,ForwardRates] = simTermStructs(nPeriods) simulates future zero curve paths using a specified HullWhite1F object. \\
[ZeroRates,ForwardRates] = simTermStructs(nPeriods, Name, Value) simulates future zero curve paths using a specified HullWhite1F object with additional options specified by one or more Name, Value pair arguments.
\end{tabular} \\
\hline \multirow[t]{8}{*}{Input Arguments} & nPeriods
\(\quad\) Number of simulation periods. \\
\hline & Name-Value Pair Arguments \\
\hline & Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN. \\
\hline & 'deltaTime' \\
\hline & Scalar time step between periods. \\
\hline & Default: 1 \\
\hline & 'nTrials' \\
\hline & Positive scalar integer number of simulated trials (sample paths) of NPERIODS observations each. If you do not specify a value for this argument, the default is 1 , indicating a single path of correlated state variables. \\
\hline
\end{tabular}

\section*{HullWhite 1 F.simTermStructs}

Default: 1

\section*{'antithetic'}

Boolean scalar flag indicating whether antithetic sampling is used to generate the Gaussian random variates that drive the zero-drift, unit-variance rate Brownian vector \(d W(t)\). For details, see simBySolution.

Default: false

\section*{'Z'}

Direct specification of the dependent random noise process used to generate the zero-drift, unit-variance rate Brownian vector \(d W(t)\) that drives the simulation. For details, see simBySolution for the HWV model.

Default: Uses default for simBySolution. If you do not specify a value for \(Z\), simBySolution generates Gaussian variates.

\section*{'Tenor'}

Numeric vector of maturities to compute at each time step.
Default: tenor of the Hullwhite1F object's zero curve

\section*{Output Arguments}

\section*{ZeroRates}
nPeriods+1-by-nTenors-by-nTrials matrix of simulated zero-rate term structures.

\section*{ForwardRates}
nPeriods+1-by-nTenors-by-nTrials matrix of simulated forward-rate term structures.

\section*{Attributes}

\section*{Examples Simulate Term Structures for the HullWhite IF Model}

Create a HW1F object.
```

Settle = datenum('15-Dec-2007');
CurveTimes = [1:5 7 10 20]';
ZeroRates = [.01 .018 .024 .029 .033 . 034 .035 .034]';
CurveDates = daysadd(Settle,360*CurveTimes,1);
irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);
alpha = .1;
sigma = .01;
HW1F = HullWhite1F(irdc,alpha,sigma)

```

HW1F =
HullWhite1F with properties:
```

ZeroCurve: [1x1 IRDataCurve]
Alpha: @(t,V)inAlpha
Sigma: @(t,V)inSigma

```

Simulate the term structures for the specified HW1F object.

\section*{HullWhite 1 F.simTermStructs}
\[
\text { SimPaths }=\text { HW1F.simTermStructs(10,'nTrials',100); }
\]

\section*{See Also \\ HullWhite1F}
\begin{tabular}{ll} 
Related & - "Price Swaptions With Interest-Rate Models Using Simulation" on \\
Examples & page \(2-105\)
\end{tabular}

\section*{Purpose}

Calibrate Hull-White tree using caps
Syntax
[Alpha, Sigma, OptimOut] = hwcalbycap(RateSpec, MarketStrike, MarketMaturity, MarketVolatility, Strike, Settle, Maturity) [Alpha, Sigma, OptimOut] = hwcalbycap(RateSpec, MarketStrike, MarketMaturity, MarketVolatility, Strike, Settle, Maturity, 'Name1', Value1...)

\section*{Arguments}
\begin{tabular}{ll} 
RateSpec & \begin{tabular}{l} 
The annualized, continuously \\
compounded rate term structure. For \\
more information, see intenvset.
\end{tabular} \\
MarketStrike & \begin{tabular}{l} 
NSTRIKES-by-1 vector of market cap \\
strikes, as a decimal number.
\end{tabular} \\
MarketMaturity & \begin{tabular}{l} 
NMATS-by-1 vector of market cap \\
maturity dates.
\end{tabular} \\
MarketVolatility & \begin{tabular}{l} 
NSTRIKES-by-NMATS matrix of market \\
flat volatilities, where NSTRIKES is \\
the number of caplet strikes from \\
MarketStrike and NMATS is the caplet \\
maturity dates from MarketMaturity.
\end{tabular} \\
Strike & \begin{tabular}{l} 
Scalar representing the rate at which \\
the cap is exercised, as a decimal \\
number.
\end{tabular} \\
Settle & \begin{tabular}{l} 
Scalar representing the settle date of \\
the cap.
\end{tabular} \\
Maturity & \begin{tabular}{l} 
Scalar representing the maturity date \\
of the cap. \\
(Optional) Scalar representing the \\
frequency of payments per year.
\end{tabular} \\
fefault is 1.
\end{tabular}

\section*{hwcalbycap}
\begin{tabular}{ll} 
Principal & (Optional) Scalar representing the \\
notional principal amount. Default is \\
100. \\
(Optional) NINST-by-1 vector \\
representing the basis used when \\
annualizing the input forward rate. \\
& - \(0=\) actual/actual (default) \\
& - \(1=30 / 360\) (SIA) \\
& - \(2=\) actual/360 \\
& - \(3=\) actual/365 \\
& - \(4=30 / 360\) (BMA) \\
& - \(5=30 / 360\) (ISDA) \\
& - \(6=30 / 360\) (European) \\
& - \(7=\) actual/365 (Japanese) \\
& - \(8=\) actual/actual (ICMA) \\
& - \(9=\) actual/360 (ICMA) \\
& - \(10=\) actual/365 (ICMA) \\
& - \(11=30 / 360 E\) (ICMA) \\
& - \(12=\) actual/actual (ISDA) \\
& - \(13=\) BUS/252 \\
& For more information, see basis on \\
page Glossary-1. \\
& (Optional) Scalar representing the \\
observation date of the investment \\
horizons. The default is the Settle \\
date.
\end{tabular}

LB

UB

X0

OptimOptions
(Optional) 2-by-1 vector of the lower bounds, defined as [LBSigma; LBAlpha], used in the search algorithm function. Default is \(L B=[0 ; 0]\). For more information, see lsqnonlin.
(Optional) 2-by-1 vector of the upper bounds, defined as [UBSigma; LBAlpha], used in the search algorithm function. Default is \(U B=[\quad]\) (unbound). For more information, see lsqnonlin.
(Optional) 2-by-1 vector of the initial values, defined as [Sigma0; Alpha0], used in the search algorithm function. Default is \(\mathrm{XO}=[0.5 ; 0.5]\). For more information, see lsqnonlin.
(Optional) Structure with optimization parameters. For more information, see optimoptions.

Note All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. You can specify parameter name/value pairs in any order. Names are case-insensitive and partial string matches are allowed provided no ambiguities exist.

\section*{Description}
[Alpha, Sigma, OptimOut] = hwcalbycap(RateSpec, MarketStrike, MarketMaturity, MarketVolatility, Strike, Settle, Maturity) [Alpha, Sigma, OptimOut] = hwcalbycap(RateSpec, MarketStrike,

\section*{hwcalbycap}

MarketMaturity, MarketVolatility, Strike, Settle, Maturity, 'Name1', Value1...)

Use hwcalbycap to estimate the Alpha (mean reversion) and Sigma (volatility) using cap market data and the Hull-White model.

The outputs are:
- Alpha - Scalar representing the mean reversion value obtained from calibrating the cap using market information.
- Sigma - Scalar representing the volatility value obtained from calibrating the cap using market information.
- OptimOut - Structure with optimization results.

\section*{Examples For an example, see "Calibrating Hull-White Model Using Market Data" on page 2-93. \\ See Also \\ capbyblk | hwcalbyfloor | hwtree | lsqnonlin | HullWhite1F}

\section*{Purpose}

Calibrate Hull-White tree using floors
Syntax
```

[Alpha, Sigma, OptimOut] = hwcalbyfloor(RateSpec,
MarketStrike, MarketMaturity, MarketVolatility, Strike,
Settle, Maturity)
[Alpha, Sigma, OptimOut] = hwcalbyfloor(RateSpec,
MarketStrike, MarketMaturity, MarketVolatility, Strike,
Settle, Maturity, 'Name1', Value1...)

```

\section*{Arguments}
\begin{tabular}{ll} 
RateSpec & \begin{tabular}{l} 
The annualized, continuously \\
compounded rate term structure. For \\
more information, see intenvset.
\end{tabular} \\
MarketStrike & \begin{tabular}{l} 
NSTRIKES-by-1 vector of market floor \\
strikes as a decimal number.
\end{tabular} \\
MarketMaturity & \begin{tabular}{l} 
NMATS-by-1 vector of market floor \\
maturity dates.
\end{tabular} \\
MarketVolatility & \begin{tabular}{l} 
NSTRIKES-by-NMATS matrix of market \\
flat volatilities.
\end{tabular} \\
Strike & \begin{tabular}{l} 
Scalar representing the rate at which \\
the floor is exercised, as a decimal \\
number.
\end{tabular} \\
Settle & \begin{tabular}{l} 
Scalar representing the settle date of \\
the floor.
\end{tabular} \\
Maturity & \begin{tabular}{l} 
Scalar representing the maturity date \\
of the floor.
\end{tabular} \\
Reset & \begin{tabular}{l} 
(Optional) Scalar representing the \\
frequency of payments per year.
\end{tabular} \\
Default is 1.
\end{tabular}

\section*{hwcalbyfloor}
\begin{tabular}{ll} 
Principal & (Optional) Scalar representing the \\
notional principal amount. Default is \\
Basis & (Optional) NINST-by-1 vector \\
representing the basis used when \\
annualizing the input forward rate. \\
& - \(0=\) actual/actual (default) \\
& - \(1=30 / 360\) (SIA) \\
& - \(2=\) actual/360 \\
& - \(3=\) actual/365 \\
& - \(4=30 / 360\) (BMA) \\
& - \(5=30 / 360\) (ISDA) \\
& - \(6=30 / 360\) (European) \\
& - \(7=\) actual/365 (Japanese) \\
& - \(8=\) actual/actual (ICMA) \\
& - \(9=\) actual/360 (ICMA) \\
& - \(10=\) actual/365 (ICMA) \\
& - \(11=30 / 360 E\) (ICMA) \\
& - \(12=\) actual/actual (ISDA) \\
& - \(13=\) BUS/252 \\
& For more information, see basis on \\
page Glossary-1. \\
& (Optional) Scalar representing the \\
observation date of the investment \\
horizons. The default is the Settle \\
date.
\end{tabular}

LB

UB

X0

OptimOptions
(Optional) 2-by-1 vector of the lower bounds, defined as [LBSigma; LBAlpha], used in the search algorithm function. Default is \(L B=[0 ; 0]\). For more information, see lsqnonlin.
(Optional) 2-by-1 vector of the upper bounds, defined as [UBSigma; LBAlpha], used in the search algorithm function. Default is \(U B=[\quad]\) (unbound). For more information, see lsqnonlin.
(Optional) 2-by-1 vector of the initial values, defined as [Sigma0; Alpha0], used in the search algorithm function. Default is \(\mathrm{XO}=[0.5 ; 0.5]\). For more information, see lsqnonlin.
(Optional) Structure with optimization parameters. For more information, see optimoptions.

Note All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. You can specify parameter name/value pairs in any order. Names are case-insensitive and partial string matches are allowed provided no ambiguities exist.

\section*{Description}
[Alpha, Sigma, OptimOut] = hwcalbyfloor(RateSpec, MarketStrike, MarketMaturity, MarketVolatility, Strike, Settle, Maturity)

\section*{hwcalbyfloor}
[Alpha, Sigma, OptimOut] = hwcalbyfloor(RateSpec, MarketStrike, MarketMaturity, MarketVolatility, Strike, Settle, Maturity, 'Name1', Value1...)

Use hwcalbyfloor to estimate the Alpha (mean reversion) and Sigma (volatility) using floor market data and the Hull-White model.

The outputs are:
- Alpha - Scalar representing the mean reversion value obtained from calibrating the floor using market information.
- Sigma - Scalar representing the volatility value obtained from calibrating the floor using market information.
- OptimOut - Structure with optimization results.

\section*{Examples For an example, see "Calibrating Hull-White Model Using Market Data" on page 2-93.}

\author{
See Also floorbyblk | hwcalbycap | hwtree | lsqnonlin | HullWhite1F
}

\section*{Purpose}

Instrument prices from Hull-White interest-rate tree
Syntax [Price, PriceTree] = hwprice(HWTree, InstSet, Options)
Arguments

\section*{Description}

HWTree Interest-rate tree structure created by hwtree.
InstSet Variable containing a collection of NINST instruments. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or string for each instrument.

Options (Optional) Derivatives pricing options structure created with derivset.
[Price, PriceTree] = hwprice(HWTree, InstSet, Options) computes arbitrage-free prices for instruments using an interest-rate tree created with hwtree. All instruments contained in a financial instrument variable, InstSet, are priced.

Price is a number of instruments (NINST)-by-1 vector of prices for each instrument. The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, NaN is returned.
PriceTree is a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node.

PriceTree. PTree contains the clean prices.
PriceTree.AITree contains the accrued interest.
PriceTree.tObs contains the observation times.
hwprice handles instrument types: 'Bond', 'CashFlow', 'OptBond', 'OptEmBond', 'OptEmBond', 'OptFloat', 'OptEmFloat', 'Fixed',

\section*{hwprice}
'Float', 'Cap', 'Floor', 'RangeFloat', 'Swap'. See instadd to construct defined types.
Related single-type pricing functions are:
- bondbyhw: Price a bond from a Hull-White tree.
- capbyhw: Price a cap from a Hull-White tree.
- cfbyhw: Price an arbitrary set of cash flows from a Hull-White tree.
- fixedbyhw: Price a fixed-rate note from a Hull-White tree.
- floatbyhw: Price a floating-rate note from a Hull-White tree.
- floorbyhw: Price a floor from a Hull-White tree.
- optbndbyhw: Price a bond option from a Hull-White tree.
- optembndbyhw: Price a bond with embedded option by a Hull-White tree.
- optfloatbybdt: Price a floating-rate note with an option from a Hull-White tree.
- optemfloatbybdt: Price a floating-rate note with an embedded option from a Hull-White tree.
- rangefloatbyhw: Price range floating note using a Hull-White tree.
- swapbyhw: Price a swap from a Hull-White tree.
- swaptionbyhw: Price a swaption from a Hull-White tree.

Examples Load the HW tree and instruments from the data file deriv.mat. Price the cap and bond instruments contained in the instrument set.
```

load deriv.mat;
HWSubSet = instselect(HWInstSet,'Type', {'Bond', 'Cap'});
instdisp(HWSubSet)
%Table of instrument portfolio partially displayed:
Index Type CouponRate Settle Maturity Period ... Name ...

```
```

1 Bond 0.04 01-Jan-2004 01-Jan-2007 1 ... 4% bond
2 Bond 0.04 01-Jan-2004 01-Jan-2008 1 ... 4% bond
Index Type Strike Settle Maturity CapReset ... Name ...
3 Cap 0.06 01-Jan-2004 01-Jan-2008 1 ... 6% Cap
[Price, PriceTree] = hwprice(HWTree, HWSubSet);
Price =
100.9188
99.3296
0.5837

```

You can use treeviewer to see the prices of these three instruments along the price tree.
```

treeviewer(PriceTree, HWSubSet)

```

\section*{hwprice}


First 4\% Bond (Maturity 2007)


Second 4\% Bond (Maturity 2008)


6\% Cap

Price the following multi-stepped coupon bonds using the following data:
```

% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
% Create a portfolio of stepped coupon bonds with different maturities
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011';'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2011' .042;'01-Jan-2012' .05; '01-Jan-2013' .06; '01-Jan-2014' .07}};
ISet = instbond(CouponRate, Settle, Maturity, 1);
instdisp(ISet)
%Table of instrument portfolio partially displayed:
Index Type CouponRate Settle Maturity Period Basis EndMonthRule ... Face
1 Bond [Cell] 01-Jan-2010 01-Jan-2011 1 0 ... 100
2 Bond [Cell] 01-Jan-2010 01-Jan-2012 1 0 0 ... 100
3 Bond [Cell] 01-Jan-2010 01-Jan-2013 1 0 0 . 100
4 Bond [Cell] 01-Jan-2010 01-Jan-2014 1 0... 100
% Build the tree with the following data
VolDates = ['1-Jan-2011'; '1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'];
VolCurve = 0.01;
AlphaDates = '01-01-2014';
AlphaCurve = 0.1;
HWVolSpec = hwvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RS.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVolSpec, RS, HWTimeSpec);

```

\section*{hwprice}
```

% Compute the price of the stepped coupon bonds
PHW = hwprice(HWT, ISet)
PHW =
100.6763
100.7368
100.9266
101.0115

```

Price a portfolio of stepped callable bonds and stepped vanilla bonds using the following data:
```

% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
%Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
% Create an instrument portfolio of 3 stepped callable bonds and three
% stepped vanilla bonds
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2011' .042;'01-Jan-2012' .05; '01-Jan-2013' .06; '01-Jan-2014' .07}};
OptSpec='call';
Strike=100;
ExerciseDates='01-Jan-2011'; %Callable in one year
% Bonds with embedded option

```
```

ISet = instoptembnd(CouponRate, Settle, Maturity, OptSpec, Strike,...
ExerciseDates, 'Period', 1);
% Vanilla bonds
ISet = instbond(ISet, CouponRate, Settle, Maturity, 1);
% Display the instrument portfolio
instdisp(ISet)
%Table of instrument portfolio partially displayed:

| Index Type CouponRate Settle | Maturity | OptSpec | Strike ExerciseDates | $\ldots$ | AmericanOpt |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | OptEmBond [Cell] | $01-J a n-2010$ | $01-J a n-2012$ | call | 100 | $01-J a n-2011$ | $\ldots$ | 0 |
| 2 | OptEmBond [Cell] | $01-J a n-2010$ | $01-J a n-2013$ | call | 100 | $01-J a n-2011$ | $\ldots$ | 0 |
| 3 | OptEmBond [Cell] | $01-J a n-2010$ | $01-J a n-2014$ | call | 100 | $01-J a n-2011$ | $\ldots$ | 0 |

```

```

% Build the tree with the following data
VolDates = ['1-Jan-2011'; '1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'];
VolCurve = 0.01;
AlphaDates = '01-01-2014';
AlphaCurve = 0.1;
HWVolSpec = hwvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RS.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVolSpec, RS, HWTimeSpec);
%The first three rows corresponds to the price of the stepped callable bonds
% and the last three rows corresponds to the price of the stepped vanilla bonds.
PHW = hwprice(HWT, ISet)

```

\section*{hwprice}
```

PHW =
100.4222
100.2275
100.0502
100.7368
100.9266
101.0115

```

Compute the price of a portfolio using the following data:
```

% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
Compounding = 1;
% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates',...
StartDates, 'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
% Create an instrument portfolio with two range notes and a floating rate
% note with the following data:
Spread = 200;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';
% First Range Note:
RateSched(1).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(1).Rates = [0.045 0.055; 0.0525 0.0675; 0.06 0.08];
% Second Range Note:
RateSched(2).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};

```
```

RateSched(2).Rates = [0.048 0.059; 0.055 0.068; 0.07 0.09];
% Create InstSet
InstSet = instadd('RangeFloat', Spread, Settle, Maturity, RateSched);
% Add a floating-rate note
InstSet = instadd(InstSet, 'Float', Spread, Settle, Maturity);
% Display the portfolio instrument
instdisp(InstSet)

| Index | Type | Spread | Settle | Maturity | RateSched | FloatReset | Basis | Principal EndMonthRule |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| Index | Type Spread Settle | Maturity | FloatReset Basis Principal EndMonthRule |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 3 | Float 200 | $01-$ Jan-2011 | $01-J a n-2014$ | 1 | 0 | 100 | 1 |

% The data to build the tree is as follows:
VolDates = ['1-Jan-2012'; '1-Jan-2013';'11-Jan-2014';'1-Jan-2015'];
VolCurve = 0.01;
AlphaDates = '01-01-2015';
AlphaCurve = 0.1;
HWVS = hwvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTS = hwtimespec(RS.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVS, RS, HWTS);
% Price the portfolio
Price = hwprice(HWT, InstSet)
Price =
99.4075

```

\section*{hwprice}
98.1003
105.5147

See Also hwsens | hwtree | instadd | intenvprice | intenvsens

\section*{Purpose}

Instrument prices and sensitivities from Hull-White interest-rate tree
Syntax
[Delta, Gamma, Vega, Price] = hwsens(HWTree, InstSet, Options)

\section*{Arguments}
\begin{tabular}{ll} 
HWTree & Interest-rate tree structure created by hwtree. \\
InstSet & \begin{tabular}{l} 
Variable containing a collection of NINST instruments. \\
Instruments are categorized by type; each type can \\
have different data fields. The stored data field is a \\
row vector or string for each instrument.
\end{tabular} \\
Options & \begin{tabular}{l} 
(Optional) Derivatives pricing options structure \\
created with derivset.
\end{tabular}
\end{tabular}

\section*{Description}
[Delta, Gamma, Vega, Price] = hwsens(HWTree, InstSet, Options) computes instrument sensitivities and prices for instruments using an interest-rate tree created with the hwtree function. NINST instruments from a financial instrument variable, InstSet, are priced. hwsens handles instrument types: 'Bond', 'CashFlow', 'OptBond', 'OptEmBond', 'OptEmBond', 'OptFloat', 'OptEmFloat', 'Fixed', 'Float', 'Cap', 'Floor', 'RangeFloat', 'Swap'. See instadd for information on instrument types.

Delta is an NINST-by- 1 vector of deltas, representing the rate of change of instrument prices with respect to changes in the interest rate. Delta is computed by finite differences in calls to hwtree. See hwtree for information on the observed yield curve.

Gamma is an NINST-by-1 vector of gammas, representing the rate of change of instrument deltas with respect to the changes in the interest rate. Gamma is computed by finite differences in calls to hwtree.

Vega is an NINST-by- 1 vector of vegas, representing the rate of change of instrument prices with respect to the changes in the volatility \(\sigma(t, T)\).

Vega is computed by finite differences in calls to hwtree. See hwvolspec for information on the volatility process.

Note All sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, divide by the respective instrument price.

Price is an NINST-by- 1 vector of prices of each instrument. The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, NaN is returned.
Delta and Gamma are calculated based on yield shifts of 100 basis points. Vega is calculated based on a \(1 \%\) shift in the volatility process.

\section*{Examples}

\section*{Compute Instrument Sensitivities Using an HW Interest-Rate Tree}

Load the tree and instruments from the deriv.mat data file. Compute Delta and Gamma for the cap and bond instruments contained in the instrument set.
```

load deriv.mat;
HWSubSet = instselect(HWInstSet,'Type', {'Bond', 'Cap'});
instdisp(HWSubSet)

| Index | Type CouponRate Settle | Maturity | Period | Basis | EndMontr |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | Bond | 0.04 | $01-$ Jan-2004 | $01-$ Jan-2007 | 1 | 0 |
| 1 |  |  |  |  |  |  |
| 2 | Bond | 0.04 | $01-$ Jan-2004 | $01-$ Jan-2008 | 1 | 0 |


| Index | Type | Strike Settle | Maturity | CapReset | Basis Principal |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 3 | Cap | 0.06 | $01-J a n-2004$ | $01-J a n-2008$ | 1 | 0 |

```

Compute the Delta and Gamma for the cap and bond instruments.
[Delta, Gamma] = hwsens(HWTree, HWSubSet)

Delta =
-291. 2580
- 374.6368 59.5474

Gamma =
\(1.0 \mathrm{e}+03\) *
0.8584
1.4609
4.8436

\section*{See Also}
hwprice | hwtree | hwvolspec | instadd

\section*{hwtimespec}

Purpose
Syntax TimeSpec = hwtimespec (ValuationDate, Maturity, Compounding)

ValuationDate Scalar date marking the pricing date and first observation in the tree. Specify as a serial date number or date string
Maturity Number of levels (depth) of the tree. A number of levels (NLEVELS)-by- 1 vector of dates marking the cash flow dates of the tree. Cash flows with these maturities fall on tree nodes. Maturity should be in increasing order.
Compounding (Optional) Scalar value representing the rate at which the input zero rates were compounded when annualized. Default =-1 (continuous compounding). This argument determines the formula for the discount factors:

Compounding \(=1,2,3,4,6,12=\mathrm{F}\)
Disc \(=(1+Z / F)^{\wedge}(-T)\), where \(F\) is the compounding frequency, \(Z\) is the zero rate, and \(T\) is the time in periodic units; for example, \(T\) \(=F\) is 1 year.
Compounding \(=365\)
Disc \(=(1+Z / F)^{\wedge}(-T)\), where \(F\) is the number of days in the basis year and \(T\) is a number of days elapsed computed by basis.

Compounding \(=-1\)
Disc \(=\exp (-T * Z)\), where \(T\) is time in years.

\section*{hwtimespec}
```

Description TimeSpec $=$ hwtimespec(ValuationDate, Maturity, Compounding) sets the number of levels and node times for a Hull-White tree and determines the mapping between dates and time for rate quoting.
TimeSpec is a structure specifying the time layout for hwtree. The state observation dates are [Settle; Maturity(1:end-1)]. Because a forward rate is stored at the last observation, the tree can value cash flows out to Maturity.

```

\section*{Examples Set the Number of Levels and Node Times for a Hull-White Tree}
```

This example shows how to specify a four-period tree with annual nodes and use annual compounding to report rates.

```
```

ValuationDate = 'Jan-1-2004';

```
ValuationDate = 'Jan-1-2004';
Maturity = ['12-31-2004'; '12-31-2005'; '12-31-2006';
Maturity = ['12-31-2004'; '12-31-2005'; '12-31-2006';
'12-31-2007'];
'12-31-2007'];
Compounding = 1;
Compounding = 1;
TimeSpec = hwtimespec(ValuationDate, Maturity, Compounding)
TimeSpec = hwtimespec(ValuationDate, Maturity, Compounding)
TimeSpec =
```


## FinObj: 'HWTimeSpec'

```
ValuationDate: 731947
Maturity: [4x1 double] Compounding: 1
Basis: 0
EndMonthRule: 1
```

See Also bktree | hwtree | hwvolspec

## Purpose Construct Hull-White interest-rate tree

Description

Input
Arguments

```
Syntax HWTree = hwtree(VolSpec, RateSpec, TimeSpec)
HWTree = hwtree(VolSpec, RateSpec, TimeSpec, Name, Value)
HWTree = hwtree(VolSpec, RateSpec, TimeSpec)
HWTree = hwtree(VolSpec, RateSpec, TimeSpec, Name,Value)
```

HWTree $=$ hwtree(VolSpec, RateSpec, TimeSpec) creates a structure containing time and interest-rate information on a recombining tree.

HWTree = hwtree(VolSpec, RateSpec, TimeSpec, Name, Value) creates a structure containing time and interest-rate information on a recombining tree with additional options specified by one or more Name, Value pair arguments.

## VolSpec

Volatility process specification. See hwvolspec for information on the volatility process.

## RateSpec

Interest-rate specification for the initial rate curve. See intenvset for information on declaring an interest-rate variable.

## TimeSpec

Tree time layout specification. Defines the observation dates of the HW tree and the compounding rule for date to time mapping and price-yield formulas. See hwtimespec for information on the tree structure.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

## 'Method'

String specifies the Hull-White method upon which the tree-node connectivity algorithm is based. Possible values are HW1996 and HW2000.

Note hwtree supports two tree-node connectivity algorithms. HW1996 is based on the original paper published in the Journal of Derivatives, and HW2000 is the general version of the algorithm, as specified in the paper published in August 2000.

Default: HW1996

## Output <br> Arguments

## Examples

## HWTree

Structure containing time and interest rate information of a trinomial recombining tree.

Using the data provided, create a Hull-White volatility specification (VolSpec), rate specification (RateSpec), and tree time layout specification (TimeSpec). Then use these specifications to create a Hull-White tree using hwtree.

```
Compounding = -1;
ValuationDate = '01-01-2004';
StartDate = ValuationDate;
VolDates = ['12-31-2004'; '12-31-2005'; '12-31-2006';
'12-31-2007'];
VolCurve = 0.01;
AlphaDates = '01-01-2008';
AlphaCurve = 0.1;
Rates = [0.0275; 0.0312; 0.0363; 0.0415];
HWVolSpec = hwvolspec(ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
RateSpec = intenvset('Compounding', Compounding,...
    'ValuationDate', ValuationDate,...
```


## hwtree

```
    'StartDates', ValuationDate,...
    'EndDates', VolDates,...
    'Rates', Rates);
HWTimeSpec = hwtimespec(ValuationDate, VolDates, Compounding);
HWTree = hwtree(HWVolSpec, RateSpec, HWTimeSpec)
HWTree =
    FinObj: 'HWFwdTree'
    VolSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
        tObs: [0 0.9973 1.9973 2.9973]
        dObs: [731947 732312 732677 733042]
        CFlowT: {[4\times1 double] [3\times1 double] [2x1 double] [3.9973]}
        Probs: {[3\times1 double] [3x3 double] [3\times5 double]}
    Connect: {[2] [2 3 4] [2 2 3 4 4 4]}
    FwdTree: {1x4 cell}
```

Use treeviewer to observe the tree you have created.
treeviewer(HWTree)


## References

See Also

Tutorials

Hull, J., and A. White, "Using Hull-White Interest Rate Trees", Journal of Derivatives, 1996.

Hull, J., and A. White, "The General Hull-White Model and Super Calibration", August 2000.
| hwcalbycap | hwcalbyfloor | hwprice | hwtimespec | hwvolspec | intenvset

- "Calibrating Hull-White Model Using Market Data" on page 2-93


## hwvolspec

Purpose
Syntax

Arguments

| ValuationDate | Scalar value representing the observation date <br> of the investment horizon. |
| :--- | :--- |
| VolDates | Number of points (NPOINTS) -by-1 vector of <br> yield volatility end dates. |
| VolCurve | NPOINTS-by-1 vector or scalar of yield volatility <br> values in decimal form. |
| AlphaDates | MPOINTS-by-1 vector of mean reversion end <br> dates. |
| AlphaCurve | MPOINTS-by-1 vector of positive mean reversion <br> values or scalar in decimal form. |
| InterpMethod | (Optional) Interpolation method. Default is <br> 'linear '. See interp1 for more information. |

Note The number of points in VolCurve and AlphaCurve do not have to be the same.

## Description

Volspec = hwvolspec(ValuationDate, VolDates, VolCurve, AlphaDates, AlphaCurve, InterpMethod) creates a structure specifying the volatility for hwtree.

The volatility process is such that the variance of $r(t+d t)-r(t)$ is defined as follows: $\mathrm{V}=($ Volatility.^2 .* (1-exp(-2*Alpha .* dt)) ) ./ ( 2 * Alpha). For more information on using Hull-White interest rate trees, see "Hull-White (HW) and Black-Karasinski (BK) Modeling" on page C-4.

## Examples Create a Structure Specifying the Volatility for hwtree

This example shows how to create a Hull-White volatility specification (VolSpec) using the following data.

```
ValuationDate = '01-01-2004';
StartDate = ValuationDate;
VolDates = ['12-31-2004'; '12-31-2005'; '12-31-2006';
'12-31-2007'];
VolCurve = 0.01;
AlphaDates = '01-01-2008';
AlphaCurve = 0.1;
HWVolSpec = hwvolspec(ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve)
```


## HWVolSpec =

FinObj: 'HWVolSpec'
ValuationDate: 731947
VolDates: [4x1 double]
VolCurve: [4x1 double]
AlphaCurve: 0.1000
AlphaDates: 733408
VolInterpMethod: 'linear'

See Also bktree | hwcalbycap | hwcalbyfloor | interp1

## impvbybjs

## Purpose

Syntax

## Arguments

Determine implied volatility using Bjerksund-Stensland 2002 option pricing model

Volatility = impvbybjs(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, OptPrice, 'Name1', Value1...)

| RateSpec | The annualized continuously compounded rate <br> term structure. For information on the interest <br> rate specification, see intenvset. |
| :--- | :--- |
| StockSpec | Stock specification. See stockspec. |
| Settle | NINST-by-1 vector of settlement or trade dates. |
| Maturity | NINST-by-1 vector of maturity dates. |
| OptSpec | NINST-by-1 cell array of strings 'call' or 'put'. |
| Strike | NINST-by-1 vector of strike price values. <br> OptPrice |
| NINST-by-1 vector of American option prices from <br> which the implied volatility of the underlying <br> asset are derived. |  |

Note All optional inputs are specified as matching parameter name/parameter value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. You can specify parameter name/parameter value pairs in any order; names are case-insensitive and partial string matches are allowed provided no ambiguities exist.

Limit (Optional) 1-by-2 positive vector representing the lower and upper bound of the implied volatility search interval. Default is [0.1 10], or $10 \%$ to 1000\% per annum.

Tolerance (Optional) Positive scalar implied volatility termination tolerance. Default is 1e-6.

## Description

Volatility = impvbybjs(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, OptPrice, 'Name1', Value1...) computes implied volatility using the Bjerksund-Stensland 2002 option pricing model.

Volatility is a NINST-by-1 vector of expected implied volatility values. If no solution is found, a NaN is returned.

Note impvbybjs computes implied volatility of American options with continuous dividend yield using the Bjerksund-Stensland option pricing model.

## Examples Compute the Implied Volatility Using the Bjerksund-Stensland 2002 Option Pricing Model

This example shows how to compute implied volatility using the Bjerksund-Stensland 2002 option pricing model. Consider three American call options with exercise prices of $\$ 100$ that expire on July 1, 2008. The underlying stock is trading at $\$ 100$ on January 1, 2008 and pays a continuous dividend yield of $10 \%$. The annualized continuously compounded risk-free rate is $10 \%$ per annum and the option prices are $\$ 4.063, \$ 6.77$ and $\$ 9.46$. Using this data, calculate the implied volatility of the stock using the Bjerksund-Stensland 2002 option pricing model.

```
AssetPrice = 100;
Settle = 'Jan-1-2008';
Maturity = 'Jul-1-2008';
```


## impvbybjs

```
Strike = 100;
DivAmount = 0.1;
Rate = 0.1;
% define the RateSpec and StockSpec
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rate, 'Compounding', -1, 'Basis', 1);
StockSpec = stockspec(NaN, AssetPrice, {'continuous'}, DivAmount);
OptSpec = {'call'};
OptionPrice = [4.063;6.77;9.46];
ImpVol = impvbybjs(RateSpec, StockSpec, Settle, Maturity, OptSpec,...
Strike, OptionPrice)
```

ImpVol =
0.1500
0.2501
0.3500

The implied volatility is $15 \%$ for the first call, and $25 \%$ and $35 \%$ for the second and third call options.

References

See Also
optstockbybjs | optstocksensbybjs

## Purpose

Determine implied volatility using Black option pricing model

## Syntax

Volatility = impvbyblk(RateSpec, StockSpec, Settle, Maturity, Strike, OptPrice, 'Name1', Value1...)

Arguments

| RateSpec | The annualized continuously compounded rate <br> term structure. For information on the interest <br> rate specification, see intenvset. |
| :--- | :--- |
| StockSpec | Stock specification. See stockspec. |
| Settle | NINST-by-1 vector of settlement or trade dates. |
| Maturity | NINST-by-1 vector of maturity dates. |
| OptSpec | NINST-by-1 cell array of strings 'call' or 'put '. |
| Strike | NINST-by-1 vector of strike price values. |
| OptPrice | NINST-by-1 vector of European option prices from <br> which the implied volatility of the underlying <br> asset are derived. |

Note All optional inputs are specified as matching parameter name/parameter value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. You can specify parameter name/parameter value pairs in any order; names are case-insensitive and partial string matches are allowed provided no ambiguities exist.

| Limit | (Optional) Positive scalar representing the upper <br> bound of the implied volatility search interval. <br> Default is 10, or $1000 \%$ per annum. |
| :--- | :--- |
| Tolerance | (Optional) Positive scalar implied volatility <br> termination tolerance. Default is 1e-6. |

## Description

## Examples

Volatility = impvbyblk(RateSpec, StockSpec, Settle, Maturity, Strike, OptPrice, 'Name1', Value1...) computes implied volatility using the Black option pricing model.

Volatility is a NINST-by-1 vector of expected implied volatility values. If no solution is found, a NaN is returned.

## Compute the Implied Volatility Using the Black Option Pricing Model

This example shows how to compute the implied volatility using the Black option pricing model. Consider a European call and put options on a futures contract with exercise prices of $\$ 30$ for the put and $\$ 40$ for the call that expire on September 1, 2008. Assume that on May 1, 2008 the contract is trading at $\$ 35$. The annualized continuously compounded risk-free rate is $5 \%$ per annum. Find the implied volatilities of the stock, if on that date, the call price is $\$ 1.14$ and the put price is $\$ 0.82$.

```
AssetPrice = 35;
Strike = [30; 40];
Rates = 0.05;
Settle = 'May-01-08';
Maturity = 'Sep-01-08';
% define the RateSpec and StockSpec
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1);
StockSpec = stockspec(NaN, AssetPrice);
% define the options
OptSpec = {'put';'call'};
Price = [1.14;0.82];
Volatility = impvbyblk(RateSpec, StockSpec, Settle, Maturity, OptSpec,...
Strike, Price)
```

```
Volatility =
    0.4052
    0.3021
```

The implied volatility is $41 \%$ and $30 \%$.

## See Also <br> optstockbyblk | optstocksensbyblk

Purpose
Syntax

Arguments

Determine implied volatility using Black-Scholes option pricing model
Volatility = impvbybls(RateSpec, StockSpec, Settle, Maturity, Strike, OptPrice, 'Name1', Value1...)

RateSpec The annualized continuously compounded rate term structure. For information on the interest rate specification, see intenvset.
StockSpec Stock specification. See stockspec.
Settle NINST-by-1 vector of settlement or trade dates.
Maturity NINST-by-1 vector of maturity dates.
OptSpec NINST-by-1 cell array of strings 'call' or 'put'.
Strike NINST-by-1 vector of strike price values.
OptPrice NINST-by-1 vector of European option prices from which the implied volatility of the underlying asset are derived.

Note All optional inputs are specified as matching parameter name/parameter value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. You can specify parameter name/parameter value pairs in any order; names are case-insensitive and partial string matches are allowed provided no ambiguities exist.

| Limit | (Optional) Positive scalar representing the upper <br> bound of the implied volatility search interval. <br> Default is 10, or $1000 \%$ per annum. |
| :--- | :--- |
| Tolerance | (Optional) Positive scalar implied volatility <br> termination tolerance. Default is 1e-6. |

## Description

Volatility = impvbybls(RateSpec, StockSpec, Settle, Maturity, Strike, OptPrice, 'Name1', Value1...) computes implied volatility using the Black-Scholes option pricing model.

Volatility is a NINST-by-1 vector of expected implied volatility values. If no solution is found, a NaN is returned.

## Examples Compute the Implied Volatility Using the Black-Scholes Option Pricing Model

This example shows how to compute the implied volatility using the Black-Scholes option pricing model. Consider a European call and put options with an exercise price of $\$ 40$ that expires on June 1, 2008. The underlying stock is trading at $\$ 45$ on January 1, 2008 and the risk-free rate is $5 \%$ per annum. The option price is $\$ 7.10$ for the call and $\$ 2.85$ for the put. Using this data, calculate the implied volatility of the European call and put using the Black-Scholes option pricing model.

```
AssetPrice = 45;
Settlement = 'Jan-01-2008';
Maturity = 'June-01-2008';
Strike = 40;
Rates = 0.05;
OptionPrice = [7.10; 2.85];
OptSpec = {'call';'put'};
% define the RateSpec and StockSpec
RateSpec = intenvset('ValuationDate', Settlement, 'StartDates', Settle
'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', 1);
StockSpec = stockspec(NaN, AssetPrice);
ImpvVol = impvbybls(RateSpec, StockSpec, Settlement, Maturity, OptSp
Strike, OptionPrice)
```


## impvbybls

> ImpvVol $=$
> 0.3175
> 0.4878

The implied volatility is $31.75 \%$ for the call and $48.78 \%$ for the put.

## See Also <br> optstockbybls | optstocksensbybls

## Purpose

Syntax
Volatility = impvbyrgw(RateSpec, StockSpec, Settle, Maturity, Strike, OptPrice, 'Name1', Value1...)

## Arguments

| RateSpec | The annualized continuously compounded rate <br> term structure. For information on the interest <br> rate specification, see intenvset. |
| :--- | :--- |
| StockSpec | Stock specification. See stockspec. |
| Settle | NINST-by-1 vector of settlement or trade dates. |
| Maturity | NINST-by-1 vector of maturity dates. |
| Strike | NINST-by-1 vector of strike price values. |
| OptPrice | NINST-by-1 vector of American call option <br> prices from which the implied volatility of the <br> underlying asset are derived. |

Note All optional inputs are specified as matching parameter name/parameter value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. You can specify parameter name/parameter value pairs in any order; names are case-insensitive and partial string matches are allowed provided no ambiguities exist.

| Limit | (Optional) Positive scalar representing the upper <br> bound of the implied volatility search interval. <br> Default is 10, or 1000\% per annum. |
| :--- | :--- |
| Tolerance | (Optional) Positive scalar implied volatility <br> termination tolerance. Default is 1e-6. |

Volatility = impvbyrgw(RateSpec, StockSpec, Settle, Maturity, Strike, OptPrice, 'Name1', Value1...) computes implied volatility using the Roll-Geske-Whaley option pricing model.

Volatility is a NINST-by-1 vector of expected implied volatility values. If no solution is found, a NaN is returned.

Note impvbyrgw computes implied volatility of American calls with a single cash dividend using the Roll-Geske-Whaley option pricing model.

## Examples

## Compute the Implied Volatility Using the Roll-Geske-Whaley Option Pricing Model

This example shows how to compute the implied volatility using the Roll-Geske-Whaley option pricing model. Assume that on July 1, 2008 a stock is trading at $\$ 13$ and pays a single cash dividend of $\$ 0.25$ on November 1, 2008. The American call option with a strike price of $\$ 15$ expires on July 1, 2009 and is trading at $\$ 1.346$. The annualized continuously compounded risk-free rate is $5 \%$ per annum. Calculate the implied volatility of the stock using the Roll-Geske-Whaley option pricing model.

```
AssetPrice = 13;
Strike = 15;
Rates = 0.05;
Settle = 'July-01-08';
Maturity = 'July-01-09';
% define the RateSpec and StockSpec
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
    'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1);
StockSpec = stockspec(NaN, AssetPrice, {'cash'}, 0.25, {'Nov 1,2008'});
Price = [1.346];
Volatility = impvbyrgw(RateSpec, StockSpec, Settle, Maturity, Strike, Pri
```

```
Volatility =
    0.3539
```

See Also optstockbyrgw | optstocksensbyrgw

## instadd

Purpose
Add types to instrument collection
Syntax
InstSet = instadd(InstSetOld, TypeString, Data1, Data2, ...)
InstSet = instadd('CashFlow', CFlowAmounts, CFlowDates, Settle, Basis)
InstSet = instadd('CashFlow', CFlowAmounts, CFlowDates, Settle, Basis)
InstSet = instadd('Barrier', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt, BarrierType, Barrier, Rebate) InstSet = instadd('Bond', CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face)
InstSet = instadd('OptEmBond', CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'AmericanOpt', AmericanOpt, 'Period', Period, 'Basis', Basis, 'EndMonthRule', EndMonthRule,'Face',Face, 'IssueDate', IssueDate, 'FirstCouponDate', FirstCouponDate, 'LastCouponDate', LastCouponDate, 'StartDate', StartDate) InstSet = instadd('OptBond', BondIndex, OptSpec, Strike, ExerciseDates, AmericanOpt)
InstSet = instadd('Cap', Strike, Settle, Maturity, Reset, Basis, Principal)
InstSet = instadd('Compound', UOptSpec, UStrike, USettle, UExerciseDates, UAmericanOpt,COptSpec, CStrike, CSettle, CExerciseDates, CAmericanOpt) InstSet = instadd('Fixed', CouponRate, Settle, Maturity, Reset, Basis, Principal, EndMonthRule)
InstSet = instadd('Float', Spread, Settle, Maturity, Reset, Basis, Principal, EndMonthRule)
InstSet = instadd('Floor', Strike, Settle, Maturity, Reset, Basis, Principal)
InstSet = instadd('Lookback', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt)
InstSet = instadd('OptFloat', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt)
InstSet = instadd('OptEmFloat', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt)
InstSet = instadd('RangeFloat', Spread, Settle, Maturity,

RateSched, Reset, Basis, Principal, EndMonthRule) InstSet = instadd('OptStock', OptSpec, Strike, Settle, Maturity, AmericanOpt)
InstSet = instadd('Swap', LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType, EndMonthRule, StartDate) InstSet = instadd('Swaption', OptSpec, Strike, ExerciseDates, Spread, Settle, Maturity, AmericanOpt, SwapReset, Basis, Principal)

## Description

InstSet $=$ instadd(InstSetOld, TypeString, Data1, Data2, ...) adds an instruments to an existing collection.
InstSet $=$ instadd('CashFlow', CFlowAmounts, CFlowDates, Settle, Basis) adds an arbitrary cash flow instrument. (See also instcf.)

InstSet $=$ instadd('CashFlow', CFlowAmounts, CFlowDates, Settle,
Basis) adds an asian instrument. (See also instasian.)
InstSet = instadd('Barrier', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt, BarrierType, Barrier, Rebate) adds a barrier instrument. (See also instbarrier.)

InstSet = instadd('Bond', CouponRate, Settle, Maturity, Period,
Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face) adds a bond instrument. (See also instbond.)

InstSet $=$ instadd('OptEmBond', CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'AmericanOpt', AmericanOpt, 'Period',
Period, 'Basis', Basis, 'EndMonthRule', EndMonthRule, 'Face', Face,
'IssueDate', IssueDate, 'FirstCouponDate', FirstCouponDate,
'LastCouponDate', LastCouponDate, 'StartDate', StartDate)

## instadd

adds a bond with embedded option instrument. (See also instoptembnd. )

InstSet = instadd('OptBond', BondIndex, OptSpec, Strike, ExerciseDates, AmericanOpt) adds a bond option instrument. (See also instoptbnd.)

InstSet = instadd('Cap', Strike, Settle, Maturity, Reset, Basis, Principal) adds a cap instrument. (See also instcap.)

InstSet = instadd('Compound', UOptSpec, UStrike, USettle, UExerciseDates, UAmericanOpt,COptSpec, CStrike, CSettle, CExerciseDates, CAmericanOpt) adds a compound instrument. (See also instcompound.)

InstSet = instadd('Fixed', CouponRate, Settle, Maturity, Reset,
Basis, Principal, EndMonthRule) adds a fixed-rate note instrument. (See also instfixed.)

InstSet = instadd('Float', Spread, Settle, Maturity, Reset, Basis, Principal, EndMonthRule) adds a floating-rate note instrument. (See also instfloat.)

InstSet = instadd('Floor', Strike, Settle, Maturity, Reset, Basis, Principal) adds a floor instrument. (See also instfloor.)

InstSet = instadd('Lookback', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt) adds a lookback instrument. (See also instlookback.)

InstSet = instadd('OptFloat', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt) adds a floating-rate option instrument. (See also instoptfloat.)

InstSet = instadd('OptEmFloat', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt) adds a floating-rate embedded option instrument. (See also instoptemfloat.)

InstSet = instadd('RangeFloat', Spread, Settle, Maturity, RateSched, Reset, Basis, Principal, EndMonthRule) adds a range floating note instrument. (See also instrangefloat.)

InstSet = instadd('OptStock', OptSpec, Strike, Settle, Maturity, AmericanOpt) adds a stock option instrument. (See also instoptstock.)

InstSet = instadd('Swap', LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType, EndMonthRule, StartDate) adds a swap instrument. (See also instswap.)

InstSet = instadd('Swaption', OptSpec, Strike, ExerciseDates, Spread, Settle, Maturity, AmericanOpt, SwapReset, Basis, Principal) adds a swaption instrument. (See also instswaption.)
instadd stores instruments of types 'Asian', 'Barrier', 'Bond', 'Cap', 'CashFlow', 'Compound', 'Fixed', 'Float', 'Floor', 'Lookback', 'OptBond', 'OptStock', 'Swap', or 'Swaption'. Financial Instruments Toolbox provides pricing and sensitivity routines for these instruments.

## Input <br> Arguments

## Output <br> Arguments

Examples Create a Portfolio with Two Cap Instruments and a 4\% Bond
Define the bond:

## instadd

```
Strike = [0.06; 0.07];
CouponRate = 0.04;
Settle = '06-Feb-2000';
Maturity = '15-Jan-2003';
```

Create a portfolio with two cap instruments and a $4 \%$ bond and then display the portfolio:

```
InstSet = instadd('Cap', Strike, Settle, Maturity);
InstSet = instadd(InstSet, 'Bond', CouponRate, Settle, Maturity);
instdisp(InstSet)
```

| Index | Type Strike Settle | Maturity | CapReset | Basis Principal |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | Cap | 0.06 | $06-$ Feb-2000 | $15-$ Jan-2003 | 1 | 0 | 100 |
| 2 | Cap | 0.07 | $06-$ Feb-2000 | $15-$ Jan-2003 | 1 | 0 | 100 |


| Index | Type CouponRate Settle | Maturity | Period Basis EndMonth |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 3 | Bond 0.04 | $06-$ Feb-2000 | $15-$ Jan-2003 | 2 | 0 |

See Also | instasian \| instbarrier | instbond | instcap | instcf | |
| :--- |
| instcompound \| instfixed | instfloat | instfloor | instlookback |
| linstoptbnd \| instoptembnd | instoptstock | instswap | |
| instswaption \| instaddfield | instdisp |

Related

- "Portfolio Creation" on page 1-7

Examples

- "Creating Instruments or Properties" on page 1-18

Concepts - "Instrument Constructors" on page 1-17

## Purpose

Add new instruments to instrument collection
Syntax

```
InstSet = instaddfield('FieldName', FieldList, 'Data',
DataList, 'Type', TypeString)
InstSet = instaddfield('FieldName', FieldList, 'FieldClass',
ClassList, 'Data', DataList, 'Type', TypeString)
InstSetNew = instaddfield(InstSet, 'FieldName', FieldList,
'Data', DataList,'Type',TypeString)
```


## Arguments

## instaddfield

| TypeString | String specifying the type of instrument added. <br> Instruments of different types can have different <br> Fieldname collections. |
| :--- | :--- |
| InstSet | Variable containing a collection of instruments. <br> Instruments are classified by type; each type can <br> have different Data fields. The stored Data field is a <br> row vector or string for each instrument. |

## Description

Use instaddfield to create your own types of instruments or to append new instruments to an existing collection. Argument value pairs can be entered in any order.

```
InstSet = instaddfield('FieldName', FieldList, 'Data',
DataList, 'Type', TypeString)
InstSet = instaddfield('FieldName', FieldList,
'FieldClass',
ClassList, 'Data', DataList, 'Type', TypeString) creates an
instrument variable.
InstSetNew = instaddfield(InstSet, 'FieldName', FieldList,
'Data', DataList,'Type',TypeString) adds instruments to an
existing instrument set, InstSet. The output InstSetNew is a new
instrument set containing the input data.
```


## Examples

Build a portfolio around July options.

```
Strike Call Put
    95 12.2 2.9
100 9.2 4.9
105 6.8 7.4
Strike = (95:5:105)'
CallP = [12.2; 9.2; 6.8]
```

Enter three call options with data fields Strike, Price, and Opt.

```
InstSet = instaddfield('Type','Option','FieldName',...
{'Strike','Price','Opt'}, 'Data',{ Strike, CallP, 'Call'});
    instdisp(InstSet)
\begin{tabular}{llcrl} 
Index & Type & Strike & Price & Opt \\
1 & Option & 95 & 12.2 & Call \\
2 & Option 100 & 9.2 & Call \\
3 & Option & 105 & 6.8 & Call
\end{tabular}
```

Add a futures contract and set the input parsing class.

```
InstSet = instaddfield(InstSet,'Type','Futures',...
'FieldName',{'Delivery','F'},'FieldClass',{'date','dble'},...
'Data' ,{'01-Jul-99',104.4 });
instdisp(InstSet)
Index Type Strike Price Opt
1 Option 95 12.2 Call
2 Option 100 9.2 Call
3 Option 105 6.8 Call
\begin{tabular}{llll} 
Index & Type & Delivery & F \\
4 & Futures & 01 -Jul-1999 & 104.4
\end{tabular}
```

Add a put option.

```
FN = instfields(InstSet,'Type','Option')
InstSet = instaddfield(InstSet,'Type','Option',...
'FieldName',FN, 'Data',{105, 7.4, 'Put'});
instdisp(InstSet)
Index Type Strike Price Opt
1 Option 95 12.2 Call
2 Option 100 9.2 Call
3 Option 105 6.8 Call
Index Type Delivery F
4 Futures 01-Jul-1999 104.4
```


## instaddfield

| Index | Type | Strike Price | Opt |
| :--- | :--- | :--- | :--- | :--- |
| 5 | Option 105 | 7.4 | Put |

Make a placeholder for another put.

```
InstSet = instaddfield(InstSet,'Type','Option',...
'FieldName','Opt','Data','Put')
instdisp(InstSet)
Index Type Strike Price Opt
1 Option 95 12.2 Call
2 Option 100 9.2 Call
3 Option 105 6.8 Call
Index Type Delivery F
4 Futures 01-Jul-1999 104.4
\begin{tabular}{lllll} 
Index & Type & Strike Price & Opt \\
5 & Option 105 & 7.4 & Put
\end{tabular}
```

Add a cash instrument.

```
InstSet = instaddfield(InstSet, 'Type', 'TBill',...
'FieldName','Price','Data',99)
instdisp(InstSet)
Index Type Strike Price Opt
1 Option 95 12.2 Call
2 Option 100 9.2 Call
3 Option 105 6.8 Call
Index Type Delivery F
4 Futures 01-Jul-1999 104.4
Index Type Strike Price Opt
5 Option 105 7.4 Put
```

| 6 | Option NaN | NaN Put |  |
| :--- | :--- | :--- | :--- |
| Index | Type Price |  |  |
| 7 | TBill 99 |  |  |

See Also instdisp | instget | instgetcell|instsetfield | instadd

## instasian

Purpose
Construct Asian option

```
Syntax InstSet = instasian(InstSet, OptSpec, Strike, Settle,
    ExerciseDates, AmericanOpt, AvgType, AvgPrice, AvgDate)
[FieldList, ClassList, TypeString] = instasian
```


## Arguments

| InstSet | Instrument variable. This argument is specified <br> only when adding asian instruments to an <br> existing instrument set. See instget for more <br> information on the InstSet variable. |
| :--- | :--- |
| OptSpec | NINST-by-1 list of string values 'Call' or 'Put' |
| Strike | NINST-by-1 vector of strike price values. Each |
| row is the schedule for one option. |  |
| ExerciseDates | NINST-by-1 vector of Settle dates. |
| For a European option (AmericanOpt = 0): |  |
|  | NINST-by-1 vector of exercise dates. Each row <br> is the schedule for one option. For a European <br> option, there is only one exercise date, the option <br> expiry date. |
| For an American option (AmericanOpt = 1): |  |
| NINST-by-2 vector of exercise date boundaries. |  |
| For each instrument, the option can be exercised <br> on any tree date between or including the pair <br> of dates on that row. If only one non-NaN date <br> is listed, or if ExerciseDates is NINST-by-1, the |  |
| option can be exercised between the valuation |  |
| date of the stock tree and the single listed |  |
| exercise date. |  |


| AmericanOpt | (Optional) If AmericanOpt $=0, N a N$, or is <br> unspecified, the option is a European option. If <br> AmericanOpt $=1$, the option is an American <br> option. |
| :--- | :--- |
| AvgType | (Optional) String $=$ 'arithmetic' for arithmetic <br> average (default) or 'geometric' for geometric <br> average. |
| AvgPrice | (Optional) Scalar representing the average price <br> of the underlying asset at Settle. This argument <br> is used when AvgDate < Settle. Default is the <br> current stock price. |
| AvgDate | (Optional) Scalar representing the date on which <br> the averaging period begins. Default = Settle. |

Data arguments are NINST-by-1 vectors, scalar, or empty. Fill unspecified entries in vectors with NaN. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices [].

## Description

```
InstSet = instasian(InstSet, OptSpec, Strike, Settle,
ExerciseDates, AmericanOpt, AvgType, AvgPrice, AvgDate)
specifies an Asian option.
```

[FieldList, ClassList, TypeString] = instasian displays the classes.

FieldList is a number of fields (NFIELDS)-by-1 cell array of strings listing the name of each data field for this instrument type.

ClassList is an NFIELDS-by- 1 cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are 'dble', 'date', and 'char'.

TypeString is a string specifying the type of instrument added. For an Asian option instrument, TypeString = 'Asian'.

## instasian

## Examples Create an Asian Option Instrument

Load the example instrument set, deriv.mat, and set the required values for an asian option instrument.
load deriv.mat
Create a subportfolio with barrier and lookback options.
CRRSubSet = instselect(CRRInstSet,'Type',\{'Barrier', 'Lookback'\});
Define the asian instrument.

```
OptSpec = 'put';
Strike = NaN;
Settle = '01-Jan-2003';
ExerciseDates = '01-Jan-2004';
```

Add a floating strike asian option to the instrument set.
InstSet = instasian(CRRSubSet, OptSpec, Strike, Settle, ExerciseDates); instdisp(InstSet)

| Index | Type | OptSpec Strike Settle | ExerciseDates | American0pt Be |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | Barrier call | 105 | $01-J a n-2003$ | $01-J a n-2006$ | 1 | ui |
|  |  |  |  |  |  |  |
| Index | Type | OptSpec | Strike | Settle | ExerciseDates | American0pt |
| 2 | Lookback call | 115 | $01-J a n-2003$ | $01-J a n-2006$ | 0 | L |
| 3 | Lookback call | 115 | $01-J a n-2003$ | $01-J a n-2007$ | 0 |  |


| Index | Type OptSpec | Strike Settle |  |
| :--- | :--- | :--- | :--- |
| 4 | Asian put | NaN | 01-Jan-2003 |

$\begin{array}{lll}\text { ExerciseDates } & \text { AmericanOpt AvgT } \\ 01-\text { Jan-2004 } & 0 & \text { arit }\end{array}$

See Also instadd | instdisp | instget

## Purpose

Construct barrier option

Syntax<br>InstSet = instbarrier(InstSet, OptSpec, Strike, Settle, ExerciseDates, AmericanOpt, BarrierSpec, Barrier, Rebate) [FieldList, ClassList, TypeString] = instbarrier

## Arguments

## instbarrier

| AmericanOpt | If AmericanOpt $=0$, NaN, or is unspecified, the <br> option is a European option. If AmericanOpt $=1$, <br> the option is an American option. |
| :--- | :--- |
| BarrierSpec | List of string values: <br>  <br> 'UI': Up Knock In |
|  | 'UO': Up Knock Out |
|  | 'DI': Down Knock In |
| Barrier | 'DO': Down Knock Out |
| Rebate | Vector of barrier values. <br> (Optional) Vector of rebate values. |

Data arguments are NINST-by-1 vectors, scalar, or empty. Fill unspecified entries in vectors with NaN. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices [].

## Description

InstSet = instbarrier(InstSet, OptSpec, Strike, Settle, ExerciseDates, AmericanOpt, BarrierSpec, Barrier, Rebate) specifies a barrier option.
[FieldList, ClassList, TypeString] = instbarrier displays the classes.

FieldList is a number of fields (NFIELDS)-by-1 cell array of strings listing the name of each data field for this instrument type.

ClassList is an NFIELDS-by- 1 cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are 'dble', 'date', and 'char'.

TypeString is a string specifying the type of instrument added. For a barrier option instrument, TypeString = 'Barrier'.

## Examples Create Two Barrier Option Instruments

Create an instrument set of two barrier options with the following data:

```
OptSpec = {'put';'call'};
Strike = 112;
Settle = '01-Jan-2012';
ExerciseDates = '01-Jan-2015';
BarrierSpec = {'do';'ui'};
Barrier = [101;102];
AmericanOpt = 0;
```

Create the instrument set (InstSet) for the two barrier options.
InstSet = instbarrier(OptSpec, Strike, Settle, ExerciseDates,American
Display the instrument set.

```
instdisp(InstSet)
```

| Index | Type | OptSpec | Strike Settle | ExerciseDates | American0p |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | Barrier put | 112 | $01-$ Jan-2012 | $01-$ Jan-2015 | 0 |
| 2 | Barrier call | 112 | 01 -Jan-2012 | $01-$ Jan-2015 | 0 |

See Also instadd | instdisp | instget

## instbond

Purpose Construct bond instrument

```
Syntax InstSet = instbond(InstSet, CouponRate, Settle, Maturity,
Period, Basis, EndMonthRule, IssueDate, FirstCouponDate,
LastCouponDate, StartDate, Face)
[FieldList, ClassList, TypeString] = instbond
```


## Arguments

\(\left.$$
\begin{array}{ll}\text { InstSet } & \begin{array}{l}\text { Instrument variable. This argument is } \\
\text { specified only when adding bond instruments } \\
\text { to an existing instrument set. See instget } \\
\text { for more information on the InstSet variable. }\end{array} \\
\text { CouponRate } & \begin{array}{l}\text { Decimal annual rate indicating the annual } \\
\text { percentage rate used to determine the } \\
\text { coupons payable on a bond. CouponRate is a } \\
\text { NINST-by-1 vector or NINST-by-1 cell array of } \\
\text { decimal annual rates, or decimal annual rate } \\
\text { schedules. For the latter case of a variable } \\
\text { coupon schedule, each element of the cell } \\
\text { array is a NumDates-by-2 cell array, where the }\end{array}
$$ <br>
first column is dates and the second column <br>
is its associated rate. The date indicates the <br>

last day that the coupon rate is valid.\end{array}\right\}\)| Settlement date. A vector of serial date |
| :--- |
| Settle |
| numbers or date strings. Settle must be |
| earlier than Maturity. |

Basis (Optional) Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual (default)
- 1 = 30/360 (SIA)
- 2 = actual/360
- $3=$ actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual $/ 365$ (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=\mathrm{actual} /$ actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
EndMonthRule (Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. $0=$ ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. $1=$ set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.

IssueDate (Optional) Date when a bond was issued.

## instbond

| FirstCouponDate | (Optional) Last coupon date of a bond before <br> the maturity date; used when bond has an <br> irregular last coupon period. In the absence <br> of a specified FirstCouponDate, a specified <br> LastCouponDate determines the coupon <br> structure of the bond. The coupon structure <br> of a bond is truncated at the LastCouponDate, <br> regardless of where it falls, and is followed <br> only by the bond's maturity cash flow date. <br> If you do not specify a LastCouponDate, the <br> cash flow payment dates are determined from <br> other inputs. <br> (Optional) Last coupon date of a bond before |
| :--- | :--- |
| the maturity date; used when bond has an |  |
| irregular last coupon period. In the absence |  |
| of a specified FirstCouponDate, a specified |  |
|  | LastCouponDate determines the coupon |
| structure of the bond. The coupon structure |  |
| of a bond is truncated at the LastCouponDate, |  |
| regardless of where it falls, and is followed |  |
| only by the bond's maturity cash flow date. |  |
| If you do not specify a LastCouponDate, the |  |

column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid. Default = 100.

Data arguments are number of instruments (NINST)-by-1 vectors, scalar, or empty. Fill unspecified entries in vectors with NaN. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices [].

## Description

## Examples

## Create a Bond Instrument

Create a new instrument variable with the following information:

```
CouponRate= [0.035;0.04];
Settle= 'Nov-1-2013';
Maturity = 'Nov-1-2014';
Period =1;
```


## instbond

```
InstSet = instbond(CouponRate, Settle, Maturity, ...
Period)
InstSet =
    FinObj: 'Instruments'
    IndexTable: [1x1 struct]
        Type: {'Bond'}
    FieldName: {{11x1 cell}}
    FieldClass: {{11x1 cell}}
    FieldData: {{11x1 cell}}
```

Display the instrument set.

```
instdisp(InstSet)
```

| Index | Type CouponRate | Settle | Maturity | Period Basis EndMontr |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | Bond 0.035 | $01-$ Nov-2013 | $01-$ Nov-2014 | 1 | 0 | 1 |
| 2 | Bond 0.04 | $01-$ Nov-2013 | $01-$ Nov-2014 | 1 | 0 | 1 |

See Also hjmprice | instaddfield | instdisp | instget | intenvprice

## Purpose Construct cap instrument

```
Syntax
InstSet = instcap(InstSet, Strike, Settle, Maturity, Reset,
Basis, Principal)
[FieldList, ClassList, TypeString] = instcap
```


## Arguments

| InstSet | Instrument variable. This argument is specified <br> only when adding cap instruments to an existing <br> instrument set. See instget for more information <br> on the InstSet variable. |
| :--- | :--- |
| Strike | Rate at which the cap is exercised, as a decimal <br> number. |
| Settle | Settlement date. Serial date number representing <br> the settlement date of the cap. |
| Maturity | Serial date number representing the maturity date of <br> the cap. <br> (Optional) NINST-by-1 vector representing the |
| Reset | frequency of payments per year. Default $=1$. |
| Basis | (Optional) Day-count basis of the instrument. A |
|  | vector of integers. |
|  | - $0=$ actual/actual (default) |
|  | - $1=30 / 360$ (SIA) |
|  | - $2=$ actual/360 |
|  | - $3=$ actual/365 |
|  | - $4=30 / 360$ (BMA) |
|  | - $5=30 / 360$ (ISDA) |
|  | - $6=30 / 360$ (European) |
|  | - $7=$ actual/365 (Japanese) |

## instcap

- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Principal (Optional) The notional principal amount. Default $=100$.

## Description

InstSet = instcap(InstSet, Strike, Settle, Maturity, Reset, Basis, Principal) creates a new instrument set containing cap instruments or adds cap instruments to an existing instrument set.
[FieldList, ClassList, TypeString] = instcap displays the classes.

FieldList is a number of fields (NFIELDS)-by- 1 cell array of strings listing the name of each data field for this instrument type.

ClassList is an NFIELDS-by- 1 cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are 'dble', 'date', and 'char'.

TypeString is a string specifying the type of instrument added. For a cap instrument, TypeString = 'Cap'.

## Examples Create Two Cap Instruments

Create a new instrument variable with the following information:

```
Strike = [0.035; 0.045];
Settle= 'Jan-1-2013';
Maturity = 'Jan-1-2014';
```

```
Reset = 1;
Basis = 1;
Principal = 1000;
```

Create the new cap instruments.
InstSet = instcap(Strike, Settle, Maturity, Reset, Basis, Principal)
InstSet =
FinObj: 'Instruments
IndexTable: [1x1 struct]
Type: $\left\{{ }^{\prime}\right.$ Cap' $\}$
FieldName: $\{\{6 \times 1$ cell $\}$
FieldClass: $\{\{6 \times 1$ cell $\}$
FieldData: $\{\{6 \times 1$ cell $\}$

Display the cap instruments.
instdisp(InstSet)

| Index | Type Strike Settle | Maturity | CapReset | Basis Princip |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | Cap | 0.035 | 01-Jan-2013 | 01 -Jan-2014 | 1 | 1 |
| 2 | Cap | 0.045 | 01-Jan-2013 | 01 -Jan-2014 | 1 | 1000 |
| 2 |  |  |  |  |  |  |

See Also
hjmprice | instaddfield | instbond | instdisp | instfloor | instswap | intenvprice

## instcf

Purpose

```
Syntax InstSet = instcf(InstSet, CFlowAmounts, CFlowDates, Settle,
Basis)
[FieldList, ClassList, TypeString] = instcf
```


## Arguments

| InstSet | Instrument variable. This argument is specified <br> only when adding cash flow instruments to an <br> existing instrument set. See instget for more <br> information on the InstSet variable. |
| :--- | :--- |
| CFlowAmounts | Number of instruments (NINST) by maximum <br> number of cash flows (MOSTCFS) matrix of cash <br> flow amounts. Each row is a list of cash flow <br> values for one instrument. If an instrument has <br> fewer than MOSTCFS cash flows, the end of the row <br> is padded with NaNs. |
| CFlowDates | NINST-by-MOSTCFS matrix of cash flow dates. <br> Each entry contains the date of the corresponding <br> cash flow in CFlowAmounts. |
| Settle | Settlement date on which the cash flows are <br> priced. |
| (Optional) Day-count basis of the instrument. A |  |
| vector of integers. |  |$\quad$| - $0=$ actual/actual (default) |
| :--- |

- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.

Only one data argument is required to create an instrument. Other arguments can be omitted or passed as empty matrices []. Dates can be input as serial date numbers or date strings.

## Description

InstSet = instcf(InstSet, CFlowAmounts, CFlowDates, Settle, Basis) creates a new instrument set from data arrays or adds instruments of type CashFlow to an instrument set.
[FieldList, ClassList, TypeString] = instcf lists field metadata for an instrument of type CashFlow.

FieldList is a number of fields (NFIELDS)-by-1 cell array of strings listing the name of each data field for this instrument type.

ClassList is an NFIELDS-by- 1 cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are 'dble', 'date', and 'char'.

TypeString specifies the type of instrument added; for example, TypeString = 'CashFlow'.

See Also instadd | instdisp | instget | intenvprice

## instcompound

Purpose
Syntax
Construct compound option

InstSet = instcompound(InstSet, UOptSpec, UStrike, USettle, UExerciseDates, UAmericanOpt, COptSpec, CStrike, CSettle, CExerciseDates, CAmericanOpt)
[FieldList, ClassList, TypeString] = instcompound

## Arguments

| InstSet | Instrument variable. This argument is specified <br> only when adding compound instruments to an <br> existing instrument set. See instget for more <br> information on the InstSet variable. |
| :--- | :--- |
| UOptSpec | String = 'Call' or 'Put '. |
| UStrike | 1-by-1 vector of strike price values. |
| USettle | 1-by-1 vector of Settle dates. |

\(\left.$$
\begin{array}{ll}\text { CStrike } & \begin{array}{l}\text { NINST-by-1 vector of strike price values. Each } \\
\text { row is the schedule for one option. }\end{array} \\
\text { CSettle } & \begin{array}{l}\text { 1-by-1 vector containing the settlement or trade } \\
\text { date. }\end{array} \\
\text { CExerciseDates } & \begin{array}{l}\text { For a European option (CAmericanOpt = 0): } \\
\\
\text { NINST-by-1 vector of exercise dates. Each row } \\
\text { is the schedule for one option. For a European } \\
\text { option, there is only one exercise date, the } \\
\text { option expiry date. }\end{array} \\
& \begin{array}{l}\text { For an American option (CAmericanOpt = 1): }\end{array} \\
& \begin{array}{l}\text { NINST-by-2 vector of exercise date boundaries. } \\
\text { For each instrument, the option can be } \\
\text { exercised on any tree date between or including } \\
\text { the pair of dates on that row. If only one }\end{array}
$$ <br>
non-NaN date is listed, or if ExerciseDates <br>
is NINST-by-1, the option can be exercised <br>

between the valuation date of the stock tree\end{array}\right\}\)| and the single listed exercise date. |
| :--- |

Description InstSet = instcompound(InstSet, UOptSpec, UStrike, USettle, UExerciseDates, UAmericanOpt, COptSpec, CStrike, CSettle, CExerciseDates, CAmericanOpt) specifies a compound option.
[FieldList, ClassList, TypeString] = instcompound displays the classes.

FieldList is a number of fields (NFIELDS)-by-1 cell array of strings listing the name of each data field for this instrument type.

## instcompound

ClassList is an NFIELDS-by- 1 cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are 'dble', 'date', and 'char'.

TypeString is a string specifying the type of instrument added. For a compound option instrument, TypeString = 'Compound'.

## Examples Create a Compound Option Instrument

Define a compound option instrument with the following data:

```
UOptSpec = 'Call';
UStrike = 130;
USettle = '01-Jan-2012';
UExerciseDates = '01-Jan-2015';
UAmericanOpt = 0;
COptSpec = 'Put';
CStrike = 5;
CSettle = '01-Jan-2012';
CExerciseDates = '01-Jan-2014';
CAmericanOpt = 0;
InstSet = instcompound(UOptSpec, UStrike, USettle,UExerciseDates, ...
UAmericanOpt, COptSpec, CStrike, CSettle,CExerciseDates, CAmericanOpt)
InstSet = instcompound(UOptSpec, UStrike, USettle,UExerciseDates, ...
UAmericanOpt, COptSpec, CStrike, CSettle,CExerciseDates)
```


## InstSet =

## FinObj: 'Instruments'

IndexTable: [1x1 struct] Type: \{'Compound'\}
FieldName: \{\{10x1 cell\}\}
FieldClass: \{\{10x1 cell\}\}
FieldData: \{\{10x1 cell\}\}

```
InstSet =
    FinObj: 'Instruments'
IndexTable: [1x1 struct]
    Type: {'Compound'}
    FieldName: {{10x1 cell}}
FieldClass: {{10x1 cell}}
    FieldData: {{10x1 cell}}
```

Display the instrument set.
instdisp(InstSet)
Index Type UOptSpec UStrike USettle UExerciseDates UAmeric

See Also instadd | instdisp | instget

## instdelete

Purpose Complement of instrument set by matching conditions

```
Syntax ISubSet = instdelete(InstSet, 'FieldName', FieldList, 'Data',
DataList, 'Index', IndexSet, 'Type', TypeList)
```


## Arguments

| InstSet | Variable containing a collection of instruments. <br> Instruments are classified by type; each type can have <br> different data fields. The stored data field is a row <br> vector or string for each instrument. |
| :---: | :--- |
| FieldList | String or number of fields (NFIELDS)-by-1 cell array <br> of strings listing the name of each data field to match <br> with data values. |
| DataList | Number of values (NVALUES)-by-M array or <br> NFIELDS-by-1 cell array of acceptable data values for <br> each field. Each row lists a data row value to search <br> for in the corresponding FieldList. The number of <br> columns is arbitrary and matching will ignore trailing |
| NaNs or spaces. |  |

> Note Argument value pairs can be entered in any order. The InstSet variable must be the first argument. 'FieldName' and 'Data' arguments must appear together or not at all.

## Description

The output argument ISubSet contains instruments not matching the input criteria. Instruments are deleted from ISubSet if all the Field, Index, and Type conditions are met. An instrument meets an individual Field condition if the stored FieldName data matches any of the rows listed in the DataList for that FieldName. See instfind for more examples on matching criteria.

## Examples

Retrieve the instrument set variable ExampleInst from the data file InstSetExamples.mat. The variable contains three types of instruments: Option, Futures, and TBill.
load InstSetExamples;
instdisp(ExampleInst)
instdisp(ExampleInst)

| Index | Type | Strike | Price | Opt | Contracts |
| :--- | :--- | :---: | :---: | :---: | :---: |
| 1 | Option | 95 | 12.2 | Call | 0 |
| 2 | Option 100 | 9.2 | Call | 0 |  |
| 3 | Option 105 | 6.8 | Call | 1000 |  |


| Index | Type | Delivery | F | Contracts |
| :--- | :--- | :--- | :--- | :--- |
| 4 | Futures | 01-Jul-1999 | 104.4 | -1000 |


| Index | Type | Strike | Price | Opt | Contracts |
| :--- | :--- | :---: | :---: | :---: | :---: |
| 5 | Option | 105 | 7.4 | Put | -1000 |
| 6 | Option | 95 | 2.9 | Put | 0 |


| Index | Type Price | Maturity | Contracts |
| :--- | :--- | :--- | :--- |
| 7 | TBill 99 | $01-J u l-1999$ | 6 |

Create a new variable, ISet, with all options deleted.

## instdelete

```
ISet = instdelete(ExampleInst, 'Type','Option');
instdisp(ISet)
\begin{tabular}{lllll} 
Index & Type & Delivery & F & Contracts \\
1 & Futures & 01-Jul-1999 & 104.4 & -1000
\end{tabular}
Index Type Price Maturity Contracts
2 TBill 99 01-Jul-1999 6
```

See Also instaddfield | instfind | instget | instselect

Purpose Display instruments
Syntax CharTable = instdisp(InstSet)
Arguments

Description
CharTable $=$ instdisp(InstSet) creates a character array displaying the contents of an instrument collection, InstSet. If instdisp is called without output arguments, the table is displayed in the Command Window.

Note When using instdisp, a value of NaN in one of the columns for an instrument indicates that the default value for that parameter will be used in the instrument's pricing function.

CharTable is a character array with a table of instruments in InstSet. For each instrument row, the Index and Type are printed along with the field contents. Field headers are printed at the tops of the columns.
$\begin{array}{ll}\text { Examples } & \begin{array}{l}\text { Retrieve the instrument set ExampleInst from the data file } \\ \text { InstSetExamples.mat. ExampleInst contains three types of } \\ \text { instruments: Option, Futures, and TBill. }\end{array} \\ \text { load InstSetExamples; } \\ \text { instdisp(ExampleInst) } \\ \text { Index Type Strike Price Opt Contracts } \\ 1 & \text { Option } 95 \\ 2 & \text { Option } 100\end{array}$

| Index | Type | Strike | Price | Opt | Contracts |
| :--- | :--- | :---: | :---: | :---: | :---: |
| 1 | Option | 95 | 12.2 | Call | 0 |
| 2 | Option 100 | 9.2 | Call | 0 |  |

## instdisp

| 3 | Option 105 | 6.8 | Call | 1000 |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Index | Type | Delivery | F | Contracts |  |
| 4 | Futures | 01 -Jul-1999 | 104.4 | -1000 |  |
|  |  |  |  |  |  |
| Index | Type | Strike | Price | Opt | Contracts |
| 5 | Option | 105 | 7.4 | Put | -1000 |
| 6 | Option | 95 | 2.9 | Put | 0 |
|  |  |  |  |  |  |
| Index | Type Price | Maturity | Contracts |  |  |
| 7 | TBill 99 | $01-J u l-1999$ | 6 |  |  |

Create a swap instrument and use instdisp to display the instrument. Notice that value of NaN in two columns for this instrument indicates that the default values for LegReset and LegType parameters will be used in the swap instrument's pricing function.

```
LegRate1 = [0.065, 0];
Settle1 = datenum('jan-1-2007');
Maturity1 = datenum('jan-1-2012');
ISet = instswap(LegRate1, Settle1, Maturity1);
instdisp(ISet)
```



```
See Also
datestr | num2str | instaddfield | instget
```



## instfields

| Index | Type | Strike Price Opt | Contracts |
| :--- | :--- | :--- | :---: | :---: |
| 5 | Option 105 | 7.4 Put | -1000 |
| 6 | Option 95 | 2.9 Put | 0 |
|  |  |  |  |
| Index | Type Price Maturity | Contracts |  |
| 7 | TBill 99 | $01-J u l-1999$ | 6 |

Get the fields listed for type 'Option'.

```
[FieldList, ClassList] = instfields(ExampleInst, 'Type',...
'Option')
FieldList =
    'Strike'
    'Price'
    'Opt'
    'Contracts'
ClassList =
    'dble'
    'dble'
    'char'
    'dble'
```

Get the fields listed for types 'Option' and 'TBill'.
FieldList $=$ instfields(ExampleInst, 'Type', \{'Option', 'TBill'\})
FieldList =
Strike'
'Opt ${ }^{\prime}$
'Price'
'Maturity'
'Contracts'

Get all the fields listed in any type in the variable.

```
FieldList = instfields(ExampleInst)
FieldList =
    'Delivery'
    'F'
    'Strike'
    'Opt'
    'Price'
    'Maturity'
    'Contracts'
```

See Also instdisp | instlength | insttypes

## instfind

```
Purpose Search instruments for matching conditions
Syntax IndexMatch = instfind(InstSet, 'FieldName', FieldList, 'Data',
DataList,'Index', IndexSet, 'Type', TypeList)
```

Arguments

| InstSet | Variable containing a collection of instruments. <br> Instruments are classified by type; each type can <br> have different data fields. The stored data field is a <br> row vector or string for each instrument. |
| :---: | :--- |
| FieldList | String or number of fields (NFIELDS)-by-1 cell array <br> of strings listing the name of each data field to <br> match with data values. |
| DataList | Number of values (NVALUES)-by-M array or <br> NFIELDS-by-1 cell array of acceptable data values <br> for each field. Each row lists a data row value to <br> search for in the corresponding FieldList. The <br> number of columns is arbitrary, and matching will <br> ignore trailing NaNs or spaces. |
| IndexSet | (Optional) Number of instruments (NINST)-by-1 <br> vector restricting positions of instruments to check <br> for matches. The default is all indices available in <br> the instrument variable. |
| TypeList | (Optional) String or number of types (NTYPES)-by-1 <br> cell array of strings restricting instruments to <br> match one of TypeList types. The default is all |
|  | types in the instrument variable. |

Argument value pairs can be entered in any order. The InstSet variable must be the first argument. 'FieldName' and 'Data' arguments must appear together or not at all.

## Description

Examples

IndexMatch = instfind(InstSet, 'FieldName', FieldList, 'Data',
DataList,'Index', IndexSet, 'Type', TypeList) returns indices of instruments matching Type, Field, or Index values.

IndexMatch is an NINST-by- 1 vector of positions of instruments matching the input criteria. Instruments are returned in IndexMatch if all the Field, Index, and Type conditions are met. An instrument meets an individual Field condition if the stored FieldName data matches any of the rows listed in the DataList for that FieldName.

Retrieve the instrument set ExampleInst from the data file InstSetExamples.mat. ExampleInst contains three types of instruments: Option, Futures, and TBill.

## load InstSetExamples;

instdisp(ExampleInst)

| Index | Type | Strike | Price | Opt | Contracts |
| :--- | :--- | :--- | :--- | :--- | :---: |
| 1 | Option | 95 | 12.2 | Call | 0 |
| 2 | Option 100 | 9.2 | Call | 0 |  |
| 3 | Option 105 | 6.8 | Call | 1000 |  |


| Index | Type | Delivery | F | Contracts |
| :--- | :--- | :--- | :--- | :--- |
| 4 | Futures | 01 -Jul-1999 | 104.4 | -1000 |


| Index | Type | Strike | Price | Opt | Contracts |
| :--- | :--- | :--- | :--- | :--- | :---: |
| 5 | Option | 105 | 7.4 | Put | -1000 |
| 6 | Option 95 | 2.9 | Put | 0 |  |


| Index | Type | Price Maturity | Contracts |
| :--- | :--- | :--- | :--- | :--- |
| 7 | TBill 99 | $01-J u l-1999$ | 6 |

Make a vector, Opt95, containing the indexes within ExampleInst of the options struck at 95 .

```
Opt95 = instfind(ExampleInst, 'FieldName','Strike','Data','95')
```


## instfind

> Opt95 $=$
> 1
> 6

Locate the futures and Treasury bill instruments within ExampleInst.
Types = instfind(ExampleInst,'Type',\{'Futures';'TBill'\})

Types =

4
7

See Also instaddfield | instget | instgetcell|instselect

## Purpose Construct fixed-rate instrument

```
Syntax InstSet = instfixed(InstSet, CouponRate, Settle, Maturity,
Reset, Basis, Principal, EndMonthRule)
[FieldList, ClassList, TypeString] = instfixed
```


## Arguments

| InstSet | Instrument variable. This argument is specified <br> only when adding fixed-rate note instruments to <br> an existing instrument set. See instget for more <br> information on the InstSet variable. |
| :--- | :--- |
| CouponRate | Decimal annual rate. <br> Settle <br>  <br> Mettlement date. Date string or serial date number <br> representing the settlement date of the fixed-rate <br> note. |
| Reset | Date string or serial date number representing the <br> maturity date of the fixed-rate note. <br> (Optional) NINST-by-1 vector representing the <br> frequency of payments per year. Default $=1$. |
| Basis | (Optional) Day-count basis of the instrument. A |
|  | vector of integers. |
|  | - $0=$ actual/actual (default) |
|  | - $1=30 / 360$ (SIA) |
|  | - $2=$ actual/360 |
|  | - $3=$ actual/365 |
|  | - $4=30 / 360$ (BMA) |
|  | - $5=30 / 360$ (ISDA) |
|  | - $6=30 / 360$ (European) |
|  | - $7=$ actual/365 (Japanese) |

## instfixed

- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Principal (Optional) NINST-by-1 of notional principal amounts or NINST-by- 1 cell array where each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid. Default is 100 .

EndMonthRule (Optional) NINST-by-1 vector representing the End-of-month rule. Default $=1$.

Data arguments are number of instruments (NINST)-by-1 vectors, scalar, or empty. Fill unspecified entries in vectors with NaN. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices [].

Description InstSet = instfixed(InstSet, CouponRate, Settle, Maturity, Reset, Basis, Principal, EndMonthRule) creates a new instrument set containing fixed-rate instruments or adds fixed-rate instruments to an existing instrument set.
[FieldList, ClassList, TypeString] = instfixed displays the classes.

FieldList is a number of fields (NFIELDS)-by-1 cell array of strings listing the name of each data field for this instrument type.

ClassList is an NFIELDS-by- 1 cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are 'dble', 'date', and 'char'.

TypeString is a string specifying the type of instrument added. For a fixed-rate instrument, TypeString = 'Fixed'.

## Examples Create a Fixed-Rate Instrument

Define the characteristics of the fixed-rate instrument.

```
CouponRate = .03;
Settle = datenum('15-Mar-2013');
Maturity = datenum('15-Mar-2018');
Reset = 4;
Basis = 1;
Principal = 1000;
EndMonthRule = 1;
```

Create the new cap instrument.

```
ISet = instfixed(CouponRate, Settle, Maturity, Reset, Basis, Principa
```

ISet =
FinObj:
IndexTable: $[1 \times 1$ struct $]$
Type: $\{$ 'Fixed' $\}$
FieldName: $\{\{7 \times 1$ cell $\}\}$
FieldClass: $\{\{7 \times 1$ cell $\}$
FieldData: $\{\{7 \times 1$ cell $\}\}$

Display the fixed-rate instrument.

```
instdisp(ISet)
```

\author{

| Index Type CouponRate Settle | Maturity | FixedReset Basis Pri |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | Fixed 0.03 | $15-$ Mar-2013 | $15-$ Mar-2018 | 4 | 1 | <br> See Also hjmprice | instaddfield | instbond | instcap | instdisp | instswap | intenvprice

}

## Purpose Construct floating-rate instrument

```
Syntax
InstSet = instfloat(InstSet, Spread, Settle, Maturity, Reset,
Basis, Principal, EndMonthRule)
InstSet = instfloat(InstSet, Spread, Settle, Maturity, Reset,
Basis, Principal, EndMonthRule, CapRate, FloorRate)
[FieldList, ClassList, TypeString] = instfloat
```


## Arguments

| InstSet | Instrument variable. This argument is specified <br> only when adding floating-rate note instruments to <br> an existing instrument set. See instget for more <br> information on the InstSet variable. |
| :--- | :--- |
| Spread | Number of basis points over the reference rate. |
| Settle | Settlement date. Date string or serial date number <br> representing the settlement date of the floating-rate <br> note. |
| Maturity | Date string or serial date number representing the <br> maturity date of the floating-rate note. |
| Reset | (Optional) NINST-by-1 vector representing the <br> frequency of payments per year. Default = 1. |
| Basis | (Optional) Day-count basis of the instrument. A |
|  | vector of integers. |
|  | - $0=$ actual/actual (default) |
|  | - $1=30 / 360$ (SIA) |
|  | - $2=$ actual/360 |
|  | - $3=$ actual/365 |
|  | - $4=30 / 360$ (BMA) |
|  | - $5=30 / 360$ (ISDA) |
|  | - $6=30 / 360$ (European) |

## instfloat

- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Principal (Optional) NINST-by-1 of notional principal amounts or NINST-by- 1 cell array where each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid. Default is 100 .
EndMonthRule (Optional) NINST-by-1 vector representing the End-of-month rule. Default $=1$.
CapRate (Optional) NINST-by-1 decimal annual rate or NINST-by- 1 cell array, where each element is a NumDates-by- 2 cell array, and the cell array first column is dates, and the second column is associated cap rates. The date indicates the last day that the cap rate is valid.

FloorRate (Optional) NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by- 2 cell array, and the cell array first column is dates, and the second column is associated floor rates. The date indicates the last day that the floor rate is valid.

Data arguments are number of instruments (NINST)-by-1 vectors, scalar, or empty. Fill unspecified entries in vectors with NaN. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices [].

## Description

## Examples

## Create a Floating-Rate Instrument

Define the characteristics of the floating-rate instrument.

```
Spread = 2;
Settle = datenum('15-Mar-2013');
Maturity = datenum('15-Mar-2018');
Reset = 4;
Basis = 1;
Principal = 1000;
```


## instfloat

```
EndMonthRule = 1;
CapRate = 0.35;
FloorRate = 0.27;
```

Create the new floating-rate instrument.
ISet = instfloat(Spread, Settle, Maturity, Reset, Basis, Principal, ... EndMonthRule, CapRate, FloorRate)

```
ISet =
FinObj: 'Instruments'
IndexTable: \([1 \times 1\) struct]
Type: \(\{\) 'Float' \(\}\)
FieldName: \(\{\{9 \times 1\) cell \(\}\}\)
FieldClass: \(\{\{9 \times 1\) cell \(\}\)
FieldData: \(\{\{9 \times 1\) cell \(\}\}\)
```

Display the floating-rate instrument.
instdisp(ISet)

| Index | Type Spread Settle | Maturity | FloatReset | Basis Princip |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | Float 2 | 15-Mar-2013 | 15-Mar-2018 | 4 | 1 |

See Also
hjmprice | instaddfield | instbond | instcap | instdisp | instswap | intenvprice

## Purpose Construct floor instrument

```
Syntax InstSet = instfloor(InstSet, Strike, Settle, Maturity, Reset,
Basis, Principal)
[FieldList, ClassList, TypeString] = instfloor
```


## Arguments

| InstSet | Instrument variable. This argument is specified only when adding floor instruments to an existing instrument set. See instget for more information on the InstSet variable. |
| :---: | :---: |
| Strike | Rate at which the floor is exercised, as a decimal number. |
| Settle | Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than Maturity |
| Maturity | Maturity date. A vector of serial date numbers or date strings. |
| Reset | (Optional) NINST-by-1 vector representing the frequency of payments per year. Default $=1$. |
| Basis | (Optional) Day-count basis of the instrument. A vector of integers. |
|  | - $0=$ actual/actual (default) |
|  | - $1=30 / 360$ (SIA) |
|  | - 2 = actual/360 |
|  | - 3 = actual/365 |
|  | - $4=30 / 360$ (BMA) |
|  | - $5=30 / 360$ (ISDA) |
|  | - $6=30 / 360$ (European) |
|  | - $7=$ actual/365 (Japanese) |

## instfloor

- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual $/ 365$ (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Principal (Optional) The notional principal amount. Default $=100$.

## Description

## Examples Create a Floor Instrument

Define the characteristics of the floor instrument.

```
Strike = 0.22;
Settle = datenum('15-Mar-2013');
```

```
Maturity = datenum('15-Mar-2018');
Reset = 4;
Basis = 1;
Principal = 1000;
Create the new floor instrument.
```

```
ISet = instfloor(Strike, Settle, Maturity, Reset, Basis, Principal)
```

ISet = instfloor(Strike, Settle, Maturity, Reset, Basis, Principal)
ISet =
FinObj: 'Instruments'
IndexTable: [1x1 struct]
Type: {'Floor'}
FieldName: {{6x1 cell}}
FieldClass: {{6x1 cell}}
FieldData: {{6x1 cell}}

```

Display the floor instrument.
```

instdisp(ISet)

```
\begin{tabular}{llllll} 
Index & Type Strike Settle & Maturity & FloorReset Basis Prir \\
1 & Floor 0.22 & \(15-\) Mar-2013 & 15-Mar-2018 & 4 & 1
\end{tabular}

See Also hjmprice | instaddfield | instbond | instcap | instdisp | instswap | intenvprice

\section*{instget}

Purpose Data from instrument variable

\author{
Syntax [Data_1, Data_2,...,Data_n] = instget(InstSet, 'FieldName', FieldList, 'Index', IndexSet, 'Type', TypeList)
}

\section*{Arguments}
\begin{tabular}{cl} 
InstSet & \begin{tabular}{l} 
Variable containing a collection of instruments. \\
Instruments are classified by type; each type can \\
have different data fields. The stored data field is a \\
row vector or string for each instrument.
\end{tabular} \\
FieldList & \begin{tabular}{l} 
(Optional) String or number of fields (NFIELDS)-by-1 \\
cell array of strings listing the name of each data \\
field to match with data values. FieldList entries \\
can also be either 'Type' or ' Index'; these return \\
type strings and index numbers respectively. The \\
default is all fields available for the returned set of \\
instruments.
\end{tabular} \\
IndexSet & \begin{tabular}{l} 
(Optional) Number of instruments (NINST)-by-1 \\
vector of positions of instruments to work on. If \\
TypeList is also entered, instruments referenced \\
must be one of TypeList types and contained in
\end{tabular} \\
TypeList & \begin{tabular}{l} 
IndexSet. The default is all indices available in the \\
instrument variable. \\
(Optional) String or number of types (NTYPES)-by-1 \\
cell array of strings restricting instruments to match
\end{tabular} \\
one of TypeList types. The default is all types in the \\
instrument variable.
\end{tabular}

Argument value pairs can be entered in any order. The InstSet variable must be the first argument.

\section*{Description}
[Data_1, Data_2,...,Data_n] = instget(InstSet, 'FieldName', FieldList, 'Index', IndexSet, 'Type', TypeList) retrieves data arrays from an instrument variable.

Data_1 is an NINST-by-M array of data contents for the first field in FieldList. Each row corresponds to a separate instrument in IndexSet. Unavailable data is returned as NaN or as spaces.

Data_n is an NINST-by-M array of data contents for the last field in FieldList.

\section*{Examples Retrieve the instrument set ExampleInst from the data file.} InstSetExamples.mat. ExampleInst contains three types of instruments: Option, Futures, and TBill.
```

load InstSetExamples;
instdisp(ExampleInst)

```
\begin{tabular}{llcccc} 
Index & Type & Strike & Price & Opt & Contracts \\
1 & Option & 95 & 12.2 & Call & 0 \\
2 & Option 100 & 9.2 & Call & 0 \\
3 & Option 105 & 6.8 & Call & 1000
\end{tabular}
\begin{tabular}{lllll} 
Index & Type & Delivery & F & Contracts \\
4 & Futures & 01 -Jul-1999 & 104.4 & -1000
\end{tabular}
\begin{tabular}{llcrc} 
Index & Type & Strike & Price & Opt \\
Contracts \\
5 & Option 105 & 7.4 Put & -1000 \\
6 & Option 95 & 2.9 Put & 0
\end{tabular}
\begin{tabular}{llcl} 
Index & Type Price Maturity & Contracts \\
7 & TBill 99 & \(01-J u l-1999\) & 6
\end{tabular}

Extract the price from all instruments.
```

P = instget(ExampleInst,'FieldName','Price')
P =

```

\section*{instget}
12.2000
9.2000
6.8000
NaN
7.4000
2.9000
99.0000

Get all the prices and the number of contracts held.
```

[P,C] = instget(ExampleInst, 'FieldName', {'Price', 'Contracts'})
P =
12.2000
9.2000
6.8000
Nan
7.4000
2.9000
99.0000
C =
0
0
1 0 0 0
-1000
-1000
0
6

```

Compute a value V. Create a new variable ISet that appends V to ExampleInst.
```

V = P.*C
ISet = instsetfield(ExampleInst, 'FieldName', 'Value', 'Data',...

```
V);
instdisp(ISet)
\begin{tabular}{llrrrrr} 
Index & Type & Strike & Price & Opt & Contracts & Value \\
1 & Option 95 & 12.2 & Call & 0 & 0 \\
2 & Option 100 & 9.2 & Call & 0 & 0 \\
3 & Option 105 & 6.8 & Call & 1000 & 6800
\end{tabular}
\begin{tabular}{llllll} 
Index & Type & Delivery & F & Contracts Value \\
4 & Futures & 01-Jul-1999 & \(104.4-1000\) & NaN
\end{tabular}
\begin{tabular}{llcccr} 
Index & Type & Strike & Price Opt & Contracts & Value \\
5 & Option 105 & 7.4 Put & -1000 & -7400 \\
6 & Option 95 & 2.9 Put & 0 & 0
\end{tabular}
\begin{tabular}{llcll} 
Index & Type Price Maturity & Contracts Value \\
7 & TBill 99 & 01-Jul-1999 & 6 & 594
\end{tabular}

Look at only the instruments that have nonzero Contracts.
```

Ind = find(C ~= 0)

```

Ind =

3
4
5
7
Get the Type and Opt parameters from those instruments. (Only options have a stored 'Opt ' field.)
```

[T,0] = instget(ExampleInst, 'Index', Ind, 'FieldName',...

```
\{'Type', 'Opt'\})
\(T=\)

Option

\section*{instget}
Futures
OptionTBill
\(0=\)
Call
Put
Create a string report of holdings Type, Opt, and Value.
rstring \(=\) [T, 0, num2str(V(Ind))]
rstring =
Option Call 6800
Futures ..... NaN
Option Put ..... - 7400
TBill ..... 594
See Also instaddfield | instdisp | instgetcell

\section*{Purpose Data and context from instrument variable}
```

Syntax

```

Arguments

\section*{Description}

Argument value pairs can be entered in any order. The InstSet variable must be the first argument.
```

[DataList, FieldList, ClassList] =

```
[DataList, FieldList, ClassList] =
instgetcell(InstSet, 'FieldName', FieldList, 'Index',
```

instgetcell(InstSet, 'FieldName', FieldList, 'Index',

```

\section*{instgetcell}

IndexSet, 'Type', TypeList) retrieves data and context from an instrument variable.

DataList is an NFIELDS-by- 1 cell array of data contents for each field. Each cell is an NINST-by-M array, where each row corresponds to a separate instrument in IndexSet. Any data which is not available is returned as NaN or as spaces.

FieldList is an NFIELDS-by-1 cell array of strings listing the name of each field in DataList.

ClassList is an NFIELDS-by- 1 cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are 'dble', 'date', and 'char'.

IndexSet is an NINST-by-1 vector of positions of instruments returned in DataList.

TypeSet is an NINST-by- 1 cell array of strings listing the type of each instrument row returned in DataList.

\section*{Examples}

Retrieve the instrument set ExampleInst from the data file InstSetExamples.mat. ExampleInst contains three types of instruments: Option, Futures, and TBill.
load InstSetExamples;
instdisp(ExampleInst)
\begin{tabular}{llcccc} 
Index & Type & Strike & Price & Opt & Contracts \\
1 & Option & 95 & 12.2 & Call & 0 \\
2 & Option 100 & 9.2 & Call & 0 \\
3 & Option 105 & 6.8 & Call & 1000
\end{tabular}
Index Type Delivery F Contracts

4 Futures 01-Jul-1999 104.4-1000
\begin{tabular}{lllrl} 
Index & Type & Strike & Price & Opt \\
Contracts \\
5 & Option & 105 & 7.4 Put & -1000 \\
6 & Option 95 & 2.9 Put & 0
\end{tabular}
```

Index Type Price Maturity Contracts
7 TBill 99 01-Jul-1999 6

```

Get the prices and contracts from all instruments.
```

FieldList = {'Price'; 'Contracts'}
DataList = instgetcell(ExampleInst, 'FieldName', FieldList )
P = DataList{1}
C = DataList{2}
P =
12.2000
9.2000
6.8000
NaN
7.4000
2.9000
99.0000
C =
0
0
1000
-1000
-1000
0
6

```

Get all the option data: Strike, Price, Opt, Contracts.
[DataList, FieldList, ClassList] = instgetcell(ExampleInst,...
'Type', 'Option')
DataList =
    [5x1 double]

\section*{instgetcell}
```

    [5x1 double]
    [5\times4 char ]
    [5x1 double]
    FieldList =
'Strike'
Price'
Opt'
Contracts'
ClassList =
dble
dble'
'char'
dble'

```

Look at the data as a comma-separated list. Type help lists for more information on cell array lists.

DataList\{:\}
ans =

95
100
105
105
95
ans =
12.2100
9.2000
6.8000
7.3900ans =
Call
Call
Call
Put
Put
ans =00
100- 100
        0
See Also instaddfield | instdisp | instget

\section*{instlength}
Purpose Count instruments

Syntax NInst = instlength(InstSet)
Arguments

\section*{Description \\ NInst = instlength(InstSet) computes NInst, the number of instruments contained in the variable, InstSet.}

\author{
See Also \\ instdisp | instfields | insttypes
}

\section*{Purpose Construct lookback option}
```

Syntax
InstSet = instlookback(InstSet, OptSpec, Strike, Settle,
ExerciseDates, AmericanOpt)
[FieldList, ClassList, TypeString] = instlookback

```

\section*{Arguments}
\begin{tabular}{ll} 
InstSet & \begin{tabular}{l} 
Instrument variable. This argument is specified \\
only when adding lookback instruments to an \\
existing instrument set. See instget for more \\
information on the InstSet variable.
\end{tabular} \\
OptSpec & \begin{tabular}{l} 
NINST-by-1 list of string values 'Call' or 'Put'. \\
Strike \\
Settle \\
ExerciseDates \\
\\
NINST-by-1 vector of strike price values. Each \\
row is the schedule for one option.
\end{tabular} \\
NINST-by-1 vector of Settle dates. \\
For a European option (AmericanOpt = 0): \\
& \begin{tabular}{l} 
NINST-by-1 vector of exercise dates. Each row \\
is the schedule for one option. For a European \\
option, there is only one exercise date, the option \\
expiry date.
\end{tabular} \\
& \begin{tabular}{l} 
For an American option (AmericanOpt = 1):
\end{tabular} \\
\begin{tabular}{l} 
NINST-by-2 vector of exercise date boundaries. \\
For each instrument, the option can be exercised \\
on any tree date between or including the pair \\
of dates on that row. If only one non-NaN date \\
is listed, or if ExerciseDates is NINST-by-1, the
\end{tabular} \\
option can be exercised between the valuation \\
date of the stock tree and the single listed \\
exercise date.
\end{tabular}

\section*{instlookback}

AmericanOpt = 1, the option is an American option.

Data arguments are number of instruments (NINST)-by-1 vectors, scalar, or empty. Fill unspecified entries in vectors with NaN. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices [].

\section*{Description}

InstSet = instlookback(InstSet, OptSpec, Strike, Settle, ExerciseDates, AmericanOpt) specifies a lookback option.
[FieldList, ClassList, TypeString] = instlookback displays the classes.

FieldList is a number of fields (NFIELDS)-by-1 cell array of strings listing the name of each data field for this instrument type.

ClassList is an NFIELDS-by- 1 cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are 'dble', 'date', and 'char'.

TypeString is a string specifying the type of instrument added. For a lookback option instrument, TypeString = 'Lookback'.

\section*{Examples Create a Lookback Option Instrument}

Define a floating strike lookback instrument with the following data:
```

OptSpec = 'call';
Strike = NaN;
Settle = '01-Jan-2012';
ExerciseDates = '01-Jan-2015';

```

Create the instrument set.
InstSet = instlookback(OptSpec, Strike, Settle, ExerciseDates);

Display the lookback instrument.
```

instdisp(InstSet)

```
\begin{tabular}{llllll} 
Index & Type & OptSpec Strike Settle & ExerciseDates & AmericanOr \\
1 & Lookback call & NaN & 01 -Jan-2012 & \(01-J a n-2015\) & 0
\end{tabular}

See Also instadd | instdisp | instget

\section*{instoptbnd}

Purpose Construct bond option
```

Syntax InstSet = instoptbnd(InstSet, BondIndex, OptSpec, Strike,
ExerciseDates)
InstSet = instoptbnd(InstSet, BondIndex, OptSpec, Strike,
ExerciseDates, AmericanOpt)
[FieldList, ClassList, TypeString] = instoptbnd

```

\section*{Arguments}
\begin{tabular}{ll} 
InstSet & \begin{tabular}{l} 
Variable containing a collection of instruments. \\
Instruments are classified by type; each type can \\
have different data fields. The stored data field \\
is a row vector or string for each instrument.
\end{tabular} \\
BondIndex & \begin{tabular}{l} 
Number of instruments (NINST)-by-1 vector of \\
indices pointing to underlying instruments of \\
Type 'Bond ' which are also stored in InstSet. \\
See instbond for information on specifying the \\
bond data.
\end{tabular} \\
OptSpec & NINST-by-1 list of string values 'Call' or 'Put '.
\end{tabular}

Note The interpretation of the Strike and ExerciseDates arguments depends upon the setting of the AmericanOpt argument. If AmericanOpt \(=0, N a N\), or is unspecified, the option is a European or Bermuda option. If AmericanOpt \(=1\), the option is an American option.
\begin{tabular}{ll} 
Strike & \begin{tabular}{l} 
European option: NINST-by-1 vector of strike \\
price values. \\
Bermuda option: NINST by number of strikes \\
(NSTRIKES) matrix of strike price values.
\end{tabular} \\
Each row is the schedule for one option. If \\
an option has fewer than NSTRIKES exercise \\
opportunities, the end of the row is padded with \\
NaNs. \\
For an American option: \\
ExerciseDates & \begin{tabular}{l} 
NINST-by-1 vector of strike price values for each \\
option.
\end{tabular} \\
\begin{tabular}{l} 
NINST-by-1 (European option) or \\
NINST-by-NSTRIKES (Bermuda option) \\
matrix of exercise dates. Each row is the \\
schedule for one option. For a European option, \\
there is only one exercise date, the option expiry \\
date.
\end{tabular} \\
\begin{tabular}{l} 
For an American option:
\end{tabular} \\
\begin{tabular}{l} 
NINST-by-2 vector of exercise date boundaries. \\
For each instrument, the option can be exercised \\
on any coupon date between or including the pair \\
of dates on that row. If only one non-NaN date \\
is listed, or if ExerciseDates is NINST-by-1, the
\end{tabular} \\
option can be exercised between the underlying \\
bond Settle and the single listed exercise date.
\end{tabular}

Data arguments are NINST-by-1 vectors, scalar, or empty. Fill unspecified entries in vectors with NaN. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices [].

\section*{instoptbnd}

Description

\section*{Examples}

InstSet = instoptbnd(InstSet, BondIndex, OptSpec, Strike, ExerciseDates) specifies a European or Bermuda option.

InstSet = instoptbnd(InstSet, BondIndex, OptSpec, Strike, ExerciseDates, AmericanOpt) specifies an American option if AmericanOpt is set to 1 . If AmericanOpt is not set to 1 , the function specifies a European or Bermuda option.
[FieldList, ClassList, TypeString] = instoptbnd displays the classes.

FieldList is a number of fields (NFIELDS)-by- 1 cell array of strings listing the name of each data field for this instrument type.
ClassList is an NFIELDS-by- 1 cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are 'dble', 'date', and 'char'.

TypeString is a string specifying the type of instrument added. For a bond option instrument, TypeString = 'OptBond'.

\section*{Create a Bond Option Instrument}

Create a new instrument variable with the following information:
```

BondIndex = 1;
OptSpec = 'call';
Strike= 85;
ExerciseDates = 'Nov-1-2014';
AmericanOpt = 1;
CouponRate= [0.035;0.04];
Settle= 'Nov-1-2013';
Maturity = 'Nov-1-2014';
Period =1;

```

Create the instrument portfolio with two bonds.
```

InstSet = instbond(CouponRate, Settle, Maturity, ...
Period)

```
```

InstSet =
FinObj: 'Instruments'
IndexTable: [1x1 struct]
Type: {'Bond'}
FieldName: {{11x1 cell}}
FieldClass: {{11x1 cell}}
FieldData: {{11x1 cell}}
Create an option on the first bond
InstSet = instoptbnd(InstSet, BondIndex, OptSpec, Strike, ExerciseDat
InstSet =
FinObj: 'Instruments'
IndexTable: [1x1 struct]
Type: {2x1 cell}
FieldName: {2x1 cell}
FieldClass: {2x1 cell}
FieldData: {2x1 cell}

```

Display the instrument set.
```

instdisp(InstSet)

```
\begin{tabular}{lllllll} 
Index & Type CouponRate & Settle & Maturity & Period Basis EndMd \\
1 & Bond 0.035 & 01-Nov-2013 & \(01-\) Nov-2014 & 1 & 0 & 1 \\
2 & Bond 0.04 & \(01-\) Nov-2013 & \(01-\) Nov-2014 & 1 & 0 & 1
\end{tabular}
\begin{tabular}{lllllll} 
Index & Type & UnderInd & OptSpec & Strike & ExerciseDates & AmericanOpt \\
3 & OptBond & 1 & call & 85 & 01 -Nov-2014 & 1
\end{tabular}

\section*{instoptbnd}

\author{
See Also hjmprice | instadd | instdisp | instget
}

\section*{Purpose Construct bond with embedded option}

\author{
Syntax \\ ```
InstSet = instoptembnd (CouponRate, Settle, Maturity, \\ OptSpec, Strike, ExerciseDates, 'AmericanOpt', \\ AmericanOpt, 'Period', Period, 'Basis', Basis, \\ 'EndMonthRule', EndMonthRule,'Face', Face, 'IssueDate', \\ IssueDate, 'FirstCouponDate', FirstCouponDate, \\ 'LastCouponDate', LastCouponDate,'StartDate',StartDate) \\ InstSet = instoptembnd(InstSetOld, CouponRate,...) \\ [FieldList, ClassList, TypeString] = instoptembnd
```

}

## Arguments

| CouponRate | Decimal annual rate indicating the annual <br> percentage rate used to determine the coupons <br> payable on a bond. CouponRate is a NINST-by-1 <br> vector or NINST-by-1 cell array of decimal annual <br> rates, or decimal annual rate schedules. For the <br> latter case of a variable coupon schedule, each <br> element of the cell array is a NumDates-by-2 cell <br> array, where the first column is dates and the <br> second column is its associated rate. The date <br> indicates the last day that the coupon rate is <br> valid. |
| :--- | :--- |
| Settle | NINST-by-1 vector of settlement dates. <br> Maturity <br> OptSpec$\quad$NINST-by-1 vector of maturity dates. |
|  | NINST-by-1 vector of string values 'Call' or <br> 'Put'. |

For a European or Bermuda option
Strike NINST-by-NSTRIKES matrix of strike price values. Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaN's.

## instoptembnd

| ExerciseDates | NINST-by-NSTRIKES matrix of exercise dates. <br> Each row is the schedule for one option. For a <br> European option, there is only one ExerciseDate <br> on the option expiry date. |
| :--- | :--- |
| AmericanOpt | (Optional) NINST-by-1 vector of flags. <br> AmericanOpt is O for each European or Bermuda <br> option. The default is 0 if AmericanOpt is NaN <br> or not entered. |
| For an American option |  |
| Strike | NINST-by-1 vector of strike price values for each <br> option. |
| ExerciseDates | NINST-by-2 vector of exercise date boundaries. <br> For each instrument, the option can be exercised <br> on any coupon date between or including the pair <br> of dates on that row. If only one non-NaN date <br> is listed, or if ExerciseDates is NINST-by-1, the <br> option can be exercised between the underlying |
| bond Settle and the single listed ExerciseDate. |  |


| Basis(Optional) Day-count basis of the instrument. <br> Basis is a vector of integers with the following <br> possible values: <br> - $0=$ actual/actual (default) <br> - $1=30 / 360$ (SIA) <br> - $2=$ actual/360 <br> - $3=$ actual/365 <br> - $4=30 / 360$ (BMA) <br> - $5=30 / 360$ (ISDA) <br> - $6=30 / 360$ (European) <br> - $7=$ actual/365 (Japanese) <br> - $8=$ actual/actual (ICMA) <br> - $9=$ actual/360 (ICMA) <br> - $10=$ actual/365 (ICMA) <br> - $11=30 / 360 E$ (ICMA) <br> - $12=$ actual/actual (ISDA) <br> - $13=$ BUS/252 <br> For more information, see basis on page <br> Glossary-1. <br> (Optional) NINST-by-1 matrix for the |
| :--- |
| end-of-month rule. This rule applies only when <br> Maturity is an end-of-month date for a month <br> having 30 or fewer days. When the value is 0, <br> the end-of-month rule is ignored, meaning that a <br> bond's coupon payment date is always the same <br> numerical day of the month. When the value is <br> 1, the end-of-month rule is set rule on (default), <br> meaning that a bond's coupon payment date is <br> always the last actual day of the month. |

## instoptembnd

| IssueDate | (Optional) NINST-by-1 matrix for the bond issue <br> date. |
| :--- | :--- |
| FirstCouponDate |  |
|  | (Optional) Date when a bond makes its <br> first coupon payment; used when bond has <br> an irregular first coupon period. When |
| FirstCouponDate and LastCouponDate are both |  |
| specified, FirstCouponDate takes precedence in |  |
| determining the coupon payment structure. If |  |
| you do not specify a FirstCouponDate, the cash |  |
| flow payment dates are determined from other |  |
| inputs. |  |
| LastCouponDate |  |
|  | (Optional) Last coupon date of a bond before the |
| maturity date; used when bond has an irregular |  |
| last coupon period. In the absence of a specified |  |

date indicates the last day that the face value is valid. Default is 100 .

Note Data arguments are NINST-by-1 vectors, scalar, or empty. Fill unspecified entries in vectors with NaN. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices [].

```
Description InstSet = instoptembnd (CouponRate, Settle, Maturity,
OptSpec, Strike, ExerciseDates, 'AmericanOpt',
AmericanOpt, 'Period', Period, 'Basis', Basis,
'EndMonthRule', EndMonthRule,'Face', Face, 'IssueDate',
IssueDate, 'FirstCouponDate', FirstCouponDate,
'LastCouponDate', LastCouponDate,'StartDate',StartDate)
creates InstSet, a variable containing a collection of instruments.
```

Note instopembnd uses optional parameter name/value pairs such that, 'Name1', Value1, 'Name2', Value2, and so on, are a variable length list of name/value pairs.

Instruments are broken down by type and each type can have different data fields. Each stored data field has a row vector or string for each instrument. See instget for more information on the InstSet variable.

InstSet = instoptembnd(InstSetOld, CouponRate,...) adds 'OptEmBond' instruments to an instrument variable.
[FieldList, ClassList, TypeString] = instoptembnd lists field metadata for the 'OptEmBond' instrument.

FieldList is a number of fields (NFIELDS)-by-1 cell array of strings listing the name of each data field for this instrument type.

## instoptembnd

ClassList is an NFIELDS-by- 1 cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are 'dble', 'date', and 'char'.

TypeString is a string specifying the type of instrument added. For a bond option instrument, TypeString = 'OptEmBond'.

## Examples Construct a Bond With an Embedded Option

This example shows how to construct a bond with an embedded option using the following data.

```
Settle = 'jan-1-2007';
Maturity = 'jan-1-2010';
CouponRate = 0.07;
OptSpec = 'call';
Strike= 100;
ExerciseDates= {'jan-1-2008' '01-Jan-2010'};
AmericanOpt=1;
Period = 1;
InstSet = instoptembnd(CouponRate, ...
Settle, Maturity, OptSpec, Strike, ExerciseDates,'AmericanOpt', Americar
'Period', Period);
% display the instrument
    instdisp(InstSet)
```

| Index | Type | CouponRate Settle | Maturity | OptSpec | Strike |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | OptEmBond | 0.07 | $01-$ Jan-2007 | $01-$ Jan-2010 | call |

See Also instadd | instdisp | instget
Purpose Create option instrument on floating-rate note or add instrument tocurrent portfolio
Syntax InstSet =

instoptfloat(FloatIndex,OptSpec,Strike,ExerciseDates)

InstSet =

instoptfloat(FloatIndex,OptSpec,Strike, ExerciseDates,

    AmericanOpt)
    InstSet = instoptfloat(InstSetOld, __ )

[FieldList,ClassList,TypeString] = instoptfloat

## Description InstSet =

instoptfloat(FloatIndex,OptSpec,Strike,ExerciseDates) to
specify an European option for a floating-rate note.
InstSet =
instoptfloat(FloatIndex,OptSpec,Strike,ExerciseDates,
AmericanOpt) to specify an American or Bermuda option for a
floating-rate note.
InstSet = instoptfloat(InstSetOld, __ ) to add instruments to
an existing portfolio.
[FieldList, ClassList,TypeString] = instoptfloat lists the field
metadata for the 'OptFloat' instrument.

## Input Arguments

## FloatIndex - Indices pointing to underlying instruments

vector of nonnegative integers
Indices pointing to underlying instruments of Type 'Float' specified by a NINST-by-1 vector. The instruments of Type 'Float' are also stored in the InstSet variable. For more information, see instfloat.

## instoptfloat

## Data Types <br> double <br> OptSpec - Definition of option <br> string | cell array of strings

Definition of option as 'call' or 'put' specified as a NINST-by-1 cell array of strings for 'call' or 'put'.

Data Types
char | cell

## Strike - Option strike price values for European, Bermuda, or American option

nonnegative integer | vector of nonnegative integers
Option strike price values for option (European, Bermuda or American) specified as nonnegative integers using as NINST-by-NSTRIKES vector of strike price values.

- For a European or Bermuda option - NINST-by-NSTRIKES matrix of strike price values. Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- For an American Option - NINST-by-1 vector of strike price values for each option.


## Data Types

single | double

## ExerciseDates - Exercise date for option (European, Bermuda, or American) <br> nonnegative integer | vector of nonnegative integers

Exercise date for option (European, Bermuda, or American) specified as nonnegative integers using a NINST-by-NSTRIKES or NINST-by-2 vector of for the option exercise dates, depending on the option type.

- For a European or Bermuda option - NINST-by-NSTRIKES matrix of exercise dates. Each row is the schedule for one option. For a

European option, there is only one ExerciseDate on the option expiry date

- For an American option - NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is NINST-by-1, the option can be exercised between the underlying bond Settle date and the single listed ExerciseDate.


## Data Types <br> char | cell

## AmericanOpt - Option type

0 if AmericanOpt is NaN or not entered (default) | scalar | vector of positive integers [0,1]

Option type specified as NINST-by-1 positive integer scalar flags with values.

- For an European or Bermuda option - AmericanOpt is 0 for each European or Bermuda option.
- For an American option - AmericanOpt is 1 for each American option. The AmericanOpt argument is required to invoke American exercise rules.


## Data Types <br> single | double

## Output Arguments

## InstSet - Variable containing a collection of instruments <br> scalar | vector

Variable containing a collection of instruments returned as a scalar or vector with the instruments broken down by type and each type can have different data fields. Each stored data field has a row vector or string for each instrument. For more information on the InstSet variable, see instget.

## FieldList - Data field for instrument type

## instoptfloat

## string | cell array

Data field for instrument type returned as a NFIELDS-by-1 cell array of strings listing the name of each data field for this instrument type.

## ClassList - Data class of each field

string with value: 'dble', 'date', 'char' | cell array with values: 'dble', 'date', 'char'

Data class of each field returned as aNFIELDS-by- 1 cell array of strings listing the data class of each field. The class determines how arguments will be parsed.

## TypeString - Type of instrument added

string with value 'OptFloat'
Type of instrument added returned as a string. The string for a floating-rate option instrument is TypeString = 'OptFloat'.

## Examples Create an Instrument Portfolio with a Call Option for a Floating-Rate Note

Define the floating-rate note:

```
Settle = 'Nov-1-2012';
Maturity = 'Nov-1-2015';
Spread = 50;
Reset = 1;
Create InstSet:
InstSet = instfloat(Spread, Settle, Maturity, Reset)
InstSet =
    FinObj: 'Instruments'
    IndexTable: [1x1 struct]
    Type: {'Float'}
```

```
    FieldName: {{9x1 cell}}
FieldClass: {{9x1 cell}}
    FieldData: {{9x1 cell}}
```

Display the instrument:

```
instdisp(InstSet)
```

| Index | Type Spread Settle | Maturity | FloatReset Basis Prir |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | Float 50 | $01-$ Nov-2012 | 01 -Nov-2015 | 1 | 0 |

Add a European call option to the instrument portfolio:
OptSpec = 'call';
Strike = 100;
ExerciseDates = 'Nov-1-2015';
Create InstSet:

```
InstSet = instoptfloat(InstSet, 1, OptSpec, Strike, ExerciseDates)
```

```
InstSet =
```

    FinObj: 'Instruments'
    IndexTable: [1x1 struct]
        Type: \(\{2 \times 1\) cell\}
        FieldName: \{2x1 cell\}
    FieldClass: \(\{2 \times 1\) cell\}
    FieldData: \{2x1 cell\}
    Display the instrument:
instdisp(InstSet)

## instoptfloat

| Index | Type Spread Settle | Maturity | FloatReset | Basis Princip |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | Float 50 | 01 -Nov-2012 | $01-$ Nov-2015 | 1 | 0 | 100 |  |
|  |  |  |  |  |  |  |  |
| Index | Type | UnderInd OptSpec | Strike ExerciseDates | AmericanOpt |  |  |  |
| 2 | OptFloat 1 | call | 100 | $01-$ Nov-2015 | 0 |  |  |

See Also instadd \| instoptemfloat

## Purpose

Syntax

Description

Create embedded option instrument on floating-rate note or add instrument to current portfolio

```
InstSet = instopemtfloat(Spread,Settle,Maturity,OptSpec,
Strike,ExerciseDates)
InstSet = instopemtfloat(___,Name,Value)
InstSet = instoptfloat(InstSetOld,Spread,Settle,Maturity,
OptSpec,Strike,ExerciseDates)
InstSet = instoptfloat(___ ,Name,Value)
[FieldList,ClassList,TypeString] = instoptfloat
```

InstSet = instopemtfloat(Spread,Settle, Maturity, OptSpec, Strike, ExerciseDates) creates an embedded option instrument for a floating-rate note.

InstSet = instopemtfloat (__ , Name, Value) creates an embedded option instrument for a floating-rate note using optional name-value pair arguments.

InstSet = instoptfloat(InstSet0ld,Spread,Settle, Maturity, OptSpec, Strike, ExerciseDates) adds embedded option instruments for a floating-rate note instruments to an existing portfolio.

InstSet = instoptfloat( __ , Name, Value) adds embedded option instruments for a floating-rate note to an existing portfolio using optional name-value pair arguments.
[FieldList, ClassList,TypeString] = instoptfloat lists field metadata for the 'OptEmFloat' instrument.

## instoptemfloat

## Input Arguments

## Spread - Number of basis points over the reference rate nonnegative integer | vector of nonnegative integers

Number of basis points over the reference rate specified as a vector nonnegative integers for the number of instruments (NINST)-by-1).

## Data Types

single | double

## Settle - Settlement dates of floating-rate note

ValuationDate of HW Tree (default) | nonnegative integer | vector of nonnegative integers

Settlement dates of floating-rate note specified as nonnegative integers using a NINST-by-1 vector of dates.

## Data Types <br> char

## Maturity - Floating-rate note maturity date

nonnegative integer | vector of nonnegative integers
Floating-rate note maturity date specified as nonnegative integers using a NINST-by-1 vector of dates.

## Data Types <br> char

## OptSpec - Definition of option

string | cell array of strings
Definition of option as 'call' or 'put' specified as a NINST-by-1 cell array of strings for 'call' or 'put'.

Data Types
char | cell

## Strike - Embedded option strike price values

nonnegative integer | vector of nonnegative integers

Embedded option strike price values for option specified as nonnegative integers using as NINST-by-NSTRIKES or NINST-by-1 vector of strike price values, depending on the type of option.

- For a European or Bermuda Option - NINST-by-NSTRIKES matrix of strike price values where each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- For an American Option - NINST-by-1 vector of strike price values for each option.


## Data Types

single | double

## ExerciseDates - Exercise date for embedded option

nonnegative integer | vector of nonnegative integers
Exercise date for embedded option specified as nonnegative integers using a NINST-by-NSTRIKES or NINST-by-2 vector of the option exercise dates, depending on the type of option.

- For a European or Bermuda Option - NINST-by-NSTRIKES of exercise dates where each row is the schedule for one option. For a European option, there is only one ExerciseDate on the option expiry date.
- For an American Option - NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is NINST-by-1, the option can be exercised between the underlying bond Settle date and the single listed ExerciseDate.


## Data Types <br> char | cell

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can

## instoptemfloat

specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, Valuen.

Example: InstSet =
instoptemfloat(Spread, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'Reset

## 'AmericanOpt' - Embedded option type

0 if AmericanOpt is NaN or not entered (default) | scalar | vector of positive integers[0,1]

Embedded option type specified as NINST-by-1 positive integer scalar flags with values.

- For a European or Bermuda option - AmericanOpt is 0 for each European or Bermuda option. The default is 0 if AmericanOpt is NaN or not entered.
- For an American option - AmericanOpt is 1 for each American option. The AmericanOpt argument is required to invoke American exercise rules.


## Data Types <br> single | double <br> 'Reset' - Frequency of payments per year

1 (default) | positive integer from the set[1,2,3,4,6, 12] | vector of positive integers from the set [1,2,3,4,6,12]

Frequency of payments per year specified as positive integers for the values $1,2,4,6,12$ ] in a NINST-by- 1 vector.

## Data Types <br> single | double <br> 'Basis' - Day-count basis of the instrument

0 (actual/actual) (default) | positive integers of the set [1...13] | vector of positive integers of the set [1...13]

Day-count basis of the instrument specified as a positive integer using a NINST-by-1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- $2=$ actual $/ 360$
- $3=$ actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- $9=$ actual/360 (ISMA)
- $10=\mathrm{actual} / 365$ (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

Data Types
single | double

## 'Principal' - Principal values

100 (default) | nonnegative integer | vector of nonnegative integers | cell array of nonnegative integers

Principal values specified as a nonnegative integer using a NINST-by-1 vector of notional principal amounts.

## Data Types <br> single | double

## 'Options' - Structure containing derivatives pricing options structure

## instoptemfloat

```
Structure containing derivatives pricing options specified using derivset.
```


## Data Types

```
struct
```


## 'EndMonthRule' - End-of-month rule flag

1 (in effect) (default) | nonnegative integer [0,1]
End-of-month rule flag is specified as a nonnegative integer [0, 1] using a NINST-by- 1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

```
Data Types
single | double
```


## 'CapRate' - Indicates the last day that the cap rate is valid

nonnegative decimal | cell array of nonnegative decimals
NINST-by- 1 decimal annual rate or NINST-by- 1 cell array where each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated cap rates.

Data Types
single | double

## 'FloorRate' - Indicates the last day that the floor rate is valid nonnegative decimal | cell array of nonnegative decimals

NINST-by- 1 decimal annual rate or NINST-by- 1 cell array where each element is a NumDates-by- 2 cell array where the first column is dates and the second column is associated floor rates.

Data Types
single | double

## Output Arguments

## Examples Create an Instrument Portfolio with a Embedded Option Floating-Rate Note

Define the embedded call option:

```
Settle = 'Nov-1-2012';
```

Settle = 'Nov-1-2012';
Maturity = 'Nov-1-2015';
Maturity = 'Nov-1-2015';
Spread = 25;
Spread = 25;
OptSpec = 'call';
OptSpec = 'call';
Strike= 100;
Strike= 100;
ExerciseDates = 'Nov-1-2015';
ExerciseDates = 'Nov-1-2015';
Reset = 1;

```
Reset = 1;
```


## instoptemfloat

```
Create InstSet:
InstSet = instoptemfloat(Spread, Settle, Maturity, OptSpec,...
Strike, ExerciseDates,'Reset', Reset)
InstSet =
    FinObj: 'Instruments'
IndexTable: [1x1 struct]
        Type: {'OptEmFloat'}
    FieldName: {{13x1 cell}}
FieldClass: {{13x1 cell}}
    FieldData: {{13x1 cell}}
```

Display the instrument:
instdisp(InstSet)

| Index | Type | Spread Settle | Maturity | OptSpec Strike Exer |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | OptEmFloat | 25 | 01-Nov-2012 | 01-Nov-2015 | call |

See Also instadd | instoptfloat
Concepts

- basis on page Glossary-1


## Purpose

Construct stock option
Syntax

```
InstSet = instoptstock(InstSet, OptSpec, Strike,
Settle, ExerciseDates)
InstSet = instoptstock(InstSet, OptSpec, Strike,
Settle, ExerciseDates, AmericanOpt)
[FieldList, ClassList, TypeString] = instoptstock
```


## Arguments

Note The interpretation of the Strike and ExerciseDates arguments depends upon the setting of the AmericanOpt argument. If AmericanOpt $=0, \mathrm{NaN}$, or is unspecified, the option is a European or Bermuda option. If AmericanOpt $=1$, the option is an American option.

| Strike | European option: NINST-by-1 vector of strike <br> price values. <br> Bermuda option: NINST by number of strikes <br> (NSTRIKES) matrix of strike price values. |
| :--- | :--- |
| Each row is the schedule for one option. If |  |
| an option has fewer than NSTRIKES exercise |  |
| opportunities, the end of the row is padded with |  |
| NaNs. |  |
| American option: NINST-by-1 vector of strike |  |
| price values for each option. |  |

## instoptstock

Settle NINST-by-1 vector of settlement dates.
ExerciseDates
NINST-by-1 (European option) or NINST-by-NSTRIKES (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.

For an American option:
NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is NINST-by-1, the option can be exercised between the underlying bond Settle and the single listed exercise date.

## Description

Data arguments are NINST-by-1 vectors, scalar, or empty. Fill unspecified entries in vectors with NaN. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices [].

InstSet = instoptstock(InstSet, OptSpec, Strike, Settle, ExerciseDates) specifies a European or Bermuda option.

InstSet = instoptstock(InstSet, OptSpec, Strike, Settle, ExerciseDates, AmericanOpt) specifies an American option if AmericanOpt is set to 1 . If AmericanOpt is not set to 1 , the function specifies a European or Bermuda option.
[FieldList, ClassList, TypeString] = instoptstock displays the classes.

FieldList is a number of fields (NFIELDS)-by-1 cell array of strings listing the name of each data field for this instrument type.

## instoptstock

ClassList is an NFIELDS-by- 1 cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are 'dble', 'date', and 'char'.

TypeString is a string specifying the type of instrument added. For a stock option instrument, TypeString = 'OptStock'.

## Examples Create a Stock Option Instrument

Create an instrument set of two stock options with the following data:

```
OptSpec = {'put';'call'};
Strike = [95;98];
Settle = '01-May-2012';
ExerciseDates = {'01-May-2014';'01-May-2015'};
AmericanOpt = [0;1];
```

Create the stock option instruments.

```
InstSet = instoptstock(OptSpec, Strike,Settle, ExerciseDates, America
```

InstSet =
FinObj: 'Instruments
IndexTable: [1x1 struct]
Type: \{'OptStock'\}
FieldName: \{\{5x1 cell\}\}
FieldClass: \{\{5x1 cell\}\}
FieldData: \{\{5x1 cell\}\}

Display the instrument set.

```
instdisp(InstSet)
```

| Index | Type | OptSpec | Strike Settle | ExerciseDates | AmericanO |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | OptStock put | 95 | 01 -May-2012 | 01 -May-2014 | 0 |

## instoptstock

2 OptStock call 98 01-May-2012 01-May-2015 1

## See Also instadd | instdisp | instget

Purpose
Construct range note instrument

## Syntax

Description

## Input <br> Arguments

 Reset, Basis, Principal, EndMonthRule) instrument from data arrays.
## Spread

Number of basis points over the reference rate.

ISet = instrangefloat(Spread,Settle, Maturity, RateSched,

ISet = instrangefloat(Spread,Settle, Maturity, RateSched, Reset, Basis, Principal, EndMonthRule) creates a new range

## Settle

NINST-by-1 vector of dates representing the settle date of the floating-rate note.

## Maturity

NINST-by- 1 vector of dates representing the maturity date of the floating-rate note.

## RateSched

NINST-by-1 vector of structures representing the range of rates within which cash flows are nonzero. Each element of the structure array contains two fields:

- RateSched. Dates - NDates-by-1 cell array of dates corresponding to the range schedule.
- RateSched. Rates -NDates-by-2 array with the first column containing the lower bound of the range and the second column containing the upper bound of the range. Cash flow for date RateSched. Dates $(n)$ is nonzero for rates in the range RateSched.Rates $(n, 1)<$ Rate < RateSched.Rate ( $n, 2$ ).


## Reset

(Optional) NINST-by-1 vector representing the frequency of payments per year.

Default: 1

## Basis

(Optional) Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual $/ 365$
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual $/ 365$ (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

## Principal

(Optional) NINST-by-1 vector of the notional principal amount.
Default: 100

## EndMonthRule

(Optional) NINST-by-1 vector for end-of-month rule. Values are 1 (in effect) and 0 (not in effect).

Default: 1 (in effect)

Note Data arguments are number of instruments NINST-by-1 vectors, scalar, or empty. Fill unspecified entries in vectors with NaN. Only one data argument is required to create the instrument. You can omit or pass the others as empty matrices []. However, you cannot price the instrument when using the range note pricing function if you are missing any of the required input arguments.

## Output Arguments

## Definitions

## ISet

Variable containing a collection of instruments. Instruments are divided by type and each type can have different data fields. Each stored data field has a row vector or string for each instrument. Values are:

- FieldList - NFIELDS-by-1 cell array of strings listing the name of each data field for this instrument type.
- ClassList - NFIELDS-by-1 cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are'dble', 'date', and 'char'.
- TypeString - String specifying the type of instrument added. TypeString = 'RangeFloat'.

For more information, on ISet see instget.

## Range Note Instrument

A range note is a structured (market-linked) security whose coupon rate is equal to the reference rate as long as the reference rate is within a certain range. If the reference rate is outside of the range, the coupon rate is 0 for that period. This type of instrument entitles the holder

## instrangefloat

to cash flows that depend on the level of some reference interest rate and are floored to be positive. The note holder gets directs exposure to the reference rate. In return for the drawback that no interest will be paid for the time the range is left, they offer higher coupon rates than comparable standard products, like vanilla floating notes.

## Examples Create a Range Note Instrument

Create an instrument portfolio with a range note.

```
Spread = 100;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';
RateSched.Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched.Rates = [0.045 0.055; 0.0525 0.0675; 0.06 0.08];
% Create InstSet
InstSet = instrangefloat(Spread, Settle, Maturity, RateSched);
% Display the portfolio instrument
instdisp(InstSet)
```

| Index | Type | Spread Settle | Maturity | RateSched FloatRese |
| :--- | :--- | :--- | :--- | :--- |
| 1 | RangeFloat 100 | $01-$ Jan-2011 | $01-$ Jan-2014 | [Struct] 1 |

Add a second range note instrument to the portfolio. Second Range Note:

```
Spread2 = 200;
```

Settle2 = 'Jan-1-2011';
Maturity2 = 'Jan-1-2013';
RateSched2.Dates = \{'Jan-1-2012'; 'Jan-1-2013'\};
RateSched2.Rates $=[0.0480 .059 ; 0.055$ 0.068];
InstSet = instrangefloat(InstSet, Spread2, Settle2, Maturity2, RateSched2

```
% Display the portfolio instrument
instdisp(InstSet)
\begin{tabular}{llllll} 
Index & Type & Spread Settle & Maturity & RateSched Float \\
1 & RangeFloat 100 & \(01-\) Jan-2011 & \(01-\) Jan-2014 & [Struct] & 1 \\
2 & RangeFloat 200 & \(01-\) Jan-2011 & \(01-\) Jan-2013 & [Struct] & 1
\end{tabular}
```


## References Jarrow, Robert, Modelling Fixed Income Securities and Interest Rate Options, Stanford Economics and Finance, 2nd edition, 2002.

See Also
| instbond | instcap | instswap | instaddfield | instdisp | intenvprice | rangefloatbybk | rangefloatbybdt | rangefloatbyhw | rangefloatbyhjm

## instselect

## Purpose Create instrument subset by matching conditions

```
Syntax InstSubSet = instselect(InstSet, 'FieldName', FieldList,
'Data', DataList, 'Index', IndexSet,'Type', TypeList)
```


## Arguments

| InstSet | Variable containing a collection of instruments. <br> Instruments are classified by type; each type can <br> have different data fields. The stored data field is a <br> row vector or string for each instrument. |
| :---: | :--- |
| FieldList | String or number of fields (NFIELDS)-by-1 cell array <br> of strings listing the name of each data field to match <br> with data values. |
| DataList | Number of values (NVALUES)-by-M array or <br> NFIELDS-by-1 cell array of acceptable data values for <br> each field. Each row lists a data row value to search <br> for in the corresponding FieldList. The number <br> of columns is arbitrary and matching will ignore <br> trailing NaNs or spaces. |
| IndexSet | (Optional) Number of instruments (NINST)-by-1 <br> vector restricting positions of instruments to check <br> for matches. The default is all indices available in <br> the instrument variable. |
| TypeList | (Optional) String or number of types (NTYPES)-by-1 <br> cell array of strings restricting instruments to match |
| one of TypeList types. The default is all types in the |  |
| instrument variable. |  |

Argument value pairs can be entered in any order. The InstSet variable must be the first argument. 'FieldName' and 'Data' arguments must appear together or not at all. 'Index' and 'Type' arguments are each optional.

## Description <br> Examples <br> InstSubSet = instselect(InstSet, 'FieldName', FieldList, 'Data', DataList, 'Index', IndexSet,'Type', TypeList) creates an instrument subset (InstSubSet) from an existing set of instruments (InstSet). <br> InstSubSet is a variable containing instruments matching the input criteria. Instruments are returned in InstSubSet if all the Field, Index, and Type conditions are met. An instrument meets an individual Field condition if the stored FieldName data matches any of the rows listed in the DataList for that FieldName. See instfind for examples on matching criteria. <br> Retrieve the instrument set ExampleInst from the data file InstSetExamples.mat. The variable contains three types of instruments: Option, Futures, and TBill. <br> ```load InstSetExamples \\ instdisp(ExampleInst)``` <br> | Index | Type | Price Maturity | Contracts |
| :--- | :--- | :--- | :--- | :--- | :---: |
| 7 | TBill 99 | 01 -Jul-1999 | 6 |

Make a new portfolio containing only options struck at 95.

```
Opt95 = instselect(ExampleInst, 'FieldName', 'Strike',...
'Data', '95')
```


## instselect

```
instdisp(Opt95)
Opt95 =
Index Type Strike Price Opt Contracts
1 Option 95 12.2 Call 0
2 Option 95 2.9 Put 0
```

Make a new portfolio containing only futures and Treasury bills.

```
FutTBill = instselect(ExampleInst,'Type',{'Futures';'TBill'})
instdisp(FutTBill) =
Index Type Delivery F Contracts
1 Futures 01-Jul-1999 104.4 -1000
Index Type Price Maturity Contracts
2 TBill 99 01-Jul-1999 6
```

See Also
instaddfield | instdelete | instfind | instget | instgetcell

## Purpose Add or reset data for existing instruments

## Syntax

```
InstSet = instsetfield(InstSet, 'FieldName', FieldList,
'Data', DataList)
InstSet = instsetfield(InstSet, 'FieldName', FieldList,
'Data', DataList, 'Index', IndexSet, 'Type', TypeList)
```


## Arguments

$\left.\begin{array}{cl}\text { InstSet } & \begin{array}{l}\text { Variable containing a collection of instruments. } \\ \text { Instruments are classified by type; each type can } \\ \text { have different data fields. The stored data field is a } \\ \text { row vector or string for each instrument. InstSet } \\ \text { must be the first argument in the list. }\end{array} \\ \text { FieldList } & \begin{array}{l}\text { String or number of fields (NFIELDS)-by-1 cell array of } \\ \text { strings listing the name of each data field. FieldList } \\ \text { cannot be named with the reserved names Type or } \\ \text { Index. }\end{array} \\ \text { DataList } & \begin{array}{l}\text { Number of instruments (NINST)-by-M array or } \\ \text { NFIELDS-by-1 cell array of data contents for each field. }\end{array} \\ \text { IndexSet } & \begin{array}{l}\text { Each row in a data array corresponds to a separate } \\ \text { instrument. Single rows are copied to apply to all } \\ \text { instruments to be worked on. The number of columns }\end{array} \\ \text { is arbitrary, and data is padded along columns. }\end{array} \quad \begin{array}{l}\text { NINST-by-1 vector of positions of instruments to } \\ \text { work on. If TypeList is also entered, instruments } \\ \text { referenced must be one of TypeList types and } \\ \text { contained in IndexSet. }\end{array}\right\}$

Argument value pairs can be entered in any order.

## instsetfield

Description
instsetfield sets data for existing instruments in a collection variable.
InstSet = instsetfield(InstSet, 'FieldName', FieldList, 'Data', DataList) resets or adds fields to every instrument.

InstSet = instsetfield(InstSet, 'FieldName', FieldList, 'Data', DataList, 'Index', IndexSet, 'Type', TypeList) resets or adds fields to a subset of instruments.

The output InstSet is a new instrument set variable containing the input data.

## Examples

Retrieve the instrument set ExampleInstSF from the data file InstSetExamples.mat. ExampleInstSF contains three types of instruments: Option, Futures, and TBill.

```
load InstSetExamples;
ISet = ExampleInstSF;
instdisp(ISet)
```

| Index | Type | Strike | Price | Opt |
| :--- | :--- | ---: | ---: | :--- |
| 1 | Option | 95 | 12.2 | Call |
| 2 | Option | 100 | 9.2 | Call |
| 3 | Option 105 | 6.8 | Call |  |


| Index | Type | Delivery | F |
| :--- | :--- | :--- | :--- |
| 4 | Futures | 01 -Jul-1999 | 104.4 |


| Index | Type | Strike | Price | Opt |
| :--- | :--- | :--- | :--- | :--- |
| 5 | Option 105 | 7.4 | Put |  |
| 6 | Option NaN | NaN | Put |  |

```
Index Type Price
7 TBill 99
```

Enter data for the option in Index 6: Price 2.9 for a Strike of 95.

```
ISet = instsetfield(ISet, 'Index',6,...
'FieldName',{'Strike','Price'}, 'Data',{ 95 , 2.9 });
```

```
instdisp(ISet)
Index Type Strike Price Opt
1 Option 95 12.2 Call
2 Option 100 9.2 Call
3 Option 105 6.8 Call
Index Type Delivery F
4 Futures 01-Jul-1999 104.4
Index Type Strike Price Opt
5 Option 105 7.4 Put
6 Option 95 2.9 Put
Index Type Price
7 TBill 99
```

Create a new field Maturity for the cash instrument.

```
MDate = datenum('7/1/99');
ISet = instsetfield(ISet, 'Type', 'TBill', 'FieldName',...
'Maturity','FieldClass', 'date', 'Data', MDate);
instdisp(ISet)
Index Type Price Maturity
7 TBill 99 01-Jul-1999
```

Create a new field Contracts for all instruments.

```
ISet = instsetfield(ISet, 'FieldName', 'Contracts', 'Data', 0);
instdisp(ISet)
Index Type Strike Price Opt Contracts
1 Option 95 12.2 Call 0
2 Option 100 9.2 Call 0
3 Option 105 6.8 Call 0
Index Type Delivery F Contracts
4 Futures 01-Jul-1999 104.4 0
```


## instsetfield

| Index | Type | Strike | Price Opt | Contracts |
| :--- | :--- | :--- | :---: | :--- | :--- |
| 5 | Option 105 | 7.4 Put | 0 |  |
| 6 | Option 95 | 2.9 Put | 0 |  |
|  |  |  |  |  |
| Index | Type Price | Maturity | Contracts |  |
| 7 | TBill 99 | $01-J u l-1999$ | 0 |  |

Set the Contracts fields for some instruments.

```
ISet = instsetfield(ISet,'Index',[3; 5; 4; 7],...
'FieldName','Contracts', 'Data', [1000; -1000; -1000; 6]);
instdisp(ISet)
Index Type Strike Price Opt Contracts
1 Option 95 12.2 Call 0
2 Option 100 9.2 Call 0
3 Option 105 6.8 Call 1000
Index Type Delivery F Contracts
4 Futures 01-Jul-1999 104.4 -1000
Index Type Strike Price Opt Contracts
5 Option 105 7.4 Put -1000
6 Option 95 2.9 Put 0
Index Type Price Maturity Contracts
7 TBill 99 01-Jul-1999 6
```

See Also instaddfield|instdisp|instget | instgetcell

```
Purpose Construct swap instrument
Syntax InstSet = instswap(InstSet, LegRate, Settle, Maturity)
InstSet = instswap(InstSet, LegRate, Settle, Maturity,
InstSet, LegReset, Basis, Principal, LegType,
EndMonthRule)
InstSet = instswap(InstSet, LegRate, Settle, Maturity, LegReset, Basis
Principal, LegType, EndMonthRule, StartDate)
[FieldList, ClassList, TypeString] = instswap
```


## Arguments

| InstSet | Instrument variable. This argument is specified only when adding a swap to an existing instrument set. See instget for more information on the InstSet variable. |
| :---: | :---: |
| LegRate | Number of instruments (NINST)-by-2 matrix, with each row defined as: |
|  | [CouponRate Spread] or [Spread CouponRate] |
|  | CouponRate is the decimal annual rate. Spread is the number of basis points over the reference rate. The first column represents the receiving leg, while the second column represents the paying leg. |
| Settle | Settlement date. NINST-by-1 vector of serial date numbers or date strings. Settle must be earlier than Maturity. |
| Maturity | Maturity date. NINST-by-1 vector of dates representing the maturity date for each swap. |
| LegReset | (Optional) NINST-by-2 matrix representing the reset frequency per year for each swap. Default = [11 1 ]. |

## instswap

> Basis (Optional) Day-count basis of the instrument. A vector of integers. - $0=$ actual/actual (default) - $1=30 / 360$ (SIA) - $2=$ actual/360 - $3=$ actual/365 - $4=30 / 360$ (BMA) - $5=30 / 360$ (ISDA) - $6=30 / 360$ (European) - $7=$ actual/365 (Japanese) - $8=$ actual/actual (ICMA) - $9=$ actual/360 (ICMA) - $10=$ actual/365 (ICMA) - $11=30 / 360 E$ (ICMA) - $12=$ actual/actual (ISDA) - $13=$ BUS/252 Frincipal For more information, see basis on page Glossary- 1. (Optional) NINST-by-1 vector or NINST-by-1 cell array of the notional principal amounts or principal value schedules. For the latter case, each element of the cell array is a NumDates-by-2 matrix where the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.. Default = 100.

| LegType | (Optional) NINST-by-2 matrix. Each row represents <br> an instrument. Each column indicates if the <br> corresponding leg is fixed (1) or floating (0). This <br> matrix defines the interpretation of the values <br> entered in LegRate. Default is [1,0] for each <br> instrument. |
| :--- | :--- |
| EndMonthRule | (Optional) NINST-by-1 vector representing the <br> End-of-month rule. Default $=1$. |
| StartDate $\quad$(Optional) NINST-by-1 vector of dates when the swaps <br> actually start. Default is Settle. |  |

Data arguments are number of instruments (NINST)-by-1 vectors, scalar, or empty. Fill unspecified entries in vectors with NaN. Only one data argument is required to create the instrument; the others may be omitted or passed as empty matrices [].

## Description

InstSet = instswap(InstSet, LegRate, Settle, Maturity) creates a new instrument set containing swap instruments or adds swap instruments to an existing instrument set.

InstSet = instswap(InstSet, LegRate, Settle, Maturity, InstSet, LegReset, Basis, Principal, LegType, EndMonthRule) uses optional input arguments to create a new instrument set containing swap instruments or adds swap instruments to an existing instrument set.

InstSet = instswap(InstSet, LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType, EndMonthRule, StartDate) to create a new forward swap instrument or to add a forward swap instrument to an existing portfolio.
[FieldList, ClassList, TypeString] = instswap displays the classes.

FieldList is a number of fields (NFIELDS)-by-1 cell array of strings listing the name of each data field for this instrument type.

## instswap

ClassList is an NFIELDS-by- 1 cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are 'dble', 'date', and 'char'.

TypeString is a string specifying the type of instrument added. For a swap instrument, TypeString = 'Swap'.

## Definitions <br> Amortizing Swap

In an amortizing swap, the notional principal decreases periodically because it is tied to an underlying financial instrument with a declining (amortizing) principal balance, such as a mortgage.

## Forward Swap

Agreement to enter into an interest-rate swap arrangement on a fixed date in future.

## Examples Create a Vanilla Swap Instrument

Create a vanilla swap using market data.
Use the following market data to create a swap instrument.

```
LegRate = [0.065, 0]
Settle = 'jan-1-2007';
Maturity = 'jan-1-2012';
LegReset = [1, 1];
Basis = 0
Principal = 100
LegType = [1, 0]
InstSet = instswap(LegRate, Settle, Maturity, LegReset, Basis, Principal,
```

LegRate =
$0.0650 \quad 0$

```
Basis =
    0
Principal =
    1 0 0
LegType =
    1 0
InstSet =
    FinObj: 'Instruments
    IndexTable: [1x1 struct]
            Type: {'Swap'}
        FieldName: {{9x1 cell}}
    FieldClass: {{9x1 cell}}
        FieldData: {{9x1 cell}}
```

View the swap instrument using instdisp.
instdisp(InstSet)

| Index Type LegRate | Settle | Maturity | LegReset Basis Pr: |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | Swap $[0.065$ | 0] 01 -Jan-2007 | 01-Jan-2012 | $\left[\begin{array}{llll}1 & 1\end{array}\right]$ | 0 |

See Also hjmprice | instaddfield | instbond | instcap | instdisp | instfloor | intenvprice

## instswaption

Purpose Construct swaption instrument

```
Syntax InstSet = instswaption(OptSpec, Strike, ExerciseDates,
Spread, Settle, Maturity)
InstSet = instswaption(OptSpec, Strike, ExerciseDates,
Spread, Settle, Maturity, AmericanOpt,
SwapReset, Basis, Principal)
InstSet = instswaption(InstSetOld, OptSpec, Strike,
ExerciseDates, Spread, ...)
[FieldList, ClassList, TypeString] = instswaption;
```


## Arguments

Fill unspecified entries in vectors with the value NaN. Only one data argument is required to create the instruments; the others may be omitted or passed as empty matrices []. Type [FieldList, ClassList] $=$ instswaption to see the classes. Dates can be input as serial date numbers or date strings.

OptSpec NINST-by-1 cell array of strings 'call' or 'put'. A 'call' swaption entitles the buyer to pay the fixed rate. A 'put' swaption entitles the buyer to receive the fixed rate.

Strike NINST-by-1 vector of strike swap rate values.
For a European option:
ExerciseDates NINST-by-1 vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one ExerciseDate on the option expiry date.

AmericanOpt NINST-by-1 vector of flags. AmericanOpt is 0 for each European option. The default is 0 if AmericanOpt is NaN or not entered.

For an American option:

ExerciseDates NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is NINST-by-1, the option can be exercised between the underlying swap Settle and the single listed ExerciseDate.

AmericanOpt NINST-by-1 vector of flags. AmericanOpt is 1 for each American option. The AmericanOpt argument is required to invoke American exercise rules.

For an American or a European option:
Spread NINST-by-1 vector representing the number of basis points over the reference rate.

Settle NINST-by-1 vector of dates representing the settle date for each swap.

Maturity NINST-by-1 vector of dates representing the maturity date for each swap.

SwapReset (Optional) NINST-by-1 vector representing the reset frequency per year for the underlying swap. Default is 1 .

Basis (Optional) Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)


## instswaption

- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=\mathrm{actual} / 365$ (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Principal (Optional) NINST-by-1 vector of the notional principal amounts. Default is 100.

## Description

To specify a European option: InstSet = instswaption(OptSpec, Strike, ExerciseDates, Spread, Settle, Maturity)

```
To specify an American option: InstSet = instswaption(OptSpec, Strike, ExerciseDates, Spread, Settle, Maturity, AmericanOpt, SwapReset, Basis, Principal)
```

To add swaption instruments to an instrument variable: InstSet = instswaption(InstSetOld, OptSpec, Strike, ExerciseDates, Spread, ...)
To list field metadata for the swaption instrument: [FieldList, ClassList, TypeString] = instswaption;

Outputs:

| InstSet | Variable containing a collection of instruments. <br> Instruments are broken down by type and each type <br> can have different data fields. Each stored data field <br> has a row vector or string for each instrument. For <br> more information on the ISet variable, see instget. |
| :--- | :--- |
| FieldList | NFIELDS-by-1 cell array of strings listing the name of <br> each data field for this instrument type. |
| ClassList | NFIELDS-by-1 cell array of strings listing the data <br> class of each field. The class determines how <br> arguments will be parsed. Valid strings are 'dble', <br> 'date', and 'char'. |
| TypeString | String specifying the type of instrument added. <br> TypeString = 'Swaption'. |

## Examples Construct Two Swaption Instruments

This example shows how to create two European swaption instruments using the the following data.

```
OptSpec = {'Call'; 'Put'};
Strike = .05;
ExerciseDates = 'jan-1-2011';
Spread=0;
Settle = 'jan-1-2007';
Maturity = 'jan-1-2012';
AmericanOpt = 0;
InstSet = instswaption(OptSpec, Strike, ExerciseDates, Spread, Settle
    AmericanOpt);
% view the European swaption instruments using instdisp
instdisp(InstSet)
```

Index Type OptSpec Strike ExerciseDates Spread Settle

## instswaption

| 1 | Swaption Call | 0.05 | $01-J a n-2011$ | 0 | $01-J a n-2007$ | 01 -Jar |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | Swaption Put | 0.05 | $01-J a n-2011$ | 0 | $01-$ Jan-2007 | 01 -Jar |

See Also instadd | instget | instdisp

## Purpose List types

Syntax $\quad$ TypeList $=$ insttypes(InstSet)

## Arguments

## Description

## Examples

TypeList = insttypes(InstSet) retrieves a list of types stored in an instrument variable.

TypeList is a number of types (NTYPES)-by-1 cell array of strings listing the Type of instruments contained in the variable.

Retrieve the instrument set variable ExampleInst from the data file InstSetExamples.mat. ExampleInst contains three types of instruments: Option, Futures, and TBill.
load InstSetExamples;
instdisp(ExampleInst)

| Index | Type | Strike | Price | Opt | Contracts |
| :--- | :--- | :---: | :---: | :---: | :---: |
| 1 | Option 95 | 12.2 | Call | 0 |  |
| 2 | Option 100 | 9.2 | Call | 0 |  |
| 3 | Option 105 | 6.8 | Call | 1000 |  |

Index Type Delivery F Contracts

4 Futures 01-Jul-1999 104.4-1000

| Index | Type | Strike | Price | Opt |
| :--- | :--- | :--- | ---: | :--- |
| Contracts |  |  |  |  |
| 5 | Option 105 | 7.4 Put | -1000 |  |
| 6 | Option 95 | 2.9 Put | 0 |  |

## instłypes

| Index | Type Price Maturity | Contracts |  |
| :--- | :--- | :---: | :--- |
| 7 | TBill 99 | $01-J u l-1999$ | 6 |

List all of the types included in ExampleInst.
TypeList $=$ insttypes(ExampleInst)

```
TypeList =
    'Futures'
    'Option'
    'TBill'
```

See Also instdisp | instfields | instlength

| Purpose | Properties of interest-rate structure |
| :--- | :--- |
| Syntax | ParameterValue $=$ intenvget (RateSpec, ' 'ParameterName ' ) |
| Arguments |  |

RateSpec A structure containing the properties of an interest-rate structure. See intenvset for information on creating RateSpec.

ParameterName String indicating the parameter name to be accessed. The value of the named parameter is extracted from the structure RateSpec. It is sufficient to type only the leading characters that uniquely identify the parameter. Case is ignored for parameter names.

## Description ParameterValue = intenvget(RateSpec,'ParameterName') obtains the value of the named parameter ParameterName extracted from RateSpec. <br> Examples Use intenvset to set the interest-rate structure. <br> ```RateSpec = intenvset('Rates', 0.05, 'StartDates',... \\ '20-Jan-2000', 'EndDates', '20-Jan-2001')```

Now use intenvget to extract the values from RateSpec.
[R, RateSpec] = intenvget(RateSpec, 'Rates')
$\mathrm{R}=$
0.0500

RateSpec =
FinObj: 'RateSpec'

## intenvget

```
Compounding: 2
Disc: 0.9518
Rates: 0.0500
EndTimes: 2
StartTimes: 0
EndDates: 730871
StartDates: 730505
ValuationDate: 730505
Basis: 0
EndMonthRule: 1
```

See Also intenvset

Purpose Price instruments from set of zero curves
Syntax Price = intenvprice(RateSpec, InstSet)
Arguments

## Description

Price = intenvprice(RateSpec, InstSet) computes arbitrage-free
prices for instruments against a set of zero coupon bond rate curves.

Price is a number of instruments (NINST) by number of curves (NUMCURVES) matrix of prices of each instrument. If an instrument cannot be priced, a NaN is returned in that entry.
intenvprice handles the following instrument types: 'Bond', 'CashFlow', 'Fixed', 'Float', 'Swap'. See instadd for information about constructing defined types.
See single-type pricing functions to retrieve pricing information.

| bondbyzero | Price bonds from a set of zero curves. |
| :--- | :--- |
| cfbyzero | Price arbitrary cash flow instrument from a set <br> of zero curves. |
| fixedbyzero | Fixed-rate note prices from a set of zero curves. |
| floatbyzero | Floating-rate note prices from a set of zero curves. |
| swapbyzero | Swap prices from a set of zero curves. |

## intenvprice

## Examples Load Zero Curves and Instruments from Data File

```
Load the zero curves and instruments.
load deriv.mat
instdisp(ZeroInstSet)
\begin{tabular}{lllllll} 
Index & Type CouponRate Settle & Maturity & Period Basis EndMontr \\
1 & Bond 0.04 & 01-Jan-2000 & 01-Jan-2003 & 1 & NaN & NaN \\
2 & Bond 0.04 & 01-Jan-2000 & 01-Jan-2004 & 2 & NaN & NaN
\end{tabular}
\begin{tabular}{llllll} 
Index Type CouponRate Settle & Maturity & FixedReset Basis Pri \\
3 & Fixed 0.04 & 01-Jan-2000 & 01-Jan-2003 & 1 & NaN NaN
\end{tabular}
\begin{tabular}{llllll} 
Index & Type Spread Settle & Maturity & FloatReset Basis Princip \\
4 & Float 20 & \(01-J a n-2000\) & \(01-J a n-2003\) & 1 & NaN
\end{tabular}
\begin{tabular}{llllll} 
Index & Type LegRate & Settle & Maturity & LegReset Basis Princi \\
5 & Swap \(\left[\begin{array}{llll}0.06 & 20\end{array}\right]\) & \(01-J a n-2000\) & \(01-J a n-2003\) & {\(\left[\begin{array}{lll}1 & 1\end{array}\right]\)} & NaN
\end{tabular}
```

Price the instruments.
Price = intenvprice(ZeroRateSpec, ZeroInstSet)

Price =
98.7159
97.5334
98.7159
100.5529
3.6923

## Price Multi-Stepped Coupon Bonds

Price the following multi-stepped coupon bonds using the following data:

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDate
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
% Create a portfolio of stepped coupon bonds with different maturities
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011';'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2011' .042;'01-Jan-2012' .05; '01-Jan-2013' .0}
% Display the instrument portfolio
ISet = instbond(CouponRate, Settle, Maturity, 1);
instdisp(ISet)
\begin{tabular}{llllllll} 
Index & Type CouponRate & Settle & Maturity & Period & Basis & EndMC \\
1 & Bond & CCll] & 01-Jan-2010 & 01-Jan-2011 & 1 & 0 & 1 \\
2 & Bond [Cell] & 01-Jan-2010 & 01-Jan-2012 & 1 & 0 & 1 \\
3 & Bond [Cell] & 01-Jan-2010 & 01-Jan-2013 & 1 & 0 & 1 \\
4 & Bond [Cell] & 01-Jan-2010 & 01-Jan-2014 & 1 & 0 & 1
\end{tabular}
```

Build a BDTTree to price the stepped coupon bonds. Assume the volatility to be $10 \%$

Sigma = 0.1;
BDTTimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length BDTT = bdttree(BDTVolSpec, RS, BDTTimeSpec);

## intenvprice

```
% Compute the price of the stepped coupon bonds
PBDT = bdtprice(BDTT, ISet)
PBDT =
    100.6763
    100.7368
    100.9266
    101.0115
```

See Also hjmprice | hjmsens | instadd | intenvsens | intenvset

## Purpose Instrument price and sensitivities from set of zero curves

```
Syntax
[Delta, Gamma, Price] = intenvsens(RateSpec, InstSet)
```


## Arguments

## Description

| RateSpec | A structure containing the properties of an <br> interest-rate structure. See intenvset for <br> information on creating RateSpec. |
| :--- | :--- |
| InstSet | Variable containing a collection of instruments. |
|  | Instruments are categorized by type; each type can <br> have different data fields. The stored data field is a <br> row vector or string for each instrument. |

[Delta, Gamma, Price] = intenvsens(RateSpec, InstSet) computes dollar prices and price sensitivities for instruments that use a zero coupon bond rate structure.

Delta is a number of instruments (NINST) by number of curves (NUMCURVES) matrix of deltas, representing the rate of change of instrument prices with respect to shifts in the observed forward yield curve. Delta is computed by finite differences.

Gamma is an NINST-by-NUMCURVES matrix of gammas, representing the rate of change of instrument deltas with respect to shifts in the observed forward yield curve. Gamma is computed by finite differences.

Note Both sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, divide by the respective instrument price.

Price is an NINST-by-NUMCURVES matrix of prices of each instrument. If an instrument cannot be priced, a NaN is returned.

## intenvsens

intenvsens handles the following instrument types: 'Bond', 'CashFlow', 'Fixed', 'Float', 'Swap'. See instadd for information about constructing defined types.

## Examples Load the tree and instruments from a data file.

```
load deriv.mat
instdisp(ZeroInstSet)
```


[Delta, Gamma] = intenvsens(ZeroRateSpec, ZeroInstSet)
Delta =
-272.6403
-347. 4386
-272.6403
-1.0445
-282.0405
Gamma =
$1.0 \mathrm{e}+003$ *

> 1.0298
> 1.6227
> 1.0298
> 0.0033
> 1.0596

See Also hjmprice | hjmsens | instadd | intenvprice | intenvset

## intenvset

Purpose
Syntax $\quad \begin{aligned} & \text { [RateSpec, RateSpec0ld] }=\text { intenvset (RateSpec, 'Argument1', } \\ & \\ & \text { Value1, 'Argument2', Value2, } . . .) \\ & \\ & \\ & \text { [RateSpec, RateSpecOld] }=\text { intenvset } \\ & \text { intenvset }\end{aligned}$

## Arguments

Arguments may be chosen from the following table and specified in any order.

Compounding Scalar value representing the rate at which the input zero rates or forward rates were compounded when annualized. Default $=2$. This argument determines the formula for the discount factors:

Compounding $=1,2,3,4,6,12$
Disc $=(1+Z / F)^{\wedge}(-T)$, where $F$ is the compounding frequency, $Z$ is the zero rate, and T is the time in periodic units; for example, T $=F$ is 1 year.

Compounding $=365$
Disc $=(1+Z / F)^{\wedge}(-T)$, where $F$ is the number of days in the basis year and $T$ is a number of days elapsed computed by basis.

Compounding $=-1$

| Disc | Number of points (NPOINTS) by number of curves (NCURVES) matrix of unit bond prices over investment intervals from StartDates, when the cash flow is valued, to EndDates, when the cash flow is received. |
| :---: | :---: |
| Rates | Number of points (NPOINTS) by number of curves (NCURVES) matrix of rates in decimal form. For example, $5 \%$ is 0.05 in Rates. Rates are the yields over investment intervals from StartDates, when the cash flow is valued, to EndDates, when the cash flow is received. |
| EndDates | NPOINTS-by-1 vector or scalar of serial maturity dates ending the interval to discount over. |
| StartDates | NPOINTS-by-1 vector or scalar of serial dates starting the interval to discount over. Default = ValuationDate. StartDates must be earlier than EndDates. |
| ValuationDate | (Optional) Scalar value in serial date number form representing the observation date of the investment horizons entered in StartDates and EndDates. Default $=$ min (StartDates). |
| Basis | (Optional) Day-count basis of the instrument. A scalar of integers. |
|  | - $0=$ actual/actual (default) |
|  | - $1=30 / 360$ (SIA) |
|  | - 2 = actual/360 |
|  | - 3 = actual/365 |
|  | - $4=30 / 360$ (BMA) |
|  | - $5=30 / 360$ (ISDA) |

## intenvset

- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- $12=$ actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
EndMonthRule (Optional) End-of-month rule. A scalar. This rule applies only when EndDates is an end-of-month date for a month having 30 or fewer days. $0=$ ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. $1=$ set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.

It is sufficient to type only the leading characters that uniquely identify the parameter. Case is ignored for argument names.

When creating a new RateSpec, the set of arguments passed to intenvset must include StartDates, EndDates, and either Rates or Disc.
Call intenvset with no input or output arguments to display a list of argument names and possible values.

## Description

[RateSpec, RateSpecOld] = intenvset(RateSpec, 'Argument1', Value1, 'Argument2', Value2, ...) creates an interest term
structure (RateSpec) in which the input argument list is specified as argument name /argument value pairs. The argument name portion of the pair must be recognized as a valid field of the output structure RateSpec; the argument value portion of the pair is then assigned to its paired field.

If the optional argument RateSpec is specified, intenvset modifies an existing interest term structure RateSpec by changing the named argument to the specified values and recalculating the arguments dependent on the new values.
[RateSpec, RateSpecOld] = intenvset creates an interest term structure RateSpec with all fields set to [].
intenvset with no input or output arguments displays a list of argument names and possible values.

RateSpecOld is a structure containing the properties of an interest-rate structure before the changes introduced by the call to intenvset.

## Examples Create a RateSpec for a Zero Curve

Use intenvset to create a RateSpec for a zero curve.

```
RateSpec = intenvset('Rates', 0.05, 'StartDates',...
'20-Jan-2000', 'EndDates', '20-Jan-2001')
```


## RateSpec =

```
            FinObj: 'RateSpec'
        Compounding: 2
            Disc: 0.9518
            Rates: 0.0500
            EndTimes: 2
        StartTimes: 0
            EndDates: 730871
        StartDates: 730505
        ValuationDate: 730505
```


## intenvset

Basis: 0<br>EndMonthRule: 1

Now change the Compounding argument to 1 (annual).

```
RateSpec = intenvset(RateSpec, 'Compounding', 1)
RateSpec =
                    FinObj: 'RateSpec'
        Compounding: 1
            Disc: 0.9518
            Rates: 0.0506
            EndTimes: 1
        StartTimes: 0
            EndDates: 730871
        StartDates: 730505
    ValuationDate: 730505
            Basis: 0
    EndMonthRule: 1
```

Calling intenvset with no input or output arguments displays a list of argument names and possible values.

```
intenvset
```

```
    Compounding: [ 1 | {2} | 3 | 4 | 6 | 12 | 365 | - 1 ]
                            Disc: [ scalar | vector (NPOINTS x 1) ]
                            Rates: [ scalar | vector (NPOINTS x 1) ]
            EndDates: [ scalar | vector (NPOINTS x 1) ]
            StartDates: [ scalar | vector (NPOINTS x 1) ]
ValuationDate: [ scalar ]
    Basis: [ {0} | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10
EndMonthRule: [ 0 | {1} ]
```


## Create a RateSpec for a Forward Curve

```
Use intenvset to create a RateSpec for a forward curve.
RateSpec = intenvset('Rates', 0.05, 'StartDates',...
'20-Jan-2001', 'EndDates', '20-Jan-2002', 'ValuationDate','20-Jan-2000
RateSpec =
    FinObj: 'RateSpec
        Compounding: 2
            Disc: 0.9518
            Rates: 0.0500
            EndTimes: 4
        StartTimes: 2
            EndDates: 731236
            StartDates: 730871
    ValuationDate: 730505
            Basis: 0
    EndMonthRule: 1
```

Now change the Compounding argument to 1 (annual).
RateSpec = intenvset(RateSpec, 'Compounding', 1)

RateSpec =
FinObj: 'RateSpec'
Compounding: 1
Disc: 0.9518
Rates: 0.0506
EndTimes: 2

## intenvset

```
    StartTimes: 1
        EndDates: 731236
    StartDates: 730871
ValuationDate: 730505
    Basis: 0
EndMonthRule: 1
```


## Create a RateSpec Using Two Curves

Define data for the interest-rate term structure and use intenvset to create a RateSpec.

```
StartDates = '01-Oct-2011';
'EndDates', EndDates, 'Compounding', 1)
RateSpec =
```

```
                    FinObj: 'RateSpec'
```

                    FinObj: 'RateSpec'
        Compounding: 1
        Compounding: 1
            Disc: [4x2 double]
            Disc: [4x2 double]
            Rates: [4x2 double]
            Rates: [4x2 double]
            EndTimes: [4x1 double]
            EndTimes: [4x1 double]
        StartTimes: [4x1 double]
        StartTimes: [4x1 double]
            EndDates: [4x1 double]
            EndDates: [4x1 double]
        StartDates: 734777
        StartDates: 734777
    ValuationDate: 734777
ValuationDate: 734777
Basis: 0
Basis: 0
EndMonthRule: 1

```
    EndMonthRule: 1
```

EndDates = ['01-Oct-2012'; '01-Oct-2013';'01-Oct-2014';'01-Oct-2015'];
Rates $=$ [ [0.0356;0.041185;0.04489;0.047741],[0.0325;0.0423;0.0437;0.0465]
RateSpec = intenvset('Rates', Rates, 'StartDates',StartDates,...

To look at the Rates for the two interest-rate curves:

RateSpec. Rates

```
ans =
    0.0356 0.0325
    0.0412 0.0423
    0.0449 0.0437
    0.0477 0.0465
```

See Also intenvget | intenvprice | intenvset

## isafin

Purpose $\quad \begin{aligned} & \text { True if input argument is financial structure type or financial object } \\ & \text { class }\end{aligned}$
Syntax IsFinObj = isafin(Obj, ClassName)
Arguments

| Obj | Name of a financial structure. |
| :--- | :--- |
| ClassName | String containing the name of a financial structure <br> class. |


| Description | IsFinObj = isafin(Obj, ClassName) returns True (1) if input <br> argument is a financial structure type or financial object class, <br> otherwise False ( 0 ) is returned. |
| :--- | :--- |

Examples load deriv.mat
IsFinObj = isafin(HJMTree, 'HJMFwdTree')

IsFinObj =
1
See Also classfin

## Purpose

Price instruments using implied trinomial tree (ITT)
Syntax
Price = ittprice(ITTTree, InstSet)
Price = ittprice(ITTTree, InstSet, Options)
[Price, PriceTree] = ittprice(ITTTree, InstSet, Options)

## Arguments

| ITTTree | Implied trinomial stock tree. See itttree for <br> information on creating the variable ITTTree. |
| :---: | :--- |
| InstSet | Variable containing a collection of NINST instruments. <br> Instruments are broken down by type and each type <br> can have different data fields. |
| Options | (Optional) Structure created using derivset <br> containing derivative pricing options. |

## Description

```
Price = ittprice(ITTTree, InstSet)
Price = ittprice(ITTTree, InstSet, Options)
[Price, PriceTree] = ittprice(ITTTree, InstSet, Options)
```

The outputs for ittprice are:

- Price is a NINST-by- 1 vector of prices of each instrument at time 0 . The prices are computed by backward dynamic programming on the stock tree. If an instrument cannot be priced, a NaN is returned in that entry.
- PriceTree is a structure containing trees of vectors of instrument prices and a vector of observation times for each node.
- PriceTree.PTree contains the prices.
- PriceTree.tObs contains the observation times.
- PriceTree.dObs contains the observation dates.


## ittprice

ittprice computes prices for instruments using an implied trinomial tree created with itttree.

Note ittprice handles the following instrument types: optstock, barrier, Asian, lookback, and compound. Use instadd to construct the defined types.

When using an implied trinomial tree, pricing of path-dependent options is done using Hull-White. Consequently, for these options there are no unique prices on the tree nodes with the exception of the root node. The corresponding nodes of the tree are populated with NaNs for these particular options. For information on single-type pricing functions to retrieve state-by-state pricing tree information, see the following:

- barrierbyitt for pricing barrier options using an ITT tree
- optstockbyitt for pricing American, European or Bermuda options using an ITT tree
- asianbyitt for pricing Asian options using an ITT tree
- lookbackbyitt for pricing lookback options using an ITT tree
- compoundbyitt for price compound options using an ITT tree


## Examples Price Instruments Using Implied Trinomial Tree (ITT)

Load the ITT tree and instruments from the data file deriv.mat.
load deriv.mat
Display the barrier and Asian options contained in the instrument set.

```
ITTSubSet = instselect(ITTInstSet,'Type', {'Barrier', 'Asian'});
```

instdisp(ITTSubSet)

| Index | Type | OptSpec Strike Settle | ExerciseDates | AmericanOp |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | Barrier call | 85 | $01-$ Jan-2006 | $31-$ Dec-2008 | 1 |

Price the barrier and Asian options contained in the instrument set.

```
[Price, PriceTree] = ittprice(ITTTree, ITTSubSet)
```

Price =
2.4074
3.2052
6.6074
PriceTree =
FinObj: 'TrinPriceTree'
PTree: \{1x5 cell\}
tobs: [ $\left.\begin{array}{lllll}0 & 1 & 2 & 3 & 4\end{array}\right]$


## See Also ittsens | itttree

Purpose
Instrument sensitivities and prices using implied trinomial tree (ITT)

## Syntax

```
[Delta, Gamma, Vega] = ittsens(ITTTree, InstSet)
[Delta, Gamma, Vega, Price] = ittsens(ITTTree, InstSet)
[Delta, Gamma, Vega, Price] = ittsens(ITTTree, InstSet,
Options)
```


## Arguments

| ITTTree | Implied trinomial stock tree. See itttree for <br> information on creating the variable ITTTree. |
| :---: | :--- |
| InstSet | Variable containing a collection of NINST instruments. <br> Instruments are broken down by type and each type <br> can have different data fields. |
| Options | (Optional) Structure created using derivset <br> containing derivative pricing options. |

## Description

[Delta, Gamma, Vega] = ittsens(ITTTree, InstSet) [Delta, Gamma, Vega, Price] = ittsens(ITTTree, InstSet) [Delta, Gamma, Vega, Price] = ittsens(ITTTree, InstSet, Options)
The outputs for ittsens are:

- Delta is a NINST-by- 1 vector of deltas, representing the rate of change of instruments prices with respect to changes in the stock price.
- Gamma is a NINST-by- 1 vector of gammas, representing the rate of change of instruments deltas with respect to changes in the stock price.
- Vega is a NINST-by-1 vector of vegas, representing the rate of change of instruments prices with respect to changes in the volatility of the stock. Vega is computed by finite differences in calls to itttree.
- Price is a NINST-by-1 vector of prices of each instrument. The prices are computed by backward dynamic programming on the stock tree. If an instrument cannot be priced, a NaN is returned.
ittsens computes dollar sensitivities and prices for instruments using an ITT tree created with itttree.

Note ittsens handles the following instrument types: optstock, barrier, Asian, lookback, and compound. Use instadd to construct the defined types.

For path-dependent options (lookbacks and Asians), Delta and Gamma are computed by finite differences in calls to ittprice. For the rest of the options (optstock, barrier, and compound), Delta and Gamma are computed from the ITT tree and the corresponding option price tree.

All sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, they must be divided by their respective instrument price.

## Examples Compute Instrument Sensitivities Using an Implied Trinomial Tree (ITT)

Load the ITT tree and instruments from the data file deriv.mat and display the vanilla options and barrier option instruments.

```
load deriv.mat
ITTSubSet = instselect(ITTInstSet,'Type', {'OptStock', 'Barrier'});
instdisp(ITTSubSet)
```

| Index | Type | OptSpec | Strike Settle | ExerciseDates | American0 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | OptStock | call | 95 | $01-$ Jan-2006 | $31-$ Dec-2008 | 1 |
| 2 | OptStock put | 80 | 01 -Jan-2006 | $01-$ Jan-2010 | 0 |  |

Index Type OptSpec Strike Settle ExerciseDates AmericanOp

## ittsens

Compute the Delta and Gamma sensitivities of vanilla options and barrier option contained in the instrument set.

```
[Delta, Gamma] = ittsens(ITTTree, ITTSubSet)
```

Warning: The option set specified in StockOptSpec was too narrow for the generated tree.
This made extrapolation necessary. Below is a list of the options that we outside of the range of those specified in StockOptSpec.

```
Option Type: 'call' Maturity: 01-Jan-2007 Strike=67.2897
```

Option Type: 'put' Maturity: 01-Jan-2007 Strike=37.1528
Option Type: 'put' Maturity: 01-Jan-2008 Strike=27.6066
Option Type: 'put' Maturity: 31-Dec-2008 Strike=20.5132
Option Type: 'call' Maturity: 01-Jan-2010 Strike=164.0157
Option Type: 'put' Maturity: 01-Jan-2010 Strike=15.2424

Delta =
0.2387
-0.4283
0.3482

Gamma =
0.0260
0.0188
0.0380

# References Chriss, Neil. and I. Kawaller, Black-Scholes and Beyond: Options Pricing Models, McGraw-Hill, 1996, pp. 308-312. 

See Also ittprice | itttree

## itttimespec

Purpose Specify time structure using implied trinomial tree (ITT)
Syntax TimeSpec = itttimespec (ValuationDate, Maturity, NumPeriods)

## Arguments

## Description

| ValuationDate | Scalar date marking the pricing date and first <br> observation in the tree. Specify ValuationDate as <br> a serial date number or date string. |
| :--- | :--- |
| Maturity | Scalar date marking the depth of the tree. |
| NumPeriods | Scalar that determines how many time steps are <br> in the tree. |

## Examples Creates the Structure Specifying the Time Layout for an ITT Tree

This example shows how to specify a four-period tree with time steps of 1 year.

ValuationDate = '1-July-2006';
Maturity = '1-July-2010';
TimeSpec = itttimespec(ValuationDate, Maturity, 4)

TimeSpec =
FinObj: 'ITTTimeSpec'
ValuationDate: 732859
Maturity: 734320
NumPeriods: 4
Basis: 0

```
EndMonthRule: 1
        tObs: [0 1 1 2 3 4]
    dObs: [732859 733224 733589 733954 734320]
```

See Also itttree | stockspec

Purpose Build implied trinomial stock tree

Syntax itttree(StockSpec, RateSpec, TimeSpec, StockOptSpec)

## Arguments

## Description

## Examples

StockSpec Stock specification. For more information, see stockspec.

RateSpec Interest rate specification of the initial risk-free rate curve. For more information on declaring an interest rate variable, see intenvset.

TimeSpec Tree time layout specification. Defines the observation dates of the implied trinomial tree. For more information on the tree structure, see itttimespec.

StockOptSpec Option stock specification. For more information, see stockoptspec.
itttree(StockSpec, RateSpec, TimeSpec, StockOptSpec) creates the itttree structure specifying stock and time information for an implied trinomial tree.

For this example, assume that the interest rate is fixed at $8 \%$ annually between the valuation date of the tree (January 1, 2006) until its maturity.

```
Rate = 0.08
ValuationDate = '01-01-2006';
EndDate = '01-01-2008';
RateSpec = intenvset('StartDates', ValuationDate, 'EndDates', EndDate, ...
    'ValuationDate', ValuationDate, 'Rates', Rate, 'Compounding', -1);
```

To build an ITTTree, create StockSpec, TimeSpec, and StockOptSpec structures.

To create theStockSpec structure:

```
Sigma = 0.20;
```

AssetPrice = 50;
DividendType = 'cash';
DividendAmounts $=$ [0.50; 0.50; 0.50; 0.50];
ExDividendDates = \{'03-Jan-2007'; '01-Apr-2007'; '05-July-2007';'01-Oct-2007'\}
StockSpec = stockspec(Sigma, AssetPrice, DividendType, ...
DividendAmounts, ExDividendDates)

StockSpec =

```
            FinObj: 'StockSpec'
            Sigma: 0.2000
            AssetPrice: 50
        DividendType: 'cash'
    DividendAmounts: [4x1 double]
    ExDividendDates: [4x1 double]
```

The syntax for building a TimeSpec structure is TimeSpec $=$ itttimespec(ValuationDate, Maturity, NumPeriods).

Consider building an ITT tree, with a valuation date of January 1, 2006; a maturity date of January 1, 2008; and four time steps.

ValuationDate = '01-01-2006';
EndDate = '01-01-2008';
NumPeriods = 4;
TimeSpec = itttimespec(ValuationDate, EndDate, NumPeriods)
TimeSpec =
FinObj: 'ITTTimeSpec'

## ittrree

```
ValuationDate: 732678
        Maturity: 733408
    NumPeriods: 4
            Basis: 0
EndMonthRule: 1
    tObs: [0 0.5000 1 1.5000 2]
    dObs: [732678 732860 733043 733225 733408]
```

The syntax for building a StockOptSpec structure is [StockOptSpec] = stockoptspec(OptPrice, Strike, Settle, Maturity, OptSpec).

```
Settle = '01/01/06';
```

Maturity = ['07/01/06';
'07/01/06';
'07/01/06';
'07/01/06';
'01/01/07';
'01/01/07';
'01/01/07';
'01/01/07';
'07/01/07';
'07/01/07';
'07/01/07';
'07/01/07';
'01/01/08';
'01/01/08';
'01/01/08';
'01/01/08'];
Strike = [113;
101;
100;
88;
128;
112;
100;

```
    78;
    144;
    112;
    100;
    69;
    162;
    112;
    100;
    61];
OptPrice =[
    0;
    4.807905472659144;
    1.306321897011867;
    0.048039195057173;
                                    0;
    2.310953054191461;
    1.421950392866235;
    0.020414826276740;
                            0;
    5.091986935627730;
    1.346534812295291;
    0.005101325584140;
                                    0;
    8.047628153217246;
    1.219653432150932;
    0.001041436654748];
OptSpec = { 'call';
    'call';
    put';
    put';
    'call';
    'call';
    put';
    'put';
    'call';
```


## ittrree

```
    'call';
    'put';
    'put';
    'call';
    'call';
    'put';
    'put'};
StockOptSpec = stockoptspec(OptPrice, Strike, Settle, Maturity, OptSpec)
StockOptSpec =
    FinObj: 'StockOptSpec
    OptPrice: [16x1 double]
        Strike: [16x1 double]
        Settle: 732678
    Maturity: [16x1 double]
        OptSpec: {16x1 cell}
    InterpMethod: 'price'
```

Note In this example, the extrapolation warnings are turned on to display warnings on the Command Window. These warnings are a consequence of having to extrapolate to find the option price of the tree nodes. In this example, the set of inputs options was too narrow for the shift in the tree nodes introduced by the disturbance used to calculate the sensitivities. As a consequence extrapolation for some of the nodes was needed.

Use the following command to turn on extrapolation warnings:

```
warning('on', 'fininst:itttree:Extrapolation');
```

Use the StockSpec, RateSpec, TimeSpec, and StockOptSpec structure to create an ITTTree.

ITTTree = itttree(StockSpec, RateSpec, TimeSpec, StockOptSpec)

Warning: The option set specified in StockOptSpec was too narrow for the generated tree. This made extrapolation necessary. Below is a list of the options that were outside of the range of those specified in StockOptSpec.

| Option | Type: | 'call' | Maturity: | 02-Jul-2006 | Strike=60.7466 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Option | Type: | 'put' | Maturity: | 02-Jul-2006 | Strike=50.0731 |
| Option | Type: | 'put' | Maturity: | 02-Jul-2006 | Strike=41.3344 |
| Option | Type: | 'call' | Maturity: | 01-Jan-2007 | Strike=73.8592 |
| Option | Type: | 'call' | Maturity: | 01-Jan-2007 | Strike=60.8227 |
| Option | Type: | 'put' | Maturity: | 01-Jan-2007 | Strike=50.1492 |
| Option | Type: | 'put' | Maturity: | 01-Jan-2007 | Strike=41.4105 |
| Option | Type: | 'put' | Maturity: | 01-Jan-2007 | Strike=34.2559 |
| Option | Type: | 'call' | Maturity: | 02-Jul-2007 | Strike=88.8310 |
| Option | Type: | 'call' | Maturity: | 02-Jul-2007 | Strike=72.9081 |
| Option | Type: | 'call' | Maturity: | 02-Jul-2007 | Strike=59.8715 |
| Option | Type: | 'put' | Maturity: | 02-Jul-2007 | Strike=49.1980 |
| Option | Type: | 'put' | Maturity: | 02-Jul-2007 | Strike=40.4594 |
| Option | Type: | 'put' | Maturity: | 02-Jul-2007 | Strike=33.3047 |
| Option | Type: | 'put' | Maturity: | 02-Jul-2007 | Strike=27.4470 |
| Option | Type: | 'call' | Maturity: | 01-Jan-2008 | Strike=107.2895 |
| Option | Type: | 'call' | Maturity: | 01-Jan-2008 | Strike=87.8412 |
| Option | Type: | 'call' | Maturity: | 01-Jan-2008 | Strike=71.9183 |
| Option | Type: | 'call' | Maturity: | 01-Jan-2008 | Strike=58.8817 |
| Option | Type: | 'put' | Maturity: | 01-Jan-2008 | Strike=48.2083 |
| Option | Type: | 'put' | Maturity: | 01-Jan-2008 | Strike=39.4696 |
| Option | Type: | 'put' | Maturity: | 01-Jan-2008 | Strike=32.3150 |
| Option | Type: | 'put' | Maturity: | 01-Jan-2008 | Strike=26.4573 |
| Option | Type: | 'put' | Maturity: | 01-Jan-2008 | Strike=21.6614 |

> In itttree>Interp0ptPrices at 675
In itttree at 277

ITTTree =

FinObj: 'ITStockTree<br>StockSpec: [1×1 struct]<br>StockOptSpec: [1x1 struct]

## itttree

```
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
    tObs: [0 0.500000000000000 1 1.500000000000000 2]
    dObs: [732678 732860 733043 733225 733408]
    STree: {1x5 cell}
    Probs: {[3\times1 double] [3\times3 double] [3\times5 double] [3\times7 double]}
```

See Also
intenvset | itttimespec | stockoptspec | stockspec

## LiborMarketModel

## Superclasses

Purpose

Create LIBOR Market Model

## Description

The LIBOR Market Model (LMM) differs from short rate models in that it evolves a set of discrete forward rates. Specifically, the lognormal LMM specifies the following diffusion equation for each forward rate

$$
\frac{d F_{i}(t)}{F_{i}}=-\mu_{i} d t+\sigma_{i}(t) d W_{i}
$$

where:
$W$ is an N -dimensional geometric Brownian motion with

$$
d W_{i}(t) d W_{j}(t)=\rho_{i j}
$$

The LMM relates the drifts of the forward rates based on no-arbitrage arguments. Specifically, under the Spot LIBOR measure, the drifts are expressed as

$$
\mu_{i}(t)=-\sigma_{i}(t) \sum_{j=q(t)}^{i} \frac{\tau_{j} \rho_{i, j} \sigma_{j}(t) F_{j}(t)}{1+\tau_{j} F_{j}(t)}
$$

where:
$\tau_{i}$ is the time fraction associated with the $i$ th forward rate $q(t)$ is an index defined by the relation

$$
T_{q(t)-1}<t<T_{q(t)}
$$

and the Spot LIBOR numeraire is defined as

$$
B(t)=P\left(t, T_{q(t)}\right) \prod_{n=0}^{q(t)-1}\left(1+\tau_{n} F_{n}\left(T_{n}\right)\right)
$$

## LiborMarketModel

Construction
OBJ = LiborMarketModel(ZeroCurve,VolFunc,Correlation) constructs a LIBOR Market Model object.

For example:

```
Settle = datenum('15-Dec-2007');
CurveTimes = [1:5 7 10 20]';
ZeroRates = [.01 .018 .024 .029 .033 .034 .035 .034]';
CurveDates = daysadd(Settle,360*CurveTimes,1);
irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);
LMMVolFunc = @(a,t) (a(1)*t + a(2)).*exp(-a(3)*t) + a(4);
LMMVolParams = [.3 -.02 .7 .14];
numRates = 20;
VolFunc(1:numRates-1) = {@(t) LMMVolFunc(LMMVolParams,t)};
```

Beta = .08;
CorrFunc = @(i,j,Beta) $\exp (-B e t a * a b s(i-j))$;
Correlation = CorrFunc(meshgrid(1:numRates-1)',meshgrid(1:numRates-1), Bet
LMM = LiborMarketModel(irdc,VolFunc,Correlation,'Period',1);

## Properties The following properties are from the LiborMarketModel class.

## ZeroCurve

ZeroCurve is specified using the output from IRDataCurve or RateSpec. This is the zero curve used to evolve the path of future interest rates.

## Attributes:

SetAccess public

GetAccess public

## VolFunc

NumRates-by-1 cell array of function handles. Each function handle must take time as an input and, return a scalar volatility.

Attributes:

SetAccess
GetAccess
public
public

## Correlation

NumRates-by-NumRates correlation matrix.

## Attributes:

SetAccess public
GetAccess public

## NumFactors

Number of Brownian factors. The default is NaN, where the number of factors is equal to the number of rates.

## Attributes:

| SetAccess | public |
| :--- | :--- |
| GetAccess | public |

## Period

Period of the forward rates. The default is 2 , meaning forward rates are spaced at $0, .5,1,1.5$ and so on. Possible values for Period are: 1, 2, 4, and 12.

Note Correlation and VolFunc are sized with NumRates-by-1 since the first rate is locked in and essentially dead.

## Attributes:

## LiborMarketModel

| SetAccess | public |
| :--- | :--- |
| GetAccess | public |

## Methods

simTermStructs

Simulate term structures for LIBOR Market Model

## Definitions <br> LIBOR Market Model

Copy
Semantics
The LIBOR Market Model, also called the BGM Model (Brace, Gatarek, Musiela Model) is a financial model of interest rates. The quantities that are modeled are a set of forward rates (also called forward LIBORs) which have the advantage of being directly observable in the market, and whose volatilities are naturally linked to traded contracts.

## Examples <br> Construct a LIBOR Market Model

Value. To learn how value classes affect copy operations, see Copying Objects in the MATLAB documentation.

Construct a LMM object.

```
Settle = datenum('15-Dec-2007');
CurveTimes = [1:5 7 10 20]';
ZeroRates = [.01 .018 .024 .029 .033 .034 .035 .034]';
CurveDates = daysadd(Settle,360*CurveTimes,1);
irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);
LMMVolFunc = @(a,t) (a(1)*t + a(2)).*exp(-a(3)*t) + a(4);
LMMVolParams = [.3 -. 02 .7 .14];
numRates = 20;
VolFunc(1:numRates-1) = {@(t) LMMVolFunc(LMMVolParams,t)};
```


## LiborMarketModel

```
Beta = .08;
CorrFunc = @(i,j,Beta) exp(-Beta*abs(i-j));
Correlation = CorrFunc(meshgrid(1:numRates-1)',meshgrid(1:numRates -1)
LMM = LiborMarketModel(irdc,VolFunc,Correlation,'Period',1);
```

Simulate the term structures for the specified LMM object.
[ZeroRates, ForwardRates] = LMM.simTermStructs(10,'nTrials',100);

## References Brigo, D. and F. Mercurio, Interest Rate Models - Theory and Practice, Springer Finance, 2006.

See Also HullWhite1F | LinearGaussian2F | simTermStructs

## Related Examples

- "Price Swaptions With Interest-Rate Models Using Simulation" on page 2-105
- Class Attributes
- Property Attributes


## LiborMarketModel.simTermStructs

Purpose Simulate term structures for LIBOR Market Model

```
Syntax
[ZeroRates,ForwardRates] = simTermStructs(nPeriods)
[ZeroRates,ForwardRates] = simTermStructs(nPeriods,
Name, Value)
```

Description

Input Arguments
[ZeroRates,ForwardRates] = simTermStructs(nPeriods) simulates future zero curve paths using a specified LiborMarketModel object.
[ZeroRates,ForwardRates] = simTermStructs(nPeriods, Name, Value) simulates future zero curve paths using a specified LiborMarketModel object with additional options specified by one or more Name, Value pair arguments.

## nPeriods

Number of simulation periods.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, Valuen.

## 'nTrials'

Positive scalar integer number of simulated trials (sample paths) of NPERIODS observations each. If you do not specify a value for this argument, the default is 1 , indicating a single path of correlated state variables.

## Default: 1

## 'antithetic'

Boolean scalar flag indicating whether antithetic sampling is used to generate the Gaussian random variates that drive the

## LiborMarketModel.simTermStructs

zero-drift, unit-variance rate Brownian vector $d W(t)$. For details, see simBySolution.

Default: false

## 'Z'

Direct specification of the dependent random noise process used to generate the zero-drift, unit-variance rate Brownian vector $d W(t)$ that drives the simulation. For details, see simBySolution for the GBM model.

Default: Uses default for simBySolution. If you do not specify a value for $Z$, simBySolution generates Gaussian variates.

## 'Tenor'

Numeric vector of maturities to compute at each time step.
Default: tenor the number of rates in the LiborMarketModel object as specified by the correlation and volatility functions

## Output

Arguments

## ZeroRates

nPeriods+1-by-nTenors-by-nTrials matrix of simulated zero-rate term structures.

## ForwardRates

nPeriods+1-by-nTenors-by-nTrials matrix of simulated forward-rate term structures.

## Attributes

| Access | public |
| :--- | :--- |
| Static | false |
| Hidden | false |

## LiborMarketModel.simTermStructs

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

## Examples Simulate Term Structures for a LIBOR Market Model

Create a LMM object.

```
Settle = datenum('15-Dec-2007');
CurveTimes = [1:5 7 10 20]';
ZeroRates = [.01 .018 .024 .029 .033 . 034 .035 .034]';
CurveDates = daysadd(Settle,360*CurveTimes,1);
irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);
LMMVolFunc = @(a,t) (a(1)*t + a(2)).*exp(-a(3)*t) + a(4);
LMMVolParams = [.3 -. 02 .7 .14];
numRates = 20;
VolFunc(1:numRates-1) = {@(t) LMMVolFunc(LMMVolParams,t)};
```

Beta = .08;
CorrFunc = @(i,j,Beta) $\exp (-B e t a * a b s(i-j)) ;$
Correlation = CorrFunc(meshgrid(1:numRates-1)',meshgrid(1:numRates-1), Bet
LMM = LiborMarketModel(irdc,VolFunc,Correlation,'Period',1)

LMM =

LiborMarketModel with properties:
ZeroCurve: [1x1 IRDataCurve]
VolFunctions: \{1x19 cell\}
Correlation: [19x19 double]
NumFactors: NaN
Period: 1

## LiborMarketModel.simTermStructs

Simulate the term structures for the specified LMM object.
[ZeroRates, ForwardRates] = LMM.simTermStructs(20,'nTrials',100);

| See Also | LiborMarketModel \| blackvolbyrebonato |
| :--- | :--- |
| Related | - "Price Swaptions With Interest-Rate Models Using Simulation" on |
| Examples | page $2-105$ |

## Superclasses

Purpose
Create two-factor additive Gaussian interest-rate model
Description The two-factor additive Gaussian interest rate-model is specified using the zero curve, $a, b$, sigma, eta, and rho parameters for these equations:

$$
\begin{aligned}
& r(t)=x(t)+y(t)+\phi(t) \\
& d x(t)=-\alpha(t) x(t) d t+\sigma(t) d W_{1}(t), x(0)=0 \\
& d y(t)=-b(t) y(t) d t+\eta(t) d W_{2}(t), y(0)=0
\end{aligned}
$$

where $d W_{1}(t) d W_{2}(t)=\rho d t$ is a two-dimensional Brownian motion with correlation $\rho$, and $\varphi$ is a function chosen to match the initial zero curve.

## Construction

## OBJ = LinearGaussian2F(ZeroCurve,a, b, sigma, eta, rho)

 constructs an object for a two-factor additive Gaussian interest-rate model.For example:

```
Settle = datenum('15-Dec-2007');
CurveTimes = [1:5 7 10 20]';
ZeroRates = [.01 .018 .024 . 029 . 033 .034 .035 .034]';
CurveDates = daysadd(Settle,360*CurveTimes,1);
irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);
a = .07;
b = .5;
sigma = .01;
eta = .006;
rho = -.7;
G2PP = LinearGaussian2F(irdc,a,b,sigma,eta,rho);
```

The following properties are from the LinearGaussian2F class.

## ZeroCurve

ZeroCurve is specified using IRDataCurve or RateSpec. This is the zero curve used to evolve the path of future interest rates.

## Attributes:

| SetAccess | public |
| :--- | ---: |
| GetAccess | public |

## a

Mean reversion for the first factor, specified either as a scalar or function handle which takes time as input and returns a scalar mean reversion value.

## Attributes:

| SetAccess | public |
| :--- | :--- |
| GetAccess | public |

b
Mean reversion for the second factor, specified either as a scalar or as a function handle which takes time as input and returns a scalar mean reversion value.

## Attributes:

```
SetAccess
    public
GetAccess
public
```


## sigma

Volatility for the first factor, specified either as a scalar or function handle which takes time as input and returns a scalar mean volatility.

## LinearGaussian2F

Attributes:
SetAccess public
GetAccess public
eta
Volatility for the second factor specified, either as a scalar or function handle which takes time as input and returns a scalar mean volatility.

## Attributes:

SetAccess public

GetAccess public
rho
Scalar correlation of the factors.
Attributes:
SetAccess public
GetAccess public

## Methods

## Definitions

simTermStructs

Simulate term structures for two-factor additive Gaussian interest-rate model

## Two-Factor Additive Gaussian Interest-Rate Model

Short-rate model based on two factors where the short rate is the sum of the two factors and a deterministic function, in this case $\varphi(t)$, which is chosen to match the initial term structure.

Copy
Semantics

## Examples

Value. To learn how value classes affect copy operations, see Copying Objects in the MATLAB documentation.

## Construct a Two-Factor Additive Gaussian Interest-Rate Model

Construct a two-factor additive Gaussian interest-rate model.

```
Settle = datenum('15-Dec-2007');
CurveTimes = [1:5 7 10 20]';
ZeroRates = [.01 .018 .024 .029 . 033 . 034 .035 .034]';
CurveDates = daysadd(Settle,360*CurveTimes,1);
irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);
a = .07;
b = .5;
sigma = .01;
eta = .006;
rho = -.7;
G2PP = LinearGaussian2F(irdc,a,b,sigma,eta,rho)
```

G2PP =

LinearGaussian2F with properties:

## ZeroCurve: [1x1 IRDataCurve]

a: @(t,V)ina
b: @(t,V)inb
sigma: @(t,V)insigma
eta: @(t,V)ineta
rho: -0.7000

## LinearGaussian2F

Use the simTermStructs method to simulate term structures based on the LinearGaussian2F model.

SimPaths = G2PP.simTermStructs(10,'nTrials',100);

\section*{References Brigo, D. and F. Mercurio, Interest Rate Models - Theory and Practice, Springer Finance, 2006. <br> See Also $\quad$| HullWhite1F \| LiborMarketModel | simTermStructs | capbylg2f | |
| :--- |
| floorbylg2f \| swaptionbylg2f | <br> Related - "Price Swaptions With Interest-Rate Models Using Simulation" on Examples page 2-105 <br> - Class Attributes <br> - Property Attributes}

## LinearGaussian2F.simTermStructs

| Purpose | Simulate term structures for two-factor additive Gaussian interest-rate model |
| :---: | :---: |
| Syntax | ```[ZeroRates,ForwardRates] = simTermStructs(nPeriods) [ZeroRates,ForwardRates] = simTermStructs(nPeriods, Name,Value)``` |
| Description | [ZeroRates,ForwardRates] = simTermStructs(nPeriods) simulates future zero curve paths using a specified LinearGaussian2F object. <br> [ZeroRates,ForwardRates] = simTermStructs(nPeriods, Name, Value) simulates future zero curve paths using a specified LinearGaussian2F object with additional options specified by one or more Name, Value pair arguments. |
| Input Arguments | nPeriods <br> Number of simulation periods. |
|  | Name-Value Pair Arguments |
|  | Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN. |
|  | 'deltaTime' |
|  | Scalar time step between periods. |
|  | Default: 1 |
|  | 'nTrials' |
|  | Positive scalar integer number of simulated trials (sample paths) of NPERIODS observations each. If you do not specify a value for this argument, the default is 1 , indicating a single path of correlated state variables. |

## LinearGaussian2F.simTermStructs

Default: 1

## 'antithetic'

Boolean scalar flag indicating whether antithetic sampling is used to generate the Gaussian random variates that drive the zero-drift, unit-variance rate Brownian vector $d W(t)$. For details, see simBySolution for the HWV model.

Default: false

## 'Z'

Direct specification of the dependent random noise process used to generate the zero-drift, unit-variance rate Brownian vector $d W(t)$ that drives the simulation. For details, see simBySolution for the HWV model.

Default: Uses default for simBySolution. If you do not specify a value for Z, simBySolution generates Gaussian variates.

## 'Tenor'

Numeric vector of maturities to compute at each time step.
Default: tenor of the LinearGaussian2F object's zero curve

## Output Arguments

## ZeroRates

nPeriods+1-by-nTenors-by-nTrials matrix of simulated zero-rate term structures.

## ForwardRates

nPeriods+1-by-nTenors-by-nTrials matrix of simulated forward-rate term structures.

## LinearGaussian2F.simTermStructs

## Attributes

## Examples Simulate Term Structures for the LinearGaussian2F Model

Create a two-factor additive Gaussian interest-rate model.

```
Settle = datenum('15-Dec-2007');
CurveTimes = [1:5 7 10 20]';
ZeroRates = [.01 .018 .024 .029 .033 .034 .035 .034]';
CurveDates = daysadd(Settle,360*CurveTimes,1);
irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);
a = .07;
b = .5;
sigma = .01;
eta = .006;
rho = -.7;
G2PP = LinearGaussian2F(irdc,a,b,sigma,eta,rho)
```

G2PP =
LinearGaussian2F with properties:
ZeroCurve: [1x1 IRDataCurve]
a: $@(t, V)$ ina
b: @(t,V)inb
sigma: @(t,V)insigma

## LinearGaussian2F.simTermStructs

```
eta: @(t,V)ineta
rho: -0.7000
```

Use the simTermStructs method to simulate term structures based on the LinearGaussian2F model.

```
SimPaths = G2PP.simTermStructs(10,'nTrials',100);
```

See Also LinearGaussian2F
Related $\quad$ "Price Swaptions With Interest-Rate Models Using Simulation" on
Examples page $2-105$

## Purpose

Price lookback option from Cox-Ross-Rubinstein tree

Syntax<br>PriceTree = lookbackbycrr(CRRTree, OptSpec, Strike, Settle, ExerciseDates, AmericanOpt)

| CRRTree | Stock tree structure created by crrtree. <br> OptSpec <br> Number of instruments (NINST)-by-1 cell array <br> of strings 'call' or 'put '. |
| :--- | :--- |
| Strike | NINST-by-1 vector of strike price values. Each <br> row is the schedule for one option. To calculate <br> the value of a floating-strike lookback option, <br> specify Strike as NaN. |
| ExerciseDates | NINST-by-1 vector of Settle dates. The settle <br> date for every lookback is set to the valuation <br> date of the stock tree. The lookback argument |
|  | Settle is ignored. |
| For a European option (AmericanOpt = 0): |  |
| NINST-by-1 vector of exercise dates. Each row <br> is the schedule for one option. For a European <br> option, there is only one exercise date, the <br> option expiry date. |  |
| For an American option (AmericanOpt = 1): |  |
| NINST-by-2 vector of exercise date boundaries. |  |
| For each instrument, the option can be <br> exercised on any tree date between or including <br> the pair of dates on that row. If only one |  |
| non-NaN date is listed, or if ExerciseDates |  |
| is NINST-by-1, the option can be exercised |  |

between the valuation date of the stock tree and the single listed exercise date.
AmericanOpt
(Optional) If AmericanOpt $=0, \mathrm{NaN}$, or is unspecified, the option is a European option. If AmericanOpt = 1, the option is an American option.

## Description

PriceTree = lookbackbycrr(CRRTree, OptSpec, Strike, Settle, ExerciseDates, American0pt) calculates the value of fixedand floating-strike lookback options. Data arguments are NINST-by-1 vectors, scalar, or empty. Fill unspecified entries in vectors with NaN. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices [].

Price is a NINST-by- 1 vector of expected option prices at time 0 .

Note lookbackbycrr calculates values of fixed and floating strike lookback options. To compute the value of a floating strike lookback option, strike should be specified as NaN. Pricing of lookback options is done using Hull-White (1993). Consequently, for these options there are not unique prices on the tree nodes with the exception of the root node.

## Examples Price a Lookback Option Using a CRR Binomial Tree

This example shows how to price a lookback option using a CRR binomial tree by loading the file deriv.mat, which provides CRRTree. The CRRTree structure contains the stock specification and time information needed to price the option.

```
load deriv.mat;
OptSpec = 'Call';
Strike = 115;
Settle = '01-Jan-2003';
```

```
ExerciseDates = '01-Jan-2006';
Price = lookbackbycrr(CRRTree, OptSpec, Strike, Settle, ...
ExerciseDates)
```

Price $=$
7.6015
References

Hull, J., and A. White, "Efficient Procedures for Valuing European
and American Path-Dependent Options," Journal of Derivatives, Fall
1993, pp. 21-31.

See Also<br>crrtree | instlookback

Purpose | Calculate prices of European lookback options using Conze-Viswanathan |
| :--- |
| and Goldman-Sosin-Gatto models |

Syntax Price = lookbackbycvgsg(RateSpec, StockSpec, OptSpec, Strike, Settle,
ExerciseDates)
Price = lookbackbycvgsg( __ , Name, Value)

## Description Price =

lookbackbycvgsg(RateSpec,StockSpec,OptSpec,Strike,Settle, ExerciseDates) returns prices of European lookback options using Conze-Viswanathan and Goldman-Sosin-Gatto models. lookbackbycvgsg calculates prices of European fixed- and floating-strike lookback options. To compute the value of a floating-strike lookback option, Strike must be specified as NaN. The Goldman-Sosin-Gatto model is used for floating-strike lookback options. The Conze-Viswanathan model is used for fixed-strike lookback options.

Price = lookbackbycvgsg( __ , Name, Value) returns prices of European lookback options using Conze-Viswanathan and Goldman-Sosin-Gatto models with optional name-value pair arguments. lookbackbycvgsg calculates prices of European fixedand floating-strike lookback options. To compute the value of a floating-strike lookback option, Strike must be specified as NaN. The Goldman-Sosin-Gatto model is used for floating-strike lookback options. The Conze-Viswanathan model is used for fixed-strike lookback options.

RateSpec - Interest-rate term structure
structure
Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from intenvset. For information on the interest-rate specification, see intenvset.

Data Types
struct

## StockSpec - Stock specification for underlying asset

structure
Stock specification for the underlying asset. For information on the stock specification, see stockspec.
stockspec handles several types of underlying assets. For example, for physical commodities the price is represented by StockSpec.Asset, the volatility is represented by StockSpec.Sigma, and the convenience yield is represented by StockSpec.DividendAmounts.

## Data Types <br> struct

## OptSpec - Definition of option

string with values 'call' or 'put ' | cell array of strings
Definition of option as 'call' or 'put', specified as a NINST-by-1 cell array of strings.

Data Types
char | cell

## Strike - Option strike price values

integer | vector of integers
Option strike price values, specified as an integer using a NINST-by-1 vector of strike price values.

## Data Types

single | double

## Settle - Settlement or trade date

nonnegative integer | vector of nonnegative integers | date string | cell array of date strings

Settlement or trade date for the lookback option, specified as a date string or as nonnegative integers using a NINST-by-1 vector or cell array of dates.

Data Types
char | cell

## lookbackbycvgsg

## ExerciseDates - European option expiry date <br> nonnegative integer | vector of nonnegative integers | date string | cell array of date strings

European option expiry date, specified as a date string or as nonnegative integers using a NINST-by-1 vector or cell array of dates.

Data Types
char | cell

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

Example: Price =
lookbackbycvgsg(RateSpec,StockSpec,OptSpec, Strike,Settle, ExerciseDates,

## Output Price - Expected prices of lookback option Arguments <br> vector

'AssetMinMax' - Maximum or minimum underlying asset price if unspecified, the lookback option is newly issued, and AssetMinMax = StockSpec.AssetPrice (default) | nonnegative integer

Maximum or minimum underlying asset price, specified as a NINST-by-1 vector.

Data Types<br>single | double

Expected prices of the lookback option, returned as a NINST-by-1 vector.

## Examples Compute the Price of a Floating Lookback Option Using the Goldman-Sosin-Gatto Model

Define the RateSpec.

```
StartDates = 'Jan-1-2013';
EndDates = 'Jan-1-2014';
Rates = 0.042;
Compounding = -1;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', Start[
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
RateSpec =
                    FinObj: 'RateSpec'
        Compounding: -1
            Disc: 0.9589
            Rates: 0.0420
            EndTimes: 1
        StartTimes: 0
            EndDates: 735600
        StartDates: 735235
ValuationDate: 735235
            Basis: 0
EndMonthRule: 1
```

Define the StockSpec.

```
AssetPrice = 50;
Sigma = 0.36;
StockSpec = stockspec(Sigma, AssetPrice)
```

StockSpec =
FinObj: 'StockSpec'
Sigma: 0.3600
AssetPrice: 50
DividendType: []
DividendAmounts: 0

## lookbackbycvgsg

ExDividendDates: []

Define the floating lookback options.

```
Settle = 'Jan-1-2013';
Maturity = 'April-1-2013';
OptSpec = {'put';'call'};
Strike = NaN;
```

Compute the price of the European floating lookback options.
Price = lookbackbycvgsg(RateSpec, StockSpec, OptSpec, Strike, Settle, Mat

Price =
7.2581
6.9777

## Compute the Price of a Fixed Lookback Option Using the Conze-Viswanathan Model

Define the RateSpec.

```
StartDates = 'Jan-1-2013';
EndDates = 'Jan-1-2014';
Rates = 0.045;
Compounding = -1;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDate
'EndDates', EndDates, 'Rates', Rates,'Compounding', Compounding)
```

RateSpec =
FinObj: 'RateSpec'

```
    Compounding: -1
        Disc: 0.9560
        Rates: 0.0450
        EndTimes: 1
        StartTimes: 0
        EndDates: 735600
    StartDates: 735235
ValuationDate: 735235
        Basis: 0
    EndMonthRule: 1
```

Define the StockSpec.

```
AssetPrice = 102;
Sigma = 0.45;
StockSpec = stockspec(Sigma, AssetPrice)
```

StockSpec =

```
                    FinObj: 'StockSpec'
                    Sigma: 0.4500
        AssetPrice: 102
        DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Define the fixed lookback options.

```
Settle = 'Jan-1-2013';
Maturity = 'July-1-2013';
OptSpec = {'put';'call'};
Strike = [98;101];
```

Price the European fixed lookback options.

## lookbackbycvgsg

Price $=$ lookbackbycvgsg(RateSpec, StockSpec, OptSpec, Strike, Settle, Mat

Price =
18.3130
30.4021

## References

Hull, J. C. Options, Futures, and Other Derivatives, 5th ed. Englewood Cliffs, NJ: Prentice Hall, 2002.

See Also lookbacksensbycvgsg | lookbackbyls | lookbacksensbyls |
Concepts • "Lookback Option" on page 3-38

## lookbacksensbycvgsg

## Purpose

Syntax

## Description

## Input Arguments

Calculate prices and sensitivities of European lookback options using Conze-Viswanathan and Goldman-Sosin-Gatto models

```
PriceSens = lookbacksensbycvgsg(RateSpec,StockSpec,OptSpec,Strike,Set
ExerciseDates)
PriceSens = lookbacksensbycvgsg(___,Name,Value)
```

PriceSens =
lookbacksensbycvgsg(RateSpec,StockSpec, OptSpec,Strike,Settle, ExerciseDates) returns prices and sensitivities of European lookback options using Conze-Viswanathan and Goldman-Sosin-Gatto models. lookbacksensbycvgsg calculates prices of European fixed- and floating-strike lookback options. To compute the value of a floating-strike lookback option, Strike must be specified as NaN. The Goldman-Sosin-Gatto model is used for floating-strike lookback options. The Conze-Viswanathan model is used for fixed-strike lookback options.

PriceSens = lookbacksensbycvgsg( __ , Name, Value) returns prices and sensitivities of European lookback options using Conze-Viswanathan and Goldman-Sosin-Gatto models with optional name-value pair arguments. lookbacksensbycvgsg calculates prices of European fixed- and floating-strike lookback options. To compute the value of a floating-strike lookback option, Strike must be specified as NaN. The Goldman-Sosin-Gatto model is used for floating-strike lookback options. The Conze-Viswanathan model is used for fixed-strike lookback options.

## RateSpec - Interest-rate term structure <br> structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from intenvset. For information on the interest-rate specification, see intenvset.

## Data Types <br> struct

## lookbacksensbycvgsg

## StockSpec - Stock specification for underlying asset <br> structure

Stock specification for the underlying asset. For information on the stock specification, see stockspec.
stockspec handles several types of underlying assets. For example, for physical commodities the price is represented by StockSpec.Asset, the volatility is represented by StockSpec.Sigma, and the convenience yield is represented by StockSpec.DividendAmounts.

## Data Types <br> struct

## OptSpec - Definition of option

string with values 'call' or 'put' | cell array of strings
Definition of option as 'call' or 'put', specified as a NINST-by-1 cell array of strings.

Data Types
char | cell

## Strike - Option strike price values

integer | vector of integers
Option strike price values, specified as an integer using a NINST-by-1 vector of strike price values.

Data Types
single | double

## Settle - Settlement or trade date

nonnegative integer | vector of nonnegative integers | date string | cell array of date strings

Settlement or trade date for the lookback option, specified as a date string or as nonnegative integers using a NINST-by-1 vector or cell array of dates.

## Data Types <br> char | cell

## ExerciseDates - European option expiry date <br> nonnegative integer | vector of nonnegative integers | date string | cell array of date strings

European option expiry date, specified as a date string or as nonnegative integers using a NINST-by- 1 vector or cell array of dates.

## Data Types <br> char | cell

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: PriceSens =
lookbacksensbycvgsg(RateSpec,StockSpec, OptSpec, Strike, Settle, Exercise

## 'AssetMinMax' - Maximum or minimum underlying asset price

if unspecified, the lookback option is newly issued, and AssetMinMax = StockSpec.AssetPrice (default) | nonnegative integer
Maximum or minimum underlying asset price, specified as a NINST-by-1 vector.

Data Types
single | double

## 'OutSpec' - Define outputs

\{'Price'\} (default) | string with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | cell array of strings with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

## lookbacksensbycvgsg


#### Abstract

Define outputs, specifying a NOUT- by-1 or a 1-by-NOUT cell array of strings with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

OutSpec = \{'All'\} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec to include each sensitivity.


```
Example: OutSpec =
\{'delta','gamma', 'vega','lambda','rho','theta','price'\}
```

```
Data Types
char | cell
```


## Output Arguments <br> PriceSens - Expected prices and sensitivities of lookback option vector

Expected prices and sensitivities of the lookback option, returned as a NINST-by-1 vector.

## Examples Compute the Price and Delta of a Floating Lookback Option Using the Goldman-Sosin-Gatto Model

Define the RateSpec.

```
StartDates = 'Jan-1-2013';
EndDates = 'Jan-1-2014';
Rates = 0.41;
Compounding = -1;
```


## RateSpec =

```
FinObj: 'RateSpec'
Compounding: -1
Disc: 0.6637
Rates: 0.4100
```

RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDate
'EndDates', EndDates, 'Rates', Rates,'Compounding', Compounding)

```
    EndTimes: 1
    StartTimes: 0
    EndDates: 735600
    StartDates: 735235
ValuationDate: 735235
        Basis: 0
EndMonthRule: 1
```

Define the StockSpec with continuous dividend yield.

```
AssetPrice = 120;
Sigma = 0.3;
Yield = 0.045;
StockSpec = stockspec(Sigma, AssetPrice, 'Continuous', Yield)
StockSpec =
                    FinObj: 'StockSpec'
                    Sigma: 0.3000
        AssetPrice: 120
    DividendType: {'continuous'}
    DividendAmounts: 0.0450
    ExDividendDates: []
```

Define the floating lookback option.

```
Settle = 'Jan-1-2013';
Maturity = 'July-1-2013';
OptSpec = 'call';
Strike = NaN;
SMinMax = 100;
```

Compute the price and delta of the European floating lookback option.

## lookbacksensbycvgsg

```
OutSpec = {'price', 'delta'};
[Price, Delta] = lookbacksensbycvgsg(RateSpec, StockSpec, OptSpec, Strike
Settle, Maturity,'AssetMinMax', SMinMax, 'OutSpec', OutSpec)
```

Price =
36.9926

Delta =
0.8659

## Compute the Price and Delta of a Fixed Lookback Option Using the Conze-Viswanathan Model

Define the RateSpec.
StartDates = 'Jan-1-2013';
EndDates = 'Jan-1-2015';
Rates = 0.1;
Compounding = -1;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDate 'EndDates', EndDates, 'Rates', Rates,'Compounding', Compounding)

## RateSpec =

FinObj: 'RateSpec'
Compounding: -1
Disc: 0.8187
Rates: 0.1000
EndTimes: 2
StartTimes: 0
EndDates: 735965

```
StartDates: 735235
ValuationDate: 735235
Basis: 0
EndMonthRule: 1
```

Define the StockSpec.

```
AssetPrice = 103;
Sigma = 0.30;
StockSpec = stockspec(Sigma, AssetPrice)
```


## StockSpec =

```
                    FinObj: 'StockSpec'
                    Sigma: 0.3000
        AssetPrice: 103
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Define the fixed lookback option.

```
Settle = 'Jan-1-2013';
Maturity = 'July-1-2013';
OptSpec = 'call';
Strike = 99;
```

Price and delta for the European fixed lookback option.
OutSpec = \{'price', 'delta'\};
[Price, Delta] = lookbacksensbyls(RateSpec, StockSpec, OptSpec,... Strike, Settle, Maturity,'OutSpec', OutSpec)

## lookbacksensbycvgsg

Price =
22.6783

Delta =
1.1345

## References

Hull, J. C. Options, Futures, and Other Derivatives, 5th ed. Englewood Cliffs, NJ: Prentice Hall, 2002.

See Also lookbackbycvgsg | lookbackbyls | lookbacksensbyls | stockspec |
Concepts

- "Lookback Option" on page 3-38


## Purpose

Price lookback option from Equal Probabilities binomial tree

Syntax

Arguments

| EQPTree | Stock tree structure created by eqptree. <br> OptSpec <br> Strike <br> Settle <br> of strings ' call' or 'put '. |
| :--- | :--- |
|  | NINST-by-1 vector of strike price values. Each <br> row is the schedule for one option. To calculate <br> the value of a floating-strike lookback option, <br> specify Strike as NaN. |
| ExerciseDates | NINST-by-1 vector of Settle dates. The settle <br> date for every lookback is set to the valuation <br> date of the stock tree. The lookback argument |
|  | Settle is ignored. |
| For a European option (AmericanOpt $=0$ ): |  |
| NINST-by-1 vector of exercise dates. Each row <br> is the schedule for one option. For a European <br> option, there is only one exercise date, the <br> option expiry date. |  |
| For an American option (AmericanOpt $=1$ ): |  |
| NINST-by-2 vector of exercise date boundaries. |  |
| For each instrument, the option can be <br> exercised on any tree date between or including <br> the pair of dates on that row. If only one |  |
| non-NaN date is listed, or if ExerciseDates |  |
| is NINST-by-1, the option can be exercised |  |

between the valuation date of the stock tree and the single listed exercise date.
AmericanOpt
(Optional) If AmericanOpt $=0, \mathrm{NaN}$, or is unspecified, the option is a European option. If AmericanOpt = 1, the option is an American option.

## Description

Price = lookbackbyeqp(EQPTree, OptSpec, Strike, ExerciseDates, AmericanOpt) calculates the value of fixed- and floating-strike lookback options. Data arguments are NINST-by-1 vectors, scalar, or empty. Fill unspecified entries in vectors with NaN. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices [ ].

Price is a NINST-by- 1 vector of expected option prices at time 0 .

Note lookbackbyeqp calculates values of fixed and floating strike lookback options. To compute the value of a floating strike lookback option, strike should be specified as NaN. Pricing of lookback options is done using Hull-White (1993). Consequently, for these options there are not unique prices on the tree nodes with the exception of the root node.

## Examples Price a Lookback Option Using an EQP Equity Tree

This example shows how to price a lookback option using an EQP equity tree by loading the file deriv.mat, which provides EQPTree. The EQPTree structure contains the stock specification and time information needed to price the option.

```
load deriv.mat
OptSpec = 'Call';
Strike = 115;
Settle = '01-Jan-2003';
```

ExerciseDates = '01-Jan-2006';Price = lookbackbyeqp(EQPTree, OptSpec, Strike, Settle, ...ExerciseDates)
Price =
8.7941
References Hull, J., and A. White, "Efficient Procedures for Valuing European and American Path-Dependent Options," Journal of Derivatives, Fall 1993, pp. 21-31.
See Also eqptree | instlookback

## lookbackbyitt

Purpose
Price lookback option using implied trinomial tree (ITT)

Syntax<br>[Price, PriceTree] = lookbackbyitt(ITTTree, OptSpec, Strike, Settle, ExerciseDates, AmericanOpt)

## Arguments

| ITTTree | Stock tree structure created by itttree. |
| :---: | :---: |
| OptSpec | Number of instruments (NINST)-by- 1 cell array of strings 'call' or 'put'. |
| Strike | NINST-by-1 vector of strike price values. Each row is the schedule for one option. To calculate the value of a floating-strike lookback option, specify Strike as NaN. |
| Settle | NINST-by- 1 vector of Settle dates. The settle date for every lookback option is set to the ValuationDate of the stock tree. The lookback argument Settle is ignored. |
| ExerciseDates | For a European option (AmericanOpt $=0$ ): <br> NINST-by-1 vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date. |
|  | For an American option (AmericanOpt = 1): <br> NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is NINST-by-1, the option can be exercised |

## lookbackbyitt

$$
\begin{array}{ll}
\text { AmericanOpt } & \text { (Optional) If AmericanOpt }=0, \mathrm{NaN} \text {, or is } \\
\text { unspecified, the option is a European option. If } \\
\text { AmericanOpt }=1 \text {, the option is an American } \\
\text { option. }
\end{array}
$$

## Description

Price $=$ lookbackbyitt(ITTTree, OptSpec, Strike, Settle, ExerciseDates, AmericanOpt) calculates the value of fixed- and floating-strike lookback options. Data arguments for lookbackbyitt are NINST-by-1 vectors, scalar, or empty. Fill unspecified entries in vectors with NaN. Only one data argument is required to create the instrument; the others may be omitted or passed as empty matrices [].

Price is a NINST-by-1 vector of expected option prices at time 0 .


#### Abstract

Note lookbackbyitt calculates values of fixed and floating strike lookback options. To compute the value of a floating strike lookback option, strike should be specified as NaN. Pricing of lookback options is done using Hull-White (1993). Consequently, for these options there are not unique prices on the tree nodes with the exception of the root node.


## Examples Price a Lookback Option Using an ITT Equity Tree

This example shows how to price a lookback option using an ITT equity tree by loading the file deriv.mat, which provides the ITTTree. The ITTTree structure contains the stock specification and time information needed to price the option.

```
load deriv.mat
OptSpec = 'Call';
Strike = 85;
Settle = '01-Jan-2006';
```


## lookbackbyitt

```
ExerciseDates = '01-Jan-2008';
Price = lookbackbyitt(ITTTree, OptSpec, Strike, Settle, ExerciseDates)
```


## Price =

$$
0.5426
$$

## References Hull, J., and A. White, "Efficient Procedures for Valuing European and American Path-Dependent Options," Journal of Derivatives, Fall 1993, pp. 21-31.

See Also<br>instlookback | itttree

## Purpose

Calculate prices of lookback options using Longstaff-Schwartz model
Syntax

Description

## Input Arguments

[Price,Paths,Times, Z] = lookbackbyls(RateSpec,StockSpec,OptSpec,Strik ExerciseDates)
[Price,Paths,Times, Z] = lookbackbyls( __ , Name, Value)
[Price,Paths,Times, Z] = lookbackbyls(RateSpec,StockSpec,OptSpec,Strike,Settle, ExerciseDates) returns prices of lookback options using the Longstaff-Schwartz model for Monte Carlo simulations. lookbackbyls computes prices of European and American lookback options. For American options, the Longstaff-Schwartz least squares method calculates the early exercise premium. lookbackbyls calculates values of fixed- and floating-strike lookback options. To compute the value of a floating-strike lookback option, Strike must be specified as NaN .
[Price,Paths,Times,Z] = lookbackbyls( __ ,Name, Value) returns prices of lookback options using the Longstaff-Schwartz model for Monte Carlo simulations with optional name-value pair arguments. lookbackbyls computes prices of European and American lookback options. For American options, the Longstaff-Schwartz least squares method calculates the early exercise premium. lookbackbyls calculates values of fixed- and floating-strike lookback options. To compute the value of a floating-strike lookback option, Strike must be specified as NaN .

RateSpec - Interest-rate term structure
structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from intenvset. For information on the interest-rate specification, see intenvset.

## Data Types <br> struct

## StockSpec - Stock specification for underlying asset

## lookbackbyls

## structure

Stock specification for the underlying asset. For information on the stock specification, see stockspec.
stockspec handles several types of underlying assets. For example, for physical commodities the price is represented by StockSpec.Asset, the volatility is represented by StockSpec.Sigma, and the convenience yield is represented by StockSpec.DividendAmounts.

## Data Types <br> struct

## OptSpec - Definition of option

string with values 'call' or 'put' | cell array of strings
Definition of option as 'call' or 'put', specified as a NINST-by-1 cell array of strings.

Data Types
char | cell

## Strike - Option strike price values

integer | vector of integers
Option strike price values, specified as an integer using a NINST-by-1 vector of strike price values.

## Data Types <br> single | double

## Settle - Settlement or trade date

nonnegative integer | vector of nonnegative integers | date string | cell array of date strings

Settlement or trade date for the lookback option, specified as a date string or as nonnegative integers using a NINST-by-1 vector or cell array of dates.

Data Types
char | cell

## ExerciseDates - Matrix of exercise callable or puttable dates for European or American options

## nonnegative integer | vector of nonnegative integers | date string | cell array of date strings

Matrix of exercise callable or puttable dates for European or American options, specified as a date string or as nonnegative integers as follows:

- European option - NINST-by-1 vector of exercise dates. For a European option, there is only one exercise date which is the option expiry date.
- American option - NINST-by-2 vector of exercise date boundaries. For each instrument, the option is exercised on any coupon date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is a NINST-by-1 vector of serial date numbers or cell array of strings, the option is exercised between Settle and the single listed exercise date.


## Data Types

char | cell

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: Price =
lookbackbyls(RateSpec, StockSpec1,StockSpec2, Settle, Maturity, OptSpec, S

## 'AmericanOpt' - Option type

0 European (default) | scalar with value [0,1]
Option type, specified as an integer scalar flag with these values:

- 0 - European
- 1 - American

For American options, the Longstaff-Schwartz least squares method calculates the early exercise premium.

Data Types<br>single | double

'NumTrials' - Scalar number of independent sample paths
1000 (default) | nonnegative scalar integer
Scalar number of independent sample paths (simulation trials), specified as a nonnegative integer.

```
Data Types
single | double
```


## 'NumPeriods' - Scalar number of simulation periods per trial 100 (default) | nonnegative scalar integer

Scalar number of simulation periods per trial, specified as a nonnegative integer. NumPeriods is considered only when pricing European lookback options. For American lookback options, NumPeriods is equal to the number of exercise days during the life of the option.

Data Types
single | double

## 'Z' - Time series array of dependent random variates

## vector

Time series array of dependent random variates, specified as a NumPeriods-by-1-by-NumTrials 3-D array. The $Z$ value generates the Brownian motion vector (i.e., Wiener processes) that drives the simulation.

Data Types
single | double

## 'Antithetic' - Indicator for antithetic sampling

false (default) | scalar logical flag with value of true or false
Indicator for antithetic sampling, specified with a value of true or false.
Data Types
logical

## Output Price - Expected price of lookback option Arguments

## Examples Compute the Price for a Floating Lookback Option Using Monte Carlo Simulation

Define the RateSpec.

## lookbackbyls

```
StartDates = 'Jan-1-2013';
EndDates = 'Jan-1-2014';
Rates = 0.042;
Compounding = -1;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDate
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
RateSpec =
```

```
                    FinObj: 'RateSpec'
```

                    FinObj: 'RateSpec'
    Compounding: -1
    Compounding: -1
            Disc: 0.9589
            Disc: 0.9589
            Rates: 0.0420
            Rates: 0.0420
        EndTimes: 1
        EndTimes: 1
    StartTimes: 0
    StartTimes: 0
        EndDates: 735600
        EndDates: 735600
    StartDates: 735235
    StartDates: 735235
    ValuationDate: 735235
ValuationDate: 735235
Basis: 0
Basis: 0
EndMonthRule: 1

```
EndMonthRule: 1
```

Define the StockSpec.
AssetPrice = 50;
Sigma = 0.36;
StockSpec = stockspec(Sigma, AssetPrice)

StockSpec =

```
                    FinObj: 'StockSpec
                    Sigma: 0.3600
        AssetPrice: 50
        DividendType: []
DividendAmounts: 0
```

Define the floating lookback option.

```
Settle = 'Jan-1-2013';
Maturity = 'April-1-2013';
OptSpec = 'put';
Strike = NaN;
```

Compute the price of the European floating lookback option.
Price $=$ lookbackbyls(RateSpec, StockSpec, OptSpec, Strike, Settle, Ma

Price =
6.5409

## Compute the Price of a Fixed Lookback Option Using Monte Carlo Simulation

Define the RateSpec.

```
StartDates = 'Jan-1-2013';
EndDates = 'Jan-1-2014';
Rates = 0.045;
Compounding = -1;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', Start[
'EndDates', EndDates, 'Rates', Rates,'Compounding', Compounding)
```

RateSpec =
FinObj: 'RateSpec'
Compounding: -1

## lookbackbyls

```
                        Disc: 0.9560
                Rates: 0.0450
        EndTimes: 1
        StartTimes: 0
        EndDates: 735600
    StartDates: 735235
ValuationDate: 735235
    Basis: 0
EndMonthRule: 1
```

Define the StockSpec.
AssetPrice = 102;
Sigma $=0.45$;
StockSpec = stockspec(Sigma, AssetPrice)

StockSpec =

```
                    FinObj: 'StockSpec
                    Sigma: 0.4500
        AssetPrice: 102
    DividendType: []
DividendAmounts: 0
ExDividendDates: []
```

Define the fixed lookback option.

```
Settle = 'Jan-1-2013';
Maturity = 'July-1-2013';
OptSpec = 'call';
Strike = 98;
```

Compute the price of the European fixed lookback option.

```
Price = lookbackbyls(RateSpec, StockSpec, OptSpec, Strike, Settle, Ma
```


## Price =

30.0917

## References

Hull, J. C. Options, Futures, and Other Derivatives, 5th ed. Englewood Cliffs, NJ: Prentice Hall, 2002.

## See Also lookbackbycvgsg | lookbacksensbycvgsg | lookbacksensbyls | stockspec | intenvset

## Concepts

## lookbacksensbyls

Purpose Calculate prices and sensitivities of lookback options using Longstaff-Schwartz model

```
Syntax [PriceSens,Paths,Times,Z] = lookbacksensbyls(RateSpec,StockSpec,OptSpec,S
ExerciseDates)
[PriceSens,Paths,Times,Z] =
lookbacksensbyls( __ ,Name,Value)
```

Description

Input Arguments
[PriceSens,Paths,Times,Z] = lookbacksensbyls(RateSpec,StockSpec,OptSpec,Strike,Settle, ExerciseDates) returns prices and sensitivities of lookback options using the Longstaff-Schwartz model for Monte Carlo simulations. lookbacksensbyls computes prices of European and American lookback options. For American options, the Longstaff-Schwartz least squares method calculates the early exercise premium. lookbacksensbyls calculates values of fixed- and floating-strike lookback options. To compute the value of a floating-strike lookback option, Strike must be specified as NaN.
[PriceSens,Paths,Times,Z] = lookbacksensbyls (__ , Name, Value) returns prices and sensitivities of lookback options using the Longstaff-Schwartz model for Monte Carlo simulations with optional name-value pair arguments. lookbacksensbyls computes prices of European and American lookback options. For American options, the Longstaff-Schwartz least squares method calculates the early exercise premium. lookbacksensbyls calculates values of fixed- and floating-strike lookback options. To compute the value of a floating-strike lookback option, Strike must be specified as NaN .

## RateSpec - Interest-rate term structure <br> structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from intenvset. For information on the interest-rate specification, see intenvset.

## lookbacksensbyls

## Data Types

struct

## StockSpec - Stock specification for underlying asset <br> structure

Stock specification for the underlying asset. For information on the stock specification, see stockspec.
stockspec can handle several types of underlying assets. For example, for physical commodities the price is represented by StockSpec.Asset, the volatility is represented by StockSpec.Sigma, and the convenience yield is represented by StockSpec.DividendAmounts.

## Data Types

struct

## OptSpec - Definition of option

string with values 'call' or 'put' | cell array of strings
Definition of option as 'call' or 'put', specified as a NINST-by-1 cell array of strings.

```
Data Types
char | cell
```


## Strike - Option strike price values

integer | vector of integers
Option strike price values, specified as an integer using a NINST-by-1 vector of strike price values.

```
Data Types
single | double
```


## Settle - Settlement or trade date <br> nonnegative integer | vector of nonnegative integers | date string | cell array of date strings

## lookbacksensbyls

Settlement or trade date for the lookback option, specified as a date string or as nonnegative integers using a NINST-by-1 vector or cell array of dates.

## Data Types <br> char | cell

## ExerciseDates - Matrix of exercise callable or puttable dates for European or American options

nonnegative integer | vector of nonnegative integers | date string | cell array of date strings

Matrix of exercise callable or puttable dates for European or American options, specified as a date string or as nonnegative integers as follows:

- European option - NINST-by-1 vector of exercise dates. For a European option, there is only one exercise date which is the option expiry date.
- American option - NINST-by-2 vector of exercise date boundaries. For each instrument, the option is exercised on any coupon date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is a NINST-by-1 vector of serial date numbers or cell array of strings, the option is exercised between Settle and the single listed exercise date.


## Data Types <br> char | cell

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

Example: PriceSens =
lookbacksensbyls(RateSpec, StockSpec1,StockSpec2,Settle, Maturity, OptSpec, s

## 'AmericanOpt' - Option type

0 European (default) | scalar with value [0,1]
Option type, specified as an integer scalar flag with these values:

- 0 - European
- 1 - American

For American options, the Longstaff-Schwartz least squares method calculates the early exercise premium.

## Data Types

single | double

## 'NumTrials' - Scalar number of independent sample paths 1000 (default) | nonnegative scalar integer

Scalar number of independent sample paths (simulation trials), specified as a nonnegative integer.

```
Data Types
single | double
```


## 'NumPeriods' - Scalar number of simulation periods per trial 100 (default) | nonnegative scalar integer

Scalar number of simulation periods per trial, specified as a nonnegative integer. NumPeriods is considered only when pricing European lookback options. For American lookback options, NumPeriod is equal to the number of exercise days during the life of the option.

```
Data Types
single | double
```


## 'Z' - Time series array of dependent random variates

 vectorTime series array of dependent random variates, specified as a NumPeriods-by-1-by-NumTrials 3-D array. The $\mathbf{Z}$ value generates the Brownian motion vector (i.e., Wiener processes) that drives the simulation.

## lookbacksensbyls

Data Types<br>single | double

## 'Antithetic' - Indicator for antithetic sampling

## false (default) | scalar logical flag with value true or false

Indicator for antithetic sampling, specified with a value of true or false.

Data Types
logical

## 'OutSpec' - Define outputs

\{'Price'\} (default) | string with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | cell array of strings with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

Define outputs, specifying a NOUT- by-1 or a 1-by-NOUT cell array of strings with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

OutSpec $=\{$ 'All' $\}$ specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec to include each sensitivity.
Example: OutSpec =
\{'delta','gamma','vega','lambda','rho','theta','price'\}
Data Types
char | cell

## Output Arguments

PriceSens - Expected price and sensitivities of lookback option
vector

Expected price and sensitivities of the lookback option, returned as a NINST-by-1 vector.

Paths - Simulated paths of correlated state variables
vector

## lookbacksensbyls

Simulated paths of correlated state variables, returned as a NumPeriods + 1-by-1-by-NumTrials 3-D time series array. Each row of Paths is the transpose of the state vector $X(t)$ at time $t$ for a given trial.

## Times - Observation times associated with simulated paths

 vectorObservation times associated with simulated paths, returned as a NumPeriods + 1-by-1 column vector of observation times associated with the simulated paths. Each element of Times is associated with the corresponding row of Paths.

## Z - Time series array of dependent random variates

## vector

Time series array of dependent random variates, returned as a NumPeriods-by-1-by-NumTrials 3-D array when $Z$ is specified as an input argument. If the $Z$ input argument is not specified, then the $Z$ output argument contains the random variates generated internally.

## Examples Compute the Price and Delta of a European Floating Lookback Option Using Monte Carlo Simulation

Define the RateSpec.

```
StartDates = 'Jan-1-2013';
EndDates = 'Jan-1-2014';
Rates = 0.41;
Compounding = -1;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', Startl
'EndDates', EndDates, 'Rates', Rates,'Compounding', Compounding)
```


## lookbacksensbyls

```
Rates: 0.4100
EndTimes: 1
StartTimes: 0
EndDates: 735600
StartDates: 735235
ValuationDate: 735235
Basis: 0
EndMonthRule: 1
```

Define the StockSpec with continuous dividend yield.

```
AssetPrice = 120;
Sigma = 0.3;
Yield = 0.045;
StockSpec = stockspec(Sigma, AssetPrice, 'Continuous', Yield)
```

StockSpec =

```
                    FinObj: 'StockSpec'
                    Sigma: 0.3000
        AssetPrice: 120
    DividendType: {'continuous'}
DividendAmounts: 0.0450
ExDividendDates: []
```

Define the floating lookback option.

```
Settle = 'Jan-1-2013';
Maturity = 'July-1-2013';
OptSpec = 'call';
Strike = NaN;
```

Compute the price and delta of the European floating lookback option.

```
OutSpec = {'price', 'delta'};
[Price, Delta] = lookbacksensbyls(RateSpec, StockSpec, OptSpec, Strik
'OutSpec', OutSpec)
Price =
```

    27.4701
    Delta =
0.2289

## Compute the Price and Delta of a European Fixed Lookback Option Using Monte Carlo Simulation

Define the RateSpec.

```
StartDates = 'Jan-1-2013';
EndDates = 'Jan-1-2015';
Rates = 0.1;
Compounding = -1;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', Startl
'EndDates', EndDates, 'Rates', Rates,'Compounding', Compounding)
```


## RateSpec =

FinObj: 'RateSpec
Compounding: -1
Disc: 0.8187
Rates: 0.1000
EndTimes: 2
StartTimes: 0
EndDates: 735965

## lookbacksensbyls

```
    StartDates: 735235
ValuationDate: 735235
    Basis: 0
    EndMonthRule: 1
Define the StockSpec.
```

```
AssetPrice = 103;
```

AssetPrice = 103;
Sigma = 0.30;
StockSpec = stockspec(Sigma, AssetPrice)
StockSpec =

```
```

                    FinObj: 'StockSpec
    ```
                    FinObj: 'StockSpec
                    Sigma: 0.3000
                    Sigma: 0.3000
        AssetPrice: 103
        AssetPrice: 103
    DividendType: []
    DividendType: []
DividendAmounts: 0
DividendAmounts: 0
ExDividendDates: []
```

ExDividendDates: []

```

Define the fixed lookback option.
```

Settle = 'Jan-1-2013';
Maturity = 'July-1-2013';
OptSpec = 'call';
Strike = 99;

```

Compute the price and delta of the European fixed lookback option.
OutSpec = \{'price', 'delta'\};
[Price, Delta] = lookbacksensbyls(RateSpec, StockSpec, OptSpec, Strike, 'OutSpec', OutSpec)

Price =
22.6783

Delta =
1.1345

\section*{References}

Hull, J. C. Options, Futures, and Other Derivatives, 5th ed. Englewood Cliffs, NJ: Prentice Hall, 2002.
See Also lookbackbycvgsg | lookbacksensbycvgsg | lookbackbyls | intenvset | stockspec
Concepts

\section*{Irtimespec}

Purpose Specify time structure for Leisen-Reimer binomial tree

Description

\section*{Input \\ Arguments}

\section*{Output \\ Arguments}

\section*{Examples}

Syntax TimeSpec = lrtimespec(ValuationDate, Maturity, NumPeriods)
TimeSpec = lrtimespec(ValuationDate, Maturity, NumPeriods) specifies a time structure for a Leisen-Reimer stock tree.

\section*{ValuationDate}

Scalar date marking the pricing date and first observation in the Leisen-Reimer stock tree. Specify ValuationDate as a serial date number or date string.

\section*{Maturity}

Scalar date marking the depth of the Leisen-Reimer stock tree.

\section*{NumPeriods}

Scalar value determining how many time steps are in the Leisen-Reimer stock tree.

Note Leisen-Reimer requires the number of steps to be an odd number.

\section*{TimeSpec}

Structure specifying the time layout for a Leisen-Reimer stock tree.

\section*{Specify the Time Structure for Leisen-Reimer Binomial Tree}

This example shows how to specify a 5 -period tree with time steps of 1 year.
```

ValuationDate = '1-July-2010';

```
Maturity = '1-July-2015';
TimeSpec = lrtimespec(ValuationDate, Maturity, 5)

TimeSpec =
```

FinObj: 'BinTimeSpec'
ValuationDate: 734320
Maturity: 736146
NumPeriods: 5
Basis: 0
EndMonthRule: 1
tObs: [ $\left.0 \begin{array}{llllll}0 & 1 & 2 & 3 & 4 & 5\end{array}\right]$
dObs: [734320 734685735050735415735780736146$]$

```

\title{
References Leisen D.P., M. Reimer, "Binomial Models for Option Valuation Examining and Improving Convergence," Applied Mathematical \\ Finance, Number 3, 1996, pp. 319-346.
}

\section*{See Also}
| stockspec | lrtree

\section*{Irłree}

\section*{Purpose Build Leisen-Reimer stock tree}

\section*{Description}

Input
Arguments
```

Syntax LRTree = lrtree(StockSpec, RateSpec, TimeSpec, Strike)
LRTree = lrtree(StockSpec, RateSpec, TimeSpec,
Strike, Name,Value)
LRTree = lrtree(StockSpec, RateSpec, TimeSpec, Strike) LRTree = lrtree(StockSpec, RateSpec, TimeSpec, Strike, Name,Value)

```

LRTree = lrtree(StockSpec, RateSpec, TimeSpec, Strike) constructs a Leisen-Reimer stock tree.

LRTree = lrtree(StockSpec, RateSpec, TimeSpec, Strike, Name, Value) constructs a Leisen-Reimer stock tree with additional options specified by one or more Name, Value pair arguments.

\section*{StockSpec}

Stock specification. For more information, see stockspec.

\section*{RateSpec}

Interest rate specification of the initial risk-free rate curve. For more information, see intenvset.

\section*{TimeSpec}

Tree time layout specification. For more information, see lrtimespec.

\section*{Strike}

Scalar defining the option strike.

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

\section*{'Method'}

String value. Use PP1 for Peizer-Pratt method 1 inversion and PP2 for Peizer-Pratt method 2 inversion. For more information on PP1 and PP2 methods, see "Leisen-Reimer Tree (LR) Modeling" on page C-7.

\section*{Default: PP1}

\section*{Output Arguments}

\section*{Examples}

\section*{LRTree}

Structure specifying stock and time information for a Leisen-Reimer tree.

\section*{Build a Leisen-Reimer Stock Tree}

This example shows how to build Leisen-Reimer stock tree. Consider a European put option with an exercise price of \(\$ 30\) that expires on June 1, 2010. The underlying stock is trading at \(\$ 30\) on January 1, 2010 and has a volatility of \(30 \%\) per annum. The annualized continuously compounded risk-free rate is \(5 \%\) per annum. Using this data, create a Leisen-Reimer tree with 101 steps using the PP1 method.
```

AssetPrice = 30;
Strike = 30;
ValuationDate = 'Jan-1-2010';
Maturity = 'June-1-2010';
% define StockSpec
Sigma = 0.3;
StockSpec = stockspec(Sigma, AssetPrice);
% define RateSpec
Rates = 0.05;
Settle = ValuationDate;
Basis = 1;
Compounding = -1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', Se
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Bas

```

\section*{Irtree}
```

% build the Leisen-Reimer (LR) tree with 101 steps
LRTimeSpec = lrtimespec(ValuationDate, Maturity, 101);
% use the PP1 method
LRMethod = 'PP1';
LRTree = lrtree(StockSpec, RateSpec, LRTimeSpec, Strike, ...
'method', LRMethod)
LRTree =

```
```

        FinObj: 'BinStockTree'
    ```
        FinObj: 'BinStockTree'
        Method: 'LR'
        Method: 'LR'
    Submethod: 'PP1'
    Submethod: 'PP1'
    Strike: 30
    Strike: 30
    StockSpec: [1x1 struct]
    StockSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    RateSpec: [1x1 struct]
            tObs: [1x102 double]
            tObs: [1x102 double]
            dObs: [1x102 double]
            dObs: [1x102 double]
            STree: {1x102 cell}
            STree: {1x102 cell}
        UpProbs: [101x1 double]
```

        UpProbs: [101x1 double]
    ```

References Leisen D.P., M. Reimer, "Binomial Models for Option Valuation Examining and Improving Convergence," Applied Mathematical Finance, Number 3, 1996, pp. 319-346.

See Also
| stockspec | lrtimespec | intenvset | optstockbylr | optstocksensbylr

> Purpose

> Syntax

Arguments
\begin{tabular}{ll} 
RateSpec & \begin{tabular}{l} 
The annualized, continuously compounded rate \\
term structure. For information on the interest \\
rate specification, see intenvset.
\end{tabular} \\
StockSpec1 & \begin{tabular}{l} 
Stock specification for asset 1. See stockspec.
\end{tabular} \\
StockSpec2 & \begin{tabular}{l} 
Stock specification for asset 2 . See stockspec. \\
Settle
\end{tabular} \\
\begin{tabular}{l} 
NINST-by-1 vector of settlement or trade dates.
\end{tabular} \\
OptSpec & NINST-by-1 vector of maturity dates. \\
Strike & NINST-by-1 cell array of strings 'call' or 'put'. \\
Corr & NINST-by-1 vector of strike price values. \\
& \begin{tabular}{l} 
NINST-by-1 vector of correlation between the \\
underlying asset prices.
\end{tabular}
\end{tabular}

\section*{Description}

\section*{Examples Compute Rainbow Option Prices Using the Stulz Option Pricing Model}

Consider a European rainbow option that gives the holder the right to buy either \(\$ 100,000\) worth of an equity index at a strike price of 1000
(asset 1) or \(\$ 100,000\) of a government bond (asset 2 ) with a strike price of \(100 \%\) of face value, whichever is worth more at the end of 12 months. On January 15, 2008, the equity index is trading at 950, pays a dividend of \(2 \%\) annually and has a return volatility of \(22 \%\). Also on January 15, 2008, the government bond is trading at 98 , pays a coupon yield of \(6 \%\), and has a return volatility of \(15 \%\). The risk-free rate is \(5 \%\). Using this data, if the correlation between the rates of return is \(-0.5,0\), and 0.5 , calculate the price of the European rainbow option.

Since the asset prices in this example are in different units, it is necessary to work in either index points (asset 1) or in dollars (asset 2). The European rainbow option allows the holder to buy the following: 100 units of the equity index at \(\$ 1000\) each (for a total of \(\$ 100,000\) ) or 1000 units of the government bonds at \(\$ 100\) each (for a total of \(\$ 100,000\) ). To convert the bond price (asset 2) to index units (asset 1), you must make the following adjustments:
- Multiply the strike price and current price of the government bond by 10 (1000/100).
- Multiply the option price by 100 , considering that there are 100 equity index units in the option.
Once these adjustments are introduced, the strike price is the same for both assets (\$1000). First, create the RateSpec:
```

Settle = 'Jan-15-2008';
Maturity = 'Jan-15-2009';
Rates = 0.05;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', Basis)

```
RateSpec =
    FinObj: 'RateSpec'
    Compounding: -1
```

                        Disc: 0.9512
            Rates: 0.0500
        EndTimes: 1
        StartTimes: 0
        EndDates: 733788
    StartDates: 733422
    ValuationDate: 733422
Basis: 1
EndMonthRule: 1

```

Create the two StockSpec definitions.
```

AssetPrice1 = 950; % Asset 1 => Equity index
AssetPrice2 = 980; % Asset 2 => Government bond
Sigma1 = 0.22;
Sigma2 = 0.15;
Div1 = 0.02;
Div2 = 0.06;

```
StockSpec1 = stockspec(Sigma1, AssetPrice1, 'continuous', Div1)
StockSpec2 = stockspec(Sigma2, AssetPrice2, 'continuous', Div2)

\section*{StockSpec1 =}

FinObj: 'StockSpec'
Sigma: 0.2200
AssetPrice: 950
DividendType: \{'continuous'\}
DividendAmounts: 0.0200
ExDividendDates: []

StockSpec2 =
FinObj: 'StockSpec'
```

                    Sigma: 0.1500
        AssetPrice: 980
        DividendType: {'continuous'}
    DividendAmounts: 0.0600
ExDividendDates: []

```

Calculate the price of the options for different correlation levels.
```

Strike = 1000 ;
Corr = [-0.5; 0; 0.5];
OptSpec = 'call';
Price = maxassetbystulz(RateSpec, StockSpec1, StockSpec2,...
Settle, Maturity, OptSpec, Strike, Corr)

```
Price =
    111.6683
    103.7715
    92.4412

These are the prices of one unit. This means that the premium is \(11166.83,10377.15\), and 9244.12 (for 100 units).

See Also
intenvset | maxassetsensbystulz | minassetbystulz | stockspec

\section*{Purpose}

\section*{Syntax}
```

PriceSens = maxassetsensbystulz(RateSpec, StockSpec1,
StockSpec2, Settle, Maturity, OptSpec, Strike, Corr)
PriceSens = maxassetsensbystulz(RateSpec, StockSpec1,
StockSpec2, Settle, Maturity, OptSpec, Strike,
Corr, OutSpec)

```

\section*{Arguments}
\begin{tabular}{|c|c|}
\hline RateSpec & The annualized, continuously compounded rate term structure. For information on the interest rate specification, see intenvset. \\
\hline StockSpec1 & Stock specification for asset 1. See stockspec. \\
\hline StockSpec2 & Stock specification for asset 2. See stockspec. \\
\hline Settle & NINST-by-1 vector of settlement or trade dates. \\
\hline Maturity & NINST-by-1 vector of maturity dates. \\
\hline OptSpec & NINST-by-1 cell array of strings 'call' or 'put'. \\
\hline Strike & NINST-by-1 vector of strike price values. \\
\hline Corr & NINST-by- 1 vector of correlation between the underlying asset prices. \\
\hline OutSpec & (Optional) All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. You can specify parameter name/value pairs in any order. Names are case-insensitive and partial string matches are allowed provided no ambiguities exist. Valid parameter names are: \\
\hline
\end{tabular}
- NOUT-by-1 or 1-by-NOUT cell array of strings indicating the nature and order of the outputs
for the function. Possible values are 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', or 'All'.

For example, OutSpec = \{'Price'; 'Lambda'; 'Rho'\} specifies that the output should be Price, Lambda, and Rho, in that order.

To invoke from a function: [Price, Lambda, Rho] = maxassetsensbystulz(..., 'OutSpec', \{'Price', 'Lambda', 'Rho'\})
OutSpec \(=\{\) 'All' \(\}\) specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec as OutSpec \(=\) \{'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', 'Price'\};.
- Default is OutSpec = \{'Price'\}.

\section*{Description}

PriceSens = maxassetsensbystulz(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr) computes rainbow option prices using the Stulz option pricing model.

PriceSens = maxassetsensbystulz(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr, OutSpec) computes rainbow option prices and sensitivities using the Stulz option pricing model.
PriceSens is a NINST-by-1 or NINST-by-2 vector of expected prices and sensitivities values.

\section*{Examples Compute Rainbow Option Prices and Sensitivities Using the Stulz Option Pricing Model}

Consider a European rainbow option that gives the holder the right to buy either \(\$ 100,000\) of an equity index at a strike price of 1000 (asset 1 ) or \(\$ 100,000\) of a government bond (asset 2) with a strike price of \(100 \%\) of face value, whichever is worth more at the end of 12 months. On January 15,2008 , the equity index is trading at 950 , pays a dividend of \(2 \%\) annually, and has a return volatility of \(22 \%\). Also on January 15, 2008, the government bond is trading at 98, pays a coupon yield of \(6 \%\), and has a return volatility of \(15 \%\). The risk-free rate is \(5 \%\). Using this data, calculate the price and sensitivity of the European rainbow option if the correlation between the rates of return is \(-0.5,0\), and 0.5 .

Since the asset prices in this example are in different units, it is necessary to work in either index points (for asset 1) or in dollars (for asset 2). The European rainbow option allows the holder to buy the following: 100 units of the equity index at \(\$ 1000\) each (for a total of \(\$ 100,000\) ) or 1000 units of the government bonds at \(\$ 100\) each (for a total of \(\$ 100,000\) ). To convert the bond price (asset 2) to index units (asset 1), you must make the following adjustments:
- Multiply the strike price and current price of the government bond by 10 (1000/100).
- Multiply the option price by 100 , considering that there are 100 equity index units in the option.

Once these adjustments are introduced, the strike price is the same for both assets (\$1000). First, create the RateSpec:
```

Settle = 'Jan-15-2008';
Maturity = 'Jan-15-2009';
Rates = 0.05;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,..
'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', Bas:

```

\section*{maxassetsensbystulz}

\section*{RateSpec =}
```

            FinObj: 'RateSpec'
        compounding: -1
            Disc: 0.9512
            Rates: 0.0500
        EndTimes: 1
        StartTimes: 0
            EndDates: 733788
        StartDates: 733422
    ValuationDate: 733422
Basis: 1
EndMonthRule: 1

```

Create the two StockSpec definitions.
```

AssetPrice1 = 950; % Asset 1 => Equity index
AssetPrice2 = 980; % Asset 2 => Government bond
Sigma1 = 0.22;
Sigma2 = 0.15;
Div1 = 0.02;
Div2 = 0.06;
StockSpec1 = stockspec(Sigma1, AssetPrice1, 'continuous', Div1)
StockSpec2 = stockspec(Sigma2, AssetPrice2, 'continuous', Div2)
StockSpec1 =
FinObj: 'StockSpec'
Sigma: 0.2200
AssetPrice: 950
DividendType: {'continuous'}
DividendAmounts: 0.0200
ExDividendDates: []

```

\section*{StockSpec2 =}
```

            FinObj: 'StockSpec'
            Sigma: 0.1500
        AssetPrice: 980
    DividendType: {'continuous'}
    DividendAmounts: 0.0600
ExDividendDates: []

```

Calculate the price and delta for different correlation levels.
```

Strike = 1000 ;
Corr = [-0.5; 0; 0.5];
OutSpec = {'price'; 'delta'};
OptSpec = 'call';
[Price, Delta] = maxassetsensbystulz(RateSpec, StockSpec1, StockSpec2
Settle, Maturity, OptSpec, Strike, Corr,'OutSpec', OutSpec)

```
Price =
    111.6683
    103.7715
    92.4412
Delta =
\begin{tabular}{ll}
0.4594 & 0.3698 \\
0.4292 & 0.3166 \\
0.4053 & 0.2512
\end{tabular}

\section*{maxassetsensbystulz}

The output Delta has two columns: the first column represents the Delta with respect to the equity index (asset 1), and the second column represents the Delta with respect to the government bond (asset 2). The value 0.4595 represents Delta with respect to one unit of the equity index. Since there are 100 units of the equity index, the overall Delta would be 45.94 ( \(100 * 0.4594\) ) for a correlation level of -0.5 . To calculate the Delta with respect to the government bond, remember that an adjusted price of 980 was used instead of 98 . Therefore, for example, the Delta with respect to government bond, for a correlation of 0.5 would be \(251.2(0.2512 * 100 * 10)\).

\section*{See Also}
intenvset | maxassetbystulz | stockspec

\section*{Purpose}

Syntax

Arguments

\section*{Description}

\section*{Examples}

Determine European rainbow option prices on minimum of two risky assets using Stulz option pricing model
```

Price = minassetbystulz(RateSpec, StockSpec1, StockSpec2,
Settle, Maturity, OptSpec, Strike, Corr)

```
\begin{tabular}{ll} 
RateSpec & \begin{tabular}{l} 
The annualized, continuously compounded rate \\
term structure. For information on the interest \\
rate specification, see intenvset.
\end{tabular} \\
StockSpec1 & \begin{tabular}{l} 
Stock specification for asset 1. See stockspec.
\end{tabular} \\
StockSpec2 & \begin{tabular}{l} 
Stock specification for asset 2 . See stockspec.
\end{tabular} \\
Settle & NINST-by-1 vector of settlement or trade dates. \\
Maturity & NINST-by-1 vector of maturity dates. \\
OptSpec & NINST-by-1 cell array of strings 'call' or 'put'. \\
Strike & \begin{tabular}{l} 
NINST-by-1 vector of strike price values. \\
Corr
\end{tabular} \\
& \begin{tabular}{l} 
NINST-by-1 vector of correlation between the \\
underlying asset prices.
\end{tabular}
\end{tabular}

Price = minassetbystulz(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr) computes option prices using the Stulz option pricing model.

Price is a NINST-by- 1 vector of expected option prices.

\section*{Compute Rainbow Option Prices Using the Stulz Option Pricing Model}

Consider a European rainbow put option that gives the holder the right to sell either stock A or stock B at a strike of 50.25 , whichever has the lower value on the expiration date May 15, 2009. On November 15,

\section*{minassetbystulz}

2008, stock A is trading at 49.75 with a continuous annual dividend yield of \(4.5 \%\) and has a return volatility of \(11 \%\). Stock B is trading at 51 with a continuous dividend yield of \(5 \%\) and has a return volatility of \(16 \%\). The risk-free rate is \(4.5 \%\). Using this data, if the correlation between the rates of return is \(-0.5,0\), and 0.5 , calculate the price of the minimum of two assets that are European rainbow put options. First, create the RateSpec:
```

Settle = 'Nov-15-2008';
Maturity = 'May-15-2009';
Rates = 0.045;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', Basis)

```
RateSpec =
```

            FinObj: 'RateSpec'
        Compounding: -1
            Disc: 0.9778
            Rates: 0.0450
        EndTimes: 0.5000
        StartTimes: 0
        EndDates: 733908
    StartDates: 733727
    ValuationDate: 733727
Basis: 1
EndMonthRule: 1

```

Create the two StockSpec definitions.
AssetPriceA = 49.75;
AssetPriceB = 51;
SigmaA = 0.11;
```

SigmaB = 0.16;
DivA = 0.045;
DivB = 0.05;
StockSpecA = stockspec(SigmaA, AssetPriceA, 'continuous', DivA)
StockSpecB = stockspec(SigmaB, AssetPriceB, 'continuous', DivB)
StockSpecA =
FinObj: 'StockSpec'
Sigma: 0.1100
AssetPrice: 49.7500
DividendType: {'continuous'}
DividendAmounts: 0.0450
ExDividendDates: []
StockSpecB =

```
```

            FinObj: 'StockSpec'
    ```
            FinObj: 'StockSpec'
                    Sigma: 0.1600
                    Sigma: 0.1600
        AssetPrice: 51
        AssetPrice: 51
        DividendType: {'continuous'}
        DividendType: {'continuous'}
    DividendAmounts: 0.0500
    DividendAmounts: 0.0500
    ExDividendDates: []
```

    ExDividendDates: []
    ```

Compute the price of the options for different correlation levels.
Strike = 50.25;
Corr = [-0.5;0;0.5];
OptSpec = 'put';
Price = minassetbystulz(RateSpec, StockSpecA, StockSpecB, Settle,... Maturity, OptSpec, Strike, Corr)

\section*{minassetbystulz}

> Price \(=\)
> 3.4320
> 3.1384
> 2.7694

The values 3.43, 3.14, and 2.77 are the price of the European rainbow put options with a correlation level of \(-0.5,0\), and 0.5 respectively.

\section*{See Also}
intenvset | maxassetbystulz | minassetsensbystulz | stockspec

\section*{Purpose}

Syntax

\section*{Arguments}

Determine European rainbow option prices and sensitivities on minimum of two risky assets using Stulz pricing model
```

PriceSens = minassetsensbystulz(RateSpec, StockSpec1,
StockSpec2, Settle, Maturity, OptSpec, Strike, Corr)
PriceSens = minassetsensbystulz(RateSpec, StockSpec1,
StockSpec2, Settle, Maturity, OptSpec, Strike,
Corr, OutSpec)

```
\begin{tabular}{ll} 
RateSpec & \begin{tabular}{l} 
The annualized continuously compounded rate \\
term structure. For information on the interest \\
rate specification, see intenvset.
\end{tabular} \\
StockSpec1 & \begin{tabular}{l} 
Stock specification for asset 1. See stockspec.
\end{tabular} \\
StockSpec2 & \begin{tabular}{l} 
Stock specification for asset 2. See stockspec. \\
Settle
\end{tabular} \\
Maturity & NINST-by-1 vector of settlement or trade dates. \\
OptSpec & NINST-by-1 vector of maturity dates. \\
Strike & NINST-by-1 cell array of strings 'call' or 'put '. \\
Corr & \begin{tabular}{l} 
NINST-by-1 vector of strike price values.
\end{tabular} \\
OutSpec & \begin{tabular}{l} 
NINST-by-1 vector of correlation between the \\
underlying asset prices. \\
(Optional) All optional inputs are specified as
\end{tabular} \\
& \begin{tabular}{l} 
matching parameter name/value pairs. The \\
parameter name is specified as a character string, \\
followed by the corresponding parameter value.
\end{tabular} \\
& \begin{tabular}{l} 
You can specify parameter name/value pairs \\
in any order. Names are case-insensitive and
\end{tabular} \\
& \begin{tabular}{l} 
partial string matches are allowed provided no \\
ambiguities exist. Valid parameter names are:
\end{tabular}
\end{tabular}
- NOUT-by-1 or 1-by-NOUT cell array of strings indicating the nature and order of the outputs

\section*{minassetsensbystulz}
for the function. Possible values are 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', or 'All'.

For example, OutSpec = \{'Price'; 'Lambda'; 'Rho'\} specifies that the output should be Price, Lambda, and Rho, in that order.

To invoke from a function: [Price, Lambda, Rho] = minassetsensbystulz(..., 'OutSpec', \{'Price', 'Lambda', 'Rho'\})
OutSpec \(=\{\) 'All' \(\}\) specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec as OutSpec \(=\) \{'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', 'Price'\};.
- Default is OutSpec = \{'Price'\}.

\section*{Description}

PriceSens = minassetsensbystulz(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr) computes rainbow option prices using the Stulz option pricing model.

PriceSens = minassetsensbystulz(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr, OutSpec) computes rainbow option prices and sensitivities using the Stulz option pricing model.
PriceSens is a NINST-by-1 or NINST-by-2 vector of expected prices and sensitivities.

\section*{Examples Compute Rainbow Option Prices and Sensitivities Using the Stulz Option Pricing Model}

Consider a European rainbow put option that gives the holder the right to sell either stock A or stock B at a strike of 50.25 , whichever has the lower value on the expiration date May 15, 2009. On November 15, 2008 , stock A is trading at 49.75 with a continuous annual dividend yield of \(4.5 \%\) and has a return volatility of \(11 \%\). Stock B is trading at 51 with a continuous dividend yield of \(5 \%\) and has a return volatility of \(16 \%\). The risk-free rate is \(4.5 \%\). Using this data, if the correlation between the rates of return is \(-0.5,0\), and 0.5 , calculate the price and sensitivity of the minimum of two assets that are European rainbow put options. First, create the RateSpec:
```

Settle = 'Nov-15-2008';
Maturity = 'May-15-2009';
Rates = 0.045;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,..
'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', Bas:

```

\section*{RateSpec =}
```

            FinObj: 'RateSpec'
        Compounding: -1
            Disc: 0.9778
            Rates: 0.0450
            EndTimes: 0.5000
        StartTimes: 0
            EndDates: 733908
        StartDates: 733727
        ValuationDate: 733727
            Basis: 1
    EndMonthRule: 1

```

\section*{minassetsensbystulz}
```

Create the two StockSpec definitions.
AssetPriceA = 49.75;
AssetPriceB = 51;
SigmaA = 0.11;
SigmaB = 0.16;
DivA = 0.045;
DivB = 0.05;
StockSpecA = stockspec(SigmaA, AssetPriceA, 'continuous', DivA)
StockSpecB = stockspec(SigmaB, AssetPriceB, 'continuous', DivB)
StockSpecA =
FinObj: 'StockSpec'
Sigma: 0.1100
AssetPrice: 49.7500
DividendType: {'continuous'}
DividendAmounts: 0.0450
ExDividendDates: []
StockSpecB =
FinObj: 'StockSpec'
Sigma: 0.1600
AssetPrice: 51
DividendType: {'continuous'}
DividendAmounts: 0.0500
ExDividendDates: []

```

Calculate price and delta for different correlation levels.
```

Strike = 50.25;

```
```

Corr = [-0.5;0;0.5];
OptSpec = 'put';
OutSpec = {'Price'; 'delta'};
[P, D] = minassetsensbystulz(RateSpec, StockSpecA, StockSpecB,...
Settle, Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec)

```
P =
    3.4320
    3.1384
    2.7694
D =
    -0.4183 -0.3496
    \(-0.3746-0.3189\)
    \(-0.3304-0.2905\)

The output Delta has two columns: the first column represents the Delta with respect to the stock A (asset 1 ), and the second column represents the Delta with respect to the stock \(B\) (asset 2 ). The value 0.4183 represents Delta with respect to the stock A for a correlation level of -0.5. The Delta with respect to stock B, for a correlation of zero is -0.3189.

\section*{See Also intenvset | minassetbystulz | stockspec}

\section*{mkbush}

Purpose
Create bushy tree
Syntax \(\quad \begin{aligned} & \text { [Tree, NumStates] } \\ & \text { NodeVal) }\end{aligned}\)
Arguments
\begin{tabular}{ll} 
NumLevels & \begin{tabular}{l} 
Number of time levels of the tree. \\
NumChild \\
1-by- number of levels (NUMLEVELS) vector with \\
number of branches (children) of the nodes in each \\
level.
\end{tabular} \\
NumPos & \begin{tabular}{l} 
1-by-NUMLEVELS vector containing the length of the \\
state vectors in each time level.
\end{tabular} \\
Trim & \begin{tabular}{l} 
(Optional) Scalar 0 or 1. If Trim \(=1\), NumPos \\
decreases by 1 when moving from one time level to \\
the next. Otherwise, if Trim \(=0\) (Default), NumPos \\
does not decrease.
\end{tabular} \\
NodeVal & \begin{tabular}{l} 
(Optional) Initial value at each node of the tree. \\
Default = NaN.
\end{tabular}
\end{tabular}

Description [Tree, NumStates] = mkbush(NumLevels, NumChild, NumPos, Trim,
NodeVal) creates a bushy tree Tree with initial values NodeVal at each node. NumStates is a 1 -by-NUMLEVELS vector containing the number of state vectors in each level.

Examples Create a tree with four time levels, two branches per node, and a vector of three elements in each node with each element initialized to NaN.
```

Tree = mkbush(4, 2, 3);
treeviewer(Tree)

```


See Also
bushpath | bushshape

\section*{mktree}

Purpose Create recombining binomial tree
```

Syntax $\quad$ Tree $=$ mktree (NumLevels, NumPos, NodeVal, IsPriceTree)

```

\section*{Arguments}
\begin{tabular}{ll} 
NumLevels & \begin{tabular}{l} 
Number of time levels of the tree.
\end{tabular} \\
NumPos & \begin{tabular}{l} 
1-by-NUMLEVELS vector containing the length of \\
the state vectors in each time level.
\end{tabular} \\
NodeVal & \begin{tabular}{l} 
(Optional) Initial value at each node of the tree. \\
Default \(=\) NaN.
\end{tabular} \\
IsPriceTree & \begin{tabular}{l} 
(Optional) Boolean determining if a final \\
horizontal branch is added to the tree. \\
Default \(=0\).
\end{tabular}
\end{tabular}

\section*{Description Tree \(=\) mktree(NumLevels, NumPos, NodeVal, IsPriceTree) creates a recombining tree Tree with initial values NodeVal at each node.}

\section*{Examples}

Create a recombining tree of four time levels with a vector of two elements in each node and each element initialized to NaN .

Tree = mktree(4, 2);
See Also treepath | treeshape
\begin{tabular}{ll} 
Purpose & Create recombining trinomial tree \\
Syntax & TrinTree \(=\) mktrintree(NumLevels, NumPos, NumStates, NodeVal) \\
Arguments &
\end{tabular}

NumLevels Number of time levels of the tree.
NumPos 1-by-NUMLEVELS vector containing the length of the state vectors in each time level.

NumStates 1-by-NUMLEVELS vector containing the number of state vectors in each time level.

NodeVal (Optional) Initial value at each node of the tree. Default \(=\mathrm{NaN}\).

\section*{Description TrinTree = mktrintree(NumLevels, NumPos, NumStates, NodeVal) creates a recombining tree Tree with initial values NodeVal at each node. \\ Examples \(\quad\) Create a recombining trinomial tree of four time levels with a vector of
two elements in each node and each element initialized to NaN. \\ TrinTree = mktrintree(4, [2 22 2], [1 3 5 7]);}

See Also trintreepath | trintreeshape

Purpose
Create money-market tree from Black-Derman-Toy interest-rate tree
Syntax MmktTree \(=\) mmktbybdt (BDTTree)

\section*{Arguments}

BDTTree Interest-rate tree structure created by bdttree.

\section*{Description}

\section*{Examples}

MMktTree \(=\) mmktbybdt (BDTTree) creates a money-market tree from an interest-rate tree structure created by bdttree.
```

load deriv.mat;
MMktTree = mmktbybdt(BDTTree);
treeviewer(MMktTree)

```


See Also bdttree

Purpose
Syntax MmktTree \(=\) mmktbyhjm(HJMTree)

\section*{Arguments}

HJMTree Forward-rate tree structure created by hjmtree.

\section*{Description}

\section*{Examples}
```

load deriv.mat;
MMktTree = mmktbyhjm(HJMTree);
treeviewer(MMktTree)

```


See Also
hjmtree

\section*{Superclasses}

Purpose Create numerix object to set up Numerix CAIL environment
Description The numerix object makes the Numerix engine directly accessible from MATLAB. To use the capabilities of Numerix CAIL, you must have CAIL client software installed on your local desktop.

> Construction \(N=\) numerix(DATADIRECTORYPATH) constructs the numerix object and sets up the Numerix CrossAsset Integration Layer (CAIL) environment given the path, DATADIRECTORYPATH.

\section*{Properties \\ The following properties are from the numerix class.}

\section*{Path}

Defines the path for DATADIRECTORYPATH. This path is the location of the templates and is created by the client installation of CAIL. At the highest level, a template defines the interface; it encapsulates the instructions for performing the calculation, the calculation's required and optional input parameters, and the calculation's outputs.

Attributes:

SetAccess
GetAccess public
public

\section*{RepositoryPath}

RepositoryPath defines the path location for the repository directory in a file system.

Attributes:

SetAccess public
GetAccess public

\section*{Repository}

Repositories are collections of templates and are defined as a directory in a file system.

Attributes:
```

SetAccess
public
GetAccess
public

```

\section*{Context}

The calculation context manages all of the CAIL information. Context contains the location of the template repository and is responsible for creating a CAIL application context in which to perform the calculations.

\section*{Attributes:}

SetAccess
public
GetAccess
public

\section*{LookupsPath}

Defines the path for the numeric instruments data types.
Attributes:
SetAccess public
GetAccess public

\section*{MarketsPath}

Defines a the path for the logical schema for naming all of the market data. MarketsPath enables you to provide a data dictionary to map business market data to CAIL to reduce the task of inputting market data into CAIL objects directly.

\section*{Attributes:}
```

SetAccess
public
GetAccess
public

```

\section*{FixingsPath}

Defines the path for the schema for naming historical fixing data for rates and prices.

Attributes:
SetAccess public
GetAccess public

\section*{TradesPath}

Defines the path to the trade instrument definitions.
Attributes:

SetAccess public
GetAccess public

\section*{Parameters}

Defines the calculation parameters and market data, if available.
Attributes:
\begin{tabular}{ll} 
SetAccess & public \\
GetAccess & public
\end{tabular}

\section*{Methods}
parseResults

Converts Numerix CAIL data to MATLAB data types

\section*{Definitions CrossAsset Integration Layer (CAIL)}

The CrossAsset Integration Layer (CAIL) is an application programming interface (API), which extends the data-driven approach of Numerix.
The calculation workflow of CAIL is:.
1 Select a trade template for a specified deal or trade from the repository. The trade template specifies a set of inputs, a set of outputs, and the dependencies on other information (model, market data, calendar, etc.).

2 Provide the input parameters to the trade template and call the calculation context. The calculation context follows the dependency path to collect the needed information and produces the output specified by the template.


Copy
Semantics

\section*{Examples}

\section*{Construct a Numerix Object}

Initialize Numerix CAIL environment.
```

import com.numerix.integration.*;
import com.numerix.integration.implementation.*;

```

Construct a numerix object.
```

n = numerix('i:\NumeriX_java_10_3_0\data')

```
\(\mathrm{n}=\)
```

            Path: 'i:\NumeriX_java_10_3_0\data
    RepositoryPath: 'i:\NumeriX_java_10_3_0\data\Repository'
Repository: [1x1 com.numerix.integration.implementation.FileSystemRepository]
Context: [1x1 com.numerix.integration.implementation.LocalCalculationContext]
LookupsPath: 'i:\NumeriX_java_10_3_0\data\Data\LookupRules'
MarketsPath: 'i:\NumeriX_java_10_3_0\data\Data\Markets
FixingsPath: 'i:\NumeriX_java_10_3_0\data\Data\Fixings
TradesPath: 'i:\NumeriX_java_10_3_0\data\Data\Trades'
Parameters: [1x1 com.numerix.integration.implementation.CalculationParameters]

```

See Also parseResults
Related - "Working with Simple Numerix Trades" on page 10-2
Examples
- "Working with Advanced Numerix Trades" on page 10-5
- "Use Numerix to Price Cash Deposits" on page 10-12
- "Use Numerix for Interest-Rate Risk Assessment" on page 10-15
- Class Attributes
- Property Attributes

External • http://www.numerix.com/cail
Web Sites

\section*{Purpose Converts Numerix CAIL data to MATLAB data types}
Syntax
R = parseResults(N,Results)

Description \(\quad R=\) parseResults ( \(N\), Results) returns Numerix CAIL data in native MATLAB data types.

\section*{Input \\ Arguments}

\section*{N}

Numerix object constructed using numerix.

\section*{Results}

Result instances for each trade instance.

\section*{Output \\ Arguments \\ R}

Results from Numerix table output represented as MATLAB data types.

\section*{Attributes}
\begin{tabular}{ll} 
Access & public \\
Static & false \\
Hidden & false
\end{tabular}

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

\section*{Examples Return Results for Numerix CAIL API to Price a Callable Reverse Floater}

Initialize Numerix environment.
```

import com.numerix.integration.*;
import com.numerix.integration.implementation.*;
n = numerix('i:\NumeriX_java_10_3_0\data')

```

\section*{numerix.parseResults}
\(n=\)
```

Path: 'i:\NumeriX_java_10_3_0\data
RepositoryPath: 'i:\Numerix_java_10_3_0\data\Repository'
Repository: [1x1 com.numerix.integration.implementation. FileSystemRepository]
Context: [1x1 com.numerix.integration.implementation.LocalCalculationContext]
LookupsPath: 'i:\NumeriX_java_10_3_0\data\Data\LookupRules'
MarketsPath: 'i:\NumeriX_java_10_3_0\data\Data\Markets'
FixingsPath: 'i:\NumeriX_java_10_3_0\data\Data\Fixings
TradesPath: 'i:\NumeriX_java_10_3_0\data\Data\Trades'
Parameters: [1x1 com.numerix.integration.implementation.CalculationParameters]

```

Create a market.
```

quotes = java.util.HashMap;
quotes.put('IR.USD-LIBOR-3M.SWAP-1Y.MID', 0.0066056);
quotes.put('IR.USD-LIBOR-3M.SWAP-10Y.MID', 0.022465005);
quotes.put('IR.USD-LIBOR-3M.SWAP-20Y.MID', 0.027544995);
market = Market('EOD_14-NOV-2011', DateExtensions.date('14-Nov-2011'), quotes.entrySet);

```

Define a trade instance based on instrument template found in the Repository.
```

tradeDescriptor = 'TRADE.IR.CALLABLEREVERSEFLOATER';
tradeParameters = java.util.HashMap;
tradeParameters.put('Trade ID','1001');
tradeParameters.put('Quote Type', 'MID');
tradeParameters.put('Currency', 'USD');
tradeParameters.put('Notional', 1000000.0);
tradeParameters.put('Effective Date', DateExtensions.date('1-Dec-2011'));
tradeParameters.put('Termination Date', DateExtensions.date('1-Dec-2021'));
tradeParameters.put('IR Index', 'LIBOR');
tradeParameters.put('IR Index Tenor', '3M');
tradeParameters.put('Structured Freq', '3M');
tradeParameters.put('Structured Side', 'Receive');
tradeParameters.put('Structured Coupon Floor', O.O);
tradeParameters.put('Structured Coupon UpBd', 0.08);
tradeParameters.put('StructuredCoupon Multiplier', 1.4);

```
```

tradeParameters.put('Structured Coupon Cap', 0.05);
tradeParameters.put('Structured Basis', 'ACT/360');
tradeParameters.put('Funding Freq', '3M');
tradeParameters.put('Funding Side', 'Pay');
tradeParameters.put('Funding Spread', 0.003);
tradeParameters.put('Funding Basis', 'ACT/360');
tradeParameters.put('Call Start Date', DateExtensions.date('1-Dec-2013'));
tradeParameters.put('Call End Date', DateExtensions.date('1-Dec-2020'));
tradeParameters.put('Option Side', 'Short');
tradeParameters.put('Option Type', 'Right to Terminate');
tradeParameters.put('Call Frequency', '3M');
tradeParameters.put('Model', 'IR.USD-LIBOR-3M.MID.DET');
tradeParameters.put('Method', 'BackwardAnalytic');

```

Create a trade instance.
```

trade = RepositoryExtensions.createTradeInstance(n.Repository, tradeDescriptor, tradeParameters);

```

Price the trades.
results = CalculationContextExtensions.calculate(n.Context, trade, market, Request.getAll);
Parse the results for MATLAB and display.
```

r = n.parseResults(results)
disp([r.Name r.Category r.Currency r.Data])

```
\(r=\)
    Category: \(\{13 \times 1\) cell \(\}\)
    Currency: \(\{13 \times 1\) cell \(\}\)
        Name: \(\{13 \times 1\) cell \(\}\)
    Data: \(\{13 \times 1\) cell \(\}\)
\begin{tabular}{llll} 
'Reporting Currency' & 'Price' & '' & 'USD' \\
'Structured Cashflow Log' & 'Cashflow' & ' & \(\{41 \times 20\) cell \} \\
'Structured Leg PV Accrued' & 'Price' & 'USD' & [ \(\quad\) 0] \\
'PV' & 'Price' & 'USD' & {\(\left[\begin{array}{lll}6.4133 e+04]\end{array}\right.\)}
\end{tabular}

\section*{numerix.parseResults}
\begin{tabular}{|c|c|c|c|}
\hline 'Structured Leg PV Clean' & 'Price \({ }^{\text {' }}\) & 'USD ' & [ 4.2637e+05] \\
\hline 'Option PV' & 'Price' & 'USD ' & [-1.3220e+05] \\
\hline 'Funding Cashflow Log' & 'Cashflow' & \(1{ }^{\prime}\) & \{41x20 cell \(\}\) \\
\hline 'Structured Leg PV' & 'Price' & 'USD ' & [ 4.2637e+05] \\
\hline 'Funding Leg PV' & 'Price' & 'USD ' & [-2.3004e+05] \\
\hline 'Funding Leg PV Accrued' & 'Price' & 'USD ' & \(0]\) \\
\hline 'Funding Leg PV Clean' & 'Price' & 'USD ' & [-2.3004e+05] \\
\hline 'Yield Risk Report' & 1 & '' & \{ \(4 \times 30\) cell \(\}\) \\
\hline 'Messages' & '1 & \({ }^{\prime}\) & \{ \(4 \times 1\) cell \(\}\) \\
\hline
\end{tabular}

See Also numerix
Related
- "Working with Simple Numerix Trades" on page 10-2 Examples
- "Working with Advanced Numerix Trades" on page 10-5
- "Use Numerix to Price Cash Deposits" on page 10-12
- "Use Numerix for Interest-Rate Risk Assessment" on page 10-15

External • http://www.numerix.com/cail
Web Sites

\section*{Purpose}

Determine option adjusted spread using Black-Derman-Toy model

Description
input
Arguments
[OAS, OAD, OAC] = oasbybdt(BDTTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates)
[OAS, OAD, OAC] = oasbybdt(BDTTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, Name, Value)
[OAS, OAD, OAC] = oasbybdt(BDTTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates) calculates option adjusted spread (OAS), duration (OAD), and convexity (OAC) using a BDT model.
[OAS, OAD, OAC] = oasbybdt(BDTTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, Name, Value) calculates option adjusted spread (OAS), duration (OAD), and convexity ( OAC ) using a BDT model with additional options specified by one or more Name, Value pair arguments.

\section*{BDTTree}

Interest-rate tree structure created by bdttree.

\section*{Price}

NINST-by-1 vector of market prices of bonds with embedded options.

\section*{CouponRate}

NINST-by-1 vector for decimal annual rate.

\section*{Settle}

NINST-by- 1 vector for settlement date.

\section*{Maturity}

\section*{oasbybdt}

NINST-by-1 vector for maturity date.

\section*{OptSpec}

NINST-by- 1 cell array of strings for 'call' or 'put'.

\section*{Strike}

Matrix of strike price values for supported option types:
- European option: NINST-by-1 vector of strike price values.
- Bermuda option: NINST by number of strikes (NSTRIKES) matrix of strike price values.
Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs .
- American option: NINST-by-1 vector of strike price values for each option.

\section*{ExerciseDates}

Matrix of exercise callable or puttable dates for supported option types:
- NINST-by-1 (European option) or NINST-by-NSTRIKES (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.
- American option: NINST-by-2 vector of exercise date boundaries. For each instrument, the option is exercised on any coupon date between or including the pair of dates on that row. If only one non- NaN date is listed, or if ExerciseDates is NINST-by-1, the option is exercised between the underlying bond Settle and the single listed exercise date.

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding
value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

\section*{'AmericanOpt'}

NINST-by-1 vector for option flags: 0 (European/Bermuda) or 1 (American).

Default: 0 (European/Bermuda)

\section*{'Basis'}

Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- 3 = actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- \(12=\) actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

\section*{oasbybdt}

\section*{'EndMonthRule'}

NINST-by-1 vector for end-of-month rule. Values are 1 (in effect) and 0 (not in effect).

Default: 1 (in effect)

\section*{'IssueDate'}

NINST-by-1 vector of bond issue date.
Default: If you do not specify an IssueDate, the cash flow payment dates are determined from other inputs.

\section*{'Face'}

NINST-by- 1 vector for face value.
Default: 100

\section*{'FirstCouponDate'}

NINST-by-1 vector. Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.

Default: If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

\section*{'LastCouponDate'}

NINST-by-1 vector. Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date.

Default: If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

\section*{'Period'}

NINST-by-1 vector for coupons per year.
Default: 2 per year

\section*{'Options'}

Structure created with derivset containing derivatives pricing options.
Default: None

\section*{Output OAS}

Arguments
NINST-by-1 vector for option adjusted spread.

\section*{OAD}

NINST-by-1 vector for option adjusted duration.

\section*{OAC}

NINST-by- 1 vector for option adjusted convexity.

\section*{Definitions Bond with Embedded Options}

A bond with embedded option allows the issuer to buy back (callable) or redeem (puttable) the bond at a predetermined price at specified future dates. Financial Instruments Toolbox software supports American, European, and Bermuda callable and puttable bonds.

The pricing for a bond with embedded options is as follows:
- Callable bond - The holder bought a bond and sold a call option to the issuer. For example, if interest rates go down by the time of the call date, the issuer is able to refinance its debt at a cheaper level and can call the bond. The price of a callable bond is:

\section*{oasbybdt}
```

Price callable bond = Price Option free bond - Price call option

```
- Puttable bond - The holder bought a bond and a put option. For example, if interest rates rise, the future value of coupon payments becomes less valuable. Therefore, the investor can sell the bond back to the issuer and then lend proceeds elsewhere at a higher rate. The price of a puttable bond is:

Price puttable bond = Price Option free bond + Price put option

\section*{Examples Compute OAS Using the Black-Derman-Toy (BDT) Model}

This example shows how to compute OAS using the Black-Derman-Toy (BDT) model with the following data.
```

ValuationDate = 'Oct-1-2010';
Rates = [0.035; 0.042; 0.047; 0.052];
StartDates = ValuationDate;
EndDates = datemnth(ValuationDate, 12:12:48)';
Compounding = 1;
% define RateSpec
RateSpec = intenvset('ValuationDate', ValuationDate,...
'StartDates', StartDates, 'EndDates', EndDates, ...
'Rates', Rates, 'Compounding', Compounding);
% specify VolSpec and TimeSpec
Sigma = 0.20;
VS = bdtvolspec(ValuationDate, EndDates, Sigma*ones(size(EndDates)));
TS = bdttimespec(ValuationDate, EndDates, Compounding);
% build the BDT tree
BDTTree = bdttree(VS, RateSpec, TS);
BDTTreenew = cvtree(BDTTree);
% instrument information

```
```

CouponRate = 0.065;
Settle = ValuationDate;
Maturity = '01-Oct-2014';
OptSpec = 'call';
Strike = 100;
ExerciseDates ='01-Oct-2011';
Period = 1;
Price = 101.58;
% compute the OAS
OAS = oasbybdt(BDTTree, Price, CouponRate, Settle, Maturity,...
OptSpec, Strike, ExerciseDates, 'Period', Period)
% use treviewer to observe the tree
treeviewer(BDTTree)

```
OAS =
    36.5591

\section*{oasbybdt}


References
Fabozzi, F., Handbook of Fixed Income Securities, McGraw-Hill, 7th edition, 2005.

Windas, T., Introduction to Option-Adjusted Spread Analysis, Bloomberg Press, 3rd edition, 2007.

See Also
| bdttree | bdtprice | instoptembnd | optembndbybdt | oasbyhjm | oasbyhw | oasbybk

\section*{Purpose}

Determine option adjusted spread using Black-Karasinski model

Description

Input
Arguments
[OAS, OAD, OAC] = oasbybk(BKTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates)
[OAS, OAD, OAC] = oasbybk(BKTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, Name, Value)
[OAS, OAD, OAC] = oasbybk(BKTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates) calculates option adjusted spread (OAS), duration (OAD), and convexity (OAC) using a BK model.
[OAS, OAD, OAC] = oasbybk(BKTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, Name, Value) calculates option adjusted spread (OAS), duration (OAD), and convexity (OAC) using a BK model with additional options specified by one or more Name, Value pair arguments.

\section*{BKTree}

Interest-rate tree structure created by bktree.

\section*{Price}

NINST-by-1 vector of market prices of bonds with embedded options.

\section*{CouponRate}

NINST-by-1 vector for decimal annual rate.

\section*{Settle}

NINST-by- 1 vector for settlement date.

\section*{Maturity}

\section*{oasbybk}

NINST-by-1 vector for maturity date.

\section*{OptSpec}

NINST-by- 1 cell array of strings for 'call' or 'put'.

\section*{Strike}

Matrix of strike price values for supported option types:
- European option: NINST-by-1 vector of strike price values.
- Bermuda option: NINST by number of strikes (NSTRIKES) matrix of strike price values.
Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- American option: NINST-by-1 vector of strike price values for each option.

\section*{ExerciseDates}

Matrix of exercise callable or puttable dates for supported option types:
- NINST-by-1 (European option) or NINST-by-NSTRIKES (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.
- American option: NINST-by-2 vector of exercise date boundaries. For each instrument, the option is exercised on any coupon date between or including the pair of dates on that row. If only one non- NaN date is listed, or if ExerciseDates is NINST-by-1, the option is exercised between the underlying bond Settle and the single listed exercise date.

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding
value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

\section*{'AmericanOpt'}

NINST-by- 1 vector for option flags: 0 (European/Bermuda) or 1 (American).

Default: 0 (European/Bermuda)

\section*{'Basis'}

Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- 3 = actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

\section*{oasbybk}

\section*{'EndMonthRule'}

NINST-by-1 vector for end-of-month rule. Values are 1 (in effect) and 0 (not in effect).

Default: 1 (in effect)

\section*{'IssueDate'}

NINST-by-1 vector of bond issue date.
Default: If you do not specify an IssueDate, the cash flow payment dates are determined from other inputs

\section*{'Face'}

NINST-by-1 vector for face value.
Default: 100

\section*{'FirstCouponDate'}

NINST-by-1 vector. Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.

Default: If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

\section*{'LastCouponDate'}

NINST-by-1 vector. Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date.

Default: If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

\section*{'Period'}

NINST-by-1 vector for coupons per year.
Default: 2 per year

\section*{'Options'}

Structure created with derivset containing derivatives pricing options.
Default: None

\section*{Output OAS}

Arguments
NINST-by-1 vector for option adjusted spread.

\section*{OAD}

NINST-by-1 vector for option adjusted duration.

\section*{OAC}

NINST-by-1 vector for option adjusted convexity.

\section*{Definitions Bond with Embedded Options}

A bond with embedded option allows the issuer to buy back (callable) or redeem (puttable) the bond at a predetermined price at specified future dates. Financial Instruments Toolbox software supports American, European, and Bermuda callable and puttable bonds.

The pricing for a bond with embedded options is as follows:
- Callable bond - The holder bought a bond and sold a call option to the issuer. For example, if interest rates go down by the time of the call date, the issuer is able to refinance its debt at a cheaper level and can call the bond. The price of a callable bond is:

\section*{oasbybk}
```

Price callable bond = Price Option free bond - Price call option

```
- Puttable bond - The holder bought a bond and a put option. For example, if interest rates rise, the future value of coupon payments becomes less valuable. Therefore, the investor can sell the bond back to the issuer and then lend proceeds elsewhere at a higher rate. The price of a puttable bond is:

Price puttable bond = Price Option free bond + Price put option

\section*{Examples Compute OAS and OAD Using the Black-Karasinski (BK) Model}

This example shows how to compute OAS and OAD using the Black-Karasinski (BK) model using the following data.
```

ValuationDate = 'Aug-2-2010';
Rates = [0.0355; 0.0382; 0.0427; 0.0489];
StartDates = ValuationDate;
EndDates = datemnth(ValuationDate, 12:12:48)';
Compounding = 1;
% define RateSpec
RateSpec = intenvset('ValuationDate', ValuationDate,...
'StartDates', StartDates,'EndDates', EndDates, ...
'Rates', Rates,'Compounding', Compounding);
% specify VolSpec and TimeSpec
Sigma = 0.10;
Alpha = 0.01;
VS = bkvolspec(ValuationDate, EndDates, Sigma*ones(size(EndDates)),...
EndDates, Alpha*ones(size(EndDates)));
TS = bktimespec(ValuationDate, EndDates, Compounding);
% build the BK tree

```
```

BKTree = bktree(VS, RateSpec, TS);
% instrument information
CouponRate = 0.045;
Settle = ValuationDate;
Maturity = '02-Aug-2014';
OptSpec = 'put';
Strike = 100;
ExerciseDates ='02-Aug-2013';
Period = 1;
AmericanOpt = 1;
Price = 101;
% compute OAS and OAD
[OAS, OAD] = oasbybk(BKTree, Price, CouponRate, Settle, Maturity,...
OptSpec, Strike, ExerciseDates, 'Period', Period, 'AmericanOpt', Amer:
% use treeviewer to observe the tree
treeviewer(BKTree)

```
OAS =
    21.0839
OAD =
1.7833

\section*{oasbybk}


References
Fabozzi, F., Handbook of Fixed Income Securities, McGraw-Hill, 7th edition, 2005.

Windas, T., Introduction to Option-Adjusted Spread Analysis, Bloomberg Press, 3rd edition, 2007.

See Also
| bktree | bkprice | instoptembnd | optembndbybk | oasbyhjm | oasbyhw | oasbybdt

Purpose
Determine option adjusted spread using Heath-Jarrow-Morton model
```

Syntax
[OAS, OAD, OAC] = oasbyhjm(HJMTree, Price, CouponRate, Settle,
Maturity, OptSpec, Strike, ExerciseDates)
[OAS, OAD, OAC] = oasbyhjm(HJMTree, Price, CouponRate, Settle,
Maturity, OptSpec, Strike, ExerciseDates,
Name,Value)

```

\section*{Description}

\section*{Input Arguments}
[OAS, OAD, OAC] = oasbyhjm(HJMTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates) calculates option adjusted spread (OAS), duration (OAD), and convexity (OAC) using an HJM model.
[OAS, OAD, OAC] = oasbyhjm(HJMTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, Name, Value) calculates option adjusted spread (OAS), duration (OAD), and convexity (OAC) using an HJM model with additional options specified by one or more Name, Value pair arguments.

\section*{HJMTree}

Interest-rate tree structure created by hjmtree.

\section*{Price}

NINST-by-1 vector of market prices of bonds with embedded options.

\section*{CouponRate}

NINST-by-1 vector for decimal annual rate.

\section*{Settle}

NINST-by- 1 vector for settlement date.

\section*{Maturity}

NINST-by-1 vector for maturity date.

\section*{OptSpec}

NINST-by-1 cell array of strings for 'call' or 'put'.

\section*{Strike}

Matrix of strike price values for supported option types:
- European option: NINST-by-1 vector of strike price values.
- Bermuda option: NINST by number of strikes (NSTRIKES) matrix of strike price values.
Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- American option: NINST-by-1 vector of strike price values for each option.

\section*{ExerciseDates}

Matrix of exercise callable or puttable dates for supported option types:
- NINST-by-1 (European option) or NINST-by-NSTRIKES (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.
- American option: NINST-by-2 vector of exercise date boundaries. For each instrument, the option is exercised on any coupon date between or including the pair of dates on that row. If only one non- NaN date is listed, or if ExerciseDates is NINST-by-1, the option is exercised between the underlying bond Settle and the single listed exercise date.

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding
value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

\section*{'AmericanOpt'}

NINST-by-1 vector for option flags: 0 (European/Bermuda) or 1 (American).

Default: 0 (European/Bermuda)

\section*{'Basis'}

Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- 3 = actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- \(12=\) actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

\section*{'EndMonthRule'}

NINST-by-1 vector for end-of-month rule. Values are 1 (in effect) and 0 (not in effect).

Default: 1 (in effect)

\section*{'IssueDate'}

NINST-by-1 vector of bond issue date.
Default: If you do not specify an IssueDate, the cash flow payment dates are determined from other inputs.

\section*{'Face'}

NINST-by-1 vector for face value.
Default: 100

\section*{'FirstCouponDate'}

NINST-by-1 vector. Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.

Default: If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

\section*{'LastCouponDate'}

NINST-by-1 vector. Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date.

Default: If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

\section*{'Period'}

NINST-by-1 vector for coupons per year.
Default: 2 per year

\section*{'Options'}

Structure created with derivset containing derivatives pricing options.
Default: None

\section*{Output OAS}

Arguments NINST-by-1 vector for option adjusted spread.

\section*{OAD}

NINST-by-1 vector for option adjusted duration.

\section*{OAC}

NINST-by- 1 vector for option adjusted convexity.

\section*{Definitions Bond with Embedded Options}

A bond with embedded option allows the issuer to buy back (callable) or redeem (puttable) the bond at a predetermined price at specified future dates. Financial Instruments Toolbox software supports American, European, and Bermuda callable and puttable bonds.

The pricing for a bond with embedded options is as follows:
- Callable bond - The holder bought a bond and sold a call option to the issuer. For example, if interest rates go down by the time of the call date, the issuer is able to refinance its debt at a cheaper level and can call the bond. The price of a callable bond is:

\section*{oasbyhim}
```

Price callable bond = Price Option free bond - Price call option

```
- Puttable bond - The holder bought a bond and a put option. For example, if interest rates rise, the future value of coupon payments becomes less valuable. Therefore, the investor can sell the bond back to the issuer and then lend proceeds elsewhere at a higher rate. The price of a puttable bond is:

Price puttable bond = Price Option free bond + Price put option

\section*{Examples Compute OAS Using the Heath-Jarrow-Morton (HJM) Model}

This example shows how to compute OAS using the Heath-Jarrow-Morton (HJM) model using the following data.
```

ValuationDate = 'Nov-1-2010';
Rates = [0.0356; 0.0427; 0.0478; 0.0529];
StartDates = ValuationDate;
EndDates = datemnth(ValuationDate, 12:12:48)';
Compounding = 1;
% define RateSpec
RateSpec = intenvset('ValuationDate', ValuationDate,...
'StartDates', StartDates,'EndDates', EndDates, ...
'Rates', Rates,'Compounding', Compounding);
% specify VolSpec and TimeSpec
Sigma = 0.02;
VS = hjmvolspec('Constant', Sigma)
TS = hjmtimespec(ValuationDate, EndDates, Compounding);
% build the HJM tree
HJMTree = hjmtree(VS, RateSpec, TS);
HJMTreenew = cvtree(HJMTree);

```
```

% instrument information
CouponRate = 0.05;
Settle = ValuationDate;
Maturity = '01-Nov-2014';
OptSpec = 'call';
Strike = 100;
ExerciseDates ='01-Nov-2011';
Period = 1;
Price = 97.5;
% compute the OAS
OAS = oasbyhjm(HJMTree, Price, CouponRate, Settle, Maturity, OptSpec,
ExerciseDates, 'Period', Period)
% use treeviewer to view the tree
treeviewer(HJMTree)
VS =
FinObj: 'HJMVolSpec'
FactorModels: {'Constant'}
FactorArgs: {{1x1 cell}}
SigmaShift: 0
NumFactors: 1
NumBranch: 2
PBranch: [0.5000 0.5000]
Fact2Branch: [-1 1]
OAS =
5.0601

```

\section*{oasbyhim}


References
Fabozzi, F., Handbook of Fixed Income Securities, McGraw-Hill, 7th edition, 2005.

Windas, T., Introduction to Option-Adjusted Spread Analysis, Bloomberg Press, 3rd edition, 2007.

See Also
| hjmtree | hjmprice | instoptembnd | optembndbyhjm | oasbybdt | oasbyhw | oasbybk

Purpose
Determine option adjusted spread using Hull-White model

\section*{Description}

\section*{Input Arguments}
[OAS, OAD, OAC] = oasbyhw(HWTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates)
[OAS, OAD, OAC] = oasbyhw(HWTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, Name, Value)
[OAS, OAD, OAC] = oasbyhw(HWTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates) calculates option adjusted spread (OAS), duration (OAD), and convexity (OAC) using an HW model.
[OAS, OAD, OAC] = oasbyhw(HWTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, Name, Value) calculates option adjusted spread (OAS), duration (OAD), and convexity (OAC) using an HW model with additional options specified by one or more Name, Value pair arguments.

\section*{HWTree}

Interest-rate tree structure created by hwtree.

\section*{Price}

NINST-by-1 vector of market prices of bonds with embedded options.

\section*{CouponRate}

NINST-by-1 vector for decimal annual rate.

\section*{Settle}

NINST-by- 1 vector for settlement date.

\section*{Maturity}

\section*{oasbyhw}

NINST-by-1 vector for maturity date.

\section*{OptSpec}

NINST-by-1 cell array of strings for 'call' or 'put'.

\section*{Strike}

Matrix of strike price values for supported option types:
- European option: NINST-by-1 vector of strike price values.
- Bermuda option: NINST by number of strikes (NSTRIKES) matrix of strike price values.

Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- American option: NINST-by-1 vector of strike price values for each option.

\section*{ExerciseDates}

Matrix of exercise callable or puttable dates for supported option types:
- NINST-by-1 (European option) or NINST-by-NSTRIKES (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.
- American option: NINST-by-2 vector of exercise date boundaries. For each instrument, the option is exercised on any coupon date between or including the pair of dates on that row. If only one non- NaN date is listed, or if ExerciseDates is NINST-by-1, the option is exercised between the underlying bond Settle and the single listed exercise date.

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding
value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

\section*{'AmericanOpt'}

NINST-by-1 vector for option flags: 0 (European/Bermuda) or 1 (American).

Default: 0 (European/Bermuda)

\section*{'Basis'}

Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- 3 = actual \(/ 365\)
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- \(12=\) actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

\section*{oasbyhw}

\section*{'EndMonthRule'}

NINST-by-1 vector for end-of-month rule. Values are 1 (in effect) and 0 (not in effect).

Default: 1 (in effect)

\section*{'Face'}

NINST-by-1 vector for face value.
Default: 100

\section*{'IssueDate'}

NINST-by-1 vector of bond issue date.
Default: If you do not specify an IssueDate, the cash flow payment dates are determined from other inputs.

\section*{'FirstCouponDate'}

NINST-by-1 vector. Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.

Default: If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

\section*{'LastCouponDate'}

NINST-by-1 vector. Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date.

Default: If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

\section*{'Period'}

NINST-by-1 vector for coupons per year.
Default: 2 per year

\section*{'Options'}

Structure created with derivset containing derivatives pricing options.
Default: None

\section*{Output OAS}

Arguments
NINST-by-1 vector for option adjusted spread.

\section*{OAD}

NINST-by-1 vector for option adjusted duration.

\section*{OAC}

NINST-by- 1 vector for option adjusted convexity.

\section*{Definitions Bond with Embedded Options}

A bond with embedded option allows the issuer to buy back (callable) or redeem (puttable) the bond at a predetermined price at specified future dates. Financial Instruments Toolbox software supports American, European, and Bermuda callable and puttable bonds.

The pricing for a bond with embedded options is as follows:
- Callable bond - The holder bought a bond and sold a call option to the issuer. For example, if interest rates go down by the time of the call date, the issuer is able to refinance its debt at a cheaper level and can call the bond. The price of a callable bond is:

\section*{oasbyhw}
```

Price callable bond = Price Option free bond - Price call option

```
- Puttable bond - The holder bought a bond and a put option. For example, if interest rates rise, the future value of coupon payments becomes less valuable. Therefore, the investor can sell the bond back to the issuer and then lend proceeds elsewhere at a higher rate. The price of a puttable bond is:

Price puttable bond = Price Option free bond + Price put option

\section*{Examples Compute OAS and OAD Using the Hull-White (HW) Model}

This example shows how to compute OAS and OAD using the Hull-White (HW) model using the following data.
```

ValuationDate = 'October-25-2010';
Rates = [0.0355; 0.0382; 0.0427; 0.0489];
StartDates = ValuationDate;
EndDates = datemnth(ValuationDate, 12:12:48)';
Compounding = 1;
% define RateSpec
RateSpec = intenvset('ValuationDate', ValuationDate,...
'StartDates', StartDates, 'EndDates', EndDates, ...
'Rates', Rates,'Compounding', Compounding);
% specify VolsSpec and TimeSpec
Sigma = 0.05;
Alpha = 0.01;
VS = hwvolspec(ValuationDate, EndDates, Sigma*ones(size(EndDates)),...
EndDates, Alpha*ones(size(EndDates)));
TS = hwtimespec(ValuationDate, EndDates, Compounding);
% build the HW tree
HWTree = hwtree(VS, RateSpec, TS);

```
```

% instrument information
CouponRate = 0.045;
Settle = ValuationDate;
Maturity = '25-October-2014';
OptSpec = 'call';
Strike = 100;
ExerciseDates = {'25-October-2010','25-October-2013'};
Period = 1;
AmericanOpt = 0;
Price = 97;
% compute the OAS
[OAS, OAD] = oasbyhw(HWTree, Price, CouponRate, Settle, Maturity,...
OptSpec, Strike, ExerciseDates, 'Period', Period, 'AmericanOpt', Amer:
% use treeviewer to observe the tree
treeviewer(HWTree)

```
OAS =
    \(-12.4436\)
OAD =
    3.3045


References
Fabozzi, F., Handbook of Fixed Income Securities, McGraw-Hill, 7th edition, 2005.

Windas, T., Introduction to Option-Adjusted Spread Analysis, Bloomberg Press, 3rd edition, 2007.

See Also
| hwtree | hwprice | instoptembnd | optembndbyhw | oasbybdt | oasbyhjm | oasbybk

Purpose
Price bond option from Black-Derman-Toy interest-rate tree
Syntax [Price, PriceTree] = optbndbybdt(BDTTree, OptSpec, Strike, ExerciseDates, AmericanOpt, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face, Options)

\section*{Arguments}

\section*{optbndbybdt}
\begin{tabular}{ll} 
ExerciseDates & \begin{tabular}{l} 
NINST-by-1 (European option) or \\
NINST-by-NSTRIKES (Bermuda option) \\
matrix of exercise dates. Each row is the \\
schedule for one option. For a European \\
option, there is only one exercise date, the \\
option expiry date.
\end{tabular} \\
& For an American option: \\
& NINST-by-2 vector of exercise date \\
boundaries. For each instrument, the \\
option can be exercised on any coupon date \\
between or including the pair of dates \\
on that or. If only one non-NaN date is \\
listed, or if ExerciseDates is NINST-by-1, \\
the option can be exercised between the \\
underlying bond Settle and the single \\
listed exercise date.
\end{tabular}

Basis

EndMonthRule

IssueDate
(Optional) Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- \(3=\) actual/ 365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- \(10=\) actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- \(12=\mathrm{actual} /\) actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
(Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. \(0=\) ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. \(1=\) set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
(Optional) Date when a bond was issued.

\section*{optbndbybdt}
\(\left.\begin{array}{ll}\text { FirstCouponDate } & \begin{array}{l}\text { (Optional) Date when a bond makes its } \\ \text { first coupon payment; used when bond has } \\ \text { an irregular first coupon period. When } \\ \text { FirstCouponDate and LastCouponDate }\end{array} \\ \text { are both specified, FirstCouponDate takes } \\ \text { precedence in determining the coupon } \\ \text { payment structure. If you do not specify a } \\ \text { FirstCouponDate, the cash flow payment } \\ \text { dates are determined from other inputs. } \\ \text { (Optional) Last coupon date of a bond } \\ \text { LastCouponDate } & \begin{array}{l}\text { before the maturity date; used when bond } \\ \\ \\ \text { has an irregular last coupon period. In the }\end{array} \\ & \text { absence of a specified FirstCouponDate, } \\ & \text { a specified LastCouponDate determines } \\ \text { the coupon structure of the bond. The } \\ \text { coupon structure of a bond is truncated at }\end{array}\right\}\)

The Settle date for every bond is set to the ValuationDate of the BDT tree. The bond argument Settle is ignored.

\section*{Description}
[Price, PriceTree] = optbndbybdt(BDTTree, OptSpec, Strike, ExerciseDates, AmericanOpt, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face, Options) computes the price of a bond option from a BDT interest-rate tree.

Price is an NINST-by- 1 matrix of expected prices at time 0 .
PriceTree is a tree structure with a vector of instrument prices at each node.

Examples Example 1. Using the BDT interest-rate tree in the deriv.mat file, price a European call option on a \(10 \%\) bond with a strike of 95 . The exercise date for the option is Jan. 01, 2002. The settle date for the bond is Jan. 01, 2000, and the maturity date is Jan. 01, 2003.
Load the file deriv.mat, which provides BDTTree. The BDTTree structure contains the time and forward-rate information needed to price the bond.
```

load deriv.mat;

```

Use optbondbybdt to compute the price of the option.
Price = optbndbybdt(BDTTree,'Call', 95,'01-Jan-2002',... 0,0.10,'01-Jan-2000', '01-Jan-2003', 1)

Price =
1.7657

Example 2. Now use optbndbybdt to compute the price of a put option on the same bond.
```

Price = optbndbybdt(BDTTree,'Put',95,'01-Jan-2002',...
0,0.10,'01-Jan-2000','01-Jan-2003',1)

```
```

Price =
0.5740

```

See Also bdtprice | bdttree | instoptbnd

\section*{Purpose}

Price bond option from Black-Karasinski interest-rate tree
Syntax [Price, PriceTree] = optbndbybk(BKTree, OptSpec, Strike, ExerciseDates, AmericanOpt, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face, Options)

\section*{Arguments}
\begin{tabular}{|c|c|}
\hline ExerciseDates & \begin{tabular}{l}
NINST-by-1 (European option) or NINST-by-NSTRIKES (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date. \\
For an American option: \\
NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non- NaN date is listed, or if ExerciseDates is NINST-by-1, the option can be exercised between the underlying bond Settle and the single listed exercise date.
\end{tabular} \\
\hline AmericanOpt & NINST-by-1 vector of flags: 0 (European/Bermuda) or 1 (American). \\
\hline CouponRate & Decimal annual rate. CouponRate is a NINST-by-1 vector. \\
\hline Settle & Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than Maturity. \\
\hline Maturity & Maturity date. A vector of serial date numbers or date strings. \\
\hline Period & (Optional) Coupons per year of the bond. A vector of integers. Allowed values are 1, 2, \(3,4,6\), and 12. Default \(=2\). \\
\hline
\end{tabular}
\begin{tabular}{|c|c|}
\hline Basis & (Optional) Day-count basis of the \\
\hline & - \(0=\) actual/actual (default) \\
\hline & - \(1=30 / 360\) (SIA) \\
\hline & - 2 = actual/360 \\
\hline & - 3 = actual/365 \\
\hline & - \(4=30 / 360\) (BMA) \\
\hline & - \(5=30 / 360\) (ISDA) \\
\hline & - \(6=30 / 360\) (European) \\
\hline & - \(7=\) actual/365 (Japanese) \\
\hline & - \(8=\) actual/actual (ICMA) \\
\hline & - 9 = actual/360 (ICMA) \\
\hline & - \(10=\mathrm{actual} / 365\) (ICMA) \\
\hline & - \(11=30 / 360 \mathrm{E}\) (ICMA) \\
\hline & - \(12=\) actual/actual (ISDA) \\
\hline & - 13 = BUS/252 \\
\hline & For more information, see basis on page Glossary-1. \\
\hline EndMonthRule & (Optional) End-of-month rule. A vector. \\
\hline & This rule applies only when Maturity is an end-of-month date for a month having 30 or \\
\hline & fewer days. \(0=\) ignore rule, meaning that a \\
\hline & bond's coupon payment date is always the \\
\hline & same numerical day of the month. \(1=\) set \\
\hline & rule on (default), meaning that a bond's \\
\hline & coupon payment date is always the last \\
\hline & \\
\hline IssueDate & (Optional) Date when a bond was issued. \\
\hline
\end{tabular}
\begin{tabular}{|c|c|}
\hline FirstCouponDate & (Optional) Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs. \\
\hline LastCouponDate & (Optional) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs. \\
\hline StartDate & (Optional) Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify StartDate, the effective start date is the Settle date. \\
\hline Face & (Optional) Face or par value. Face is a NINST-by-1 vector. Default \(=100\). \\
\hline Options & (Optional) Derivatives pricing options structure created with derivset. \\
\hline
\end{tabular}

The Settle date for every bond is set to the ValuationDate of the BK tree. The bond argument Settle is ignored.

\section*{Description}
[Price, PriceTree] = optbndbybk(BKTree, OptSpec, Strike, ExerciseDates, AmericanOpt, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face, Options) computes the price of a bond option from a Black-Karasinski interest rate tree.

Price is an NINST-by- 1 matrix of expected prices at time 0 .
PriceTree is a tree structure with a vector of instrument prices at each node.

Examples Example 1. Using the BK interest rate tree in the deriv.mat file, price a European call option on a \(4 \%\) bond with a strike of 96 . The exercise date for the option is Jan. 01, 2006. The settle date for the bond is Jan. 01, 2005, and the maturity date is Jan. 01, 2009.
Load the file deriv.mat, which provides BKTree. The BKTree structure contains the time and forward-rate information needed to price the bond.
```

load deriv.mat;

```

Use optbondbybk to compute the price of the option.
```

Price = optbndbybk(BKTree,'Call',96,'01-Jan-2006',...
0,0.04,'01-Jan-2005','01-Jan-2009')
Warning: OptBonds are valued at Tree ValuationDate rather than Settle
> In optbndbytrintree at 43
In optbndbybk at }8
Warning: Not all cash flows are aligned with the tree. Result will be
approximated.
> In optbndbytrintree at 151
In optbndbybk at }8

```
Price =

\subsection*{0.1512}

Example 2. Now use optbndbybk to compute the price of a put option on the same bond.
```

Price = optbndbybk(BKTree,'Put',96,'01-Jan-2006',...
0,0.04,'01-Jan-2005','01-Jan-2009')
Warning: OptBonds are valued at Tree ValuationDate rather than Settle
> In optbndbytrintree at 43
In optbndbybk at 88
Warning: Not all cash flows are aligned with the tree. Result will be
approximated.
> In optbndbytrintree at 151
In optbndbybk at 88
Price =
0.0272

```

See Also bkprice | bktree | instoptbnd

\section*{Purpose}

Price bond option from Heath-Jarrow-Morton interest-rate tree
Syntax
[Price, PriceTree] = optbndbyhjm(HJMTree, OptSpec, Strike, ExerciseDates, AmericanOpt, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face, Options)

\section*{Arguments}
\begin{tabular}{ll} 
HJMTree & \begin{tabular}{l} 
Forward-rate tree structure created by \\
hjmtree.
\end{tabular} \\
OptSpec & \begin{tabular}{l} 
Number of instruments (NINST)-by-1 cell \\
array of string values 'Call' or 'Put '.
\end{tabular} \\
& \begin{tabular}{l} 
European option: NINST-by-1 vector of strike \\
price values.
\end{tabular} \\
\begin{tabular}{l} 
Bermuda option: NINST by number of strikes \\
(NSTRIKES) matrix of strike price values.
\end{tabular} \\
& \begin{tabular}{l} 
Each row is the schedule for one option. If \\
an option has fewer than NSTRIKES exercise \\
opportunities, the end of the row is padded \\
with NaNs.
\end{tabular} \\
For an American option: \\
FINST-by-1 vector of strike price values for \\
each option.
\end{tabular}
\begin{tabular}{|c|c|}
\hline ExerciseDates & \begin{tabular}{l}
NINST-by-1 (European option) or NINST-by-NSTRIKES (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date. \\
For an American option: \\
NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is NINST-by-1, the option can be exercised between the underlying bond Settle and the single listed exercise date.
\end{tabular} \\
\hline AmericanOpt & NINST-by-1 vector of flags: 0 (European/Bermuda) or 1 (American). \\
\hline CouponRate & Decimal annual rate. CouponRate is a NINST-by-1 vector. \\
\hline Settle & Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than Maturity. \\
\hline Maturity & Maturity date. A vector of serial date numbers or date strings. \\
\hline Period & (Optional) Coupons per year of the bond. A vector of integers. Allowed values are 1, 2, 3, 4, 6, and 12. Default \(=2\). \\
\hline
\end{tabular}
Basis \begin{tabular}{ll} 
(Optional) Day-count basis of the instrument. \\
A vector of integers. \\
- \(0=\) actual/actual (default) \\
- \(1=30 / 360\) (SIA) \\
- \(2=\operatorname{actual/360~}\) \\
- \(3=\) actual/365 \\
- \(4=30 / 360\) (BMA) \\
- \(5=30 / 360\) (ISDA) \\
- \(6=30 / 360\) (European) \\
- \(7=\) actual/365 (Japanese) \\
- \(8=\) actual/actual (ICMA) \\
- \(9=\) actual/360 (ICMA) \\
- \(10=\) actual/365 (ICMA) \\
- \(11=30 / 360 E\) (ICMA) \\
- \(12=\) actual/actual (ISDA) \\
- \(13=\) BUS/252
\end{tabular}

For more information, see basis on page Glossary-1.

EndMonthRule (Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. \(0=\) ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. \(1=\) set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.

IssueDate (Optional) Date when a bond was issued.
\(\left.\begin{array}{ll}\text { FirstCouponDate } & \begin{array}{l}\text { (Optional) Date when a bond makes its } \\ \text { first coupon payment; used when bond has } \\ \text { an irregular first coupon period. When }\end{array} \\ & \text { FirstCouponDate and LastCouponDate } \\ \text { are both specified, FirstCouponDate takes } \\ \text { precedence in determining the coupon } \\ \text { payment structure. If you do not specify a } \\ & \text { FirstCouponDate, the cash flow payment } \\ \text { dates are determined from other inputs. } \\ \text { LastCouponDate } & \text { (Optional) Last coupon date of a bond before } \\ & \text { the maturity date; used when bond has } \\ & \text { an irregular last coupon period. In the } \\ & \text { absence of a specified FirstCouponDate, } \\ & \text { a specified LastCouponDate determines } \\ & \text { the coupon structure of the bond. The } \\ & \text { coupon structure of a bond is truncated at } \\ & \text { the LastCouponDate, regardless of where } \\ \text { it falls, and is followed only by the bond's } \\ & \text { maturity cash flow date. If you do not specify } \\ \text { a LastCouponDate, the cash flow payment } \\ \text { dates are determined from other inputs. }\end{array}\right\}\)

The Settle date for every bond is set to the ValuationDate of the HJM tree. The bond argument Settle is ignored.

\section*{Description}

\section*{Examples Compute the Price of a Bond Option From an HJM Forward-Rate Tree}

Using the HJM forward-rate tree in the deriv.mat file, price a European call option on a \(4 \%\) bond with a strike of 96 . The exercise date for the option is Jan. 01, 2003. The settle date for the bond is Jan. 01, 2000, and the maturity date is Jan. 01, 2004. Load the file deriv.mat, which provides HJMTree. The HJMTree structure contains the time and forward-rate information needed to price the bond.
```

load deriv.mat;

```

Use the function | optbondbyhjm \(\mid\) to compute the price of the option.
Price = optbndbyhjm(HJMTree,'Call',96,'01-Jan-2003',... 0,0.04,'01-Jan-2000', '01-Jan-2004')

Warning: Not all cash flows are aligned with the tree. Result will be approximated.

Price =
2.2410

See Also hjmprice | hjmtree | instoptbnd

\section*{optbndbyhw}

Purpose Price bond option from Hull-White interest-rate tree

\author{
Syntax \\ [Price, PriceTree] = optbndbyhw(HWTree, OptSpec, Strike, ExerciseDates, AmericanOpt, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face, Options)
}

\section*{Arguments}
\begin{tabular}{ll} 
HWTree & \begin{tabular}{l} 
Forward-rate tree structure created by \\
hwtree.
\end{tabular} \\
OptSpec & \begin{tabular}{l} 
Number of instruments (NINST)-by-1 cell \\
array of string values 'Call' or 'Put '.
\end{tabular} \\
Strike & \begin{tabular}{l} 
European option: NINST-by-1 vector of \\
strike price values.
\end{tabular} \\
& \begin{tabular}{l} 
Bermuda option: NINST by number of \\
strikes (NSTRIKES) matrix of strike price \\
values.
\end{tabular} \\
& \begin{tabular}{l} 
Each row is the schedule for one option. \\
If an option has fewer than NSTRIKES \\
exercise opportunities, the end of the row
\end{tabular} \\
is padded with NaNs. \\
& For an American option: \\
& NINST-by-1 vector of strike price values \\
for each option.
\end{tabular}
\begin{tabular}{|c|c|}
\hline ExerciseDates & \begin{tabular}{l}
NINST-by-1 (European option) or NINST-by-NSTRIKES (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date. \\
For an American option: \\
NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non- NaN date is listed, or if ExerciseDates is NINST-by-1, the option can be exercised between the underlying bond Settle and the single listed exercise date.
\end{tabular} \\
\hline AmericanOpt & NINST-by-1 vector of flags: 0 (European/Bermuda) or 1 (American). \\
\hline CouponRate & Decimal annual rate. CouponRate is a NINST-by-1 vector. \\
\hline Settle & Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than Maturity. \\
\hline Maturity & Maturity date. A vector of serial date numbers or date strings. \\
\hline Period & (Optional) Coupons per year of the bond. A vector of integers. Allowed values are 1, 2, 3, 4, 6, and 12. Default \(=2\). \\
\hline
\end{tabular}

\section*{optbndbyhw}
Basis \(\quad\)\begin{tabular}{l} 
(Optional) Day-count basis of the \\
instrument. A vector of integers. \\
- \(0=\mathrm{actual} / \mathrm{actual}\) (default) \\
- \(1=30 / 360\) (SIA) \\
- \(2=\mathrm{actual} / 360\) \\
- \(3=\mathrm{actual} / 365\) \\
- \(4=30 / 360\) (BMA) \\
- \(5=30 / 360\) (ISDA) \\
- \(6=30 / 360\) (European) \\
- \(7=\mathrm{actual} / 365\) (Japanese) \\
- \(8=\mathrm{actual} / \mathrm{actual}\) (ICMA) \\
- \(9=\mathrm{actual} / 360\) (ICMA) \\
- \(10=\mathrm{actual} / 365\) (ICMA) \\
- \(11=30 / 360 \mathrm{E}\) (ICMA) \\
- \(12=\mathrm{actual} / \mathrm{actual}\) (ISDA) \\
- \(13=\mathrm{BUS} / 252\)
\end{tabular}

EndMonthRule

IssueDate
(Optional) Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- \(3=\) actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- \(10=\) actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
(Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. \(0=\) ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. \(1=\) set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
(Optional) Date when a bond was issued.
\begin{tabular}{|c|c|}
\hline FirstCouponDate & (Optional) Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs. \\
\hline LastCouponDate & (Optional) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs. \\
\hline StartDate & (Optional) Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify StartDate, the effective start date is the Settle date. \\
\hline Face & (Optional) Face or par value. Face is a NINST-by-1 vector. Default \(=100\). \\
\hline Options & (Optional) Derivatives pricing options structure created with derivset. \\
\hline
\end{tabular}

\section*{optbndbyhw}

The Settle date for every bond is set to the ValuationDate of the HW tree. The bond argument Settle is ignored.

\section*{Description}
[Price, PriceTree] = optbndbyhw(HWTree, OptSpec, Strike, ExerciseDates, AmericanOpt, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face, Options) computes the price of a bond option from a Hull-White interest rate tree.
Price is an NINST-by- 1 matrix of expected prices at time 0 .
PriceTree is a tree structure with a vector of instrument prices at each node.

Examples Example 1. Using the HW interest rate tree in the deriv.mat file, price a European call option on a \(4 \%\) bond with a strike of 96 . The exercise date for the option is Jan. 01, 2006. The settle date for the bond is Jan. 01, 2005, and the maturity date is Jan. 01, 2009.
Load the file deriv.mat, which provides HWTree. The HWTree structure contains the time and forward-rate information needed to price the bond.
load deriv.mat;
Use optbondbyhw to compute the price of the option.
Price \(=\) optbndbyhw(HWTree, 'Call', 96,'01-Jan-2006', \(\ldots\)
0,0.04,'01-Jan-2005','01-Jan-2009')

Warning: OptBonds are valued at Tree ValuationDate rather than Settle Warning: Not all cash flows are aligned with the tree. Result will be approximated.

Price =
1.1556

Example 2. Now use optbondbyhw to compute the price of a put option on the same bond.
```

Price = optbndbyhw(HWTree,'Put',96,'01-Jan-2006',...
0,0.04,'01-Jan-2005','01-Jan-2009')
Warning: OptBonds are valued at Tree ValuationDate rather than Settle
Warning: Not all cash flows are aligned with the tree. Result will
be approximated.
Price =
1.0150

```
```

See Also
hwprice | hwtree | instoptbnd

```

\section*{optembndbybdt}

Purpose

Syntax
[Price, PriceTree] = optembndbybdt(BDTTree, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'Name1', Value1, 'Name2', Value2, ...)

\section*{Arguments}
\begin{tabular}{ll} 
BDTTree & \begin{tabular}{l} 
Interest-rate tree structure created by \\
bdttree.
\end{tabular} \\
CouponRate & \begin{tabular}{l} 
Decimal annual rate. CouponRate is a \\
NINST-by-1 vector or NINST-by-1 cell array of \\
decimal annual rates, or decimal annual rate \\
schedules. For the latter case of a variable \\
coupon schedule, each element of the cell \\
array is a NumDates-by-2 cell array, where the \\
first column is dates and the second column \\
is its associated rate. The date indicates the
\end{tabular} \\
last day that the coupon rate is valid.
\end{tabular}
\begin{tabular}{ll} 
Strike & \begin{tabular}{l} 
European option: NINST-by-1 vector of strike \\
price values.
\end{tabular} \\
& \begin{tabular}{l} 
Bermuda option: NINST by number of strikes \\
(NSTRIKES) matrix of strike price values.
\end{tabular} \\
& Each row is the schedule for one option. If \\
an option has fewer than NSTRIKES exercise \\
opportunities, the end of the row is padded \\
with NaNs.
\end{tabular}

\section*{optembndbybdt}
be specified in any order; names are case insensitive and partial string matches are allowed provided no ambiguities exist. Valid parameter names are as follows:
- AmericanOpt is a NINST-by- 1 matrix for flags options: 0 (European/Bermuda) or 1 (American). Default is 0 .
- Period is a NINST-by- 1 matrix for coupons per year. The default value is 2 .
- Basis is a day-count basis of the instrument. Basis is a vector of integers with the following possible values:
- 0 = actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- 3 = actual \(/ 365\)
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- \(10=\) actual \(/ 365\) (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
- EndMonthRule is a NINST-by-1 matrix for the end-of-month rule. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. When the value is 0 the end-of-month rule is ignored; this means that a bond's coupon payment date is always the same numerical day of the month. Use 1 to set the rule on; this is the default value and means that a bond's coupon payment date is always the last actual day of the month.
- IssueDate is a NINST-by-1 matrix for the bond issue date.
- FirstCouponDate is a NINST-by-1 matrix for a date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.
- LastCouponDate is a NINST-by-1 matrix for a last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow

\section*{optembndbybdt}
payment dates are determined from other inputs.
- StartDate is a NINST-by-1 matrix for date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify StartDate, the effective start date is the Settle date.
- Face value. Face is a NINST-by-1 vector or NINST-by- 1 cell array of face values, or face value schedules. For the latter case, each element of the cell array is a NumDates-by- 2 cell array, where the first column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid. Default is 100 .
- Options is a derivatives pricing options structure created with derivset.

Note The Settle date for every bond with an embedded option is set to the ValuationDate of the BDT tree; the bond's argument for Settle date is ignored.

\section*{Description}
[Price, PriceTree] = optembndbybdt(BDTTree, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'Name1', Value1, 'Name2', Value2, ...) prices bonds with embedded options using a BDT interest-rate tree.

Price is a number of instruments (NINST)-by-1 matrix of expected prices at time 0 .

PriceTree is a MATLAB structure of trees containing vectors of instrument prices and observation times for each node. Within PriceTree
- PriceTree. PTree contains the clean prices.
- PriceTree.tObs contains the observation times.
optembndbybdt computes prices of vanilla bonds with embedded options, stepped coupon bonds with embedded options, and bonds with sinking fund option provisions.

\section*{Definitions Vanilla Bond with Embedded Option}

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time. The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment. A vanilla bond with an embedded option is where an option contract has an underlying asset of a vanilla bond.

\section*{Stepped Coupon Bond with Callable and Puttable Features}

A step-up and step-down bond is a debt security with a predetermined coupon structure over time. With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond. Stepped coupon bonds can have options features (call and puts).

\section*{Sinking Fund Bond with Embedded Option}

A sinking fund bond is a coupon bond with a sinking fund provision. This provision obligates the issuer to amortize portions of the principal prior to maturity, affecting bond prices since the time of the principal repayment changes. This means that investors receive the coupon and a portion of the principal paid back over time. These types of bonds reduce credit risk, since it lowers the probability of investors not receiving their principal payment at maturity.
The bond may have a sinking fund option provision allowing the issuer to retire the sinking fund obligation either by purchasing the bonds to

\section*{optembndbybdt}
be redeemed from the market or by calling the bond via a sinking fund call, whichever is cheaper. If interest rates are high, then the issuer will buy back the requirement amount of bonds from the market since bonds will be cheap, but if interest rates are low (bond prices are high), then most likely the issuer will be buying the bonds at the call price. Unlike a call feature, however, if a bond has a sinking fund option provision, it is an obligation, not an option, for the issuer to buy back the increments of the issue as stated. Because of this, a sinking fund bond trades at a lower price than a non-sinking fund bond.

\section*{Examples}

\section*{Price a Callable Bond Using a BDT Interest-Rate Tree Model}

Create a BDTTree with the following data:
```

ZeroRates = [ 0.035;0.04;0.045];
Compounding = 1;
StartDates = ['jan-1-2007';'jan-1-2008';'jan-1-2009'];
EndDates = ['jan-1-2008';'jan-1-2009';'jan-1-2010'];
ValuationDate = 'jan-1-2007';
Create a RateSpec.

```
```

RateSpec = intenvset('Rates', ZeroRates, 'StartDates', ValuationDate,

```
RateSpec = intenvset('Rates', ZeroRates, 'StartDates', ValuationDate,
EndDates, 'Compounding', Compounding, 'ValuationDate', ValuationDate)
```

RateSpec =
FinObj: 'RateSpec'
Compounding: 1
Disc: $[3 \times 1$ double]
Rates: $[3 \times 1$ double $]$
EndTimes: $[3 \times 1$ double $]$
StartTimes: $[3 \times 1$ double $]$
EndDates: $[3 \times 1$ double]
StartDates: 733043
ValuationDate: 733043

Basis: 0<br>EndMonthRule: 1

Create a VolSpec.
Volatility = 0.10 * ones (3,1);
VolSpec = bdtvolspec(ValuationDate, EndDates, Volatility)

VolSpec =

```
                    FinObj: 'BDTVolSpec'
    ValuationDate: 733043
        VolDates: [3x1 double]
        VolCurve: [3x1 double]
VolInterpMethod: 'linear'
```

Create a TimeSpec.
TimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);
Build the BDTTree.
BDTTree = bdttree(VolSpec, RateSpec, TimeSpec)

BDTTree =

```
            FinObj: 'BDTFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tObs: [0 1 2]
        dObs: [733043 733408 733774]
        TFwd: {[3x1 double] [2x1 double] [2]}
```


## optembndbybdt

```
CFlowT: \{[3x1 double] [2x1 double] [3]\}
FwdTree: \{[1.0350] [1.0406 1.0495] [1.0447 1.0546 1.0667]\}
```

To compute the price of an American callable bond that pays a $5.25 \%$ annual coupon, matures in Jan-1-2010, and is callable on Jan-1-2008 and 01-Jan-2010.

```
BondSettlement = 'jan-1-2007';
BondMaturity = 'jan-1-2010';
CouponRate = 0.0525;
Period = 1;
OptSpec = 'call';
Strike = [100];
ExerciseDates = {'jan-1-2008' '01-Jan-2010'};
AmericanOpt = 1;
```

PriceCallBond = optembndbybdt(BDTTree, CouponRate, BondSettlement, BondMa
OptSpec, Strike, ExerciseDates, 'Period', 1,'AmericanOp', 1)
PriceCallBond =
101.4750

## Price Single Stepped Callable Bonds Using a BDT Interest-Rate Tree Model

Price the following single stepped callable bonds using the following data: The data for the interest rate term structure is as follows:

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
```

```
% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDat
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
% Instrument
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2012' .0425;'01-Jan-2014' .0750}};
OptSpec='call';
Strike=100;
ExerciseDates='01-Jan-2012'; %Callable in two years
% Build the tree
% Assume the volatility to be 10%
Sigma = 0.1;
BDTTimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length
BDTT = bdttree(BDTVolSpec, RS, BDTTimeSpec);
% The first row corresponds to the price of the callable bond with ma
% of three years. The second row corresponds to the price of the calla
% with maturity of four years.
PBDT= optembndbybdt(BDTT, CouponRate, Settle, Maturity ,OptSpec, Str:
ExerciseDates, 'Period', 1)
```

PBDT =

## optembndbybdt

## Price a Sinking Fund Bond Using a BDT Interest-Rate Tree Model

A corporation issues a three year bond with a sinking fund obligation requiring the company to sink $1 / 3$ of face value after the first year and $1 / 3$ after the second year. The company has the option to buy the bonds in the market or call them at $\$ 98$. The following data describes the details needed for pricing the sinking fund bond:

```
Rates = [0.1;0.1;0.1;0.1];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
Compounding = 1;
% Create RateSpec
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates',...
StartDates, 'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compouno
% Build the BDT tree
% Assume the volatility to be 10%
Sigma = 0.1;
BDTTimeSpec = bdttimespec(ValuationDate, EndDates);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length(Eno
BDTT = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec);
% Instrument
% The bond has a coupon rate of 9%, a period of one year and matures in
% 1-Jan-2014. Face decreases 1/3 after the first year and 1/3 after the
% second year.
CouponRate = 0.09;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';
Period = 1;
Face = { ...
{'Jan-1-2012' 100; ...
    'Jan-1-2013' 66.6666; ...
```

```
    'Jan-1-2014' 33.3333};
    };
% Option provision
OptSpec = 'call';
Strike = [98 98];
ExerciseDates ={'Jan-1-2012', 'Jan-1-2013'};
% Price of non-sinking fund bond.
PNSF = bondbybdt(BDTT, CouponRate, Settle, Maturity, Period)
```

PNSF =
97.5131

Price of the bond with the option sinking provision.
PriceSF = optembndbybdt(BDTT, CouponRate, Settle, Maturity,... OptSpec, Strike, ExerciseDates,'Period', Period, 'Face', Face)

## PriceSF =

96.8364

## See Also <br> bdtprice | bdttree | cfamounts | instoptembnd

## optembndbybk

Purpose

Syntax
[Price, PriceTree] = optembndybk(BKTree, CouponRate, Settle, Maturity, OptSpec, Strike, Strike, ExerciseDates, 'Name1', Value1, 'Name2', Value2, ...)

## Arguments

| BKTree | Interest-rate tree structure created by <br> bktree. |
| :--- | :--- |
| CouponRate | Decimal annual rate. CouponRate is a <br> NINST-by-1 vector or NINST-by-1 cell array of <br> decimal annual rates, or decimal annual rate <br> schedules. For the latter case of a variable <br> coupon schedule, each element of the cell <br> array is a NumDates-by-2 cell array, where the <br> first column is dates and the second column <br> is its associated rate. The date indicates the |
| last day that the coupon rate is valid. |  |

$\left.\begin{array}{ll}\text { Strike } & \begin{array}{l}\text { European option: NINST-by-1 vector of strike } \\ \text { price values. }\end{array} \\ & \text { Bermuda option: NINST by number of strikes } \\ \text { (NSTRIKES) matrix of strike price values. }\end{array}\right\}$

## optembndbybk

be specified in any order; names are case insensitive and partial string matches are allowed provided no ambiguities exist. Valid parameter names are as follows:

- AmericanOpt is a NINST-by- 1 matrix for flags options: 0 (European/Bermuda) or 1 (American). Default is 0 .
- Period is a NINST-by- 1 matrix for coupons per year. Default is 2 .
- Basis is a day-count basis of the instrument. Basis is a vector of integers with the following supported values:
- 0 = actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual $/ 365$
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- $10=$ actual $/ 365$ (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.

- EndMonthRule is a NINST-by- 1 matrix for the end-of-month rule. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. When the value is 0 , the end-of-month rule is ignored, meaning that a bond's coupon payment date is always the same numerical day of the month. When the value is 1 , the end-of-month rule is set on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
- IssueDate is a NINST-by- 1 matrix for the bond issue date.
- FirstCouponDate is a NINST-by-1 matrix for a date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.
- LastCouponDate is a NINST-by-1 matrix for a last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not


## optembndbybk

specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

- StartDate is a NINST-by- 1 matrix for date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify StartDate, the effective start date is the Settle date.
- Face value. Face is a NINST-by-1 vector or NINST-by- 1 cell array of face values, or face value schedules. For the latter case, each element of the cell array is a NumDates-by-2 cell array, where the first column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid. Default is 100 .
- Options is a derivatives pricing options structure created with derivset.

Note The Settle date for every bond with embedded option is set to the ValuationDate of the BKTree; the bond's argument for Settle date is ignored.

## Description

[Price, PriceTree] = optembndybk(BKTree, CouponRate, Settle, Maturity, OptSpec, Strike, Strike, ExerciseDates, 'Name1', Value1, 'Name2', Value2, ...) prices bonds with embedded options by a BK interest-rate tree.

Price is a number of instruments (NINST)-by-1 matrix of expected prices at time 0 .

PriceTree is a structure of trees containing vectors of instrument prices and observation times for each node. Within PriceTree:

- PriceTree. PTree contains the clean prices.
- PriceTree.tObs contains the observation times.
optembndbybk computes prices of vanilla bonds with embedded options, stepped coupon bonds with embedded options, and bonds with sinking fund option provisions.


## Definitions

## Vanilla Bond with Embedded Option

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time. The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment. A vanilla bond with an embedded option is where an option contract has an underlying asset of a vanilla bond.

## Stepped Coupon Bond with Callable and Puttable Features

A step-up and step-down bond is a debt security with a predetermined coupon structure over time. With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond. Stepped coupon bonds can have options features (call and puts).

## Sinking Fund Bond with Embedded Option

A sinking fund bond is a coupon bond with a sinking fund provision. This provision obligates the issuer to amortize portions of the principal prior to maturity, affecting bond prices since the time of the principal repayment changes. This means that investors receive the coupon and a portion of the principal paid back over time. These types of bonds reduce credit risk, since it lowers the probability of investors not receiving their principal payment at maturity.

## optembndbybk

The bond may have a sinking fund option provision allowing the issuer to retire the sinking fund obligation either by purchasing the bonds to be redeemed from the market or by calling the bond via a sinking fund call, whichever is cheaper. If interest rates are high, then the issuer will buy back the requirement amount of bonds from the market since bonds will be cheap, but if interest rates are low (bond prices are high), then most likely the issuer will be buying the bonds at the call price. Unlike a call feature, however, if a bond has a sinking fund option provision, it is an obligation, not an option, for the issuer to buy back the increments of the issue as stated. Because of this, a sinking fund bond trades at a lower price than a non-sinking fund bond.

## Examples

## Price a Callable Bond Using a BK Interest-Rate Tree Model

Create a BKTree with the following data:

```
ZeroRates = [ 0.035;0.04;0.045];
Compounding = 1;
StartDates = ['jan-1-2007';'jan-1-2008';'jan-1-2009'];
EndDates = ['jan-1-2008';'jan-1-2009';'jan-1-2010'];
ValuationDate = 'jan-1-2007';
Create a RateSpec.
RateSpec = intenvset('Rates', ZeroRates, 'StartDates', ValuationDate, EndDates, 'Compounding', Compounding, 'ValuationDate', ValuationDate)
```

RateSpec =
FinObj:
Compounding:
Disc:
Rates:

```
StartDates: 733043
ValuationDate: 733043
Basis: 0
EndMonthRule: 1
```

Create a VolSpec.

```
VolDates = ['jan-1-2008';'jan-1-2009';'jan-1-2010'];
```

VolCurve = 0.01;
AlphaDates = 'jan-1-2010';
AlphaCurve = 0.1;
BKVolSpec = bkvolspec(ValuationDate, VolDates, VolCurve, AlphaDates,
BKVolSpec =
FinObj: 'BKVolSpec'
ValuationDate: 733043
VolDates: [3x1 double]
VolCurve: [3x1 double]
AlphaCurve: 0.1000
AlphaDates: 734139
VolInterpMethod: 'linear'

Create a TimeSpec.
BKTimeSpec = bktimespec(ValuationDate, EndDates, Compounding);
Build the BKTree.
BKTree = bktree(BKVolSpec, RateSpec, BKTimeSpec)

BKTree $=$

## optembndbybk

```
    FinObj: 'BKFwdTree'
    VolSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
    tObs: [0 1 2]
    dObs: [733043 733408 733774]
    CFlowT: {[3x1 double] [2x1 double] [3]}
    Probs: {[3x1 double] [3x3 double]}
Connect: {[2] [2 3 4]}
FwdTree: {[1.0350] [1.0458 1.0450 1.0443] [1x5 double]}
```

To compute the price of an American puttable bond that pays an annual coupon of $5.25 \%$, matures on January 1, 2010, and is callable on January 1, 2008 and January 1, 2010.

```
BondSettlement = 'jan-1-2007';
BondMaturity = 'jan-1-2010';
CouponRate = 0.0525;
Period = 1;
OptSpec = 'put';
Strike = [100];
ExerciseDates = {'jan-1-2008' '01-Jan-2010'};
AmericanOpt = 1;
PricePutBondBK = optembndbybk(BKTree, CouponRate, BondSettlement, BondMat
OptSpec, Strike, ExerciseDates,'Period', 1, 'AmericanOpt', 1)
PricePutBondBK =
```

    102.3820
    
## Price Single Stepped Callable Bonds Using a BK Interest-Rate Tree Model

Price the following single stepped callable bonds using the following data: The data for the interest rate term structure is as follows:

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDat
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
% Instrument
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2012' .0425;'01-Jan-2014' .0750}};
OptSpec='call';
Strike=100;
ExerciseDates='01-Jan-2012'; %Callable in two years
% Build the tree with the following data
VolDates = ['1-Jan-2011'; '1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'];
VolCurve = 0.01;
AlphaDates = '01-01-2014';
AlphaCurve = 0.1;
```

BKVolSpec = bkvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
BKTimeSpec = bktimespec(RS.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVolSpec, RS, BKTimeSpec);
\% The first row corresponds to the price of the callable bond with mat
\% of three years. The second row corresponds to the price of the callable \% with maturity of four years.
PBK= optembndbybk(BKT, CouponRate, Settle, Maturity ,OptSpec, Strike,... ExerciseDates, 'Period', 1)

PBK =
100.0945
100.0945

See Also bkprice | cfamounts | bktree | instoptembnd

## Purpose

Syntax

## Arguments

$\left.\begin{array}{ll}\text { Strike } & \begin{array}{l}\text { European option: NINST-by-1 vector of strike } \\ \text { price values. }\end{array} \\ & \text { Bermuda option: NINST by number of strikes } \\ \text { (NSTRIKES) matrix of strike price values. }\end{array}\right\}$
be specified in any order; names are case insensitive and partial string matches are allowed provided no ambiguities exist. Valid parameter names are as follows:

- AmericanOpt is a NINST-by-1 matrix for flags options: 0 (European/Bermuda) or 1 (American). Default is 0 .
- Period is a NINST-by- 1 matrix for coupons per year. Default is 2 .
- Basis is a day-count basis of the instrument. Basis is a vector of integers with the following supported values:
- 0 = actual/actual (default)
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.

## optembndbyhim

- EndMonthRule is a NINST-by-1 matrix for the end-of-month rule. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. When the value is 0 , the end-of-month rule is ignored, meaning that a bond's coupon payment date is always the same numerical day of the month. When the value is 1 , the end-of-month rule is set on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
- IssueDate is a NINST-by-1 matrix for the bond issue date.
- FirstCouponDate is a NINST-by-1 matrix for a date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.
- LastCouponDate is a NINST-by-1 matrix for a last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not
specify a LastCouponDate, the cash flow payment dates are determined from other inputs.
- StartDate is a NINST-by-1 matrix for date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify StartDate, the effective start date is the Settle date.
- Face value. Face is a NINST-by-1 vector or NINST-by- 1 cell array of face values, or face value schedules. For the latter case, each element of the cell array is a NumDates-by- 2 cell array, where the first column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid. Default is 100 .
- Options is a derivatives pricing options structure created with derivset.

Note The Settle date for every bond with embedded option is set to the ValuationDate of the HJM tree; the bond's argument for Settle date is ignored.

## Description

[Price, PriceTree] = optembndbyhjm(HJMTree, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'Name1', Value1, 'Name2', Value2, ...) prices bonds with embedded options by an HJM interest-rate tree.

## optembndbyhim

Price is a number of instruments (NINST)-by-1 matrix of expected prices at time 0.

PriceTree is a structure of trees containing vectors of instrument prices and observation times for each node. Within PriceTree:

- PriceTree. PBush contains the clean prices.
- PriceTree.tObs contains the observation times.
optembndbyhjm computes prices of vanilla bonds with embedded options, stepped coupon bonds with embedded options, and bonds with sinking fund option provisions.


## Definitions

## Vanilla Bond with Embedded Option

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time. The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment. A vanilla bond with an embedded option is where an option contract has an underlying asset of a vanilla bond.

## Stepped Coupon Bond with Callable and Puttable Features

A step-up and step-down bond is a debt security with a predetermined coupon structure over time. With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond. Stepped coupon bonds can have options features (call and puts).

## Sinking Fund Bond with Embedded Option

A sinking fund bond is a coupon bond with a sinking fund provision. This provision obligates the issuer to amortize portions of the principal prior to maturity, affecting bond prices since the time of the principal repayment changes. This means that investors receive the coupon and a portion of the principal paid back over time. These types of bonds reduce credit risk, since it lowers the probability of investors not receiving their principal payment at maturity.

The bond may have a sinking fund option provision allowing the issuer to retire the sinking fund obligation either by purchasing the bonds to be redeemed from the market or by calling the bond via a sinking fund call, whichever is cheaper. If interest rates are high, then the issuer will buy back the requirement amount of bonds from the market since bonds will be cheap, but if interest rates are low (bond prices are high), then most likely the issuer will be buying the bonds at the call price. Unlike a call feature, however, if a bond has a sinking fund option provision, it is an obligation, not an option, for the issuer to buy back the increments of the issue as stated. Because of this, a sinking fund bond trades at a lower price than a non-sinking fund bond.

## Examples

## Price a Callable Bond Using an HJM Interest-Rate Tree Model

Create a HJMTree with the following data:

```
Rates = [0.05;0.06;0.07];
Compounding = 1;
StartDates = ['jan-1-2007';'jan-1-2008';'jan-1-2009'];
EndDates = ['jan-1-2008';'jan-1-2009';'jan-1-2010'];
ValuationDate = 'jan-1-2007';
Create a RateSpec.
RateSpec = intenvset('Rates', Rates, 'StartDates', ValuationDate, 'Enc
EndDates, 'Compounding', Compounding, 'ValuationDate', ValuationDate)
```

RateSpec =

## FinObj: 'RateSpec'

Compounding: 1
Disc: [3x1 double]
Rates: [3x1 double]
EndTimes: [3x1 double]
StartTimes: [3×1 double]
EndDates: [3×1 double]

```
    StartDates: 733043
ValuationDate: 733043
    Basis: 0
    EndMonthRule: 1
Create a VolSpec.
VolSpec = hjmvolspec('Constant', 0.01)
VolSpec =
    FinObj: 'HJMVolSpec'
    FactorModels: {'Constant'}
        FactorArgs: {{1x1 cell}}
        SigmaShift: 0
        NumFactors: 1
        NumBranch: 2
            PBranch: [0.5000 0.5000]
    Fact2Branch: [-1 1]
Create a TimeSpec.
TimeSpec = hjmtimespec(ValuationDate, EndDates, Compounding)
TimeSpec =
    FinObj: 'HJMTimeSpec'
    ValuationDate: 733043
        Maturity: [3\times1 double]
    Compounding: 1
            Basis: 0
    EndMonthRule: 1
```

Build the HJMTree.

```
HJMTree = hjmtree(VolSpec, RateSpec, TimeSpec)
HJMTree =
    FinObj: 'HJMFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tObs: [0 1 2]
        dObs: [733043 733408 733774]
        TFwd: {[3x1 double] [2x1 double] [2]}
    CFlowT: {[3\times1 double] [2x1 double] [3]}
FwdTree: {[3x1 double] [2x1x2 double] [1x2x2 double]}
```

To compute the price of an American callable bond that pays a 6\% annual coupon and matures and is callable on January 1, 2010.

```
BondSettlement = 'jan-1-2007';
BondMaturity = 'jan-1-2010';
CouponRate = 0.06;
Period = 1;
OptSpec = 'call';
Strike = [98];
ExerciseDates = '01-Jan-2010';
AmericanOpt = 1;
[PriceCallBond,PT] = optembndbyhjm(HJMTree, CouponRate, BondSettlemen
OptSpec, Strike, ExerciseDates, 'Period', 1,'AmericanOp',1)
```


## optembndbyhim

95.9492

```
PT =
```

```
FinObj: 'HJMPriceTree'
    tObs: [00 1 2 3]
    PBush: {[95.9492] [1x1x2 double] [1x2x2 double] [98 98 98 98]}
```


## Price Single Stepped Callable Bonds Using an HJM Interest-Rate Tree Model

Price the following single stepped callable bonds using the following data: The data for the interest rate term structure is as follows:

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,.
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
% Instrument
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2012' .0425;'01-Jan-2014' .0750}};
OptSpec='call';
Strike=100;
ExerciseDates='01-Jan-2012'; %Callable in two years
% Build the tree with the following data
Volatility = [.2; .19; .18; .17];
CurveTerm = [ 1; 2; 3; 4];
```

```
HJMTimeSpec = hjmtimespec(ValuationDate, EndDates);
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec, RS, HJMTimeSpec);
% The first row corresponds to the price of the callable bond with ma
% of three years. The second row corresponds to the price of the call
% bond with maturity of four years.
PHJM= optembndbyhjm(HJMT, CouponRate, Settle, Maturity ,OptSpec, Str:
ExerciseDates, 'Period', 1)
```


## PHJM =

100.0484
99.8009

## See Also <br> hjmprice | cfamounts | hjmtree | instoptembnd

## optembndbyhw

Purpose Price bonds with embedded options by Hull-White interest-rate tree

Syntax<br>[Price, PriceTree] = optembndbyhw(HWTree, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'Name1', Value1, 'Name2', Value2, ...)

## Arguments

\(\left.$$
\begin{array}{ll}\text { HWTree } & \begin{array}{l}\text { Interest-rate tree structure created by } \\
\text { hwtree. }\end{array} \\
\text { CouponRate } & \begin{array}{l}\text { Decimal annual rate. CouponRate is a } \\
\text { NINST-by-1 vector or NINST-by-1 cell array of } \\
\text { decimal annual rates, or decimal annual rate } \\
\text { schedules. For the latter case of a variable } \\
\text { coupon schedule, each element of the cell } \\
\text { array is a NumDates-by-2 cell array, where the }\end{array}
$$ <br>
first column is dates and the second column <br>
is its associated rate. The date indicates the <br>

last day that the coupon rate is valid.\end{array}\right\}\)| NINST-by-1 matrix for the settlement date. A |
| :--- |
| Settle |
| vector of serial date numbers or date strings. |

$\left.\begin{array}{ll}\text { Strike } & \begin{array}{l}\text { European option: NINST-by-1 vector of strike } \\ \text { price values. }\end{array} \\ & \text { Bermuda option: NINST by number of strikes } \\ \text { (NSTRIKES) matrix of strike price values. }\end{array}\right\}$

## optembndbyhw

be specified in any order; names are case insensitive and partial string matches are allowed provided no ambiguities exist. Valid parameter names are as follows:

- AmericanOpt is a NINST-by- 1 matrix for flags options: 0 (European/Bermuda) or 1 (American). Default is 0 .
- Period is a NINST-by- 1 matrix for coupons per year. Default is 2 .
- Basis is a day-count basis of the instrument. Basis is a vector of integers with the following supported values:
- 0 = actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual $/ 365$
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- $10=$ actual $/ 365$ (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.

- EndMonthRule is a NINST-by-1 matrix for the end-of-month rule. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. When the value is 0 , the end-of-month rule is ignored, meaning that a bond's coupon payment date is always the same numerical day of the month. When the value is 1 , the end-of-month rule is set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
- IssueDate is a NINST-by-1 matrix for the bond issue date.
- FirstCouponDate is a NINST-by-1 matrix for a date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.
- LastCouponDate is a NINST-by-1 matrix for a last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not


## optembndbyhw

specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

- StartDate is a NINST-by- 1 matrix for date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify StartDate, the effective start date is the Settle date.
- Face value. Face is a NINST-by-1 vector or NINST-by- 1 cell array of face values, or face value schedules. For the latter case, each element of the cell array is a NumDates-by-2 cell array, where the first column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid. Default is 100 .
- Options is a derivatives pricing options structure created with derivset.

Note The Settle date for every bond with embedded option is set to the ValuationDate of the HW tree; the bond's argument for Settle date is ignored.

## Description

[Price, PriceTree] = optembndbyhw(HWTree, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'Name1', Value1, 'Name2', Value2, ...) prices bonds with embedded options by a HW interest-rate tree.

Price is a number of instruments (NINST)-by-1 matrix of expected prices at time 0 .

PriceTree is a structure of trees containing vectors of instrument prices and observation times for each node. Within PriceTree:

- PriceTree. PTree contains the clean prices.
- PriceTree.tObs contains the observation times.
optembndbyhw computes prices of vanilla bonds with embedded options, stepped coupon bonds with embedded options, and bonds with sinking fund option provisions.


## Definitions

## Vanilla Bond with Embedded Option

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time. The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment. A vanilla bond with an embedded option is where an option contract has an underlying asset of a vanilla bond.

## Stepped Coupon Bond with Callable and Puttable Features

A step-up and step-down bond is a debt security with a predetermined coupon structure over time. With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond. Stepped coupon bonds can have options features (call and puts).

## Sinking Fund Bond with Embedded Option

A sinking fund bond is a coupon bond with a sinking fund provision. This provision obligates the issuer to amortize portions of the principal prior to maturity, affecting bond prices since the time of the principal repayment changes. This means that investors receive the coupon and a portion of the principal paid back over time. These types of bonds reduce credit risk, since it lowers the probability of investors not receiving their principal payment at maturity.

## optembndbyhw

The bond may have a sinking fund option provision allowing the issuer to retire the sinking fund obligation either by purchasing the bonds to be redeemed from the market or by calling the bond via a sinking fund call, whichever is cheaper. If interest rates are high, then the issuer will buy back the requirement amount of bonds from the market since bonds will be cheap, but if interest rates are low (bond prices are high), then most likely the issuer will be buying the bonds at the call price. Unlike a call feature, however, if a bond has a sinking fund option provision, it is an obligation, not an option, for the issuer to buy back the increments of the issue as stated. Because of this, a sinking fund bond trades at a lower price than a non-sinking fund bond.

## Examples

## Price a Callable Bond Using an HW Interest-Rate Tree Model

Create a HWTree with the following data:

```
ZeroRates = [ 0.035;0.04;0.045];
Compounding = 1;
StartDates = ['jan-1-2007';'jan-1-2008';'jan-1-2009'];
EndDates = ['jan-1-2008';'jan-1-2009';'jan-1-2010'];
ValuationDate = 'jan-1-2007';
Create a RateSpec.
RateSpec = intenvset('Rates', ZeroRates, 'StartDates', ValuationDate, EndDates, 'Compounding', Compounding, 'ValuationDate', ValuationDate)
```

RateSpec =
FinObj:
Compounding:
Disc:
Rates:

```
StartDates: 733043
ValuationDate: 733043
Basis: 0
EndMonthRule: 1
```

Create a VolSpec.
VolDates = ['jan-1-2008';'jan-1-2009';'jan-1-2010'];
VolCurve = 0.01;
AlphaDates = 'jan-1-2010';
AlphaCurve = 0.1;
HWVolSpec = hwvolspec(ValuationDate, VolDates, VolCurve, AlphaDates,

HWVolSpec =
FinObj: 'HWVolSpec'
ValuationDate: 733043
VolDates: [3x1 double]
VolCurve: [3x1 double]
AlphaCurve: 0.1000
AlphaDates: 734139
VolInterpMethod: 'linear'

Create a TimeSpec.
HWTimeSpec = hwtimespec(ValuationDate, EndDates, Compounding)

HWTimeSpec =
FinObj: 'HWTimeSpec'
ValuationDate: 733043
Maturity: [3x1 double]
Compounding: 1

11-847

## Basis: 0 <br> EndMonthRule: 1

Build the HWTree.

```
HWTree = hwtree(HWVolSpec, RateSpec, HWTimeSpec)
```


## HWTree =

```
        FinObj: 'HWFwdTree'
    VolSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
        tObs: [l0 1 2]
        dObs: [733043 733408 733774]
    CFlowT: {[3x1 double] [2x1 double] [3]}
        Probs: {[3x1 double] [3x3 double]}
    Connect: {[2] [2 3 4]}
    FwdTree: {[1.0350] [1.0624 1.0451 1.0280] [1x5 double]}
```

Compute the price of an American puttable bond that pays an annual coupon of $5.25 \%$, matures on January 1, 2010, and is puttable from January 1, 2008 to January 1, 2010.

```
BondSettlement = 'jan-1-2007';
BondMaturity = 'jan-1-2010';
CouponRate = 0.0525;
Period = 1;
OptSpec = 'put';
Strike = [100];
ExerciseDates = {'jan-1-2008' '01-Jan-2010'};
AmericanOpt = 1;
PricePutBondHW = optembndbyhw(HWTree, CouponRate, BondSettlement, BondMat
```

OptSpec, Strike, ExerciseDates,'Period', 1, 'AmericanOpt', 1)

PricePutBondHW =
102.8801

## Price Single Stepped Callable Bonds Using an HW Interest-Rate Tree Model

Price the following single stepped callable bonds using the following data: The data for the interest rate term structure is as follows:

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDat
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
% Instrument
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2012' .0425;'01-Jan-2014' .0750}};
OptSpec='call';
Strike=100;
ExerciseDates='01-Jan-2012'; %Callable in two years
% Build the tree with the following data
VolDates = ['1-Jan-2011'; '1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'];
VolCurve = 0.01;
AlphaDates = '01-01-2014';
AlphaCurve = 0.1;
```


## optembndbyhw

```
HWVolSpec = hwvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RS.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVolSpec, RS, HWTimeSpec);
% The first row corresponds to the price of the callable bond with maturi
% of three years. The second row corresponds to the price of the callable
% bond with maturity of four years.
PHW= optembndbyhw(HWT, CouponRate, Settle, Maturity,OptSpec, Strike,...
ExerciseDates, 'Period', 1)
```


## PHW =

100.0521
99.8322

## Price a Sinking Fund Bond Using an HW Interest-Rate Tree Model

A corporation issues a two year bond with a sinking fund obligation requiring the company to sink $1 / 3$ of face value after the first year. The company has the option to buy the bonds in the market or call them at $\$ 99$. The following data describes the details needed for pricing the sinking fund bond:

Rates $=[0.1 ; 0.1 ; 0.1 ; 0.1] ;$
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = \{'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'\};
Compounding = 1;
\% Create RateSpec

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates',
StartDates, 'EndDates', EndDates,'Rates', Rates, 'Compounding', Compol
% Build the HW tree
% The data to build the tree is as follows:
VolDates = ['1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014';'1-Jan-2015'];
VolCurve = 0.01;
AlphaDates = '01-01-2015';
AlphaCurve = 0.1;
HWVolSpec = hwvolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RateSpec.ValuationDate, VolDates, Compounding)
HWT = hwtree(HWVolSpec, RateSpec, HWTimeSpec);
```


## \% Instrument

```
% The bond has a coupon rate of 9%, a period of one year and matures
% 1-Jan-2013. Face decreases 1/3 after the first year.
CouponRate = 0.09;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2013';
Period = 1;
Face = { ...
    {'Jan-1-2012' 100; ...
    'Jan-1-2013' 66.6666}; ...
    };
% Option provision
OptSpec = 'call';
Strike = 99;
ExerciseDates = 'Jan-1-2012';
% Price of non-sinking fund bond.
PNSF = bondbyhw(HWT, CouponRate, Settle, Maturity, Period)
```

```
PNSF =
98.2645
```

Price of the bond with the option sinking provision.
PriceSF = optembndbyhw(HWT, CouponRate, Settle, Maturity,... OptSpec, Strike, ExerciseDates, 'Period', Period, 'Face', Face)

PriceSF =
98.1594

See Also hwprice | cfamounts | hwtree | instoptembnd

Purpose
Price embedded option on floating-rate note BDT interest-rate tree

```
Syntax
Price = optemfloatbybdt(BDTTree,Spread,Settle,Maturity,
OptSpec,Strike,ExerciseDates)
Price = optemfloatbybdt(BDTTree,Spread,Settle,Maturity,OptSpec,
Strike,ExerciseDates,AmericanOpt,Reset,Basis,Principal,
Options, EndMonthRule)
[Price,PriceTree] = optemfloatbybdt( ___ )
Price = optemfloatbybdt(BDTTree,Spread,Settle,Maturity,OptSpec,
Strike,ExerciseDates)
Price = optemfloatbybdt(
```

$\qquad$

``` , Name, Value)
[Price,PriceTree] = optemfloatbybdt( ___ )
```

Description
Price = optemfloatbybdt(BDTTree,Spread, Settle, Maturity, OptSpec, Strike, ExerciseDates) returns the embedded option price of the instrument(s) at valuation date.

Price =
optemfloatbybdt(BDTTree, Spread, Settle, Maturity, OptSpec, Strike, ExerciseDates, AmericanOpt, Reset, Basis, Principal, Options, EndMonthRule) returns the embedded option price of the instrument(s) at valuation date using optional arguments.
[Price,PriceTree] = optemfloatbybdt (__ ) returns the option price of the instrument(s) at valuation date and a structure of trees containing vectors of option prices at each node.

Price =
optemfloatbybdt(BDTTree, Spread, Settle, Maturity, OptSpec, Strike, ExerciseDates) returns the embedded option price of the instrument(s) at valuation date.

## optemfloatbybdt

Price $=$ optemfloatbybdt( __ , Name, Value) returns the embedded option price of the instrument(s) at valuation date using optional name-value pair arguments.
[Price,PriceTree] = optemfloatbybdt (__ ) returns the embedded option price of the instrument(s) at valuation date and a structure of trees containing vectors of option prices at each node using optional name-value pair arguments.

## Input Arguments

## BDTTree - Interest-rate tree structure

binomial tree structure
Interest-rate tree specified as a structure by using bdttree.

## Data Types

struct

## Spread - Number of basis points over the reference rate nonnegative integer | vector of nonnegative integers

Number of basis points over the reference rate specified as a vector nonnegative integers for the number of instruments (NINST)-by-1).

```
Data Types
single | double
```


## Settle - Settlement dates of floating-rate note

ValuationDate of HW Tree (default) | nonnegative integer | vector of nonnegative integers
Settlement dates of floating-rate note specified as nonnegative integers using a NINST-by-1 vector of dates.

[^5]
## Data Types <br> char <br> OptSpec - Definition of option

string \| cell array of strings
Definition of option as 'call' or 'put' specified as a NINST-by-1 cell array of strings for 'call' or 'put'.

Data Types
cell | char

## Strike - Option strike price values

nonnegative integer | vector of nonnegative integers
Option strike price values specified nonnegative integers using as NINST-by-NSTRIKES vector of strike price values.

Data Types
single | double

## ExerciseDates - Exercise date for option (European, Bermuda, or American)

nonnegative integer | vector of nonnegative integers
Exercise date for option (European, Bermuda, or American) specified as nonnegative integers using a NINST-by-NSTRIKES or NINST-by-2 vector of for the option exercise dates.

- If a European or Bermuda option, the ExerciseDates is a 1-by-1 (European) or 1-by-NSTRIKES (Bermuda) vector of exercise dates. For a European option, there is only one ExerciseDate on the option expiry date.
- If an American option, then ExerciseDates is a 1-by-2 vector of exercise date boundaries. The option exercises on any date between or including the pair of dates on that row. If there is only one non- NaN date, or if ExerciseDates is 1 -by-1, the option exercises between the Settle date and the single listed ExerciseDate.


## optemfloatbybdt

## Data Types <br> char | cell

## AmericanOpt - (Optional) Option type

scalar | vector of positive integers [0,1]
Option type specified as NINST-by-1 positive integer scalar flags with values:

- 0 - European/Bermuda
- 1 - American

Data Types
single | double
Reset - (Optional) Frequency of payments per year
1 (default) | positive integer from the set[1,2,3,4,6,12] | vector of positive integers from the set [1, 2, 3, 4, 6, 12]

Frequency of payments per year specified as positive integers for the values [ $1,2,3,4,6,12$ ] in a NINST-by- 1 vector.

Note Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for a FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates can not be uniquely determined because there will be more than one possible path for connecting the two payment dates.

## Data Types <br> double

## Basis - (Optional) Day-count basis of the instrument

```
0 (actual/actual) (default)| positive integers of the set [1...13] |
vector of positive integers of the set [1...13]
```

Day-count basis of the instrument specified as a positive integer using a NINST-by- 1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- $10=\mathrm{actual} / 365$ (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252


## Data Types

double

## Principal - (Optional) Principal values

100 (default) | vector of nonnegative values | cell array of nonnegative values

Principal values specified as nonnegative values using a NINST-by-1 vector or NINST-by-1 cell array of notional principal amounts. When using a NINST-by- 1 cell array, each element is a NumDates-by- 2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

## optemfloatbybdt

## Data Types <br> double | cell <br> Options - (Optional) Structure containing derivatives pricing options

structure
Structure containing derivatives pricing options specified using derivset.

Data Types
struct

## EndMonthRule - (Optional) End-of-month rule flag

1 (in effect) (default) | nonnegative integer [0,1]
End-of-month rule flag is specified as nonnegative integer [0, 1] using a NINST-by- 1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Data Types

double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: Price =
optemfloatbybdt(BDTTree, Spread, Settle, Maturity, OptSpec, Strike, ExerciseDat

## 'AmericanOpt' - Option type

```
scalar | vector of positive integers[0,1]
```

Option type specified as NINST-by-1 positive integer scalar flags with values:

- 0 - European/Bermuda
- 1 - American

Data Types
single | double

## 'Reset' - Frequency of payments per year

1 (default) | positive integer from the set[1,2,3,4,6,12] | vector of positive integers from the set [1, 2, 3, 4, 6, 12]

Frequency of payments per year specified as positive integers for the values [ $1,2,3,4,6,12$ ] in a NINST-by- 1 vector.

Note Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for a FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates can not be uniquely determined because there will be more than one possible path for connecting the two payment dates.

```
Data Types
char
'Basis' - Day-count basis of the instrument
0 (actual/actual) (default) | positive integers of the set [1...13] |
vector of positive integers of the set [1...13]
```

Day-count basis of the instrument specified as a positive integer using a NINST-by- 1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

## optemfloatbybdt

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- $9=$ actual/360 (ISMA)
- $10=\mathrm{actual} / 365$ (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252


## Data Types <br> char

## 'Principal' - (Optional) Principal values

100 (default) | vector of nonnegative values | cell array of nonnegative values

Principal values specified as nonnegative values using a NINST-by-1 vector or NINST-by- 1 cell array of notional principal amounts. When using a NINST-by- 1 cell array, each element is a NumDates-by- 2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

```
Data Types
double | cell
```


## 'Options' - Structure containing derivatives pricing options structure

Structure containing derivatives pricing options specified using derivset.

## Data Types

struct

## 'EndMonthRule' - End-of-month rule flag

1 (in effect) (default) | nonnegative integer [0,1]
End-of-month rule flag is specified as a nonnegative integer [ 0,1 ] using a NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Data Types <br> char

## Output Price - Expected prices of the floating-rate note embedded Arguments <br> Price - Expected prices of the floating-rate note embedded option at time 0 scalar | vector

Expected prices of the floating-rate note embedded option at time 0 is returned as a scalar or an NINST-by-1 vector.

## PriceTree - Structure of trees containing vectors of embedded option prices at each node

tree structure
Structure of trees containing vectors of instrument prices and accrued interest and a vector of observation times for each node returned as:

- PriceTree. PTree contains embedded option prices.


## optemfloatbybdt

## Examples Price Callable Embedded Option for Floating-Rate Note

Define the interest-rate term structure.

```
Rates = [0.03;0.034;0.038;0.04];
ValuationDate = 'Jan-1-2012';
StartDates = ValuationDate;
EndDates = {'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'; 'Jan-1-2016'};
Compounding = 1;
```

Create the RateSpec.
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates',... StartDates, 'EndDates', EndDates,'Rates', Rates, 'Compounding', Compoundi

## RateSpec =

```
            FinObj: 'RateSpec'
        Compounding: 1
            Disc: [4x1 double]
            Rates: [4x1 double]
        EndTimes: [4\times1 double]
    StartTimes: [4x1 double]
        EndDates: [4x1 double]
    StartDates: 734869
ValuationDate: 734869
            Basis: 0
    EndMonthRule: 1
```

Build the BDT tree and assume a volatility of $10 \%$.

```
Sigma = 0.1;
BDTTimeSpec = bdttimespec(ValuationDate, EndDates);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length(Eno
BDTT = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec);
```

Define the floater instruments with the embedded call option.

```
Spread = 10;
Settle = 'Jan-1-2012';
Maturity = {'Jan-1-2015';'Jan-1-2016'};
Period = 1;
OptSpec = {'call'};
Strike = 101;
ExerciseDates = 'Jan-1-2015';
```

Compute the price of the floaters with the embedded call.

```
Price= optemfloatbybdt(BDTT, Spread, Settle, Maturity, OptSpec, Strik
ExerciseDates)
```

Price =
100.2800
100.3655

| See Also | optembndbybdt \| optemfloatbyhjm | optemfloatbybk | <br> optemfloatbyhw \| instoptemfloat |
| :--- | :--- |
| Concepts | - basis on page Glossary-1 |

## optemfloatbybk

| Purpose | Price embedded option on floating-rate note BK interest-rate tree |
| :---: | :---: |
| Syntax | ```Price = optemfloatbybk(BKTree,Spread,Settle,Maturity, OptSpec,Strike,ExerciseDates) Price = optemfloatbybk(BKTree,Spread,Settle,Maturity,OptSpec, Strike,ExerciseDates,AmericanOpt,Reset,Basis,Principal, Options,EndMonthRule) [Price,PriceTree] = optemfloatbybk(___)``` |
|  | ```Price = optemfloatbybk(BKTree,Spread,Settle,Maturity,OptSpec, Strike,ExerciseDates) Price = optemfloatbybk(``` $\qquad$ <br> ```, Name,Value) [Price,PriceTree] = optemfloatbybk( __ )``` |
| Description | Price = optemfloatbybk(BKTree,Spread,Settle, Maturity, OptSpec,Strike,ExerciseDates) returns the embedded option price of the instrument(s) at valuation date. |
|  | Price = <br> optemfloatbybk(BKTree, Spread, Settle, Maturity, OptSpec, Strike, ExerciseDates, AmericanOpt, Reset, Basis, Principal, Options, EndMonthRule) returns the embedded option price of the instrument(s) at valuation date using optional arguments. |
|  | [Price,PriceTree] = optemfloatbybk( $\qquad$ ) returns the option price of the instrument(s) at valuation date and a structure of trees containing vectors of option prices at each node. |
|  | Price = <br> optemfloatbybk(BKTree, Spread, Settle, Maturity, OptSpec, Strike, ExerciseDates) returns the embedded option price of the instrument(s) at valuation date. |

Price $=$ optemfloatbybk( $\qquad$ , Name, Value) returns the embedded option price of the instrument(s) at valuation date using optional name-value pair arguments.
[Price,PriceTree] = optemfloatbybk( __ ) returns the embedded option price of the instrument(s) at valuation date and a structure of trees containing vectors of option prices at each node using optional name-value pair arguments.

## Input Arguments

BKTree - Interest-rate tree structure
binomial tree structure
Interest-rate tree specified as a structure by using bktree.

## Data Types

struct

## Spread - Number of basis points over the reference rate nonnegative integer | vector of nonnegative integers

Number of basis points over the reference rate specified as a vector nonnegative integers for the number of instruments (NINST)-by-1).

Data Types
single | double

## Settle - Settlement dates of floating-rate note <br> ValuationDate of HW Tree (default) | nonnegative integer | vector of nonnegative integers

Settlement dates of floating-rate note specified as nonnegative integers using a NINST-by- 1 vector of dates.

[^6]
## optemfloatbybk

## Data Types <br> char <br> OptSpec - Definition of option <br> string | cell array of strings

Definition of option as 'call' or 'put' specified as a NINST-by-1 cell array of strings for 'call' or 'put'.

Data Types
cell | char

## Strike - Option strike price values

nonnegative integer | vector of nonnegative integers
Option strike price values specified nonnegative integers using as NINST-by-NSTRIKES vector of strike price values.

Data Types
single | double

## ExerciseDates - Exercise date for option (European, Bermuda, or American) <br> nonnegative integer | vector of nonnegative integers

Exercise date for option (European, Bermuda, or American) specified as nonnegative integers using a NINST-by-NSTRIKES or NINST-by-2 vector of for the option exercise dates.

- If a European or Bermuda option, the ExerciseDates is a 1-by-1 (European) or 1-by-NSTRIKES (Bermuda) vector of exercise dates. For a European option, there is only one ExerciseDate on the option expiry date.
- If an American option, then ExerciseDates is a 1-by-2 vector of exercise date boundaries. The option exercises on any date between or including the pair of dates on that row. If there is only one non-NaN date, or if ExerciseDates is 1-by-1, the option exercises between the Settle date and the single listed ExerciseDate.


## Data Types <br> char | cell

## AmericanOpt - (Optional) Option type

scalar | vector of positive integers [0,1]
Option type specified as NINST-by-1 positive integer scalar flags with values:

- 0 - European/Bermuda
- 1 - American

Data Types
single | double
Reset - (Optional) Frequency of payments per year
1 (default) | positive integer from the set[1,2,3,4,6,12] | vector of positive integers from the set [1,2,3, 4, 6, 12]

Frequency of payments per year specified as positive integers for the values [1, $2,3,4,6,12$ ] in a NINST-by- 1 vector. .

Note Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for a FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates can not be uniquely determined because there will be more than one possible path for connecting the two payment dates.

## Data Types

double
Basis - (Optional) Day-count basis of the instrument
0 (actual/actual) (default) | positive integers of the set [1...13] | vector of positive integers of the set [1...13]

## optemfloatbybk

Day-count basis of the instrument specified as a positive integer using a NINST-by- 1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- $10=\mathrm{actual} / 365$ (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252


## Data Types

double

## Principal - (Optional) Principal values

100 (default) | vector of nonnegative values | cell array of nonnegative values

Principal values specified as nonnegative values using a NINST-by-1 vector or NINST-by- 1 cell array of notional principal amounts. When using a NINST-by-1 cell array, each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

# Data Types <br> double | cell <br> <br> Options - (Optional) Structure containing derivatives pricing <br> <br> Options - (Optional) Structure containing derivatives pricing options 

structure
Structure containing derivatives pricing options specified using derivset.

## Data Types

struct

## EndMonthRule - (Optional) End-of-month rule flag

1 (in effect) (default) | nonnegative integer [0,1]
End-of-month rule flag is specified as nonnegative integer [ 0,1 ] using a NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Data Types <br> double <br> Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

Example: Price =
optemfloatbybk(BKTree, Spread, Settle, Maturity, OptSpec, Strike, ExerciseD

## 'AmericanOpt' - Option type

## optemfloatbybk

## scalar | vector of positive integers [ 0,1 ]

Option type specified as NINST-by-1 positive integer scalar flags with values:

- 0 - European/Bermuda
- 1 - American

Data Types
single | double
'Reset' - Frequency of payments per year
1 (default) | positive integer from the set[1,2,3,4,6,12] | vector of positive integers from the set [1, 2, 3, 4, 6, 12]
Frequency of payments per year specified as positive integers for the values [ $1,2,3,4,6,12$ ] in a NINST-by- 1 vector. .

Note Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for a FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates can not be uniquely determined because there will be more than one possible path for connecting the two payment dates.

## Data Types <br> char

## 'Basis' - Day-count basis of the instrument

0 (actual/actual) (default) | positive integers of the set [1...13] | vector of positive integers of the set [1...13]
Day-count basis of the instrument specified as a positive integer using a NINST-by-1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- $10=$ actual/365 (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252


## Data Types <br> char

## 'Principal' - (Optional) Principal values

100 (default) | vector of nonnegative values | cell array of nonnegative values

Principal values specified as nonnegative values using a NINST-by-1 vector or NINST-by- 1 cell array of notional principal amounts. When using a NINST-by- 1 cell array, each element is a NumDates-by- 2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

## Data Types <br> double | cell

## optemfloatbybk

## 'Options' - Structure containing derivatives pricing options structure

Structure containing derivatives pricing options specified using derivset.

## Data Types

struct

## 'EndMonthRule' - End-of-month rule flag <br> 1 (in effect) (default) | nonnegative integer [0,1]

End-of-month rule flag is specified as a nonnegative integer [ 0,1 ] using a NINST-by- 1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Data Types <br> char

## Output Arguments

## Price - Expected prices of the floating-rate note embedded option at time 0 <br> scalar | vector

Expected prices of the floating-rate note embedded option at time 0 is returned as a scalar or an NINST-by-1 vector.

## PriceTree - Structure of trees containing vectors of embedded option prices at each node

## tree structure

Structure of trees containing vectors of instrument prices and accrued interest and a vector of observation times for each node returned as:

- PriceTree. PTree contains embedded option prices.


## Examples Price European Callable Embedded Option for Floating-Rate Note

Define the interest-rate term structure.

```
Rates = [0.03;0.034;0.038;0.04];
ValuationDate = 'Jan-1-2012';
StartDates = ValuationDate;
EndDates = {'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'; 'Jan-1-2016'};
Compounding = 1;
```

Create the RateSpec.

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates',
StartDates, 'EndDates', EndDates,'Rates', Rates, 'Compounding', Compol
```

RateSpec =

```
                    FinObj: 'RateSpec'
        Compounding: 1
            Disc: [4x1 double]
            Rates: [4x1 double]
            EndTimes: [4x1 double]
        StartTimes: [4x1 double]
            EndDates: [4x1 double]
    StartDates: 734869
ValuationDate: 734869
            Basis: 0
EndMonthRule: 1
```

Build the BK tree.

```
VolDates = ['1-Jan-2013'; '1-Jan-2014'; '1-Jan-2015';'1-Jan-2016'];
VolCurve = 0.01;
AlphaDates = '01-01-2016';
AlphaCurve = 0.1;
```


## optemfloatbybk

```
BKVolSpec = bkvolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
    AlphaDates, AlphaCurve);
BKTimeSpec = bktimespec(RateSpec.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVolSpec, RateSpec, BKTimeSpec)
```

BKT =

```
        FinObj: 'BKFwdTree'
        VolSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
        tObs: [[0 1 2 3 3]
        dObs: [734869 735235 735600 735965]
    CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}
Connect: {[2] [2 3 4] [2 2 3 4 4]}
FwdTree: {[1.0300] [1.0387 1.0380 1.0374] [1x5 double] [1x5 doub]
```

The floater instrument has a spread of 15 , a period of one year, and matures and is callable on Jan-1-2015.

```
Spread = 15;
Settle = 'Jan-1-2012';
Maturity = 'Jan-1-2015';
Period = 1;
OptSpec = {'call'};
Strike =101;
ExerciseDates = 'Jan-1-2015';
```

Compute the price of the floater with the embedded call.
Price= optemfloatbybk(BKT, Spread, Settle, Maturity,... OptSpec, Strike, ExerciseDates)

```
Price =
    100.4201
```

See Also optembndbybk | optemfloatbyhjm | optemfloatbybdt | optemfloatbyhw | instoptemfloat
Concepts - basis on page Glossary-1

## optemfloatbyhjm

| Purpose | Price embedded option on floating-rate note HJM interest-rate tree |
| :---: | :---: |
| Syntax | ```Price = optemfloatbyhjm(HJMTree,Spread,Settle,Maturity, OptSpec,Strike,ExerciseDates) Price = optemfloatbyhjm(HJMTree,Spread,Settle,Maturity,OptSpec, Strike,ExerciseDates,AmericanOpt,Reset,Basis,Principal, Options,EndMonthRule) [Price,PriceTree] = optemfloatbyhjm(___)``` |
|  | ```Price = optemfloatbyhjm(HJMTree,Spread,Settle,Maturity,OptSpec, Strike,ExerciseDates) Price = optemfloatbyhjm(___,Name,Value) [Price,PriceTree] = optemfloatbyhjm(___)``` |
| Description | Price $=$ optemfloatbyhjm(HJMTree, Spread, Settle, Maturity, OptSpec,Strike,ExerciseDates) returns the embedded option price of the instrument(s) at the valuation date. |
|  | Price = <br> optemfloatbyhjm(HJMTree, Spread, Settle, Maturity, OptSpec, Strike, ExerciseDates, AmericanOpt, Reset, Basis, Principal, Options, EndMonthRule) returns the embedded option price of the instrument(s) at the valuation date using optional arguments. |
|  | [Price, PriceTree] = optemfloatbyhjm( _ _ ) returns the option price of the instrument(s) at the valuation date and a structure of trees containing vectors of option prices at each node. |
|  | Price = optemfloatbyhjm(HJMTree,Spread,Settle,Maturity,OptSpec, Strike, ExerciseDates) returns the embedded option price of the instrument(s) at the valuation date. |

Price $=$ optemfloatbyhjm( __ , Name, Value) returns the embedded option price of the instrument(s) at the valuation date using optional name-value pair arguments.
[Price, PriceTree] = optemfloatbyhjm( __ ) returns the embedded option price of the instrument(s) at the valuation date and a structure of trees containing vectors of option prices at each node using optional name-value pair arguments.

## Input Arguments

HJMTree - Interest-rate tree structure
binomial tree structure
Interest-rate tree, specified as a structure by using hjmtree.
Data Types
struct

## Spread - Number of basis points over reference rate nonnegative integer | vector of nonnegative integers

Number of basis points over the reference rate, specified as a vector of nonnegative integers for the number of instruments (NINST)-by-1).

Data Types
single | double

## Settle - Settlement dates of floating-rate note <br> ValuationDate of HW tree (default) | nonnegative integer | vector of nonnegative integers

Settlement dates of floating-rate note, specified as nonnegative integers using a NINST-by- 1 vector of dates.

[^7]
## optemfloatbyhim

Data Types
char

## OptSpec - Option definition

string | cell array of strings
Option definition specified as a 'call' or 'put' using a NINST-by-1 cell array of strings.

Data Types
cell | char

## Strike - Option strike price values

nonnegative integer | vector of nonnegative integers
Option strike price values, specified as nonnegative integers using a NINST-by-NSTRIKES vector of strike price values.

Data Types
single | double

## ExerciseDates - Exercise date for European, Bermuda, or American option

nonnegative integer | vector of nonnegative integers
Exercise date for European, Bermuda, or American option, specified as nonnegative integers using a NINST-by-NSTRIKES or NINST-by-2 vector of option exercise dates.

- For a European or Bermuda option, ExerciseDates is a 1-by-1 (European) or 1-by-NSTRIKES (Bermuda) vector of exercise dates. For a European option, there is only one ExerciseDate on the option expiry date.
- For an American option, ExerciseDates is a 1-by-2 vector of exercise date boundaries. The option exercises on any date between or including the pair of dates on that row. If there is only one non- NaN date, or if ExerciseDates is 1 -by-1, the option exercises between the Settle date and the single listed ExerciseDate.


## optemfloatbyhim

## Data Types <br> char | cell

## AmericanOpt - (Optional) Option type

scalar | vector of positive integers [0,1]
Option type, specified as NINST-by-1 positive integer flags with values:

- 0 - European/Bermuda
- 1 - American

Data Types
single | double
Reset - (Optional) Frequency of payments per year
1 (default) | positive integer from the set [1,2,3,4,6,12] | vector of positive integers from the set [1, $2,3,4,6,12]$

Frequency of payments per year, specified as positive integers for the values [ $1,2,3,4,6,12$ ] in a NINST-by- 1 vector.

> Note Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for a FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there will be more than one possible path for connecting the two payment dates.

## Data Types

double
Basis - (Optional) Day-count basis of instrument
0 (actual/actual) (default) | positive integers of the set [1...13] |
vector of positive integers of the set [1...13]

## optemfloatbyhjm

Day-count basis of the instrument, specified as a positive integer using a NINST-by- 1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- $10=\mathrm{actual} / 365$ (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252


## Data Types

double

## Principal - (Optional) Principal values

100 (default) | vector of nonnegative values | cell array of nonnegative values

Principal values specified as nonnegative values using a NINST-by-1 vector or NINST-by- 1 cell array of notional principal amounts. When using a NINST-by-1 cell array, each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

## optemfloatbyhjm

## Data Types <br> double | cell <br> Options - (Optional) Structure containing derivatives pricing options

structure
Structure containing derivatives pricing options, specified using derivset.

## Data Types

struct

## EndMonthRule - (Optional) End-of-month rule flag

1 (in effect) (default) | nonnegative integer [0,1]
End-of-month rule flag, specified as nonnegative integer 0 or 1 using a NINST-by- 1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Data Types <br> double <br> Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: Price =
optemfloatbyhjm(HJMTree, Spread, Settle, Maturity, OptSpec, Strike, Exercis

## 'AmericanOpt' - Option type

## optemfloatbyhjm

## scalar | vector of positive integers $[0,1]$

Option type, specified as NINST-by-1 positive integer scalar flags with values:

- 0 - European/Bermuda
- 1 - American

Data Types
single | double

## 'Reset' - Frequency of payments per year

1 (default) | positive integer from the set [1,2,3,4,6,12] | vector of positive integers from the set [1,2,3,4,6,12]

Frequency of payments per year, specified as positive integers for the values [ $1,2,3,4,6,12$ ] in a NINST-by- 1 vector.

Note Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for a FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there will be more than one possible path for connecting the two payment dates.

```
Data Types
char
```


## 'Basis' - Day-count basis of instrument

```
0 (actual/actual) (default) | positive integers of the set [1...13] | vector of positive integers of the set [1...13]
```

Day-count basis of the instrument, specified as a positive integer using a NINST-by- 1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=\mathrm{actual} / 365$ (Japanese)
- $8=$ actual/actual (ISMA)
- $9=$ actual/360 (ISMA)
- $10=\mathrm{actual} / 365$ (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252


## Data Types <br> char

## 'Principal' - (Optional) Principal values

100 (default) | vector of nonnegative values | cell array of nonnegative values

Principal values specified as nonnegative values using a NINST-by-1 vector or NINST-by-1 cell array of notional principal amounts. When using a NINST-by- 1 cell array, each element is a NumDates-by- 2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

## Data Types <br> double | cell

## optemfloatbyhim

## 'Options' - Structure containing derivatives pricing options structure

Structure containing derivatives pricing options specified using derivset.

## Data Types

struct

## 'EndMonthRule' - End-of-month rule flag

1 (in effect) (default) | nonnegative integer [0,1]
End-of-month rule flag, specified as a nonnegative integer [ 0,1 ] using a NINST-by- 1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Data Types <br> char

## Output Arguments <br> Price - Expected prices of floating-rate note embedded option at time 0 <br> scalar | vector

Expected prices of the floating-rate note embedded option at time 0 , returned as a scalar or a NINST-by- 1 vector.

## PriceTree - Structure of trees containing vectors of embedded option prices at each node

tree structure
Structure of trees containing vectors of instrument prices and accrued interest and a vector of observation times for each node, returned as:

- PriceTree. PTree contains embedded option prices.


## optemfloatbyhjm

## Examples Price European Callable Embedded Option for a Floating-Rate Note

Define the interest-rate term structure.

```
Rates = [0.03;0.035;0.040;0.045];
ValuationDate = 'Jan-1-2012';
StartDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'}
EndDates = {'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'; 'Jan-1-2016'};
Compounding = 1;
```

Create the RateSpec.

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates',
StartDates, 'EndDates', EndDates,'Rates', Rates, 'Compounding', Compoו
```

RateSpec =

```
                    FinObj: 'RateSpec'
        Compounding: 1
            Disc: [4x1 double]
            Rates: [4x1 double]
            EndTimes: [4x1 double]
        StartTimes: [4x1 double]
            EndDates: [4x1 double]
    StartDates: [4x1 double]
ValuationDate: 734869
            Basis: 0
EndMonthRule: 1
```

Build the HJM tree.

```
VolSpec = hjmvolspec('Constant', 0.01);
TimeSpec = hjmtimespec(RateSpec.ValuationDate, EndDates, Compounding)
HJMTree = hjmtree(VolSpec, RateSpec, TimeSpec)
```


## optemfloatbyhim

```
HJMTree =
    FinObj: 'HJMFwdTree'
    VolSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
        tObs: [0 1 2 3]
        dObs: [734869 735235 735600 735965]
        TFwd: {[4x1 double] [3x1 double] [2x1 double] [3]}
    CFlowT: {[4x1 double] [3\times1 double] [2x1 double] [4]}
    FwdTree: {[4x1 double] [3x1\times2 double] [2x2x2 double] [1\times4\times2 doub]
```

The floater instrument has a spread of 15 , a period of one year, and matures and is callable on Jan-1-2016.

```
Spread = 15;
Settle = 'Jan-1-2012';
Maturity = 'Jan-1-2016';
Period = 1;
OptSpec = {'call'};
Strike = 95;
ExerciseDates = 'Jan-1-2016';
```

Compute the price of the floater with the embedded call.
Price = optemfloatbyhjm(HJMTree, Spread, Settle, Maturity,... OptSpec, Strike, ExerciseDates)

Price =
96.2355

```
See Also optembndbyhjm | optemfloatbyhw | optemfloatbybdt |
optemfloatbybk | instoptemfloat
```

Concepts - basis on page Glossary-1

## optemfloatbyhw

| Purpose | Price embedded option on floating-rate note HW interest-rate tree |
| :---: | :---: |
| Syntax | ```Price = optemfloatbyhw(HWTree,Spread,Settle,Maturity, OptSpec,Strike,ExerciseDates) Price = optemfloatbyhw(HWTree,Spread,Settle,Maturity,OptSpec, Strike,ExerciseDates,AmericanOpt,Reset,Basis,Principal, Options,EndMonthRule) [Price,PriceTree] = optemfloatbyhw(___)``` |
|  | ```Price = optemfloatbyhw(HWTree,Spread,Settle,Maturity,OptSpec, Strike,ExerciseDates) Price = optemfloatbyhw(___,Name,Value) [Price,PriceTree] = optemfloatbyhw(___)``` |
| Description | Price = optemfloatbyhw(HWTree,Spread,Settle, Maturity, OptSpec,Strike,ExerciseDates) returns the embedded option price of the instrument(s) at the valuation date. |
|  | Price = <br> optemfloatbyhw(HWTree, Spread, Settle, Maturity, OptSpec, Strike,ExerciseDates,AmericanOpt, Reset, Basis, Principal, Options, EndMonthRule) returns the embedded option price of the instrument(s) at the valuation date using optional arguments. |
|  | [Price,PriceTree] = optemfloatbyhw( $\qquad$ ) returns the option price of the instrument(s) at the valuation date and a structure of trees containing vectors of option prices at each node. |
|  | ```Price = optemfloatbyhw(HWTree,Spread,Settle,Maturity,OptSpec, Strike,ExerciseDates) returns the embedded option price of the instrument(s) at the valuation date.``` |

Price $=$ optemfloatbyhw (__ , Name, Value) returns the embedded option price of the instrument(s) at the valuation date using optional name-value pair arguments.
[Price, PriceTree] = optemfloatbyhw (__ ) returns the embedded option price of the instrument(s) at the valuation date and a structure of trees containing vectors of option prices at each node using optional name-value pair arguments.

## Input Arguments

HWTree - Interest-rate tree structure

## binomial tree structure

Interest-rate tree, specified as a structure by using hwtree.

## Data Types

struct

## Spread - Number of basis points over reference rate nonnegative integer | vector of nonnegative integers

Number of basis points over the reference rate, specified as a vector of nonnegative integers for the number of instruments (NINST)-by-1).

Data Types
single | double

## Settle - Settlement dates of floating-rate note <br> ValuationDate of HW tree (default) | nonnegative integer | vector of nonnegative integers

Settlement dates of the floating-rate note, specified as nonnegative integers using a NINST-by- 1 vector of dates.

[^8]
## optemfloatbyhw

## Data Types <br> char <br> OptSpec - Option definition <br> string | cell array of strings

Option definition, specified as 'call' or 'put' using a NINST-by-1 cell array of strings.

Data Types
cell | char

## Strike - Option strike price values

nonnegative integer | vector of nonnegative integers
Option strike price values specified as nonnegative integers using a NINST-by-NSTRIKES vector of strike price values.

Data Types
single | double

## ExerciseDates - Exercise date for option European, Bermuda, or American option

nonnegative integer | vector of nonnegative integers
Exercise date for European, Bermuda, or American option, specified as nonnegative integers using a NINST-by-NSTRIKES or NINST-by-2 vector of option exercise dates.

- For a European or Bermuda option, ExerciseDates is a 1-by-1 (European) or 1-by-NSTRIKES (Bermuda) vector of exercise dates. For a European option, there is only one ExerciseDate on the option expiry date.
- For an American option, ExerciseDates is a 1-by-2 vector of exercise date boundaries. The option exercises on any date between or including the pair of dates on that row. If there is only one non- NaN date, or if ExerciseDates is 1-by-1, the option exercises between the Settle date and the single listed ExerciseDate.


## Data Types <br> char | cell

## AmericanOpt - (Optional) Option type

scalar | vector of positive integers [ 0,1 ]
Option type, specified as NINST-by-1 positive integer scalar flags with values:

- 0 - European/Bermuda
- 1 - American

Data Types
single | double

## Reset - (Optional) Frequency of payments per year

1 (default)| positive integer from the set [1,2,3,4,6,12] | vector of positive integers from the set [1, 2, 3, 4, 6, 12]

Frequency of payments per year specified as positive integers for the values [ $1,2,3,4,6,12$ ] in a NINST-by- 1 vector.

Note Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for a FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there will be more than one possible path for connecting the two payment dates.

## Data Types

double
Basis - (Optional) Day-count basis of instrument
0 (actual/actual) (default) | positive integers of the set [1...13] | vector of positive integers of the set [1...13]

## optemfloatbyhw

Day-count basis of the instrument, specified as a positive integer using a NINST-by- 1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- $10=\mathrm{actual} / 365$ (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252


## Data Types

double

## Principal - (Optional) Principal values

100 (default) | vector of nonnegative values | cell array of nonnegative values

Principal values specified as nonnegative values using a NINST-by-1 vector or NINST-by-1 cell array of notional principal amounts. When using a NINST-by-1 cell array, each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

## optemfloatbyhw

## Data Types <br> double | cell <br> Options - (Optional) Structure containing derivatives pricing options

structure
Structure containing derivatives pricing options, specified using derivset.

## Data Types

struct

## EndMonthRule - (Optional) End-of-month rule flag

1 (in effect) (default) | nonnegative integer [0,1]
End-of-month rule flag, specified as nonnegative integer [ 0,1 ] using a NINST-by- 1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Data Types <br> double <br> Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

Example: Price =
optemfloatbyhw(HWTree, Spread, Settle, Maturity, OptSpec, Strike, ExerciseD

## 'AmericanOpt' - Option type

## optemfloatbyhw

## scalar | vector of positive integers [0, 1]

Option type, specified as NINST-by-1 positive integer scalar flags with values:

- 0 - European/Bermuda
- 1 - American

Data Types
single | double
'Reset' - Frequency of payments per year
1 (default) | positive integer from the set [1,2,3,4,6,12]| vector of positive integers from the set [1,2,3,4,6,12]
Frequency of payments per year, specified as positive integers for the values [ $1,2,3,4,6,12$ ] in a NINST-by- 1 vector.

Note Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for a FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there will be more than one possible path for connecting the two payment dates.

```
Data Types
char
```


## 'Basis' - Day-count basis of instrument

```
0 (actual/actual) (default) | positive integers of the set [1...13] | vector of positive integers of the set [1...13]
```

Day-count basis of the instrument, specified as a positive integer using a NINST-by-1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- $2=$ actual $/ 360$
- $3=$ actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- $9=$ actual/360 (ISMA)
- $10=$ actual/365 (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- $13=\mathrm{BUS} / 252$


## Data Types <br> char

## 'Principal' - (Optional) Principal values

100 (default) | vector of nonnegative values | cell array of nonnegative values

Principal values specified as nonnegative values using a NINST-by-1 vector or NINST-by- 1 cell array of notional principal amounts. When using a NINST-by-1 cell array, each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

## Data Types <br> double | cell

## optemfloatbyhw

## 'Options' - Structure containing derivatives pricing options structure

Structure containing derivatives pricing options, specified using derivset.

## Data Types

struct

## 'EndMonthRule' - End-of-month rule flag

1 (in effect) (default) | nonnegative integer [0,1]
End-of-month rule flag, specified as a nonnegative integer [ 0,1 ] using a NINST-by- 1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Data Types <br> char

## Output Arguments <br> Price - Expected prices of floating-rate note embedded option at time 0 <br> scalar | vector

Expected prices of the floating-rate note embedded option at time 0 , returned as a scalar or an NINST-by-1 vector.

## PriceTree - Structure of trees containing vectors of embedded option prices at each node

tree structure
Structure of trees containing vectors of instrument prices and accrued interest and a vector of observation times for each node, returned as:

- PriceTree. PTree contains embedded option prices.


## Examples Price European Callable Embedded Option for Floating-Rate Note

Define the interest-rate term structure.

```
Rates = [0.03;0.034;0.038;0.04];
ValuationDate = 'Jan-1-2012';
StartDates = ValuationDate;
EndDates = {'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'; 'Jan-1-2016'};
Compounding = 1;
```

Create the RateSpec.

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates',
StartDates, 'EndDates', EndDates,'Rates', Rates, 'Compounding', Compoו
```

RateSpec =

```
                    FinObj: 'RateSpec'
        Compounding: 1
            Disc: [4x1 double]
            Rates: [4x1 double]
            EndTimes: [4x1 double]
        StartTimes: [4x1 double]
            EndDates: [4x1 double]
    StartDates: 734869
ValuationDate: 734869
            Basis: 0
EndMonthRule: 1
```

Build the HW tree using the following:

```
VolDates = ['1-Jan-2013'; '1-Jan-2014'; '1-Jan-2015';'1-Jan-2016'];
VolCurve = 0.01;
AlphaDates = '01-01-2016';
AlphaCurve = 0.1;
```


## optemfloatbyhw

```
HWVolSpec = hwvolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
    AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RateSpec.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVolSpec, RateSpec, HWTimeSpec)
```

HWT =

```
    FinObj: 'HWFwdTree'
    VolSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
        tObs: [00 1 2 3]
        dObs: [734869 735235 735600 735965]
    CFlowT: {[4\times1 double] [3\times1 double] [2\times1 double] [4]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}
Connect: {[2] [2 3 4] [2 2 3 4 4]}
FwdTree: {[1.0300] [1.0553 1.0381 1.0211] [1x5 double] [1\times5 doub]
```

Define the floater instruments with the embedded call option.

```
Spread = 10;
Settle = 'Jan-1-2012';
Maturity = {'Jan-1-2015';'Jan-1-2016'};
Period = 1;
OptSpec = {'call'};
Strike = 101;
ExerciseDates = 'Jan-1-2015';
```

Compute the price of the floaters with the embedded call.
Price= optemfloatbyhw(HWT, Spread, Settle, Maturity, OptSpec, Strike,... ExerciseDates)

```
Price =
    100.2800
    100.3655
```

See Also optembndbyhw | optemfloatbyhjm | optemfloatbybdt | optemfloatbybk | instoptemfloat
Concepts - basis on page Glossary-1

## optfloatbybdt

Purpose
Price options on floating-rate notes for BDT interest-rate tree

```
Syntax Price = optfloatbybdt(BDTTree,OptSpec,Strike,ExerciseDates,AmericanOpt,
Spread,Settle,Maturity)
Price = optfloatbybdt(BDTTree,OptSpec,Strike,ExerciseDates,AmericanOpt,
Spread,Settle,Maturity,Reset,Basis,Principal,Options,EndMonthRule)
[Price,PriceTree] = optfloatbybdt(
Price = optfloatbybdt(BDTTree,OptSpec,Strike,ExerciseDates,AmericanOpt,
Spread,Settle,Maturity)
Price = optfloatbybdt(___, Name,Value)
[Price,PriceTree] = optfloatbybdt(___)
```


## Description

Price =
optfloatbybdt(BDTTree, OptSpec, Strike, ExerciseDates, AmericanOpt, Spread, Settle, Maturity) returns the option price of the instrument(s) at valuation date.

Price =
optfloatbybdt(BDTTree, OptSpec, Strike, ExerciseDates, AmericanOpt, Spread,Settle,Maturity, Reset, Basis, Principal,Options, EndMonthRule) returns the option price of the instrument(s) at valuation date using optional arguments.
[Price,PriceTree] = optfloatbybdt( __ ) returns the option price of the instrument(s) at valuation date and a structure of trees containing vectors of option prices at each node.

Price =
optfloatbybdt(BDTTree,OptSpec,Strike, ExerciseDates,AmericanOpt, Spread, Settle, Maturity) returns the option price of the instrument(s) at valuation date.

Price $=$ optfloatbybdt (__ , Name, Value) returns the option price of the instrument(s) at valuation date using optional Name-Value pairs.

## Input Arguments

[Price,PriceTree] = optfloatbybdt( _ _ ) returns the option price of the instrument(s) at valuation date and a structure of trees containing vectors of option prices at each node using optional name-value pair arguments.

## BDTTree - Interest-rate tree structure

binomial tree structure
Interest-rate tree specified as a structure by using bdttree.

## Data Types

struct

## OptSpec - Definition of option

string | cell array of strings
Definition of option as 'call' or 'put' specified as a NINST-by-1 cell array of strings for 'call' or 'put'.

## Data Types

cell | char

## Strike - Option strike price values

nonnegative integer | vector of nonnegative integers
Option strike price values specified nonnegative integers using as NINST-by-NSTRIKES vector of strike price values.

## Data Types

single | double

## ExerciseDates - Exercise date for option (European, Bermuda, or American)

## nonnegative integer | vector of nonnegative integers

Exercise date for option (European, Bermuda, or American) specified as nonnegative integers using a NINST-by-NSTRIKES or NINST-by-2 vector of for the option exercise dates.

- If a European or Bermuda option, the ExerciseDates is a 1-by-1 (European) or 1-by-NSTRIKES (Bermuda) vector of exercise dates. For


## optfloatbybdt

a European option, there is only one ExerciseDate on the option expiry date.

- If an American option, then ExerciseDates is a 1-by-2 vector of exercise date boundaries. The option exercises on any date between or including the pair of dates on that row. If there is only one non- NaN date, or if ExerciseDates is 1-by-1, the option exercises between the Settle date and the single listed ExerciseDate.


## Data Types <br> char | cell

## AmericanOpt - Option type

scalar | vector of positive integers [ 0,1 ]
Option type specified as NINST-by-1 positive integer scalar flags with values:

- 0 - European/Bermuda
- 1 - American

Data Types
single | double

## Spread - Number of basis points over the reference rate nonnegative integer | vector of nonnegative integers

Number of basis points over the reference rate specified as a vector nonnegative integers for the number of instruments (NINST)-by-1).

```
Data Types
single | double
```


## Settle - Settlement dates of floating-rate note

ValuationDate of HW Tree (default) | nonnegative integer | vector of nonnegative integers

Settlement dates of floating-rate note specified as nonnegative integers using a NINST-by-1 vector of dates.

> Note The Settle date for every floating-rate note is set to the ValuationDate of the HW Tree. The floating-rate note argument Settle is ignored.

Data Types<br>char<br>\section*{Maturity - Floating-rate note maturity date}<br>nonnegative integer | vector of nonnegative integers

Floating-rate note maturity date specified as nonnegative integers using a NINST-by- 1 vector of dates.

## Data Types

char

## Reset - (Optional) Frequency of payments per year

1 (default) | positive integer from the set[1,2,3,4,6,12] | vector of positive integers from the set [1, $2,3,4,6,12$ ]

Frequency of payments per year specified as positive integers for the values $[1,2,3,4,6,12]$ in a NINST-by- 1 vector.

Note Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for a FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates can not be uniquely determined because there will be more than one possible path for connecting the two payment dates.

Data Types<br>double

## Basis - (Optional) Day-count basis of the instrument

0 (actual/actual) (default) | positive integers of the set [1...13] | vector of positive integers of the set [1...13]

Day-count basis of the instrument specified as a positive integer using a NINST-by- 1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- $2=$ actual $/ 360$
- $3=$ actual $/ 365$
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- $9=$ actual/360 (ISMA)
- $10=$ actual/365 (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252


## Data Types

double

## Principal - (Optional) Principal values

100 (default) | vector of nonnegative values | cell array of nonnegative values

Principal values specified as nonnegative values using a NINST-by-1 vector or NINST-by- 1 cell array of notional principal amounts. When
using a NINST-by-1 cell array, each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

## Data Types <br> double | cell

## Options - (Optional) Structure containing derivatives pricing options

structure
Structure containing derivatives pricing options specified using derivset.

## Data Types

struct

## EndMonthRule - (Optional) End-of-month rule flag

1 (in effect) (default) | nonnegative integer [0,1]
End-of-month rule flag is specified as nonnegative integer [ 0,1 ] using a NINST-by- 1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Data Types <br> double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

## optfloatbybdt

> Example: Price=
> optfloatbybdt (BDTTree, OptSpec, Strike, ExerciseDates, American0pt, Spread, Set

'Reset' - Frequency of payments per year
1 (default) | positive integer from the set[1,2,3,4,6,12] | vector of positive integers from the set [1,2,3,4,6,12]

Frequency of payments per year specified as positive integers for the values $1,2,3,4,6,12]$ in a NINST-by- 1 vector.

Note Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for a FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates can not be uniquely determined because there will be more than one possible path for connecting the two payment dates.

## Data Types <br> char

## 'Basis' - Day-count basis of the instrument

0 (actual/actual) (default) | positive integers of the set [1...13] | vector of positive integers of the set [1...13]

Day-count basis of the instrument specified as a positive integer using a NINST-by-1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- $9=$ actual/360 (ISMA)
- $10=\mathrm{actual} / 365$ (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252


## Data Types

char

## 'Principal' - (Optional) Principal values

100 (default) | vector of nonnegative values | cell array of nonnegative values

Principal values specified as nonnegative values using a NINST-by-1 vector or NINST-by-1 cell array of notional principal amounts. When using a NINST-by- 1 cell array, each element is a NumDates-by- 2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.
Data Types
double | cell

## 'Principal' - Principal values

100 (default) | nonnegative integer | vector of nonnegative integers
Principal values specified as a nonnegative integer using a NINST-by-1 vector of notional principal amounts.

## Data Types <br> char

## optfloatbybdt

## 'Options' - Structure containing derivatives pricing options structure

Structure containing derivatives pricing options specified using derivset.

## Data Types

struct

## 'EndMonthRule' - End-of-month rule flag <br> 1 (in effect) (default) | nonnegative integer [0,1]

End-of-month rule flag is specified as a nonnegative integer [ 0,1 ] using a NINST-by- 1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Data Types <br> char

## Output Prise - Expected prices of the floating-rate note option at time 0 Arguments scalar | vector

Expected prices of the floating-rate note option at time 0 is returned as a scalar or an NINST-by-1 vector.

## PriceTree - Structure of trees containing vectors of option prices at each node

tree structure
Structure of trees containing vectors of instrument prices and accrued interest and a vector of observation times for each node returned as:

- PriceTree.PTree contains option prices.


## Examples Compute the Price of American Call and Put Options on a Floating-Rate Note

Define the interest-rate term structure.

```
Rates = [0.03;0.034;0.038;0.04];
ValuationDate = 'Jan-1-2012';
StartDates = ValuationDate;
EndDates = {'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'; 'Jan-1-2016'};
Compounding = 1;
```

Create the RateSpec.

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', St
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```


## RateSpec =

```
                    FinObj: 'RateSpec'
        Compounding: 1
            Disc: [4\times1 double]
            Rates: [4x1 double]
            EndTimes: [4x1 double]
        StartTimes: [4x1 double]
            EndDates: [4x1 double]
        StartDates: 734869
ValuationDate: 734869
            Basis: 0
EndMonthRule: 1
```

Build the BDT tree and assume a volatility of $10 \%$.

```
Sigma = 0.1;
```

BDTTimeSpec = bdttimespec(ValuationDate, EndDates);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length
BDTT = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec)

## optfloatbybdt

```
BDTT =
    FinObj: 'BDTFwdTree'
    VolSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
        tObs: [0 1 2 3]
        dObs: [734869 735235 735600 735965]
        TFwd: {[4\times1 double] [3x1 double] [2x1 double] [3]}
        CFlowT: {[4\times1 double] [3\times1 double] [2\times1 double] [4]}
FwdTree: {1x4 cell}
```

The floater instrument has a spread of 10 , a period of one year, and matures on Jan-1-2016.

Spread = 10;
Settle = 'Jan-1-2012';
Maturity = 'Jan-1-2016';
Period = 1;
Define the option for the floating-rate note.

```
OptSpec = {'call'; 'put'};
Strike = [100;101];
ExerciseDates = 'Jan-1-2015';
AmericanOpt = 1;
```

Compute the price of the call and put options.
Price= optfloatbybdt(BDTT, OptSpec, Strike, ExerciseDates,AmericanOpt, Settle, Maturity)

Price =
0.3655
0.8087
$\begin{array}{ll}\text { See Also } & \begin{array}{l}\text { bdttree | cfbybdt | capbybdt | swapbybdt | floorbybdt | } \\ \text { floatbybdt | bondbybdt | instoptfloat }\end{array} \\ \text { Concepts } & \text { - basis on page Glossary-1 }\end{array}$

## optfloatbybk



## Input Arguments

[Price, PriceTree] = optfloatbybk( __ ) returns the option price of the instrument(s) at valuation date and a structure of trees containing vectors of option prices at each node using optional name-value pair arguments.

## BKTree - Interest-rate tree structure

binomial tree structure
Interest-rate tree specified as a structure by using bktree.

## Data Types

struct

## OptSpec - Definition of option

string | cell array of strings
Definition of option as 'call' or 'put' specified as a NINST-by-1 cell array of strings for 'call' or 'put'.

## Data Types

cell | char

## Strike - Option strike price values

nonnegative integer | vector of nonnegative integers
Option strike price values specified nonnegative integers using as NINST-by-NSTRIKES vector of strike price values.
Data Types
single | double

## ExerciseDates - Exercise date for option (European, Bermuda, or American)

nonnegative integer | vector of nonnegative integers
Exercise date for option (European, Bermuda, or American) specified as nonnegative integers using a NINST-by-NSTRIKES or NINST-by-2 vector of for the option exercise dates.

- If a European or Bermuda option, the ExerciseDates is a 1-by-1 (European) or 1-by-NSTRIKES (Bermuda) vector of exercise dates. For


## optfloatbybk

a European option, there is only one ExerciseDate on the option expiry date.

- If an American option, then ExerciseDates is a 1-by-2 vector of exercise date boundaries. The option exercises on any date between or including the pair of dates on that row. If there is only one non- NaN date, or if ExerciseDates is 1-by-1, the option exercises between the Settle date and the single listed ExerciseDate.


## Data Types <br> char | cell

## AmericanOpt - Option type

scalar | vector of positive integers [ 0,1 ]
Option type specified as NINST-by-1 positive integer scalar flags with values:

- 0 - European/Bermuda
- 1 - American


## Data Types <br> single | double

## Spread - Number of basis points over the reference rate nonnegative integer | vector of nonnegative integers

Number of basis points over the reference rate specified as a vector nonnegative integers for the number of instruments (NINST)-by-1).

```
Data Types
single | double
```


## Settle - Settlement dates of floating-rate note

ValuationDate of HW Tree (default) | nonnegative integer | vector of nonnegative integers

Settlement dates of floating-rate note specified as nonnegative integers using a NINST-by-1 vector of dates.

> Note The Settle date for every floating-rate note is set to the ValuationDate of the HW Tree. The floating-rate note argument Settle is ignored.

Data Types<br>char<br>\section*{Maturity - Floating-rate note maturity date}<br>nonnegative integer | vector of nonnegative integers

Floating-rate note maturity date specified as nonnegative integers using a NINST-by- 1 vector of dates.

## Data Types

char

## Reset - (Optional) Frequency of payments per year

1 (default) | positive integer from the set[1,2,3,4,6,12] | vector of positive integers from the set [1, 2, 3, 4, 6, 12]
Frequency of payments per year specified as positive integers for the values [ $1,2,3,4,6,12$ ] in a NINST-by- 1 vector.

Note Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for a FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates can not be uniquely determined because there will be more than one possible path for connecting the two payment dates.

## Data Types <br> double

## Basis - (Optional) Day-count basis of the instrument

0 (actual/actual) (default) | positive integers of the set [1...13] | vector of positive integers of the set [1...13]

Day-count basis of the instrument specified as a positive integer using a NINST-by- 1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- $2=$ actual $/ 360$
- $3=$ actual $/ 365$
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- $9=$ actual/360 (ISMA)
- $10=\mathrm{actual} / 365$ (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252


## Data Types

double

## Principal - (Optional) Principal values

100 (default) | vector of nonnegative values | cell array of nonnegative values

Principal values specified as nonnegative values using a NINST-by-1 vector or NINST-by- 1 cell array of notional principal amounts. When
using a NINST-by-1 cell array, each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

## Data Types <br> double | cell

## Options - (Optional) Structure containing derivatives pricing options

structure
Structure containing derivatives pricing options specified using derivset.

## Data Types

struct

## EndMonthRule - (Optional) End-of-month rule flag

1 (in effect) (default) | nonnegative integer [0,1]
End-of-month rule flag is specified as nonnegative integer [ 0,1 ] using a NINST-by- 1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Data Types <br> double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

## optfloatbybk

> Example: Price=
> optfloatbybk(BKTTree, OptSpec, Strike, ExerciseDates, AmericanOpt, Spread, Sett

'Reset' - Frequency of payments per year
1 (default) | positive integer from the set[1,2,3,4,6,12] | vector of positive integers from the set [ $1,2,3,4,6,12$ ]

Frequency of payments per year specified as positive integers for the values $1,2,3,4,6,12]$ in a NINST-by- 1 vector.

Note Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for a FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates can not be uniquely determined because there will be more than one possible path for connecting the two payment dates.

## Data Types char

## 'Basis' - Day-count basis of the instrument

0 (actual/actual) (default) | positive integers of the set [1...13] | vector of positive integers of the set [1...13]

Day-count basis of the instrument specified as a positive integer using a NINST-by-1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)


## optfloatbybk

- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- $9=$ actual/360 (ISMA)
- $10=\mathrm{actual} / 365$ (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- $13=\mathrm{BUS} / 252$


## Data Types

char

## 'Principal' - (Optional) Principal values

100 (default) | vector of nonnegative values | cell array of nonnegative values

Principal values specified as nonnegative values using a NINST-by-1 vector or NINST-by-1 cell array of notional principal amounts. When using a NINST-by- 1 cell array, each element is a NumDates-by- 2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Data Types
double | cell
'Options' - Structure containing derivatives pricing options
structure
Structure containing derivatives pricing options specified using derivset.

Data Types
struct

## optfloatbybk

## 'EndMonthRule' - End-of-month rule flag <br> 1 (in effect) (default) | nonnegative integer [0,1]

End-of-month rule flag is specified as a nonnegative integer [ 0,1 ] using a NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Data Types <br> char

## Output Price - Expected prices of the floating-rate note option at time 0 Arguments scalar | vector

Expected prices of the floating-rate note option at time 0 is returned as a scalar or an NINST-by-1 vector.

## PriceTree - Structure of trees containing vectors of option prices at each node <br> tree structure

Structure of trees containing vectors of instrument prices and accrued interest and a vector of observation times for each node returned as:

- PriceTree.PTree contains option prices.


## Examples Compute the Price of American and European Call Options on a Floating-Rate Note

Define the interest-rate term structure.

```
Rates = [0.03;0.034;0.038;0.04];
ValuationDate = 'Jan-1-2012';
StartDates = ValuationDate;
```

```
EndDates = {'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'; 'Jan-1-2016'};
Compounding = 1;
Create the RateSpec.
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', Sta
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
RateSpec =
    FinObj: 'RateSpec'
    Compounding: 1
            Disc: [4x1 double]
            Rates: [4x1 double]
            EndTimes: [4x1 double]
        StartTimes: [4\times1 double]
            EndDates: [4x1 double]
        StartDates: 734869
ValuationDate: 734869
            Basis: 0
EndMonthRule: 1
```

Build the BK tree.

```
VolDates = ['1-Jan-2013'; '1-Jan-2014'; '1-Jan-2015';'1-Jan-2016'];
VolCurve = 0.01;
AlphaDates = '01-01-2016';
AlphaCurve = 0.1;
HWVolSpec = hwvolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
    AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RateSpec.ValuationDate, VolDates, Compounding)
HWT = hwtree(HWVolSpec, RateSpec, HWTimeSpec)
```


## optfloatbybk

HWT =

```
    FinObj: 'HWFwdTree'
    VolSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
        tObs: [0 1 2 3]
        dObs: [734869 735235 735600 735965]
    CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4]}
    Probs: {[3\times1 double] [3x3 double] [3\times5 double]}
Connect: {[2] [2 3 4] [[2 2 3 4 4]}
FwdTree: {[1.0300] [1.0553 1.0381 1.0211] [1x5 double] [1x5 doub]
```

The floater instrument has a spread of 10 , a period of one year, and matures on Jan-1-2016.

```
Spread = 10;
Settle = 'Jan-1-2012';
Maturity = 'Jan-1-2016';
Period = 1;
```

Define the option for the floating-rate note.
OptSpec = \{'call'\};
Strike = 95;
ExerciseDates = 'Jan-1-2016';
AmericanOpt = [0;1];
Compute the price of the call options.
Price= optfloatbyhw(HWT, OptSpec, Strike, ExerciseDates,AmericanOpt,... Spread, Settle, Maturity)

Price =
4.2740

$$
5.3655
$$

See Also bktree | cfbybk | capbybk | swapbybk | floorbybk | floatbybk | bondbybk | instoptfloat
Concepts - basis on page Glossary-1

## optfloatbyhim

Purpose
Price options on floating-rate notes for HJM interest-rate tree

```
Syntax Price = optfloatbyhjm(HJMTree,OptSpec,Strike,ExerciseDates,AmericanOpt,
Spread,Settle,Maturity)
Price = optfloatbyhjm(HJMTree,OptSpec,Strike,ExerciseDates,AmericanOpt,
Spread,Settle,Maturity,Reset,Basis,Principal,Options,EndMonthRule)
[Price,PriceTree] = optfloatbyhjm(
```

$\qquad$

```
Price = optfloatbyhjm(HJMTree,OptSpec,Strike,ExerciseDates,AmericanOpt,
Spread,Settle,Maturity)
Price = optfloatbyhjm(___ Name,Value)
[Price,PriceTree] = optfloatbyhjm(___)
```


## Description

Price =
optfloatbyhjm(HJMTree,OptSpec,Strike, ExerciseDates, AmericanOpt, Spread, Settle, Maturity) returns the option price of the instrument(s) at valuation date.

Price =
optfloatbyhjm(HJMTree,OptSpec,Strike, ExerciseDates,AmericanOpt, Spread,Settle, Maturity, Reset, Basis, Principal, Options, EndMonthRule) returns the option price of the instrument(s) at valuation date using optional arguments.
[Price,PriceTree] = optfloatbyhjm( _ _ ) returns the option price of the instrument(s) at valuation date and a structure of trees containing vectors of option prices at each node.

Price =
optfloatbyhjm(HJMTree, OptSpec, Strike, ExerciseDates,AmericanOpt, Spread, Settle, Maturity) returns the option price of the instrument(s) at valuation date.

Price $=$ optfloatbyhjm(__ , Name, Value) returns the option price of the instrument(s) at valuation date using optional Name-Value pairs.

## Input Arguments

[Price,PriceTree] = optfloatbyhjm( _ _ ) returns the option price of the instrument(s) at valuation date and a structure of trees containing vectors of option prices at each node using optional name-value pair arguments.

## HJMTree - Interest-rate tree structure <br> binomial tree structure

Interest-rate tree specified as a structure by using hjmtree.

## Data Types

struct

## OptSpec - Definition of option

string | cell array of strings
Definition of option as 'call' or 'put' specified as a NINST-by-1 cell array of strings for 'call' or 'put'.

## Data Types

cell | char

## Strike - Option strike price values

nonnegative integer | vector of nonnegative integers
Option strike price values specified nonnegative integers using as NINST-by-NSTRIKES vector of strike price values.

## Data Types

single | double

## ExerciseDates - Exercise date for option (European, Bermuda, or American)

nonnegative integer | vector of nonnegative integers
Exercise date for option (European, Bermuda, or American) specified as nonnegative integers using a NINST-by-NSTRIKES or NINST-by-2 vector of for the option exercise dates.

- If a European or Bermuda option, the ExerciseDates is a 1-by-1 (European) or 1-by-NSTRIKES (Bermuda) vector of exercise dates. For
a European option, there is only one ExerciseDate on the option expiry date.
- If an American option, then ExerciseDates is a 1-by-2 vector of exercise date boundaries. The option exercises on any date between or including the pair of dates on that row. If there is only one non- NaN date, or if ExerciseDates is 1-by-1, the option exercises between the Settle date and the single listed ExerciseDate.


## Data Types <br> char | cell

## AmericanOpt - Option type

scalar | vector of positive integers [ 0,1 ]
Option type specified as NINST-by-1 positive integer scalar flags with values:

- 0 - European/Bermuda
- 1 - American

Data Types
single | double

## Spread - Number of basis points over the reference rate nonnegative integer | vector of nonnegative integers

Number of basis points over the reference rate specified as a vector nonnegative integers for the number of instruments (NINST)-by-1).

```
Data Types
single | double
```


## Settle - Settlement dates of floating-rate note

ValuationDate of HW Tree (default) | nonnegative integer | vector of nonnegative integers

Settlement dates of floating-rate note specified as nonnegative integers using a NINST-by- 1 vector of dates.

> Note The Settle date for every floating-rate note is set to the ValuationDate of the HW Tree. The floating-rate note argument Settle is ignored.

Data Types<br>char<br>\section*{Maturity - Floating-rate note maturity date}<br>nonnegative integer | vector of nonnegative integers

Floating-rate note maturity date specified as nonnegative integers using a NINST-by-1 vector of dates.

## Data Types

char

## Reset - (Optional) Frequency of payments per year

1 (default) | positive integer from the set[1,2,3,4,6,12] | vector of positive integers from the set [1, 2, 3, 4, 6, 12]

Frequency of payments per year specified as positive integers for the values $[1,2,3,4,6,12]$ in a NINST-by- 1 vector.

Note Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for a FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates can not be uniquely determined because there will be more than one possible path for connecting the two payment dates.

Data Types<br>double

## Basis - (Optional) Day-count basis of the instrument

0 (actual/actual) (default) | positive integers of the set [1...13] | vector of positive integers of the set [1...13]

Day-count basis of the instrument specified as a positive integer using a NINST-by- 1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual $/ 365$
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- $9=$ actual/360 (ISMA)
- $10=\mathrm{actual} / 365$ (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252


## Data Types

double

## Principal - (Optional) Principal values

100 (default) | vector of nonnegative values | cell array of nonnegative values

Principal values specified as nonnegative values using a NINST-by-1 vector or NINST-by-1 cell array of notional principal amounts. When
using a NINST-by-1 cell array, each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

## Data Types <br> double | cell

## Options - (Optional) Structure containing derivatives pricing options

structure
Structure containing derivatives pricing options specified using derivset.

## Data Types

struct

## EndMonthRule - (Optional) End-of-month rule flag

1 (in effect) (default) | nonnegative integer [0,1]
End-of-month rule flag is specified as nonnegative integer [ 0,1 ] using a NINST-by- 1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Data Types <br> double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

## optfloatbyhim

> Example: Price=
> optfloatbyhjm(HJMTree, OptSpec, Strike, ExerciseDates, American0pt, Spread, Set

'Reset' - Frequency of payments per year
1 (default) | positive integer from the set[1,2,3,4,6,12] | vector of positive integers from the set [ $1,2,3,4,6,12$ ]

Frequency of payments per year specified as positive integers for the values [ $1,2,3,4,6,12$ ] in a NINST-by-1 vector. .

Note Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for a FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates can not be uniquely determined because there will be more than one possible path for connecting the two payment dates.

## Data Types char

## 'Basis' - Day-count basis of the instrument

0 (actual/actual) (default) | positive integers of the set [1...13] | vector of positive integers of the set [1...13]

Day-count basis of the instrument specified as a positive integer using a NINST-by-1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- $10=\mathrm{actual} / 365$ (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252


## Data Types

char

## 'Principal' - (Optional) Principal values

100 (default) | vector of nonnegative values | cell array of nonnegative values

Principal values specified as nonnegative values using a NINST-by-1 vector or NINST-by- 1 cell array of notional principal amounts. When using a NINST-by- 1 cell array, each element is a NumDates-by- 2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Data Types
double | cell
'Options' - Structure containing derivatives pricing options
structure
Structure containing derivatives pricing options specified using derivset.

Data Types
struct

## optfloatbyhim

## 'EndMonthRule' - End-of-month rule flag <br> 1 (in effect) (default) | nonnegative integer [0,1]

End-of-month rule flag is specified as a nonnegative integer [ 0,1 ] using a NINST-by- 1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Data Types <br> char

## Output Prise - Expected prices of the floating-rate note option at time 0 Arguments scalar | vector

Expected prices of the floating-rate note option at time 0 is returned as a scalar or an NINST-by-1 vector.

## PriceTree - Structure of trees containing vectors of option prices at each node <br> tree structure

Structure of trees containing vectors of instrument prices and accrued interest and a vector of observation times for each node returned as:

- PriceTree.PTree contains option prices.


## Examples Compute the Price of American and European Call Options on a Floating-Rate Note

Define the interest-rate term structure.

```
Rates = [0.03;0.035;0.040;0.045];
ValuationDate = 'Jan-1-2012';
StartDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
```

```
EndDates = {'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'; 'Jan-1-2016'};
Compounding = 1;
Create the RateSpec.
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', St
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
RateSpec =
```

```
                    FinObj: 'RateSpec'
```

                    FinObj: 'RateSpec'
        Compounding: 1
        Compounding: 1
            Disc: [4\times1 double]
            Disc: [4\times1 double]
            Rates: [4\times1 double]
            Rates: [4\times1 double]
            EndTimes: [4x1 double]
            EndTimes: [4x1 double]
        StartTimes: [4x1 double]
        StartTimes: [4x1 double]
            EndDates: [4x1 double]
            EndDates: [4x1 double]
        StartDates: [4x1 double]
        StartDates: [4x1 double]
    ValuationDate: 734869
ValuationDate: 734869
Basis: 0
Basis: 0
EndMonthRule: 1

```
EndMonthRule: 1
```

Build the HJM tree.

```
VolSpec = hjmvolspec('Constant', 0.01);
TimeSpec = hjmtimespec(RateSpec.ValuationDate, EndDates, Compounding)
HJMTree = hjmtree(VolSpec, RateSpec, TimeSpec)
```

HJMTree =

```
    FinObj: 'HJMFwdTree'
    VolSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
```

```
    tObs: [0 1 2 3]
    dObs: [734869 735235 735600 735965]
    TFwd: {[4\times1 double] [3\times1 double] [2x1 double] [3]}
CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4]}
FwdTree: {[4x1 double] [3x1x2 double] [2x2x2 double] [1x4x2 doub]
```

The floater instrument has a spread of 10 , a period of one year, and matures on Jan-1-2015.

```
Spread = 10;
Settle = 'Jan-1-2012';
Maturity = 'Jan-1-2015';
Period = 1;
```

Define the option for the floating-rate note.

```
OptSpec = {'call'};
Strike = 95;
ExerciseDates = 'Jan-1-2015';
AmericanOpt = [0;1];
```

Compute the price of the call options.
Price= optfloatbyhjm(HJMTree, OptSpec, Strike, ExerciseDates,AmericanOpt Spread, Settle, Maturity)

Price =
4.5098
5.2811

## See Also

hjmtree | cfbyhjm \| capbyhjm\| swapbyhjm \| floorbyhw \| floatbyhjm | bondbyhjm | instoptfloat

[^9]
## optfloatbyhw

Purpose
Price options on floating-rate notes for HW interest-rate tree

```
Syntax Price = optfloatbyhw(HWTree,OptSpec,Strike,ExerciseDates,AmericanOpt,
Spread,Settle,Maturity)
Price= optfloatbyhw(HWTree,OptSpec,Strike,ExerciseDates,AmericanOpt,
Spread,Settle,Maturity,Reset,Basis,Principal,Options,EndMonthRule)
[Price,PriceTree] = optfloatbyhw(
```

$\qquad$

```
Price = optfloatbyhw(HWTree,OptSpec,Strike,ExerciseDates,AmericanOpt,
Spread,Settle,Maturity)
Price = optfloatbyhw( __, Name,Value)
[Price,PriceTree] = optfloatbyhw(___)
```


## Description Price =

optfloatbyhw(HWTree, OptSpec, Strike, ExerciseDates, AmericanOpt, Spread, Settle, Maturity) returns the option price of the instrument(s) at valuation date.

Price=
optfloatbyhw(HWTree,OptSpec,Strike, ExerciseDates,AmericanOpt, Spread, Settle, Maturity, Reset, Basis, Principal, Options, EndMonthRule) returns the option price of the instrument(s) at valuation date using optional arguments.
[Price, PriceTree] = optfloatbyhw (__ ) returns the option price of the instrument(s) at valuation date and a structure of trees containing vectors of option prices at each node.

Price =
optfloatbyhw(HWTree,OptSpec,Strike, ExerciseDates,AmericanOpt, Spread, Settle, Maturity) returns the option price of the instrument(s) at valuation date.

Price = optfloatbyhw( $\qquad$ , Name, Value) returns the option price of the instrument(s) at valuation date using optional Name-Value pairs.

## Input Arguments

[Price, PriceTree] = optfloatbyhw( _ _ ) returns the option price of the instrument(s) at valuation date and a structure of trees containing vectors of option prices at each node using optional Name-Value pairs.

## HWTree - Interest-rate tree structure

binomial tree structure
Interest-rate tree specified as a structure by using hwtree.

## Data Types

struct

## OptSpec - Definition of option

string | cell array of strings
Definition of option as 'call' or 'put' specified as a NINST-by-1 cell array of strings for 'call' or 'put'.
Data Types
cell | char

## Strike - Option strike price values

nonnegative integer | vector of nonnegative integers
Option strike price values specified nonnegative integers using as NINST-by-NSTRIKES vector of strike price values.

Data Types
single | double

## ExerciseDates - Exercise date for option (European, Bermuda, or American)

nonnegative integer | vector of nonnegative integers
Exercise date for option (European, Bermuda, or American) specified as nonnegative integers using a NINST-by-NSTRIKES or NINST-by-2 vector of for the option exercise dates.

- If a European or Bermuda option, the ExerciseDates is a 1-by-1 (European) or 1-by-NSTRIKES (Bermuda) vector of exercise dates. For


## optfloatbyhw

a European option, there is only one ExerciseDate on the option expiry date.

- If an American option, then ExerciseDates is a 1-by-2 vector of exercise date boundaries. The option exercises on any date between or including the pair of dates on that row. If there is only one non-NaN date, or if ExerciseDates is 1 -by-1, the option exercises between the Settle date and the single listed ExerciseDate.


## Data Types <br> char | cell

## AmericanOpt - Option type

scalar | vector of positive integers [ 0,1 ]
Option type specified as NINST-by-1 positive integer scalar flags with values:

- 0 - European/Bermuda
- 1 - American

Data Types
single | double

## Spread - Number of basis points over the reference rate nonnegative integer | vector of nonnegative integers

Number of basis points over the reference rate specified as a vector nonnegative integers for the number of instruments (NINST)-by-1).

```
Data Types
single | double
```


## Settle - Settlement dates of floating-rate note

ValuationDate of HW Tree (default) | nonnegative integer | vector of nonnegative integers

Settlement dates of floating-rate note specified as nonnegative integers using a NINST-by-1 vector of dates.

> Note The Settle date for every floating-rate note is set to the ValuationDate of the HW Tree. The floating-rate note argument Settle is ignored.

Data Types<br>char<br>\section*{Maturity - Floating-rate note maturity date}<br>nonnegative integer | vector of nonnegative integers

Floating-rate note maturity date specified as nonnegative integers using a NINST-by-1 vector of dates.

## Data Types

char

## Reset - (Optional) Frequency of payments per year

1 (default) | positive integer from the set[1,2,3,4,6,12] | vector of positive integers from the set [1, 2, 3, 4, 6, 12]

Frequency of payments per year specified as positive integers for the values [ $1,2,3,4,6,12$ ] in a NINST-by- 1 vector.

Note Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for a FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates can not be uniquely determined because there will be more than one possible path for connecting the two payment dates.

## Data Types <br> double

## Basis - (Optional) Day-count basis of the instrument

0 (actual/actual) (default) | positive integers of the set [1...13] | vector of positive integers of the set [1...13]

Day-count basis of the instrument specified as a positive integer using a NINST-by- 1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- $9=$ actual/360 (ISMA)
- $10=$ actual/365 (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252


## Data Types

double

## Principal - (Optional) Principal values

100 (default) | vector of nonnegative values | cell array of nonnegative values

Principal values specified as nonnegative values using a NINST-by-1 vector or NINST-by- 1 cell array of notional principal amounts. When
using a NINST-by-1 cell array, each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

## Data Types <br> double | cell

## Options - (Optional) Structure containing derivatives pricing options

structure
Structure containing derivatives pricing options specified using derivset.

## Data Types

struct

## EndMonthRule - (Optional) End-of-month rule flag

1 (in effect) (default) | nonnegative integer [0,1]
End-of-month rule flag is specified as nonnegative integer [ 0,1 ] using a NINST-by- 1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Data Types <br> double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

## optfloatbyhw

> Example: Price=
> optfloatbyhw(HWTree, OptSpec, Strike, ExerciseDates, AmericanOpt, Spread, Settl

'Reset' - Frequency of payments per year
1 (default) | positive integer from the set[1,2,3,4,6,12] | vector of positive integers from the set [ $1,2,3,4,6,12$ ]

Frequency of payments per year specified as positive integers for the values [ $1,2,3,4,6,12$ ] in a NINST-by- 1 vector.

Note Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for a FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates can not be uniquely determined because there will be more than one possible path for connecting the two payment dates.

## Data Types <br> char

## 'Basis' - Day-count basis of the instrument

0 (actual/actual) (default) | positive integers of the set [1....13] | vector of positive integers of the set [1...13]

Day-count basis of the instrument specified as a positive integer using a NINST-by- 1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- $10=\mathrm{actual} / 365$ (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- $13=\mathrm{BUS} / 252$


## Data Types

char

## 'Principal' - (Optional) Principal values

100 (default) | vector of nonnegative values | cell array of nonnegative values

Principal values specified as nonnegative values using a NINST-by-1 vector or NINST-by-1 cell array of notional principal amounts. When using a NINST-by- 1 cell array, each element is a NumDates-by- 2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Data Types
double | cell
'Options' - Structure containing derivatives pricing options
structure
Structure containing derivatives pricing options specified using derivset.

Data Types
struct

## optfloatbyhw

## 'EndMonthRule' - End-of-month rule flag <br> 1 (in effect) (default) | nonnegative integer [0,1]

End-of-month rule flag is specified as a nonnegative integer [ 0,1 ] using a NINST-by- 1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Data Types <br> char

## Output Price - Expected prices of the floating-rate note option at time 0 Arguments scalar | vector

Expected prices of the floating-rate note option at time 0 is returned as a scalar or an NINST-by-1 vector.

## PriceTree - Structure of trees containing vectors of option prices at each node <br> tree structure

Structure of trees containing vectors of instrument prices and accrued interest and a vector of observation times for each node returned as:

- PriceTree.PTree contains option prices.


## Examples Compute the Price of American and European Call Options on a Floating-Rate Note

Define the interest-rate term structure.

```
Rates = [0.03;0.034;0.038;0.04];
ValuationDate = 'Jan-1-2012';
StartDates = ValuationDate;
```

```
EndDates = {'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'; 'Jan-1-2016'};
Compounding = 1;
Create the RateSpec.
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', Sta
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```


## RateSpec =

```
                    FinObj: 'RateSpec'
```

                    FinObj: 'RateSpec'
        Compounding: 1
        Compounding: 1
            Disc: [4\times1 double]
            Disc: [4\times1 double]
            Rates: [4x1 double]
            Rates: [4x1 double]
            EndTimes: [4x1 double]
            EndTimes: [4x1 double]
        StartTimes: [4x1 double]
        StartTimes: [4x1 double]
            EndDates: [4x1 double]
            EndDates: [4x1 double]
        StartDates: 734869
        StartDates: 734869
    ValuationDate: 734869
ValuationDate: 734869
Basis: 0
Basis: 0
EndMonthRule: 1

```
EndMonthRule: 1
```

Build the HW tree using the following:

```
VolDates = ['1-Jan-2013'; '1-Jan-2014'; '1-Jan-2015';'1-Jan-2016'];
VolCurve = 0.01;
AlphaDates = '01-01-2016';
AlphaCurve = 0.1;
HWVolSpec = hwvolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
    AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RateSpec.ValuationDate, VolDates, Compounding)
HWT = hwtree(HWVolSpec, RateSpec, HWTimeSpec)
```


## optfloatbyhw

```
HWT =
    FinObj: 'HWFwdTree'
    VolSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
        tObs: [0 1 2 3]
        dObs: [734869 735235 735600 735965]
    CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}
Connect: {[2] [2 3 4] [2 2 2 3 4 4]}
FwdTree: {[1.0300] [1.0553 1.0381 1.0211] [1x5 double] [1x5 doub]
```

The floater instrument has a spread of 10, a period of one year, and matures on Jan-1-2016.

```
Spread = 10;
Settle = 'Jan-1-2012';
Maturity = 'Jan-1-2016';
Period = 1;
```

Define the option for the floating-rate note.
OptSpec = \{'call'\};
Strike = 95;
ExerciseDates = 'Jan-1-2016';
AmericanOpt = [0;1];
Compute the price of the call options.
Price= optfloatbyhw(HWT, OptSpec, Strike, ExerciseDates,AmericanOpt,... Spread, Settle, Maturity)

Price =
4.2740

$$
5.3655
$$

See Also hwtree | cfbyhw | capbyhw | swapbyhw | floorbyhw | floatbyhw | bondbyhw | instoptfloat
Concepts - basis on page Glossary-1

## optstockbybjs

## Purpose

Syntax $\begin{aligned} & \text { Price }=\text { optstockbybjs(RateSpec, StockSpec, Settle, Maturity, } \\ & \text { OptSpec, Strike })\end{aligned}$
Arguments

| RateSpec | The annualized continuously compounded rate <br> term structure. For information on the interest <br> rate specification, see intenvset. |
| :--- | :--- |
| StockSpec | Stock specification. See stockspec. |
| Settle | NINST-by-1 vector of settlement or trade dates. |
| Maturity | NINST-by-1 vector of maturity dates. |
| OptSpec | NINST-by-1 cell array of strings 'call' or 'put '. |
| Strike | NINST-by-1 vector of strike price values. |

## Description

Price American options using Bjerksund-Stensland 2002 option pricing model

Settle NINST-by-1 vector of settlement or trade dates
Maturity NINST-by-1 vector of maturity dates.
OptSpec NINST-by-1 cell array of strings 'call' or 'put'.
Strike NINST-by-1 vector of strike price values.

Price = optstockbybjs(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike) computes American option prices with continuous dividend yield using the Bjerksund-Stensland 2002 option pricing model.

Price is a NINST-by-1 vector of expected option prices.

Note optstockbybjs computes prices of American options with continuous dividend yield using the Bjerksund-Stensland option pricing model.

## Examples Compute the American Option Prices With Continuous Dividend Yield Using the Bjerksund-Stensland 2002 Option Pricing Model

This example shows how to compute the American option prices with continuous dividend yield using the Bjerksund-Stensland 2002 option pricing model. Consider two American stock options (a call and a put) with an exercise price of $\$ 100$. The options expire on April 1, 2008. Assume the underlying stock pays a continuous dividend yield of $4 \%$ as of January 1, 2008. The stock has a volatility of $20 \%$ per annum and the annualized continuously compounded risk-free rate is $8 \%$ per annum. Using this data, calculate the price of the American call and put, assuming the following current prices of the stock: $\$ 90$ (for the call) and $\$ 120$ (for the put).

```
Settle = 'Jan-1-2008';
Maturity = 'April-1-2008';
Strike = 100;
AssetPrice = [90;120];
DivYield = 0.04;
Rate = 0.08;
Sigma = 0.20;
% define the RateSpec and StockSpec
StockSpec = stockspec(Sigma, AssetPrice, {'continuous'}, DivYield);
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,..
'EndDates', Maturity, 'Rates', Rate, 'Compounding', -1);
% define the option type
OptSpec = {'call'; 'put'};
Price = optstockbybjs(RateSpec, StockSpec, Settle, Maturity, OptSpec,
```

The first element of the Price vector represents the price of the call ( $\$ 0.84$ ); the second element represents the price of the put option (\$0.11).

References Bjerksund, P. and G. Stensland, Closed-Form Approximation of American Options, Scandinavian Journal of Management, 1993, Vol. 9, Suppl., pp. S88-S99.<br>Bjerksund, P. and G. Stensland, Closed Form<br>Valuation of American Options, Discussion paper 2002<br>(http://brage.bibsys.no/nhh/bitstream/URN:NBN:no-bibsys_brage_22301/1/bjerksund\% ${ }^{2}$

## See Also <br> impvbybjs | intenvset | optstocksensbybjs | stockspec

## Purpose

Price options on futures using Black option pricing model

```
Syntax
Price = optstockbyblk(RateSpec, StockSpec, Settle, Maturity,
OptSpec, Strike)
```


## Arguments

## Description

## Examples

The annualized continuously compounded rate term structure. For information on the interest rate specification, see intenvset.

StockSpec Stock specification. See stockspec.
Settle NINST-by-1 vector of settlement or trade dates.
Maturity NINST-by-1 vector of maturity dates.
OptSpec NINST-by-1 cell array of strings 'call' or 'put'.
Strike NINST-by-1 vector of strike price values.

| RateSpec | The annualized continuously compounded rate <br> term structure. For information on the interest <br> rate specification, see intenvset. |
| :--- | :--- |
| StockSpec | Stock specification. See stockspec. |
| Settle | NINST-by-1 vector of settlement or trade dates. |
| Maturity | NINST-by-1 vector of maturity dates. |
| OptSpec | NINST-by-1 cell array of strings 'call' or 'put '. |
| Strike | NINST-by-1 vector of strike price values. |

Price = optstockbyblk(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike) computes option prices on futures using the Black option pricing model.

Price is a NINST-by- 1 vector of expected option prices.

## Compute Option Prices on Futures Using the Black Option Pricing Model

This example shows how to compute option prices on futures using the Black option pricing model. Consider two European call options on a futures contract with exercise prices of $\$ 20$ and $\$ 25$ that expire on September 1, 2008. Assume that on May 1, 2008 the contract is trading at $\$ 20$, and has a volatility of $35 \%$ per annum. The risk-free rate is $4 \%$ per annum. Using this data, calculate the price of the call futures options using the Black model.

```
Strike = [20; 25];
AssetPrice = 20;
Sigma = .35;
Rates = 0.04;
Settle = 'May-01-08';
Maturity = 'Sep-01-08';
% define the RateSpec and StockSpec
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
    'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1);
StockSpec = stockspec(Sigma, AssetPrice);
% define the call options
OptSpec = {'call'};
Price = optstockbyblk(RateSpec, StockSpec, Settle, Maturity,...
OptSpec, Strike)
Price =
    1.5903
    0.3037
```


## See Also

impvbyblk | intenvset | optstocksensbyblk | stockspec

## Purpose

Price options using Black-Scholes option pricing model

Syntax<br>Price = optstockbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike)

## Arguments

| RateSpec | The annualized continuously compounded rate <br> term structure. For information on the interest <br> rate specification, see intenvset. |
| :--- | :--- |
| StockSpec | Stock specification. See stockspec. |
| Settle | NINST-by-1 vector of settlement or trade dates. |
| Maturity | NINST-by-1 vector of maturity dates. |
| OptSpec | NINST-by-1 cell array of strings 'call' or 'put '. |
| Strike | NINST-by-1 vector of strike price values. |

```
Description Price = optstockbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike)
```

Price is a NINST-by- 1 vector of expected option prices.

## optstockbybls

Note When using StockSpec with optstockbybls, you can modify StockSpec to handle other types of underliers when pricing instruments that use the Black-Scholes model.

When pricing Futures (Black model), enter the following in StockSpec:
DivType = 'Continuous';
DivAmount = RateSpec.Rates;

When pricing Foreign Currencies (Garman-Kohlhagen model), enter the following in StockSpec:

DivType = 'Continuous';
DivAmount = ForeignRate;
where ForeignRate is the continuously compounded, annualized risk free interest rate in the foreign country.

## Examples <br> Compute Option Prices Using the Black-Scholes Option Pricing Model

This example shows how to compute option prices using the Black-Scholes option pricing model. Consider two European options, a call and a put, with an exercise price of $\$ 29$ on January 1, 2008. The options expire on May 1, 2008. Assume that the underlying stock for the call option provides a cash dividend of $\$ 0.50$ on February 15, 2008. The underlying stock for the put option provides a continuous dividend yield of $4.5 \%$ per annum. The stocks are trading at $\$ 30$ and have a volatility of $25 \%$ per annum. The annualized continuously compounded risk-free rate is $5 \%$ per annum. Using this data, compute the price of the options using the Black-Scholes model.

Strike = 29;
AssetPrice = 30;
Sigma = .25;

```
Rates = 0.05;
Settle = 'Jan-01-2008';
Maturity = 'May-01-2008';
% define the RateSpec and StockSpec
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,
Maturity, 'Rates', Rates, 'Compounding', -1);
DividendType = {'cash';'continuous'};
DividendAmounts = [0.50; 0.045];
ExDividendDates = {'Feb-15-2008';NaN};
StockSpec = stockspec(Sigma, AssetPrice, DividendType, DividendAmounts
ExDividendDates);
OptSpec = {'call'; 'put'};
Price = optstockbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec,
Price =
    2.2030
    1.2025
```

impvbybls | intenvset | optstocksensbybls | stockspec

## optstockbycrr

Purpose Price stock option from Cox-Ross-Rubinstein tree

```
Syntax [Price, PriceTree] = optstockbycrr(CRRTree, OptSpec, Strike,
Settle, ExerciseDates, AmericanOpt)
```


## Arguments

CRRTree Stock tree structure created by crrtree.
OptSpec Number of instruments (NINST)-by-1 cell array of strings 'call' or 'put'.

Note The interpretation of the Strike and ExerciseDates arguments depends upon the setting of the AmericanOpt argument. If AmericanOpt $=0, \mathrm{NaN}$, or is unspecified, the option is a European or Bermuda option. If AmericanOpt $=1$, the option is an American option.

Strike European option: NINST-by-1 vector of strike price values.

Bermuda option: NINST by number of strikes (NSTRIKES) matrix of strike price values.

Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.

American option: NINST-by-1 vector of strike price values for each option.

Settle NINST-by-1 vector of settlement or trade dates.
ExerciseDates
NINST-by-1 (European option) or NINST-by-NSTRIKES (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.
For an American option:
NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is NINST-by-1, the option can be exercised between the underlying bond Settle and the single listed exercise date.

Data arguments are NINST-by-1 vectors, scalar, or empty. Fill unspecified entries in vectors with NaN. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices [].

## Description

[Price, PriceTree] = optstockbycrr(CRRTree, OptSpec, Strike, Settle, ExerciseDates, AmericanOpt) computes the price of a European, Bermuda, or American stock option.
Price is a NINST-by-1 vector of expected option prices at time 0 .
PriceTree is a tree structure with a vector of instrument prices at each node.

## Examples

Price an American Stock Option Using a CRR Binomial Tree
This example shows how to price an American stock option using a CRR binomial tree by loading the file deriv.mat, which provides CRRTree. The CRRTree structure contains the stock specification and time information needed to price the American option.

## optstockbycrr

```
load deriv.mat;
OptSpec = 'Call';
Strike = 105;
Settle = '01-Jan-2003';
ExerciseDates = '01-Jan-2005';
AmericanOpt = 1;
Price = optstockbycrr(CRRTree, OptSpec, Strike, Settle, ...
ExerciseDates, AmericanOpt)
Price =
    8.2863
```


## Price a Bermudan Stock Option Using a CRR Binomial Tree

Load the file deriv.mat, which provides CRRTree. The CRRTree structure contains the stock specification and time information needed to price the Bermudan option.
load deriv.mat;
\% Option
OptSpec = 'Call';
Strike = 105;
Settle = '01-Jan-2003';
ExerciseDatesBerm=\{'01-Jan-2004', '01-Jul-2004','01-Jan-2005','01-Jul-200
Price the Bermudan option.
Price = optstockbycrr(CRRTree, OptSpec, Strike, Settle, ExerciseDatesBern
Warning: Some ExerciseDates are not aligned with tree nodes. Result will approximated.

```
Price
    9.6381
```

See Also crrtree | instoptstock

## optstockbyeqp

Purpose

Arguments

Syntax $\begin{aligned} & \text { [Price, PriceTree] }=\text { optstockbyeqp(EQPTree, OptSpec, Strike, } \\ & \text { Settle, ExerciseDates, AmericanOpt) }\end{aligned}$
Price stock option from Equal Probabilities binomial tree

EQPTree Stock tree structure created by eqptree.
OptSpec Number of instruments (NINST)-by-1 cell array of strings 'call' or 'put'.

Note The interpretation of the Strike and ExerciseDates arguments depends upon the setting of the AmericanOpt argument. If AmericanOpt $=0, \mathrm{NaN}$, or is unspecified, the option is a European or Bermuda option. If AmericanOpt $=1$, the option is an American option.

Strike European option: NINST-by-1 vector of strike price values.

Bermuda option: NINST by number of strikes (NSTRIKES) matrix of strike price values.

Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.

American option: NINST-by-1 vector of strike price values for each option.

Settle NINST-by-1 vector of settlement or trade dates.
ExerciseDates
NINST-by-1 (European option) or

NINST-by-NSTRIKES (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.

For an American option:
NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non- NaN date is listed, or if ExerciseDates is NINST-by-1, the option can be exercised between the underlying bond Settle and the single listed exercise date.

## Description

[Price, PriceTree] = optstockbyeqp(EQPTree, OptSpec, Strike,
Settle, ExerciseDates, AmericanOpt) computes the price of a European/Bermuda or American stock option.

Price is a NINST-by- 1 vector of expected option prices at time 0 .
PriceTree is a tree structure with a vector of instrument prices at each node.

## Examples

Price an American Stock Option Using an EQP Equity Tree

This example shows how to price an American stock option using an EQP equity tree by loading the file deriv.mat, which provides

EQPTree. The EQPTree structure contains the stock specification and time information needed to price the American option.

```
load deriv.mat
OptSpec = 'Call';
Strike = 105;
Settle = '01-Jan-2003';
ExerciseDates = '01-Jan-2006';
AmericanOpt = 1;
Price = optstockbyeqp(EQPTree, OptSpec, Strike, Settle, ...
ExerciseDates, AmericanOpt)
```

Price =
12.2632

## Price a Bermudan Stock Option Using a EQP Equity Tree

Load the file deriv.mat, which provides EQPTree. The EQPTree structure contains the stock specification and time information needed to price the Bermudan option.

```
load deriv.mat;
% Option
OptSpec = 'Call';
Strike = 105;
Settle = '01-Jan-2003';
ExerciseDatesBerm={'15-Jan-2004','15-Jul-2004','15-Jan-2005','15-Jul-2005
Price the Bermudan option.
Price= optstockbyeqp(EQPTree, OptSpec, Strike, Settle, ExerciseDatesBerm)
```

Warning: Some ExerciseDates are not aligned with tree nodes. Result w: approximated.

Price =
12.0255

## See Also <br> eqptree | instoptstock

## optstockbyitt

## Purpose

Price options on stocks using implied trinomial tree (ITT)

```
Syntax [Price, PriceTree] = optstockbyitt(ITTTree, OptSpec, Strike,
Settle, ExerciseDates, AmericanOpt)
```


## Arguments

ITTTree Stock tree structure created by itttree.
OptSpec Number of instruments (NINST)-by-1 cell array of strings 'call' or 'put'.

Note The interpretation of the Strike and ExerciseDates arguments depends on the setting of the AmericanOpt argument. If AmericanOpt $=0, \mathrm{NaN}$, or is unspecified, the option is a European or Bermuda option. If AmericanOpt $=1$, the option is an American option.
$\left.\begin{array}{ll}\text { Strike } & \begin{array}{l}\text { European option: NINST-by-1 vector of strike } \\ \text { price values. }\end{array} \\ \begin{array}{l}\text { Bermuda option: NINST by number of strikes } \\ \text { (NSTRIKES) matrix of strike price values. }\end{array} \\ \text { Each row is the schedule for one option. If } \\ \text { an option has fewer than NSTRIKES exercise } \\ \text { opportunities, the end of the row is padded with }\end{array}\right\}$
$\left.\begin{array}{ll}\text { ExerciseDates } & \begin{array}{l}\text { For a European or Bermuda option: } \\ \\ \text { NINST-by-1 (European option) or } \\ \text { NINST-by-NSTRIKES (Bermuda option) } \\ \text { matrix of exercise dates. Each row is the } \\ \text { schedule for one option. For a European option, } \\ \text { there is only one exercise date, the option expiry } \\ \text { date. }\end{array} \\ \text { For an American option: } \\ & \text { NINST-by-2 vector of exercise date boundaries. } \\ & \begin{array}{l}\text { For each instrument, the option can be exercised } \\ \text { on any coupon date between or including the pair } \\ \text { of dates on that row. If only one non-NaN date } \\ \text { is listed, or if ExerciseDates is NINST-by-1, the }\end{array} \\ \text { option can be exercised between the underlying }\end{array}\right\}$

Note Data arguments for optstockbyitt are NINST-by-1 vectors, scalar, or empty. Fill unspecified entries in vectors with NaN. Only one data argument is required to create the instrument; the others may be omitted or passed as empty matrices [].

## Description

[Price, PriceTree] = optstockbyitt(ITTTree, OptSpec, Strike,
Settle, ExerciseDates, AmericanOpt) computes the price of a European/Bermuda or American stock option.

Price is a NINST-by-1 vector of expected option prices at time 0 .

## optstockbyitt

PriceTree is a tree structure with a vector of instrument prices at each node.

> Note The Settle date for every option is set to the ValuationDate of the stock tree. The option argument, Settle, is ignored.

## Examples Price an American Stock Option Using an ITT Equity Tree

This example shows how to price an American stock option using an ITT equity tree by loading the file deriv.mat, which provides the ITTTree. The ITTTree structure contains the stock specification and time information needed to price the American option.

```
load deriv.mat
```

OptSpec = 'Put';
Strike = 30;
Settle = '01-Jan-2006';
ExerciseDates = ' 01-Jan-2010 ';
AmericanOpt = 1;
Price = optstockbyitt(ITTTree, OptSpec, Strike, Settle,ExerciseDates, Ame
Price =
0.1271

## Price a Bermudan Stock Option Using an ITT Equity Tree

Load the file deriv.mat, which provides an ITTTree. The ITTTree structure contains the stock specification and time information needed to price the Bermudan option.

```
load deriv.mat;
% Option
OptSpec = 'Put';
Strike = 30;
Settle = '01-Jan-2006';
ExerciseDatesBerm={'1-Jan-2007','1-Jul-2007','1-Jan-2008','1-Jul-2008'};
```

Price the Bermudan option.

```
Price = optstockbyitt(ITTTree, OptSpec, Strike, Settle, ExerciseDates
```

Warning: Some ExerciseDates are not aligned with tree nodes. Result will be approximated.
> In procoptions at 171
In optstockbystocktree at 22
In optstockbyitt at 68
Price $=$
0.0664

## References <br> Chriss, Neil A., E. Derman, and I. Kani, "Implied trinomial trees of the volatility smile," Journal of Derivatives, 1996.

See Also<br>instoptstock | itttree | stockoptspec

## optstockbylr

Purpose Price options on stocks using Leisen-Reimer binomial tree model

```
Syntax [Price, PriceTree] = optstockbylr(LRTree, OptSpec, Strike,
Settle, ExerciseDates)
[Price, PriceTree] = optstockbylr(LRTree, OptSpec, Strike,
Settle, ExerciseDates, Name, Value)
[Price, PriceTree] = optstockbylr(LRTree, OptSpec, Strike, Settle, ExerciseDates)
[Price, PriceTree] = optstockbylr(LRTree, OptSpec, Strike, Settle, ExerciseDates, Name, Value)
```


## Description

## Input <br> Arguments

[Price, PriceTree] = optstockbylr(LRTree, OptSpec, Strike, Settle, ExerciseDates) computes option prices on stocks using the Leisen-Reimer binomial tree model.
[Price, PriceTree] = optstockbylr(LRTree, OptSpec, Strike, Settle, ExerciseDates, Name, Value) computes option prices on stocks using the Leisen-Reimer binomial tree model with additional options specified by one or more Name, Value pair arguments.

## LRTree

Stock tree structure created by lrtree.

## OptSpec

NINST-by-1 cell array of strings 'call' or 'put'.

## Strike

NINST-by-1 (European/American) or NINST-by-NSTRIKES (Bermuda) matrix of strike price values. Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.

## Settle

NINST-by- 1 matrix of settlement or trade dates.

Note The settle date for every option is set to the ValuationDate of the stock tree. The option argument, Settle, is ignored.

## ExerciseDates

NINST-by-1 (European/American) or NINST-by-NSTRIKEDATES (Bermuda) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one ExerciseDate on the option expiry date. For the American type, the option can be exercised on any tree data between the ValuationDate and tree maturity. The last element of each row must be the same as the maturity of the tree.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

## 'AmericanOpt'

NINST-by-1 flags with a value of 0 (European/Bermuda) or 1 (American).

## Default: 0

## Output Arguments

## Examples

## Price

NINST-by- 1 expected prices at time 0 .

## PriceTree

Tree structure with a vector of instrument prices at each node.

## Price Options on Stocks Using the Leisen-Reimer Binomial Tree Model

This example shows how to price options on stocks using the Leisen-Reimer binomial tree model. Consider European call and put options with an exercise price of $\$ 95$ that expire on July 1, 2010. The underlying stock is trading at $\$ 100$ on January 1, 2010, provides a continuous dividend yield of $3 \%$ per annum and has a volatility of $20 \%$ per annum. The annualized continuously compounded risk-free rate is

8\% per annum. Using this data, compute the price of the options using the Leisen-Reimer model with a tree of 15 and 55 time steps.

```
AssetPrice = 100;
Strike = 95;
ValuationDate = 'Jan-1-2010';
Maturity = 'July-1-2010';
% define StockSpec
Sigma = 0.2;
DividendType = 'continuous';
DividendAmounts = 0.03;
StockSpec = stockspec(Sigma, AssetPrice, DividendType, DividendAmounts);
% define RateSpec
Rates = 0.08;
Settle = ValuationDate;
Basis = 1;
Compounding = -1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', Settle
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Basis'
% build the Leisen-Reimer (LR) tree with 15 and 55 time steps
LRTimeSpec15 = lrtimespec(ValuationDate, Maturity, 15);
LRTimeSpec55 = lrtimespec(ValuationDate, Maturity, 55);
% use the PP2 method
LRMethod = 'PP2';
LRTree15 = lrtree(StockSpec, RateSpec, LRTimeSpec15, Strike, 'method', 
LRTree55 = lrtree(StockSpec, RateSpec, LRTimeSpec55, Strike, 'method', L
% price the call and the put options using the LR model:
OptSpec = {'call'; 'put'};
```

```
PriceLR15 = optstockbylr(LRTree15, OptSpec, Strike, Settle, Maturity)
PriceLR55 = optstockbylr(LRTree55, OptSpec, Strike, Settle, Maturity)
% calculate price using the Black-Scholes model (BLS) to compare value
% the LR model:
PriceBLS = optstockbybls(RateSpec, StockSpec, Settle, Maturity, OptSp
% compare values of BLS and LR
[PriceBLS PriceLR15 PriceLR55]
% use treeviewer to display LRTree of 15 time steps
treeviewer(LRTree15)
ans =
\begin{tabular}{lll}
9.7258 & 9.7252 & 9.7257 \\
2.4896 & 2.4890 & 2.4895
\end{tabular}
```


## optstockbylr




References
Leisen D.P., M. Reimer, "Binomial Models for Option Valuation Examining and Improving Convergence," Applied Mathematical Finance, Number 3, 1996, pp. 319-346.

See Also
| instoptstock | lrtree | optstocksensbylr

| Purpose | Price European, Bermudan, or American vanilla options using Longstaff-Schwartz model |
| :---: | :---: |
| Syntax | ```Price = optstockbyls(RateSpec,StockSpec,OptSpec,StrikeSettle, ExerciseDates) Price = optstockbyls(___,Name,Value) [Price,Path,Times,Z] = optstockbyls(RateSpec,StockSpec,OptSpec,Strikes ExerciseDates) [Price,Path,Times,Z] = optstockbyls( __ ,Name,Value)``` |
| Description | Price = <br> optstockbyls(RateSpec,StockSpec, OptSpec, StrikeSettle, <br> ExerciseDates) returns vanilla option prices using the <br> Longstaff-Schwartz model. optstockbyls computes prices of European, Bermudan, and American vanilla options. For American and Bermudan options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium. |
|  | Price $=$ optstockbyls( $\qquad$ , Name, Value) returns vanilla option prices using the Longstaff-Schwartz model using optional name-value pair arguments. optstockbyls computes prices of European, Bermudan, and American vanilla options. For American and Bermudan options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium. |
|  | [Price,Path,Times, Z] = optstockbyls(RateSpec,StockSpec, OptSpec,StrikeSettle, ExerciseDates) returns vanilla option prices using the Longstaff-Schwartz model. optstockbyls computes prices of European, Bermudan, and American vanilla options. For American and Bermudan options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium. |

## optstockbyls

## Input Arguments

[Price,Path,Times, Z] = optstockbyls( __ , Name, Value) returns vanilla option prices using the Longstaff-Schwartz model using optional name-value pair arguments. optstockbyls computes prices of European, Bermudan, and American vanilla options. For American and Bermudan options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium.

## RateSpec - Interest-rate term structure <br> structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from intenvset. For information on the interest-rate specification, see intenvset.

## Data Types <br> struct

## StockSpec - Stock specification for underlying asset <br> structure

Stock specification for underlying asset, specified using StockSpec obtained from stockspec. For information on the stock specification, see stockspec.
stockspec can handle other types of underlying assets. For example, stocks, stock indices, and commodities.

## Data Types

struct

## OptSpec - Definition of option

string with value 'call' or 'put'
Definition of option, specified as 'call' or 'put' using a string.
Data Types
char

## Strike - Option strike price values

nonnegative scalar integer

Option strike price value, specified with nonnegative scalar integer:

- For a European option, use a scalar of strike price.
- For a Bermuda option, use a 1 -by-NSTRIKES vector of strike prices.
- For an American option, use a scalar of strike price.


## Data Types

single | double

## Settle - Settlement date or trade date

date string | nonnegative scalar integer
Settlement date or trade date for the vanilla option, specified as a date string or nonnegative scalar integer.

## Data Types

char

## ExerciseDates - Option exercise dates

date string | nonnegative integer
Option exercise dates, specified as a date string or nonnegative scalar integer:

- For a European option, use a 1-by-1 vector of dates. For a European option, there is only one ExerciseDates on the option expiry date.
- For a Bermuda option, use a 1-by-NSTRIKES vector of dates.
- For an American option, use a 1-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is a 1-by-1 vector of serial date numbers or cell array of strings, the option can be exercised between Settle and the single listed ExerciseDates.


## Data Types

char

## optstockbyls

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

Example: Price =
optstockbyls(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates, 'Amer

## 'AmericanOpt' - Option type

0 European or Bermuda (default) | scalar with values [0,1]
Option type, specified as positive integer scalar flags with values:

- 0 - European or Bermuda
- 1 - American

Data Types
single | double

## 'NumTrials' - Simulation trials

## 1000 (default) | scalar

Simulation trials, specified as a scalar number of independent sample paths.

## Data Types

double

## 'NumPeriods' - Simulation periods per trial <br> 100 (default) | scalar

Simulation periods per trial, specified as a scalar number. NumPeriods is considered only when pricing European vanilla options. For American and Bermuda vanilla options, NumPeriod is equal to the number of Exercise days during the life of the option.

## Data Types <br> double

## ' $\mathbf{Z}$ ' - Dependent random variates

## TBD (default) | scalar | nonnegative integer

Dependent random variates used to generate the Brownian motion vector (i.e., Wiener processes) that drive the simulation, specified as a NumPeriods-by-2-by-NumTrials 3-D time series array.
Data Types
single | double

## 'Antithetic' - Indicator for antithetic sampling

false (default) | logical flag with value of true or false
Indicator for antithetic sampling, specified with a value of true or false.

## Data Types

logical

## Output Arguments

## Price - Expected price of vanilla option

scalar
Expected price of the vanilla option, returned as a 1-by-1 scalar.

## Path - Simulated paths of correlated state variables <br> vector

Simulated paths of correlated state variables, returned as a
(NumPeriods +1 )-by-2-by-NumTrials 3-D time series array. Each row of Paths is the transpose of the state vector $X(t)$ at time $t$ for a given trial.

## Times - Observation times associated with simulated paths vector

Observation times associated with simulated paths, returned as a (NumPeriods +1 )-by- 1 column vector of observation times associated with the simulated paths. Each element of Times is associated with the corresponding row of Paths.

## Z-Dependent random variates

## optstockbyls

## vector

Dependent random variates, if $Z$ is specified as an optional input argument, the same value is returned. Otherwise, $Z$ contains the random variates generated internally.

## Examples Compute the Price of a Vanilla Option

Define the RateSpec.

```
StartDates = 'Jan-1-2013';
EndDates = 'Jan-1-2015';
Rates = 0.05;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDate
'EndDates', EndDates, 'Rates', Rates)
```

RateSpec =

```
            FinObj: 'RateSpec'
        Compounding: 2
            Disc: 0.9060
            Rates: 0.0500
            EndTimes: 4
        StartTimes: 0
            EndDates: 735965
            StartDates: 735235
ValuationDate: 735235
            Basis: 0
    EndMonthRule: 1
```

Define the StockSpec for the asset.

```
AssetPrice = 100;
```

Sigma = 0.1;
StockSpec = stockspec(Sigma, AssetPrice)

## StockSpec =

FinObj: 'StockSpec'<br>Sigma: 0.1000<br>AssetPrice: 100<br>DividendType: []<br>DividendAmounts: 0<br>ExDividendDates: []

Define the vanilla option.

```
OptSpec = 'put';
Settle = 'Jan-1-2013';
ExerciseDates = 'Jan-1-2015';
Strike = 105;
```

Compute the vanilla option price using the Longstaff-Schwartz model.
Antithetic = true;
Price = optstockbyls(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates, 'Antithetic', Antithetic)

Price =
3.2292

See Also
optstocksensbyls

## Concepts

- "Vanilla Option" on page 3-36


## optstocksensbyls



## Input Arguments

[PriceSens, Path, Times, Z] = optstocksensbyls(__, Name, Value) returns vanilla option prices and sensitivities using the Longstaff-Schwartz model and using optional name-value pair arguments. optstockbyls computes prices of European, Bermudan, and American vanilla options. For American and Bermudan options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium.

## RateSpec - Interest-rate term structure <br> structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from intenvset. For information on the interest-rate specification, see intenvset.

## Data Types

struct

## StockSpec - Stock specification for underlying asset

structure
Stock specification for underlying asset, specified using StockSpec obtained from stockspec. For information on the stock specification, see stockspec.
stockspec can handle other types of underlying assets. For example, stocks, stock indices, and commodities.

## Data Types

struct

## OptSpec - Definition of option

string values 'call' or 'put'
Definition of option, specified as 'call' or 'put' using a string.
Data Types
char

## Strike - Option strike price value

## optstocksensbyls

nonnegative scalar integer
Option strike price value, specified with a nonnegative scalar integer:

- For a European option, use a scalar of strike price.
- For a Bermuda option, use a 1-by-NSTRIKES vector of strike price.
- For an American option, use a scalar of strike price.


## Data Types

single | double

## Settle - Settlement date or trade date

date string | nonnegative scalar integer
Settlement date or trade date for the vanilla option, specified as a date string or a nonnegative integer.

## Data Types

char

## ExerciseDates - Option exercise date

date string | nonnegative scalar integer
Option exercise date, specified as a date string or nonnegative scalar integer:

- For a European option, use a 1-by-1 vector of dates. For a European option, there is only one ExerciseDates on the option expiry date.
- For a Bermuda option, use a 1-by-NSTRIKES vector of dates.
- For an American option, use a 1-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non- NaN date is listed, or if ExerciseDates is a 1-by-1 vector of serial date numbers or cell array of strings, the option can be exercised between Settle and the single listed ExerciseDates.


## Data Types <br> char

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

Example: Price = optstocksensbyls(RateSpec,StockSpec, OptSpec,Strike,Settle,ExerciseDates,'AmericanOpt', '1', 'NumTrials', '200

## 'AmericanOpt' - Option type

0 European or Bermuda (default) | scalar with values [ 0,1 ]
Option type, specified as a positive integer scalar flag with values:

- 0 - European or Bermuda
- 1 - American

Data Types
single | double

## 'NumTrials' - Simulation trials

1000 (default) | scalar
Simulation trials, specified as a scalar number of independent sample paths.

Data Types
double

## 'NumPeriods' - Simulation periods per trial <br> 100 (default) | scalar

Simulation periods per trial, specified as a scalar number. NumPeriods is considered only when pricing European vanilla options. For American and Bermuda vanilla options, NumPeriod is equal to the number of Exercise days during the life of the option.

## Data Types

double

## optstocksensbyls

## 'Z' - Dependent random variates

## scalar | nonnegative integer

Dependent random variates used to generate the Brownian motion vector (i.e., Wiener processes) that drive the simulation, specified as a NumPeriods-by-2-by-NumTrials 3-D time series array.

## Data Types

single | double

## 'Antithetic' - Indicator for antithetic sampling

false (default) | logical flag with value of true or false
Indicator for antithetic sampling, specified with a value of true or false.

## Data Types

logical

## 'OutSpec' - Define outputs

\{'Price'\} (default) | string with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | cell array of strings with values: 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

Define outputs specifying NOUT- by-1 or 1-by-NOUT cell array of strings with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta' and 'All'.

OutSpec = \{'All'\} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec to include each sensitivity:

Example: OutSpec =
\{'delta','gamma', 'vega','lambda','rho','theta','price'\}

## Data Types <br> char | cell

## Output Arguments

## Examples Compute the Price and Sensitivities of a Vanilla Option

## Define the RateSpec.

```
StartDates = 'Jan-1-2013';
EndDates = 'Jan-1-2015';
Rates = 0.05;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', Start[
'EndDates', EndDates, 'Rates', Rates)
```

```
RateSpec =
    FinObj: 'RateSpec'
    Compounding: 2
            Disc: 0.9060
            Rates: 0.0500
        EndTimes: 4
        StartTimes: 0
        EndDates: 735965
        StartDates: 735235
ValuationDate: 735235
            Basis: 0
EndMonthRule: 1
```

Define the StockSpec for the asset.

```
AssetPrice = 100;
Sigma = 0.1;
DivType = 'continuous';
DivAmounts = 0.04;
StockSpec = stockspec(Sigma, AssetPrice, DivType, DivAmounts)
```

StockSpec =

```
                    FinObj: 'StockSpec'
                    Sigma: 0.1000
        AssetPrice: 100
    DividendType: {'continuous'}
    DividendAmounts: 0.0400
    ExDividendDates: []
```

Define the vanilla option.

```
OptSpec = 'call';
```

```
Settle = 'jan-1-2013';
ExerciseDates = 'jan-1-2015';
Strike = 105;
```

Compute the Delta sensitivity for the vanilla option using the Longstaff-Schwartz model.

```
Antithetic = true;
OutSpec = {'Delta'};
PriceSens = optstocksensbyls(RateSpec, StockSpec, OptSpec, Strike,
Settle, ExerciseDates,'Antithetic', Antithetic, 'OutSpec', OutSpec)
```


## PriceSens =

0.3945

## See Also <br> optstockbyls

Concepts • "Vanilla Option" on page 3-36

## optstockbyrgw

| Purpose | Determine American call option prices using Roll-Geske-Whaley option <br> pricing model |
| :--- | :--- |
| Syntax | Price = optstockbyrgw(RateSpec, StockSpec, Settle, <br> Maturity, Strike) |

Arguments

| RateSpec | The annualized continuously compounded rate <br> term structure. For information on the interest <br> rate specification, see intenvset. |
| :--- | :--- |
| StockSpec | Stock specification. See stockspec. |
| Settle | NINST-by-1 vector of settlement or trade dates. |
| Maturity | NINST-by-1 vector of maturity dates. |
| Strike | NINST-by-1 vector of strike price values. |

## Description Price = optstockbyrgw(RateSpec, StockSpec, Settle, Maturity, Strike) computes the American call option prices using the Roll-Geske-Whaley option pricing model.

Price is a NINST-by-1 vector of expected call option prices.

Note optstockbyrgw computes prices of American calls with a single cash dividend using the Roll-Geske-Whaley option pricing model.

## Examples Determine American Call Option Prices Using Roll-Geske-Whaley Option Pricing Model

This example shows how to determine American call option prices using Roll-Geske-Whaley option pricing model. Consider an American call option with an exercise price of $\$ 22$ that expires on February 1, 2009. The underlying stock is trading at $\$ 20$ on June 1, 2008 and has a
volatility of $20 \%$ per annum. The annualized continuously compounded risk-free rate is $6.77 \%$ per annum. The stock pays a single dividend of $\$ 2$ on September 1, 2008. Using this data, compute price of the American call option using the Roll-Geske-Whaley option pricing model.

```
Settle = 'Jun-01-2008';
Maturity = 'Feb-01-2009';
AssetPrice = 20;
Strike = 22;
Sigma = 0.2;
Rate = 0.0677;
DivAmount = 2;
DivDate = 'Sep-01-2008';
% define the RateSpec and StockSpec
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,
Maturity, 'Rates', Rate, 'Compounding', -1, 'Basis', 0);
StockSpec = stockspec(Sigma, AssetPrice, {'cash'}, DivAmount, DivDate)
Price = optstockbyrgw(RateSpec, StockSpec, Settle, Maturity,Strike)
```

Price =
0.3359

See Also
impvbyrgw | intenvset | optstocksensbyrgw | stockspec

## optstocksensbybjs

## Purpose

Syntax
PriceSens = optstocksensbybjs(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, 'Name1', Value1...)

## Arguments

| RateSpec | The annualized continuously compounded rate term structure. For information on the interest rate specification, see intenvset. |
| :---: | :---: |
| StockSpec | Stock specification. See stockspec. |
| Settle | NINST-by-1 vector of settlement or trade dates. |
| Maturity | NINST-by-1 vector of maturity dates. |
| OptSpec | NINST-by-1 cell array of strings 'call' or 'put'. |
| Strike | NINST-by-1 vector of strike price values. |
| OutSpec | (Optional) All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string followed by the corresponding parameter value. Parameter name/value pairs may be specified in any order; names are case-insensitive and partial string matches are allowed provided no ambiguities exist. Valid parameter names are: <br> - NOUT-by-1 or 1-by-NOUT cell array of strings indicating the nature and order of the outputs for the function. Possible values are: 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', or 'All'. |
|  | For example, OutSpec = \{'Price'; 'Lambda'; 'Rho'\} specifies that the output should be Price, Lambda, and Rho, in that order. |

```
To invoke from a function: [Price, Lambda, Rho] = optstocksensbybjs(..., 'OutSpec', \{'Price', 'Lambda', 'Rho'\})
OutSpec = \{'All'\} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec as OutSpec = \{'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', 'Price'\};.
```

- Default is OutSpec $=\{$ 'Price' $\}$.


## Description

PriceSens = optstocksensbybjs(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, 'Name1', Value1...) computes American option prices and sensitivities using the Bjerksund-Stensland 2002 option pricing model.
optstocksensbybjs can be used to compute six sensitivities for the Bjerksund-Stensland 2002 model: delta, gamma, vega, lambda, rho, and theta. This function is also capable of returning the price of the option. The selection of output parameters and their order is determined by the optional input parameter OutSpec. This parameter is a cell array of strings, each one specifying a desired output parameter. The order in which these output parameters are returned by the function is the same as the order of the strings contained in OutSpec.

PriceSens is a NINST-by-1 vector of expected prices or sensitivities values.

## optstocksensbybjs

Note optstocksensbybjs computes prices of American options with continuous dividend yield using the Bjerksund-Stensland option pricing model. All sensitivities are evaluated by computing a discrete approximation of the partial derivative. This means that the option is revalued with a fractional change for each relevant parameter, and the change in the option value divided by the increment, is the approximated sensitivity value.

## Examples

## Compute American Option Prices and Sensitivities Using the Bjerksund-Stensland 2002 Option Pricing Model

This example shows how to compute American option prices and sensitivities using the Bjerksund-Stensland 2002 option pricing model. Consider four American put options with an exercise price of $\$ 100$. The options expire on October 1, 2008. Assume the underlying stock pays a continuous dividend yield of $4 \%$ and has a volatility of $40 \%$ per annum. The annualized continuously compounded risk-free rate is $8 \%$ per annum. Using this data, calculate the delta, gamma, and price of the American put options, assuming the following current prices of the stock on July 1, 2008: $\$ 90, \$ 100, \$ 110$ and $\$ 120$.

```
Settle = 'July-1-2008';
Maturity = 'October-1-2008';
Strike = 100;
AssetPrice = [90;100;110;120];
Rate = 0.08;
Sigma = 0.40;
DivYield = 0.04;
% define the RateSpec and StockSpec
StockSpec = stockspec(Sigma, AssetPrice, {'continuous'}, DivYield);
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rate, 'Compounding', -1);
```

```
% define the option type
OptSpec = {'put'};
OutSpec = {'Delta', 'Gamma', 'Price'};
[Delta, Gamma, Price] = optstocksensbybjs(RateSpec, StockSpec, Settle
OptSpec, Strike, 'OutSpec', OutSpec)
Delta =
    -0.6572
    -0.4434
    -0.2660
    -0.1442
Gamma =
    0.0217
    0.0202
    0.0150
    0.0095
Price =
    12.9467
    7.4571
    3.9539
    1.9495
```

References Bjerksund, P. and G. Stensland, Closed-Form Approximation of American Options, Scandinavian Journal of Management, 1993, Vol. 9, Suppl., pp. S88-S99.

Bjerksund, P. and G. Stensland, Closed Form
Valuation of American Options, Discussion paper 2002
(http://brage.bibsys.no/nhh/bitstream/URN:NBN:no-bibsys_brage_22301/1/bjerksund\%ء
See Also impvbybjs | intenvset | optstockbybjs | stockspec

## Purpose

Syntax

Arguments

Determine option prices and sensitivities on futures using Black pricing model

```
PriceSens = optstocksensbyblk(RateSpec, StockSpec, Settle,
Maturity, OptSpec, Strike, 'Name1', Value1...)
```

| RateSpec | The annualized continuously compounded rate term structure. For information on the interest rate specification, see intenvset. |
| :---: | :---: |
| StockSpec | Stock specification. See stockspec. |
| Settle | NINST-by-1 vector of settlement or trade dates. |
| Maturity | NINST-by-1 vector of maturity dates. |
| OptSpec | NINST-by-1 cell array of strings 'call' or 'put'. |
| Strike | NINST-by-1 vector of strike price values. |
| OutSpec | (Optional) All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. Parameter name/value pairs may be specified in any order; names are case-insensitive and partial string matches are allowed provided no ambiguities exist. Valid parameter names are: |

- NOUT-by-1 or 1-by-NOUT cell array of strings indicating the nature and order of the outputs for the function. Possible values are: 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', or 'All'.

For example, OutSpec = \{'Price'; 'Lambda'; 'Rho'\} specifies that the output should be Price, Lambda, and Rho, in that order.

## optstocksensbyblk

To invoke from a function: [Price, Lambda, Rho] = optstocksensbyblk(..., 'OutSpec', \{'Price', 'Lambda', 'Rho'\})

OutSpec = \{'All'\} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec as OutSpec = \{'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', 'Price'\};.

- Default is OutSpec $=\{$ 'Price' $\}$.

Description PriceSens = optstocksensbyblk(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, 'Name1', Value1...) computes option prices and sensitivities on futures using the Black pricing model.

PriceSens is a NINST-by-1 vector of expected future prices or sensitivities values.

## Examples Compute Option Prices and Sensitivities on Futures Using the Black Pricing Model

This example shows how to compute option prices and sensitivities on futures using the Black pricing model. Consider a European put option on a futures contract with an exercise price of $\$ 60$ that expires on June 30, 2008. On April 1, 2008 the underlying stock is trading at $\$ 58$ and has a volatility of $9.5 \%$ per annum. The annualized continuously compounded risk-free rate is $5 \%$ per annum. Using this data, compute delta, gamma, and the price of the put option.

```
AssetPrice = 58;
Strike = 60;
Sigma = .095;
Rates = 0.05;
Settle = 'April-01-08';
Maturity = 'June-30-08';
```

```
% define the RateSpec and StockSpec
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,
Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', 1);
StockSpec = stockspec(Sigma, AssetPrice);
% define the options
OptSpec = {'put'};
OutSpec = {'Delta','Gamma','Price'};
[Delta, Gamma, Price] = optstocksensbyblk(RateSpec, StockSpec, Settle
Maturity, OptSpec, Strike,'OutSpec', OutSpec)
Delta =
    -0.7469
Gamma =
    0.1130
    Price =
    2.3569
```

See Also
impvbyblk | intenvset | optstockbyblk | stockspec

## optstocksensbybls

## Purpose

Syntax

## Arguments

Determine option prices and sensitivities using Black-Scholes option pricing model

PriceSens = optstocksensbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, 'Name1', Value1...)

RateSpec The annualized continuously compounded rate term structure. For information on the interest rate specification, see intenvset.

StockSpec Stock specification. See stockspec.

Settle
Maturity
OptSpec
Strike
OutSpec

NINST-by-1 vector of settlement or trade dates.
NINST-by- 1 vector of maturity dates.
NINST-by-1 cell array of strings 'call' or 'put'.
NINST-by-1 vector of strike price values.
(Optional) All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. Parameter name/value pairs may be specified in any order; names are case-insensitive and partial string matches are allowed provided no ambiguities exist. Valid parameter names are:

- NOUT-by-1 or 1-by-NOUT cell array of strings indicating the nature and order of the outputs for the function. Possible values are: 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', or 'All'.

For example, OutSpec = \{'Price'; 'Lambda'; 'Rho'\} specifies that the output should be Price, Lambda, and Rho, in that order.

> To invoke from a function: [Price, Lambda, Rho] = optstocksensbybls(..., 'OutSpec', \{'Price', 'Lambda', 'Rho'\})
> OutSpec = \{'All'\} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec as OutSpec = \{'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', 'Price'\};.

- Default is OutSpec = \{'Price'\}.

Description
PriceSens = optstocksensbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, 'Name1', Value1...) computes option prices and sensitivities using the Black-Scholes option pricing model.

PriceSens is a NINST-by-1 vector of expected prices or sensitivities values.

## optstocksensbybls

Note When using StockSpec with optstocksensbybls, you can modify StockSpec to handle other types of underliers when pricing instruments that use the Black-Scholes model.

When pricing Futures (Black model), enter the following in StockSpec:
DivType = 'Continuous';
DivAmount = RateSpec.Rates;

When pricing Foreign Currencies (Garman-Kohlhagen model), enter the following in StockSpec:

DivType = 'Continuous';
DivAmount = ForeignRate;
where ForeignRate is the continuously compounded, annualized risk free interest rate in the foreign country.

## Examples Compute Option Prices and Sensitivities Using the Black-Scholes Option Pricing Model

This example shows how to compute option prices and sensitivities using the Black-Scholes option pricing model. Consider a European call and put options with an exercise price of $\$ 30$ that expires on June 1, 2008. The underlying stock is trading at $\$ 30$ on January 1, 2008 and has a volatility of $30 \%$ per annum. The annualized continuously compounded risk-free rate is $5 \%$ per annum. Using this data, compute the delta, gamma, and price of the options using the Black-Scholes model.

AssetPrice = 30;
Strike = 30;
Sigma = .30;
Rates = 0.05;
Settle = 'January-01-2008';
Maturity = 'June -01-2008';

```
% define the RateSpec and StockSpec
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,
Maturity, 'Rates', Rates, 'Compounding',-1, 'Basis', 1);
StockSpec = stockspec(Sigma, AssetPrice);
% define the options
OptSpec = {'call', 'put'};
OutSpec = {'Delta','Gamma','Price'};
[Delta, Gamma, Price] = optstocksensbybls(RateSpec, StockSpec, Settle
Maturity, OptSpec, Strike,'OutSpec', OutSpec)
Delta =
    0.5810
    -0.4190
```

    Gamma =
    0.0673
    0.0673
    Price =
    2.6126
    1.9941
    See Also
impvbybls | intenvset | optstockbybls | stockspec

## optstocksensbylr

| Purpose | Determine option prices and sensitivities using Leisen-Reimer binomial <br> tree model |
| :--- | :--- |
| Syntax | PriceSens = optstocksensbylr(LRTree, OptSpec, Strike, Settle, <br> ExerciseDates) <br> PriceSens = optstocksensbylr(LRTree, OptSpec, Strike, Settle, <br> ExerciseDates, Name, Value) |
| Description | PriceSens = optstocksensbylr(LRTree, OptSpec, Strike, <br> Settle, <br> ExerciseDates) calculates option prices and sensitivities using a <br> Leisen-Reimer binomial tree model. |
|  | PriceSens = optstocksensbylr(LRTree, OptSpec, Strike, <br> Settle, |
| ExerciseDates, Name, Value) constructs a Leisen-Reimer stock tree |  |
| with additional options specified by one or more Name, Value pair |  |
| arguments. |  |

NINST-by-1 cell array of strings 'call' or 'put'.

## Strike

NINST-by-1 (European/American) or NINST-by-NSTRIKES (Bermuda) matrix of strike price values. Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.

## Settle

NINST-by-1 matrix of settlement or trade dates.

## ExerciseDates

NINST-by-1(European/American) or NINST-by-NSTRIKEDATES (Bermuda) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one ExerciseDate on the option expiry date. For the American type, the option can be exercised on any tree data between the ValuationDate and tree maturity. The last element of each row must be the same as the maturity of the tree.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

## 'AmericanOpt'

NINST-by- 1 flags with values of 0 (European/Bermuda) or 1 (American).
Default: 0

## 'OutSpec'

NOUT-by-1 or 1-by-NOUT cell array of strings indicating the nature and order of the outputs for the function. Possible values are Price, Delta, Gamma, Vega, Lambda, Rho, and All.

Default: Price

## Output Arguments

## Examples

## PriceSens

NINST-by-1 expected prices or sensitivities values.

## Compute Option Prices and Sensitivities Using a Leisen-Reimer Binomial Tree Model

This example shows how to compute option prices and sensitivities using a Leisen-Reimer binomial tree model. Consider European call and put options with an exercise price of $\$ 100$ that expire on December 1, 2010. The underlying stock is trading at $\$ 100$ on June 1, 2010 and has a

## optstocksensbylr

volatility of $30 \%$ per annum. The annualized continuously compounded risk-free rate is $7 \%$ per annum. Using this data, compute the price, delta and gamma of the options using the Leisen-Reimer model with a tree of 25 time steps and the PP2 method.

```
AssetPrice = 100;
```

Strike = 100;
ValuationDate = 'June-1-2010';
Maturity = 'December-1-2010';
\% define StockSpec
Sigma = 0.3;
StockSpec = stockspec(Sigma, AssetPrice);
\% define RateSpec
Rates = 0.07;
Settle = ValuationDate;
Basis = 1;
Compounding = -1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', Settle
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Basis'
\% build the Leisen-Reimer (LR) tree with 25 time steps
LRTimeSpec = lrtimespec(ValuationDate, Maturity, 25);
\% use the PP2 method
LRMethod = 'PP2';
TreeLR = lrtree(StockSpec, RateSpec, LRTimeSpec, Strike, 'method', LRMetr
\% compute prices and sensitivities using the LR model:
OptSpec = \{'call'; 'put'\};
OutSpec = \{'Price', 'Delta', 'Gamma'\};

```
[Price, Delta, Gamma] = optstocksensbylr(TreeLR, OptSpec, Strike, Set Maturity, 'OutSpec', OutSpec)
```

Price =
10.1332
6.6937

Delta =
0.6056
$-0.3944$

Gamma =
0.0185
0.0185

## References Leisen D.P., M. Reimer, "Binomial Models for Option Valuation Examining and Improving Convergence," Applied Mathematical Finance, Number 3, 1996, pp. 319-346.

See Also<br>| optstockbylr | lrtree

## optstocksensbyrgw

| Purpose | Determine American call option prices and sensitivities using <br> Roll-Geske-Whaley option pricing model |
| :--- | :--- |
| Syntax | PriceSens = optstocksensbyrgw(RateSpec, StockSpec, Settle, <br> Maturity, OptSpec, Strike, 'Name1', Value1...) |

## Arguments

| RateSpec | The annualized continuously compounded rate term structure. For information on the interest rate specification, see intenvset. |
| :---: | :---: |
| StockSpec | Stock specification. See stockspec. |
| Settle | NINST-by-1 vector of settlement or trade dates. |
| Maturity | NINST-by-1 vector of maturity dates. |
| OptSpec | NINST-by-1 cell array of strings 'call' or 'put'. |
| Strike | NINST-by-1 vector of strike price values. |
| OutSpec | (Optional) All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string followed by the corresponding parameter value. Parameter name/value pairs may be specified in any order; names are case-insensitive and partial string matches are allowed provided no ambiguities exist. Valid parameter names are: <br> - NOUT-by-1 or 1-by-NOUT cell array of strings indicating the nature and order of the outputs for the function. Possible values are: 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', or 'All'. |
|  | For example, OutSpec = \{'Price'; 'Lambda'; 'Rho'\} specifies that the output should be Price, Lambda, and Rho, in that order. |

```
To invoke from a function: [Price, Lambda, Rho] = optstocksensbyrgw(..., 'OutSpec', \{'Price', 'Lambda', 'Rho'\}) OutSpec = \{'All'\} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec as OutSpec \(=\) \{'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', 'Price'\};.
```

- Default is OutSpec $=\{$ 'Price' $\}$.


## Description

PriceSens = optstocksensbyrgw(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, 'Name1', Value1...) computes American call option prices and sensitivities using the Roll-Geske-Whaley option pricing model.

PriceSens is a NINST-by- 1 vector of expected prices or sensitivities values.

Note optstocksensbyrgw computes prices of American calls with a single cash dividend using the Roll-Geske-Whaley option pricing model. All sensitivities are evaluated by computing a discrete approximation of the partial derivative. This means that the option is revalued with a fractional change for each relevant parameter, and the change in the option value divided by the increment, is the approximated sensitivity value.

## Examples Compute American Call Option Prices and Sensitivities Using the Roll-Geske-Whaley Option Pricing Model

This example shows how to compute American call option prices and sensitivities using the Roll-Geske-Whaley option pricing model.

Consider an American stock option with an exercise price of $\$ 82$ on January 1, 2008 that expires on May 1, 2008. Assume the underlying stock pays dividends of $\$ 4$ on April 1, 2008. The stock is trading at $\$ 80$ and has a volatility of $30 \%$ per annum. The risk-free rate is $6 \%$ per annum. Using this data, calculate the price and the value of delta and gamma of the American call using the Roll-Geske-Whaley option pricing model.

```
AssetPrice = 80;
Settle = 'Jan-01-2008';
Maturity = 'May-01-2008';
Strike = 82;
Rate = 0.06;
Sigma = 0.3;
DivAmount = 4;
DivDate = 'Apr-01-2008';
% define the RateSpec and StockSpec
StockSpec = stockspec(Sigma, AssetPrice, {'cash'}, DivAmount, DivDate);
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rate, 'Compounding', -1, 'Basis', 1);
% define the OutSpec
OutSpec = {'Price', 'Delta', 'Gamma'};
[Price, Delta, Gamma] = optstocksensbyrgw(RateSpec, StockSpec, Settle,.
Maturity, Strike,'OutSpec', OutSpec)
```

Price =
4.3860

See Also impvbyrgw \| intenvset \| optstockbyrgw \| stockspec

## optpricebysim

## Purpose Price option given simulated underlying values

```
Syntax Price = optpricebysim(RateSpec,SimulatedPrices,Times,OptSpec,
Strike,ExerciseTimes)
Price = optpricebysim(___,Name,Value)
```


## Description

## Input Arguments

Price =
optpricebysim(RateSpec,SimulatedPrices,Times,OptSpec, Strike, ExerciseTimes) calculates the price of European, American, and Berumdan call/put options based on risk-neutral simulation of the underlying asset. For American and Bermudan options, the Longstaff-Schwartz least squares method is calculates the early exercise premium.

Price = optpricebysim( __ , Name, Value) calculates the price of European, American, and Berumdan call/put options based on risk-neutral simulation of the underlying asset using optional name-value pair arguments. For American options, the Longstaff-Schwartz least squares method calculates the early exercise premium.

## RateSpec - Interest-rate term structure of risk-free rates structure

Interest-rate term structure of risk-free rates (annualized and continuously compounded), specified by the RateSpec obtained from intenvset. The valuation date must be at the settlement date of the option, and the day-count basis and end-of-month rule must be the same as those used to calculate the Times input. For information on the interest-rate specification, see intenvset.

## Data Types

struct

## SimulatedPrices - Simulated prices

matrix

Simulated prices, specified using a (NumPeriods + 1)-by-NumTrials matrix of risk-neutral simulated prices. The first element of SimulatedPrices is the initial value at time 0 .

## Data Types

single | double

## Times - Annual time factors associated with simulated prices vector

Annual time factors associated with simulated prices, specified using a (NumPeriods +1 )-by- 1 column vector. Each element of Times is associated with the corresponding row of SimulatedPrices. The first element of Times must be 0 (current time).

## Data Types

single | double

## OptSpec - Definition of option

string with values 'call' or 'put'
Definition of option as 'call' or 'put', specified as a string.
Data Types
char

## Strike - Option strike price values

scalar | function handle
Option strike price values, specified as a scalar value Strike price. Strike for Bermudan options can be specified as a 1-by-NSTRIKES vector or a function handle that returns the value of the strike given the time of the strike.

Data Types
single | double | function_handle

## ExerciseTimes - Exercise time for option

nonnegative integer | vector of nonnegative integers | date string | cell array of date strings

## optpricebysim

Exercise time for the option, specified as a date string or a nonnegative integer as follows:

- For a European or Bermudan option, ExerciseTimes is a 1-by-1 (European) or 1-by-NSTRIKES (Bermudan) vector of exercise times. For a European option, there is only one ExerciseTimes on the option expiry date.
- For an American option, ExerciseTimes is a 1-by-2 vector of exercise time boundaries. The option exercises on any date between, or including, the pair of times on that row. If ExerciseTimes is 1-by-1, the option exercises between time 0 and the single listed ExerciseTimes.


## Data Types <br> char | cell

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

Example: Price = optpricebysim(RateSpec, Prices, Times, OptSpec, Settle, Strike, ExerciseTimes,

## 'AmericanOpt' - Option type

0 European or Bermudan (default) | scalar flag with value [0,1]
Option type, specified as an integer scalar flag with these values:

- 0 - European or Bermudan
- 1 - American

For American options, the Longstaff-Schwartz least squares method calculates the early exercise premium.
Data Types
single | double

## Output Price - Price of option

Arguments

## Examples

scalar
Price of the option, returned as a scalar value.

## Compute the Price of an American Option Using Monte Carlo Simulation Based on Geometric Brownian Motion

Define the option.

```
SO = 100; % Initial price of underlying asset
Sigma = .2; % Volatility of underlying asset
Strike = 110; % Strike
OptSpec = 'call'; % Call option
Settle = '1-Jan-2013'; % Settlement date of option
Maturity = '1-Jan-2014'; % Maturity date of option
r = .05; % Risk-free rate (annual, continuous compounding)
Compounding = -1; % Continuous compounding
Basis = 0; % Act/Act day count convention
T = yearfrac(Settle, Maturity, Basis); % Time to expiration in years
```

Set up the gbm object and run the Monte Carlo simulation based on Geometric Brownian Motion (GBM) using the simBySolution method from Financial Toolbox ${ }^{\text {TM }}$.

```
NTRIALS = 1000;
NPERIODS = daysact(Settle, Maturity);
dt = T/NPERIODS;
OptionGBM = gbm(r, Sigma, 'StartState', SO);
[Paths, Times, Z] = OptionGBM.simBySolution(NPERIODS, ...
'NTRIALS',NTRIALS, 'DeltaTime',dt,'Antithetic',true);
```

Create the interest-rate term structure to define RateSpec.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,
```


## optpricebysim

```
    'EndDates', Maturity, 'Rate', r, 'Compounding', Compounding,
    'Basis', Basis)
RateSpec =
    FinObj: 'RateSpec'
    Compounding: -1
        Disc: 0.9512
            Rates: 0.0500
        EndTimes: 1
    StartTimes: 0
        EndDates: 735600
        StartDates: 735235
        ValuationDate: 735235
        Basis: 0
    EndMonthRule: 1
```

Price an American option.
SimulatedPrices = squeeze(Paths);
OptPrice = optpricebysim(RateSpec, SimulatedPrices, Times, OptSpec, ... Strike, T, 'AmericanOpt', 1)

OptPrice =
6.2028

## Compute the Price of an American Asian Option Using Monte Carlo Simulation Based on Geometric Brownian Motion

Define the option.

```
SO = 100; % Initial price of underlying asset
```

```
Sigma = .2; % Volatility of underlying asset
Strike = 110; % Strike
OptSpec = 'call'; % Call option
Settle = '1-Jan-2013'; % Settlement date of option
Maturity = '1-Jan-2014'; % Maturity date of option
r = .05; % Risk-free rate (annual, continuous compounding)
Compounding = -1; % Continuous compounding
Basis = 0; % Act/Act day count convention
T = yearfrac(Settle, Maturity, Basis); % Time to expiration in years
```

Set up the gbm object and run the Monte Carlo simulation based on Geometric Brownian Motion (GBM) using the simBySolution method from Financial Toolbox ${ }^{\mathrm{TM}}$.

NTRIALS = 1000;
NPERIODS = daysact(Settle, Maturity); dt = T/NPERIODS;
OptionGBM = gbm(r, Sigma, 'StartState', SO);
[Paths, Times, Z] = OptionGBM.simBySolution(NPERIODS, ... 'NTRIALS',NTRIALS, 'DeltaTime',dt,'Antithetic',true);

Create the interest-rate term structure to define RateSpec.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,
    'EndDates', Maturity, 'Rate', r, 'Compounding', Compoundin!
    'Basis', Basis)
```

RateSpec =
FinObj: 'RateSpec'
Compounding: -1
Disc: 0.9512
Rates: 0.0500
EndTimes: 1
StartTimes: 0
EndDates: 735600

## optpricebysim

StartDates: 735235
ValuationDate: 735235
Basis: 0
EndMonthRule: 1

```
Price an American Asian option (arithmetic mean) by finding the
average price over periods.
AvgPrices = zeros(NPERIODS+1, NTRIALS);
    for i = 1:NPERIODS+1
    AvgPrices(i,:) = mean(squeeze(Paths(1:i,:,:)));
    end
    AsianPrice = optpricebysim(RateSpec, AvgPrices, Times, OptSpec, ...
    Strike, T, 'AmericanOpt', 1)
```


## AsianPrice =

1.8540

## Compute the Price of an American Lookback Option Using Monte Carlo Simulation Based on Geometric Brownian Motion

Define the option.

```
SO = 100; % Initial price of underlying asset
Sigma = .2; % Volatility of underlying asset
Strike = 110; % Strike
OptSpec = 'call'; % Call option
Settle = '1-Jan-2013'; % Settlement date of option
Maturity = '1-Jan-2014'; % Maturity date of option
r = .05; % Risk-free rate (annual, continuous compounding)
Compounding = -1; % Continuous compounding
Basis = 0; % Act/Act day count convention
```

```
T = yearfrac(Settle, Maturity, Basis); % Time to expiration in years
```

Set up the gbm object and run the Monte Carlo simulation based on Geometric Brownian Motion (GBM) using the simBySolution method from Financial Toolbox ${ }^{\mathrm{TM}}$.

NTRIALS = 1000;
NPERIODS = daysact(Settle, Maturity); dt = T/NPERIODS;
OptionGBM = gbm(r, Sigma, 'StartState', SO);
[Paths, Times, Z] = OptionGBM.simBySolution(NPERIODS, ... 'NTRIALS',NTRIALS, 'DeltaTime',dt,'Antithetic',true);

Create the interest-rate term structure to define RateSpec.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,
    'EndDates', Maturity, 'Rate', r, 'Compounding', Compoundins
    'Basis', Basis)
```

RateSpec =
FinObj: 'RateSpec'
Compounding: -1
Disc: 0.9512
Rates: 0.0500
EndTimes: 1
StartTimes: 0
EndDates: 735600
StartDates: 735235
ValuationDate: 735235
Basis: 0
EndMonthRule: 1

Price an American lookback option by finding the maximum price over periods.

## optpricebysim

```
MaxPrices = zeros(NPERIODS+1, NTRIALS);
    LastPrice = squeeze(Paths(1,:,:))';
    for i = 1:NPERIODS+1;
        MaxPrices(i,:) = max([LastPrice; Paths(i,:)]);
        LastPrice = MaxPrices(i,:);
    end
    LookbackPrice = optpricebysim(RateSpec, MaxPrices, Times, OptSpec,
        Strike, T, 'AmericanOpt', 1)
```

LookbackPrice =
10.4084

## Compute the Price of a Bermudan Option Using Monte Carlo Simulation Based on Geometric Brownian Motion

Define the option.

```
SO = 80; % Initial price of underlying asset
Sigma = .3; % Volatility of underlying asset
Strike = 75; % Strike
OptSpec = 'put'; % Put option
Settle = '1-Jan-2013'; % Settlement date of option
Maturity = '1-Jan-2014'; % Maturity date of option
ExerciseDates = {'1-Jun-2013', '1-Jan-2014'}; % Exercise dates of option
r = .05; % Risk-free rate (annual, continuous compounding)
Compounding = -1; % Continuous compounding
Basis = 0; % Act/Act day count convention
T = yearfrac(Settle, Maturity, Basis); % Time to expiration in years
ExerciseTimes = yearfrac(Settle, ExerciseDates, Basis)'; % Exercise times
```

Set up the gbm object and run the Monte Carlo simulation based on Geometric Brownian Motion (GBM) using the simBySolution method from Financial Toolbox ${ }^{\text {TM }}$.

```
NTRIALS = 1000;
NPERIODS = daysact(Settle, Maturity);
dt = T/NPERIODS;
OptionGBM = gbm(r, Sigma, 'StartState', SO);
[Paths, Times, Z] = OptionGBM.simBySolution(NPERIODS, ...
'NTRIALS',NTRIALS, 'DeltaTime',dt,'Antithetic',true);
```

Create the interest-rate term structure to define RateSpec.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,
        'EndDates', Maturity, 'Rate', r, 'Compounding', Compounding
        'Basis', Basis)
```

RateSpec =
FinObj: 'RateSpec'
Compounding: -1
Disc: 0.9512
Rates: 0.0500
EndTimes: 1
StartTimes: 0
EndDates: 735600
StartDates: 735235
ValuationDate: 735235
Basis: 0
EndMonthRule: 1

Price the Bermudan option.
SimulatedPrices = squeeze(Paths);
BermudanPrice = optpricebysim(RateSpec, SimulatedPrices, Times, ... OptSpec, Strike, ExerciseTimes)

BermudanPrice =

## Compute the Price of an American Spread Option Using Monte Carlo Simulation Based on Geometric Brownian Motion

Define the option.

```
S1 = 110; % Price of first underlying asset
S2 = 100; % Price of second underlying asset
Sigma1 = .1; % Volatility of first underlying asset
Sigma2 = .15; % Volatility of second underlying asset
Strike = 15; % Strike
Rho = .3; % Correlation between underlyings
OptSpec = 'put'; % Put option
Settle = '1-Jan-2013'; % Settlement date of option
Maturity = '1-Jan-2014'; % Maturity date of option
r = .05; % Risk-free rate (annual, continuous compounding)
Compounding = -1; % Continuous compounding
Basis = 0; % Act/Act day count convention
T = yearfrac(Settle, Maturity, Basis); % Time to expiration in years
```

Set up the gbm object and run the Monte Carlo simulation based on Geometric Brownian Motion (GBM) using the simBySolution method from Financial Toolbox ${ }^{\text {TM }}$.

```
NTRIALS = 1000;
NPERIODS = daysact(Settle, Maturity);
dt = T/NPERIODS;
SpreadGBM = gbm(r*eye(2), diag([Sigma1;Sigma2]),'Correlation',...
[1 Rho;Rho 1],'StartState',[S1;S2]);
[Paths, Times, Z] = SpreadGBM.simBySolution(NPERIODS,'NTRIALS',NTRIALS,
'DeltaTime',dt,'Antithetic',true);
```

Create the interest-rate term structure to define RateSpec.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,
            'EndDates', Maturity, 'Rate', r, 'Compounding', Compoundin
                        'Basis', Basis)
RateSpec =
                    FinObj: 'RateSpec
        Compounding: -1
            Disc: 0.9512
            Rates: 0.0500
            EndTimes: 1
        StartTimes: 0
            EndDates: 735600
        StartDates: 735235
ValuationDate: 735235
            Basis: 0
    EndMonthRule: 1
```

Price the American spread option.

```
Spread = squeeze(Paths(:,1,:) - Paths(:,2,:));
SpreadPrice = optpricebysim(RateSpec, Spread, Times, OptSpec, Strike,
T, 'AmericanOpt', 1)
```


## SpreadPrice =

9.0007

| See Also | intenvset \| gbm | simBySolution |
| :--- | :--- |
| Concepts | - "Creating Geometric Brownian Motion (GBM) Models" |

## rangefloatbybdt

Purpose Price range floating note using Black-Derman-Toy tree
Syntax Price = rangefloatbybdt(BDTTree,Spread,Settle,Maturity, RateSched)

    [Price,PriceTree] = rangefloatbybdt(BDTTree,Spread,Settle,
    Maturity, RateSched, Name,Value)

## Description

## Input

Arguments
Price = rangefloatbybdt(BDTTree,Spread,Settle, Maturity, RateSched) calculates the price of the range note instrument at the valuation date using a BDT model.
[Price,PriceTree] = rangefloatbybdt(BDTTree,Spread,Settle, Maturity, RateSched, Name, Value) calculates the price of the range note instrument at the valuation date and the price evolution for one or more range instruments using a BDT model with additional options specified by one or more Name, Value pair arguments.

## BDTTree

Interest-rate tree structure created by bdttree.

## Spread

NINST-by- 1 vector of the number of basis points over the reference rate.

## Settle

NINST-by- 1 vector of dates representing the settle date of the range floating note.

Note The Settle date for every range floating instrument is set to the ValuationDate of the BDT tree. The range floating note argument Settle is ignored.

## Maturity

NINST-by- 1 vector of dates representing the maturity date of the floating-rate note.

## RateSched

NINST-by- 1 vector of structures representing the range of rates within which cash flows are nonzero. Each element of the structure array contains two fields:

- RateSched.Dates - NDates-by-1 cell array of dates corresponding to the range schedule.
- RateSched.Rates - NDates-by-2 array with the first column containing the lower bound of the range and the second column containing the upper bound of the range. Cash flow for date RateSched. Dates $(n)$ is nonzero for rates in the range RateSched.Rates $(n, 1)<$ Rate $<$ RateSched.Rate ( $n, 2$ ).


## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

## 'Basis'

NINST-by- 1 vector representing the day-count basis used when annualizing the input forward rate tree.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=\mathrm{actual} / 365$ (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

## 'EndMonthRule'

NINST-by-1 vector for end-of-month rule. Values are 1 (in effect) and 0 (not in effect).

Default: 1 (in effect)

## 'Options'

Structure created with derivset containing derivatives pricing options.
Default: None

## 'Principal'

NINST-by-1 vector of the notional principal amount.
Default: 100

## 'Reset'

NINST-by-1 vector representing the frequency of payments per year.

Default: 1

## Output Arguments

## Definitions

## Examples

## Price

NINST-by- 1 vector for expected prices at time 0.

## PriceTree

Structure containing trees of vectors of instrument prices and accrued interest, and a vector of observation times for each node. Values are:

- PriceTree. PTree contains the clean prices.
- PriceTree.AITree contains the accrued interest.
- PriceTree.tObs contains the observation times.


## Range Note Instrument

A range note is a structured (market-linked) security whose coupon rate is equal to the reference rate as long as the reference rate is within a certain range. If the reference rate is outside of the range, the coupon rate is 0 for that period. This type of instrument entitles the holder to cash flows that depend on the level of some reference interest rate and are floored to be positive. The note holder gets directs exposure to the reference rate. In return for the drawback that no interest will be paid for the time the range is left, they offer higher coupon rates than comparable standard products, like vanilla floating notes.

## Compute the Price of a Range Note Using a Black-Derman-Toy Tree

This example shows how to compute the price of a range note using a Black-Derman-Toy tree with the following interest-rate term structure data.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
```


## rangefloatbybdt

```
Compounding = 1;
% define RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,.
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);
% range note instrument matures in Jan-1-2014 and has the following Rates
Spread = 100;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';
RateSched(1).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(1).Rates = [0.045 0.055 ; 0.0525 0.0675; 0.06 0.08];
% data to build the tree is as follows:
% assume the volatility is 10%.
Sigma = 0.1;
BDTTS = bdttimespec(ValuationDate, EndDates, Compounding);
BDTVS = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length(EndDates
BDTT = bdttree(BDTVS, RS, BDTTS);
% price the instrument
Price = rangefloatbybdt(BDTT, Spread, Settle, Maturity, RateSched)
```

Price =
97.5267

## References Jarrow, Robert, Modelling Fixed Income Securities and Interest Rate

 Options, Stanford Economics and Finance, 2nd edition, 2002.
## See Also <br> | bdttree | cfbybdt | floatbybdt | swapbybdt | floorbybdt | fixedbybdt | bondbybdt | rangefloatbyhjm | rangefloatbybk | instrangefloat | rangefloatbyhw

Purpose Price range floating note using Black-Karasinski tree
Syntax Price = rangefloatbybk(BKTree,Spread,Settle,Maturity,RateSched)
[Price,PriceTree] = rangefloatbybk(BKTree,Spread,Settle,
Maturity, RateSched, Name,Value)

## Input

Arguments
Price = rangefloatbybk(BKTree,Spread,Settle, Maturity, RateSched) calculates the price of the range note instrument at the valuation date using a BK model.
[Price,PriceTree] = rangefloatbybk(BKTree,Spread,Settle, Maturity, RateSched, Name, Value) calculates the price of the range note instrument at the valuation date and the price evolution for one or more range instruments using a BK model with additional options specified by one or more Name, Value pair arguments.

## BKTree

Interest-rate tree structure created by bktree.

## Spread

NINST-by-1 vector of the number of basis points over the reference rate.

## Settle

NINST-by- 1 vector of dates representing the settle date of the range floating note.

> Note The Settle date for every range floating instrument is set to the ValuationDate of the BK tree. The range floating note argument Settle is ignored.

## Maturity

NINST-by- 1 vector of dates representing the maturity date of the floating-rate note.

## RateSched

NINST-by- 1 vector of structures representing the range of rates within which cash flows are nonzero. Each element of the structure array contains two fields:

- RateSched.Dates - NDates-by-1 cell array of dates corresponding to the range schedule.
- RateSched.Rates - NDates-by-2 array with the first column containing the lower bound of the range and the second column containing the upper bound of the range. Cash flow for date RateSched. $\operatorname{Dates}(n)$ is nonzero for rates in the range RateSched.Rates $(n, 1)<$ Rate < RateSched. Rate ( $n, 2$ ).


## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

## 'Basis'

NINST-by-1 vector representing the day-count basis used when annualizing the input forward rate tree.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=\mathrm{actual} / 365$ (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

## 'EndMonthRule'

NINST-by-1 vector for end-of-month rule. Values are 1 (in effect) and 0 (not in effect).

Default: 1 (in effect)

## 'Options'

Structure created with derivset containing derivatives pricing options.
Default: None

## 'Principal'

NINST-by-1 vector of the notional principal amount.
Default: 100

## 'Reset'

NINST-by-1 vector representing the frequency of payments per year.

Default: 1

## Output <br> Arguments

## Definitions

## Examples

## Price

NINST-by-1 vector for expected prices at time 0.

## PriceTree

Structure containing trees of vectors of instrument prices and accrued interest, and a vector of observation times for each node. Values are:

- PriceTree. PTree contains the clean prices.
- PriceTree.AITree contains the accrued interest.
- PriceTree.tObs contains the observation times.


## Range Note Instrument

A range note is a structured (market-linked) security whose coupon rate is equal to the reference rate as long as the reference rate is within a certain range. If the reference rate is outside of the range, the coupon rate is 0 for that period. This type of instrument entitles the holder to cash flows that depend on the level of some reference interest rate and are floored to be positive. The note holder gets directs exposure to the reference rate. In return for the drawback that no interest will be paid for the time the range is left, they offer higher coupon rates than comparable standard products, like vanilla floating notes.

## Compute the Price of a Range Note Using a Black-Karasinski Tree

This example shows how to compute the price of a range note using a Black-Karasinski tree with the following interest-rate term structure data.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
```

```
Compounding = 1;
% define RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDat
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);
% range note instrument matures in Jan-1-2014 and has the following Ra
Spread = 100;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';
RateSched(1).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(1).Rates = [0.045 0.055 ; 0.0525 0.0675; 0.06 0.08];
% data to build the tree is as follows:
VolDates = ['1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014';'1-Jan-2015'];
VolCurve = 0.01;
AlphaDates = '01-01-2015';
AlphaCurve = 0.1;
BKVS = bkvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
BKTS = bktimespec(RS.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVS, RS, BKTS);
% price the instrument
Price = rangefloatbybk(BKT, Spread, Settle, Maturity, RateSched)
Price =
    102.7574
```


## References Jarrow, Robert, Modelling Fixed Income Securities and Interest Rate Options, Stanford Economics and Finance, 2nd edition, 2002.

## rangefloatbybk

See Also $\quad \begin{aligned} & \text { | bktree | cfbybk | capbybk | swapbybk | floorbybk | fixedbybk | } \\ & \text { bondbybk | rangefloatbyhjm | rangefloatbybdt | rangefloatbyhw } \\ & \text { | instrangefloat }\end{aligned}$
Description
Input
Arguments
Purpose Price range floating note using Heath-Jarrow-Morton tree

Price range floating note using Heath-Jarrow-Morton tree
Syntax Price = rangefloatbyhjm(HJMTree,Spread,Settle,Maturity,

```RateSched)
    [Price,PriceTree] = rangefloatbyhjm(HJMTree,Spread,Settle,
Maturity, RateSched, Name,Value)
rangefloatbyhjm(HJMTree,Spread,Settle,Maturity,
RateSched)
```

Price = rangefloatbyhjm(HJMTree,Spread,Settle, Maturity, RateSched) calculates the price of the range note instrument at the valuation date using an HJM model.
[Price,PriceTree] = rangefloatbyhjm(HJMTree,Spread,Settle, Maturity, RateSched, Name, Value) calculates the price of the range note instrument at the valuation date and the price evolution for one or more range instruments using an HJM model with additional options specified by one or more Name, Value pair arguments.

## HJMTree

Interest-rate tree structure created by hjmtree.

## Spread

NINST-by-1 vector of the number of basis points over the reference rate.

## Settle

NINST-by- 1 vector of dates representing the settle date of the range floating note.

Note The Settle date for every range floating instrument is set to the ValuationDate of the HJM tree. The range floating note argument Settle is ignored.

## Maturity

NINST-by-1 vector of dates representing the maturity date of the floating-rate note.

## RateSched

NINST-by- 1 vector of structures representing the range of rates within which cash flows are nonzero. Each element of the structure array contains two fields:

- RateSched.Dates - NDates-by-1 cell array of dates corresponding to the range schedule.
- RateSched.Rates - NDates-by-2 array with the first column containing the lower bound of the range and the second column containing the upper bound of the range. Cash flow for date RateSched. $\operatorname{Dates}(n)$ is nonzero for rates in the range RateSched.Rates $(n, 1)<$ Rate < RateSched. Rate ( $n, 2$ ).


## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

## 'Basis'

NINST-by-1 vector representing the day-count basis used when annualizing the input forward rate tree.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=\mathrm{actual} / 365$ (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

## 'EndMonthRule'

NINST-by-1 vector for end-of-month rule. Values are 1 (in effect) and 0 (not in effect).

Default: 1 (in effect)

## 'Options'

Structure created with derivset containing derivatives pricing options.
Default: None

## 'Principal'

NINST-by-1 vector of the notional principal amount.
Default: 100

## 'Reset'

NINST-by-1 vector representing the frequency of payments per year.

## Default: 1

## Output <br> Arguments

## Definitions

## Examples

## Price

NINST-by-1 vector for expected prices at time 0.

## PriceTree

Structure containing trees of vectors of instrument prices and accrued interest, and a vector of observation times for each node. Values are:

- PriceTree. PTree contains the clean prices.
- PriceTree.AITree contains the accrued interest.
- PriceTree.tObs contains the observation times.


## Range Note Instrument

A range note is a structured (market-linked) security whose coupon rate is equal to the reference rate as long as the reference rate is within a certain range. If the reference rate is outside of the range, the coupon rate is 0 for that period. This type of instrument entitles the holder to cash flows that depend on the level of some reference interest rate and are floored to be positive. The note holder gets directs exposure to the reference rate. In return for the drawback that no interest will be paid for the time the range is left, they offer higher coupon rates than comparable standard products, like vanilla floating notes.

## Compute the Price of a Range Note Using a Heath-Jarrow-Morton Tree

This example shows how to compute the price of a range note using a Heath-Jarrow-Morton tree with the following interest-rate term structure data.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
```

```
Compounding = 1;
% define RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDat
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);
% range note instrument matures in Jan-1-2014 and has the following R
Spread = 100;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';
RateSched(1).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(1).Rates = [0.045 0.055 ; 0.0525 0.0675; 0.06 0.08];
% data to build the tree is as follows:
Volatility = [.2; .19; .18; .17];
CurveTerm = [ 1; 2; 3; 4];
MaTree = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
HJMTS = hjmtimespec(ValuationDate, MaTree);
HJMVS = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVS, RS, HJMTS);
% price the instrument
Price = rangefloatbyhjm(HJMT, Spread, Settle, Maturity, RateSched)
```


## Price =

90.2348

## References Jarrow, Robert, Modelling Fixed Income Securities and Interest Rate

 Options, Stanford Economics and Finance, 2nd edition, 2002.See Also

| hjmtree | cfbyhjm | floatbyhjm | swapbyhjm | floorbyhjm | fixedbyhjm | bondbyhjm | rangefloatbybk | rangefloatbybdt | rangefloatbyhw | instrangefloat
Purpose Price range floating note using Hull-White tree
Syntax

Price = rangefloatbyhw(HWTree,Spread,Settle,Maturity,

RateSched)

[Price,PriceTree] = rangefloatbyhw(HWTree,Spread,Settle,

Maturity, RateSched, Name,Value)

## Description

## Input

Arguments
Price = rangefloatbyhw(HWTree,Spread,Settle,Maturity, RateSched) calculates the price of the range note instrument at the valuation date using an HW model.
[Price,PriceTree] = rangefloatbyhw(HWTree,Spread,Settle, Maturity, RateSched, Name, Value) calculates the price of the range note instrument at the valuation date and the price evolution for one or more range instruments using an HW model with additional options specified by one or more Name, Value pair arguments.
HWTree
Interest-rate tree structure created by hwtree.

## Spread

NINST-by-1 vector of the number of basis points over the reference rate.

## Settle

NINST-by- 1 vector of dates representing the settle date of the range floating note.

Note The Settle date for every range floating instrument is set to the ValuationDate of the HW tree. The range floating note argument Settle is ignored.

## Maturity

NINST-by- 1 vector of dates representing the maturity date of the floating-rate note.

## RateSched

NINST-by-1 vector of structures representing the range of rates within which cash flows are nonzero. Each element of the structure array contains two fields:

- RateSched.Dates - NDates-by-1 cell array of dates corresponding to the range schedule.
- RateSched.Rates - NDates-by-2 array with the first column containing the lower bound of the range and the second column containing the upper bound of the range. Cash flow for date RateSched. Dates $(n)$ is nonzero for rates in the range RateSched.Rates $(n, 1)<$ Rate $<$ RateSched.Rate ( $n, 2$ ).


## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

## 'Basis'

NINST-by- 1 vector representing the day-count basis used when annualizing the input forward rate tree.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

## 'EndMonthRule'

NINST-by-1 vector for end-of-month rule. Values are 1 (in effect) and 0 (not in effect).

Default: 1 (in effect)

## 'Options'

Structure created with derivset containing derivatives pricing options.
Default: None

## 'Principal'

NINST-by-1 vector of the notional principal amount.
Default: 100

## 'Reset'

NINST-by-1 vector representing the frequency of payments per year.

Default: 1

## Price

NINST-by- 1 vector for expected prices at time 0.

## PriceTree

Structure containing trees of vectors of instrument prices and accrued interest, and a vector of observation times for each node. Values are:

- PriceTree. PTree contains the clean prices.
- PriceTree.AITree contains the accrued interest.
- PriceTree.tObs contains the observation times.


## Definitions Range Note Instrument

A range note is a structured (market-linked) security whose coupon rate is equal to the reference rate as long as the reference rate is within a certain range. If the reference rate is outside of the range, the coupon rate is 0 for that period. This type of instrument entitles the holder to cash flows that depend on the level of some reference interest rate and are floored to be positive. The note holder gets directs exposure to the reference rate. In return for the drawback that no interest will be paid for the time the range is left, they offer higher coupon rates than comparable standard products, like vanilla floating notes.

## Examples

## Compute the Price of a Range Note Using a Hull-White Tree

This example shows how to compute the price of a range note using a Hull-White tree with the following interest-rate term structure data.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
Compounding = 1;
```

```
% define RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);
% range note instrument matures in Jan-1-2014 and has the following Rates
Spread = 100;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';
RateSched(1).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(1).Rates = [0.045 0.055 ; 0.0525 0.0675; 0.06 0.08];
% data to build the tree is as follows:
VolDates = ['1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014';'1-Jan-2015'];
VolCurve = 0.01;
AlphaDates = '01-01-2015';
AlphaCurve = 0.1;
HWVS = hwvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTS = hwtimespec(RS.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVS, RS, HWTS);
% price the instrument
Price = rangefloatbyhw(HWT, Spread, Settle, Maturity, RateSched)
Price =
```

96.6501

References Jarrow, Robert, Modelling Fixed Income Securities and Interest Rate Options, Stanford Economics and Finance, 2nd edition, 2002.

See Also $\quad \begin{aligned} & \text { | hwtree | cfbyhw | capbyhw | swapbyhw | floorbyhw | fixedbyhw | } \\ & \text { bondbyhw | rangefloatbybk | rangefloatbybdt | rangefloatbyhjm } \\ & \text { | instrangefloat }\end{aligned}$

Purpose
Discount factors from interest rates
Syntax
Usage 1: Interval points are input as times in periodic units.
Disc = rate2disc(Compounding, Rates, EndTimes) Disc = rate2disc(Compounding, Rates, EndTimes, StartTimes)

Usage 2: ValuationDate is passed and interval points are input as dates.
[Disc, EndTimes, StartTimes] = rate2disc(Compounding, Rates, EndDates, StartDates, ValuationDate)
[Disc, EndTimes, StartTimes] = rate2disc(Compounding, Rates, EndDates, StartDates, ValuationDate, Basis, EndMonthRule)

## Arguments

Compounding Scalar value representing the rate at which the input zero rates were compounded when annualized. This argument determines the formula for the discount factors:

Compounding $=1,2,3,4,6,12$
Disc $=(1+Z / F)^{\wedge}(-T)$, where $F$ is the compounding frequency, $Z$ is the zero rate, and T is the time in periodic units; for example, $\mathrm{T}=$ $F$ is 1 year.

Compounding $=365$
Disc $=(1+Z / F)^{\wedge}(-T)$, where $F$ is the number of days in the basis year and $T$ is a number of days elapsed computed by basis.
Compounding $=-1$
Disc $=\exp (-T * Z)$, where $T$ is time in years.
Rates
Number of points (NPOINTS) by number of curves (NCURVES) matrix of rates in decimal form. For example, $5 \%$ is 0.05 in Rates. Rates are the yields over investment intervals from

|  | StartTimes, when the cash flow is valued, to EndTimes, when the cash flow is received. |
| :---: | :---: |
| EndTimes | NPOINTS-by- 1 vector or scalar of times in periodic units ending the interval to discount over. |
| StartTimes | (Optional) NPOINTS-by-1 vector or scalar of times in periodic units starting the interval to discount Note Dehant ValduationDate is not passed, the third and fourth arguments (EndTimes and |
| EndDates |  dates ending the interval to discount over. |
|  | Note : When ValuationDate is passed, the third and fourth arguments (EndDates and StartDates) are interpreted as dates. The date ValuationDate is used as the zero point for computing the times. |
| StartDates | (Optional) NPOINTS-by-1 vector or scalar of serial dates starting the interval to discount over. StartDates must be earlier than EndDates. |
|  | Default = ValuationDate. |
| ValuationDate | Scalar value in serial date number form representing the observation date of the investment horizons entered in StartDates and EndDates. Required in Usage 2. Omitted or passed as an empty matrix to invoke Usage 1. |

Basis (Optional) Day-count basis of the instrument when using Usage 2. A vector of integers.

- $0=$ actual/actual (default)
- 1 = 30/360 (SIA)
- 2 = actual/360
- $3=$ actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
EndMonthRule (Optional) End-of-month rule when using Usage 2. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. $0=$ ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. $1=$ set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.

## Description

Usage 1: Disc = rate2disc(Compounding, Rates, EndTimes) or Disc = rate2disc(Compounding, Rates, EndTimes, StartTimes) where interval points are input as times in periodic units.

Usage 2: [Disc, EndTimes, StartTimes] = rate2disc(Compounding, Rates, EndDates, StartDates, ValuationDate) or [Disc, EndTimes, StartTimes] = rate2disc(Compounding, Rates, EndDates, StartDates, ValuationDate, Basis, EndMonthRule) where ValuationDate is passed and interval points are input as dates.
rate2disc computes the discounts over a series of NPOINTS time intervals given the annualized yield over those intervals. NCURVES different rate curves can be translated at once if they have the same time structure. The time intervals can represent a zero curve or a forward curve.

The output Disc is an NPOINTS-by-NCURVES column vector of discount factors in decimal form representing the value at time StartTime of a unit cash flow received at time EndTime.

You can specify the investment intervals either with input times (Usage 1) or with input dates (Usage 2). Entering ValuationDate invokes the date interpretation; omitting ValuationDate invokes the default time interpretations.

## For Usage 1:

- StartTimes is an NPOINTS-by- 1 column vector of times starting the interval to discount over, measured in periodic units.
- EndTimes is an NPOINTS-by- 1 column vector of times ending the interval to discount over, measured in periodic units.


## For Usage 2:

- StartDates is an NPOINTS-by-1 column vector of serial dates starting the interval to discount over, measured in days.
- EndDates is an NPOINTS-by-1 column vector of serial dates ending the interval to discount over, measured in days.

If Compounding = 365 (daily), StartDates and EndDates are measured in days as in Usage 2. Otherwise, in Usage 1, the arguments contain values, $T$, computed from SIA semiannual time factors, Tsemi, by the formula $T=T s e m i / 2 * F$, where $F$ is the compounding frequency.

## Examples

Example 1 demonstrates Usage 1. Compute discounts from a zero curve at 6 months, 12 months, and 24 months. The times to the cash flows are 1,2 , and 4 . You are computing the present value (at time 0 ) of the cash flows.

```
Compounding = 2;
Rates = [0.05; 0.06; 0.065];
EndTimes = [1; 2; 4];
Disc = rate2disc(Compounding, Rates, EndTimes)
Disc =
    0.9756
    0.9426
    0.8799
```

Example 2 demonstrates Usage 2. Compute discounts from a zero curve at 6 months, 12 months, and 24 months. Use dates to specify the ending time horizon.

```
Compounding = 2;
Rates = [0.05; 0.06; 0.065];
EndDates = ['10/15/97'; '04/15/98'; '04/15/99'];
ValuationDate = '4/15/97';
Disc = rate2disc(Compounding, Rates, EndDates, [], ValuationDate)
Disc =
    0.9756
    0.9426
    0.8799
```

Example 3 demonstrates Usage 1. Compute discounts from the 1-year forward rates beginning now, in 6 months, and in 12 months. Use
monthly compounding. The times to the cash flows are $12,18,24$, and the forward times are $0,6,12$.

Compounding = 12;
Rates $=$ [0.05; 0.04; 0.06];
EndTimes = [12; 18; 24];
StartTimes = [0; 6; 12];
Disc = rate2disc(Compounding, Rates, EndTimes, StartTimes)
Disc =
0.9513
0.9609
0.9419

See Also
disc2rate | ratetimes

Purpose
Syntax
[Rates, EndTimes, StartTimes] = ratetimes(Compounding, RefRates, RefEndTimes, RefStartTimes, EndTimes, StartTimes) [Rates, EndTimes, StartTimes] = ratetimes(Compounding, RefRates, RefEndDates, RefStartDates, EndDates, StartDates, ValuationDate)

Usage 1: ValuationDate not passed; third through sixth arguments are interpreted as times.
[Rates, EndTimes, StartTimes] = ratetimes(Compounding, RefRates, RefEndTimes, RefStartTimes, EndTimes, StartTimes)
Usage 2: ValuationDate passed and interval points input as dates. [Rates, EndTimes, StartTimes] = ratetimes(Compounding, RefRates, RefEndDates, RefStartDates, EndDates, StartDates, ValuationDate)

## Arguments

Compounding Scalar value representing the rate at which the input zero rates were compounded when annualized. This argument determines the formula for the discount factors:

Compounding $=1,2,3,4,6,12$
Disc $=(1+Z / F)^{\wedge}(-T)$, where $F$ is the compounding frequency, $Z$ is the zero rate, and T is the time in periodic units; for example, $\mathrm{T}=$ $F$ is 1 year.

Compounding $=365$
Disc $=(1+Z / F)^{\wedge}(-T)$, where $F$ is the number of days in the basis year and $T$ is a number of days elapsed computed by basis.
Compounding $=-1$

|  | Disc $=\exp (-T * Z)$, where T is time in years. |
| :--- | :--- |
| RefRates | NREFPTS-by-NCURVES matrix of reference rates <br> in decimal form. RefRates are the yields over <br> investment intervals from RefStartTimes, when <br> the cash flow is valued, to RefEndTimes, when <br> the cash flow is received. |
| RefEndTimes | NREFPTS-by-1 vector or scalar of times in periodic <br> units ending the intervals corresponding to <br> RefRates. |
| RefStartTimes | (Optional) NREFPTS-by-1 vector or scalar of <br> times in periodic units starting the intervals <br> corresponding to RefRates. Default = 0. |
| EndTimes | NPOINTS-by-1 vector or scalar of times in periodic <br> units ending the interval to discount over. |
| StartTimes | (Optional) NPOINTS-by-1 vector or scalar of times <br> in periodic units starting the interval to discount <br> over. Default = 0. |
| RefEndDates | NREFPTS-by-1 vector or scalar of serial dates <br> ending the intervals corresponding to RefRates. |
| RefStartDates | (Optional) NREFPTS-by-1 vector or scalar of <br> serial dates starting the intervals corresponding <br> to RefRates. Default = ValuationDate. |
| EndDates | NPOINTS-by-1 vector or scalar of serial maturity <br> dates ending the interval to discount over. |


| StartDates | (Optional) NPOINTS-by-1 vector or scalar of serial <br> dates starting the interval to discount over. <br> StartDates must be earlier than EndDates. |
| :--- | :--- |
| ValuationDate | Default = ValuationDate. |
| Scalar value in serial date number form <br> representing the observation date of the <br> investment horizons entered in StartDates and |  |
|  | EndDates. Required in Usage 2. Omitted or <br> passed as an empty matrix to invoke Usage 1. |

## Description

[Rates, EndTimes, StartTimes] = ratetimes(Compounding, RefRates, RefEndTimes, RefStartTimes, EndTimes, StartTimes) and [Rates, EndTimes, StartTimes] = ratetimes(Compounding, RefRates, RefEndDates, RefStartDates, EndDates, StartDates, ValuationDate) change time intervals defining an interest-rate environment.
ratetimes takes an interest-rate environment defined by yields over one collection of time intervals and computes the yields over another set of time intervals. The zero rate is assumed to be piece-wise linear in time.

Rates is an NPOINTS-by-NCURVES matrix of rates implied by the reference interest-rate structure and sampled at new intervals.
StartTimes is an NPOINTS-by- 1 column vector of times starting the new intervals where rates are desired, measured in periodic units.
EndTimes is an NPOINTS-by- 1 column vector of times ending the new intervals, measured in periodic units.

If Compounding = 365 (daily), StartTimes and EndTimes are measured in days. The arguments otherwise contain values, T, computed from SIA semiannual time factors, Tsemi, by the formula $T=T s e m i / 2$ * F , where $F$ is the compounding frequency.

You can specify the investment intervals either with input times (Usage 1) or with input dates (Usage 2). Entering the argument ValuationDate invokes the date interpretation; omitting ValuationDate invokes the default time interpretations.

## Examples <br> Example 1. The reference environment is a collection of zero rates at 6,12 , and 24 months. Create a collection of 1-year forward rates beginning at 0,6 , and 12 months. <br> ```RefRates = [0.05; 0.06; 0.065]; \\ RefEndTimes = [1; 2; 4]; \\ StartTimes = [0; 1; 2]; \\ EndTimes = [2; 3; 4]; \\ Rates = ratetimes(2, RefRates, RefEndTimes, 0, EndTimes,... \\ StartTimes)``` <br> Rates $=$ <br> 0.0600 <br> 0.0688 <br> 0.0700

Example 2. Interpolate a zero yield curve to different dates. Zero curves start at the default date of ValuationDate.

```
RefRates = [0.04; 0.05; 0.052];
RefDates = [729756; 729907; 730121];
Dates = [730241; 730486];
ValuationDate = 729391;
Rates = ratetimes(2, RefRates, RefDates, [], Dates, [],...
ValuationDate)
```

Rates $=$
0.0520
0.0520

See Also
disc2rate | rate2disc

## Purpose Price European spread options using Kirk pricing model

Syntax Price = spreadbykirk(RateSpec,StockSpec1,StockSpec2,Settle, Maturity, OptSpec,Strike,Corr)

## Description

Input
Arguments

Price =
spreadbykirk(RateSpec,StockSpec1,StockSpec2, Settle, Maturity, OptSpec,Strike, Corr) returns the price for a European spread option using the Kirk pricing model.

## RateSpec - Interest-rate term structure

structure
Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from intenvset. For information on the interest-rate specification, see intenvset.

## Data Types <br> struct

## StockSpec 1 - Stock specification for underlying asset 1

## structure

Stock specification for underlying asset 1. For information on the stock specification, see stockspec.
stockspec can handle other types of underlying assets. For example, for physical commodities the price is represented by StockSpec.Asset, the volatility is represented by StockSpec.Sigma, and the convenience yield is represented by StockSpec.DividendAmounts.

## Data Types <br> struct

## StockSpec2-Stock specification for underlying asset 2

structure
Stock specification for underlying asset 2. For information on the stock specification, see stockspec.
stockspec can handle other types of underlying assets. For example, for physical commodities the price is represented by StockSpec.Asset, the volatility is represented by StockSpec.Sigma, and the convenience yield is represented by StockSpec.DividendAmounts.

## Data Types

struct

## Settle - Settlement dates for spread option

nonnegative integer | vector of nonnegative integers | date string | cell array of date strings

Settlement dates for the spread option, specified as a date string or as nonnegative integers using a NINST-by-1 vector or cell array of dates.

## Data Types

char | cell

## Maturity - Maturity date for spread option

nonnegative integer | vector of nonnegative integers | date string | cell array of date strings

Maturity date for spread option, specified as a date string or as nonnegative integers using a NINST-by-1 vector or cell array of dates.

## Data Types

char | cell

## OptSpec - Definition of option

string with values 'call' or 'put' | cell array of strings
Definition of option as 'call' or 'put', specified as a NINST-by-1 cell array of strings.

## Data Types

cell | char

## Strike - Option strike price values

integer | vector of integers

Option strike price values, specified as an integer using a NINST-by-1 vector of strike price values.
If Strike is equal to 0 , the function computes the price of an exchange option.

```
Data Types
single | double
```


## Corr - Correlation between underlying asset prices

integer | vector of integers
Correlation between underlying asset prices, specified as an integer using a NINST-by-1 vector.

Data Types<br>single | double

## Output Arguments

## Examples

## Price - Expected prices of spread option

vector
Expected prices of the spread option, returned as a NINST-by-1 vector.

## Compute the Price of a Spread Option Using the Kirk Model

Define the spread option dates.

```
Settle = '01-Jan-2012';
Maturity = '01-April-2012';
```

Define asset 1. Price and volatility of RBOB gasoline

```
Price1gallon = 2.85; % $/gallon
Price1 = Price1gallon * 42; % $/barrel
Vol1 = 0.29;
```

Define asset 2. Price and volatility of WTI crude oil

```
Price2 = 93.20; % $/barrel
Vol2 = 0.36;
```

Define the correlation between the underlying asset prices of asset 1 and asset 2.

```
Corr = 0.42;
```

Define the spread option.

```
OptSpec = 'call';
```

Strike = 20;

Define the RateSpec.

```
rates = 0.05;
```

Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,
'EndDates', Maturity, 'Rates', rates, ...
'Compounding', Compounding, 'Basis', Basis)
RateSpec =
FinObj: 'RateSpec'
Compounding: -1
Disc: 0.9876
Rates: 0.0500
EndTimes: 0.2500
StartTimes: 0
EndDates: 734960
StartDates: 734869
ValuationDate: 734869
Basis: 1
EndMonthRule: 1

Define the StockSpec for the two assets.

```
StockSpec1 = stockspec(Vol1, Price1)
StockSpec2 = stockspec(Vol2, Price2)
StockSpec1 =
                    FinObj: 'StockSpec'
                    Sigma: 0.2900
        AssetPrice: 119.7000
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
StockSpec2 =
                    FinObj: 'StockSpec'
                    Sigma: 0.3600
        AssetPrice: 93.2000
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Compute the European spread option price based on the Kirk model.

```
Price = spreadbykirk(RateSpec, StockSpec1, StockSpec2, Settle, ...
Maturity, OptSpec, Strike, Corr)
```

Price =
11.1904

## References

Carmona, R., Durrleman, V., "Pricing and Hedging Spread Options," SIAM Review, Vol. 45, No. 4, pp. 627-685, Society for Industrial and Applied Mathematics, 2003.
See Also spreadsensbykirk \| spreadbybjs | spreadbyfd | spreadbyls
Related - "Pricing European and American Spread Options" on page 3-41Examples- "Hedging Strategies Using Spread Options" on page 3-57
Concepts - "Spread Option" on page 3-37

| Purpose | Price European spread options using Bjerksund-Stensland pricing model |
| :---: | :---: |
| Syntax | ```Price = spreadbybjs(RateSpec,StockSpec1,StockSpec2,Settle,Maturity, OptSpec,Strike,Corr)``` |
| Description | ```Price = spreadbybjs(RateSpec,StockSpec1,StockSpec2,Settle,Maturity, OptSpec,Strike, Corr) returns the price for a European spread option using the Bjerksund-Stensland pricing model.``` |
| Input Arguments | RateSpec - Interest-rate term structure structure |
|  | Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from intenvset. For information on the interest-rate specification, see intenvset. |
|  | Data Types struct |
|  | StockSpec 1 - Stock specification for underlying asset 1 structure |
|  | Stock specification for underlying asset 1. For information on the stock specification, see stockspec. |
|  | stockspec can handle other types of underlying assets. For example, for physical commodities the price is represented by StockSpec.Asset, the volatility is represented by StockSpec.Sigma, and the convenience yield is represented by StockSpec.DividendAmounts. |
|  | Data Types struct |
|  | StockSpec2-Stock specification for underlying asset 2 structure |

Stock specification for underlying asset 2. For information on the stock specification, see stockspec.
stockspec can handle other types of underlying assets. For example, for physical commodities the price is represented by StockSpec.Asset, the volatility is represented by StockSpec.Sigma, and the convenience yield is represented by StockSpec.DividendAmounts.

Data Types
struct

## Settle - Settlement dates for spread option

nonnegative integer | vector of nonnegative integers | date string | cell array of date strings
Settlement dates for the spread option, specified as a date string or as nonnegative integers using a NINST-by- 1 vector or cell array of dates.

## Data Types

char | cell

## Maturity - Maturity date for spread option

nonnegative integer | vector of nonnegative integers | date string | cell array of date strings

Maturity date for spread option, specified as a date string or as nonnegative integers using a NINST-by-1 vector or cell array of dates.

## Data Types

char | cell

## OptSpec - Definition of option

string with values 'call' or 'put' | cell array of strings
Definition of option as 'call' or 'put', specified as a NINST-by-1 cell array of strings.

## Data Types

char | cell

## Strike - Option strike price values

## integer | vector of integers

Option strike price values, specified as an integer using a NINST-by-1 vector of strike price values.

If Strike is equal to zero the function computes the price of an exchange option.

Data Types
single | double

## Corr - Correlation between underlying asset prices

integer | vector of integers
Correlation between underlying asset prices, specified as an integer using a NINST-by-1 vector.

Data Types
single | double

## Output Arguments

## Price - Expected prices of spread option

vector
Expected prices of the spread option, returned as a NINST-by-1 vector.

## Examples Compute the Price of a Spread Option Using the Bjerksund-Stensland Model

Define the spread option dates.

```
Settle = '01-Jan-2012';
Maturity = '01-April-2012';
```

Define asset 1. Price and volatility of RBOB gasoline

```
Price1gallon = 2.85; % $/gallon
Price1 = Price1gallon * 42; % $/barrel
Vol1 = 0.29;
```

Define asset 2. Price and volatility of WTI crude oil

```
Price2 = 93.20; % $/barrel
Vol2 = 0.36;
```

Define the correlation between the underlying asset prices of asset 1 and asset 2.

```
Corr = 0.42;
```

Define the spread option.

```
OptSpec = 'call';
Strike = 20;
```

Define the RateSpec.

```
rates = 0.05;
```

Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,
'EndDates', Maturity, 'Rates', rates, ...
'Compounding', Compounding, 'Basis', Basis)

## RateSpec =

```
                    FinObj: 'RateSpec'
        Compounding: -1
            Disc: 0.9876
            Rates: 0.0500
            EndTimes: 0.2500
        StartTimes: 0
            EndDates: 734960
        StartDates: 734869
            ValuationDate: 734869
            Basis: 1
            EndMonthRule: 1
```

```Define the StockSpec for the two assets.
StockSpec1 = stockspec(Vol1, Price1)
StockSpec2 = stockspec(Vol2, Price2)
StockSpec1 =
                    FinObj: 'StockSpec'
                    Sigma: 0.2900
        AssetPrice: 119.7000
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
StockSpec2 =
FinObj: 'StockSpec'
Sigma: 0.3600
AssetPrice: 93.2000
DividendType: []
DividendAmounts: 0
ExDividendDates: []
Compute the European spread option price based on the Bjerksund-Stensland model.
Price = spreadbybjs(RateSpec, StockSpec1, StockSpec2, Settle, ...
Maturity, OptSpec, Strike, Corr)
Price =
11.2000
```


## References

Carmona, R., Durrleman, V., "Pricing and Hedging Spread Options," SIAM Review, Vol. 45, No. 4, pp. 627-685, Society for Industrial and Applied Mathematics, 2003.
Bjerksund, Petter, Stensland, Gunnar, "Closed form spread option valuation," Department of Finance, NHH, 2006.
See Also spreadsensbykirk | spreadbybjs | spreadbyfd \| spreadbyls
Related- "Pricing European and American Spread Options" on page 3-41
Examples- "Hedging Strategies Using Spread Options" on page 3-57
Concepts - "Spread Option" on page 3-37

Purpose

Syntax

## Description

Price European or American spread options using finite difference method

```
Price = spreadbyfd(RateSpec,StockSpec1,StockSpec2,Settle,Maturity,
OptSpec,Strike,Corr)
Price = spreadbyfd(___ ,Name,Value)
[Price,PriceGrid,AssetPrice1,AssetPrice2,Times] = spreadbyfd(RateSpec,Stc
OptSpec,Strike,Corr)
[Price,PriceGrid,AssetPrice1,AssetPrice2,Times]
= spreadbyfd(__ ,Name,
    Value)
```

Price =
spreadbyfd(RateSpec,StockSpec1,StockSpec2,Settle, Maturity,
OptSpec, Strike, Corr) returns the price of European or American call
or put spread options using the Alternate Direction Implicit (ADI) finite
difference method. The spread is between the asset defined in
StockSpec1 minus the asset defined in StockSpec2.

Price $=\operatorname{spreadbyfd}(\ldots$, , Name, Value) returns the price of European or American call or put spread options using the Alternate Direction Implicit (ADI) finite difference method using optional name-value pair arguments. The spread is between the asset defined in StockSpec1 minus the asset defined in StockSpec2.
[Price,PriceGrid,AssetPrice1,AssetPrice2,Times] = spreadbyfd(RateSpec,StockSpec1,StockSpec2,Settle, Maturity, OptSpec,Strike,Corr) returns the Price, PriceGrid, AssetPrice1, AssetPrice2, and Times for a European or American call or put spread options using the Alternate Direction Implicit (ADI) finite difference method. The spread is between the asset defined in StockSpec1 minus the asset defined in StockSpec2.
[Price,PriceGrid,AssetPrice1,AssetPrice2,Times] = spreadbyfd( __, Name, Value) returns the Price, PriceGrid, AssetPrice1, AssetPrice2, and Times for a European or American call or put spread options using the Alternate Direction Implicit (ADI) finite difference method using optional name-value pair arguments. The spread is between the asset defined in StockSpec 1 minus the asset defined in StockSpec2.

## Input Arguments

## RateSpec - Interest-rate term structure <br> structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from intenvset. For information on the interest-rate specification, see intenvset.

## Data Types

struct

## StockSpec 1 - Stock specification for underlying asset 1

structure
Stock specification for underlying asset 1. For information on the stock specification, see stockspec.
stockspec can handle other types of underlying assets. For example, for physical commodities the price is represented by StockSpec.Asset, the volatility is represented by StockSpec.Sigma, and the convenience yield is represented by StockSpec.DividendAmounts.

## Data Types

struct

## StockSpec2 - Stock specification for underlying asset 2

## structure

Stock specification for underlying asset 2. For information on the stock specification, see stockspec.
stockspec can handle other types of underlying assets. For example, for physical commodities the price is represented by StockSpec.Asset,
the volatility is represented by StockSpec.Sigma, and the convenience yield is represented by StockSpec.DividendAmounts.

## Data Types <br> struct

## Settle - Settlement dates for spread option

nonnegative integer | vector of nonnegative integers | date string | cell array of date strings

Settlement dates for the spread option, specified as a date string or as nonnegative integers using a NINST-by-1 vector or cell array of dates.

## Data Types <br> char | cell

## Maturity - Maturity date for spread option

## nonnegative integer | vector of nonnegative integers | date string | cell array of date strings

Maturity date for spread option, specified as a date string or as nonnegative integers using a NINST-by- 1 vector or cell array of dates.

## Data Types <br> char | cell

## OptSpec - Definition of option

```
string with values 'call' or 'put' | cell array of strings
```

Definition of option as 'call' or 'put', specified as a NINST-by-1 cell array of strings.

## Data Types <br> char | cell

## Strike - Option strike price values

## integer | vector of integers

Option strike price values, specified as an integer using a NINST-by-1 vector of strike price values.

Data Types
single | double

## Corr - Correlation between underlying asset prices

integer | vector of integers
Correlation between underlying asset prices, specified as an integer using a NINST-by-1 vector.

Data Types
single | double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

Example: [Price,PriceGrid,AssetPrice1,AssetPrice2,Times] = spreadbyfd(RateSpec, StockSpec1,StockSpec2, Settle, Maturity, OptSpec, Str:

## 'AssetPriceMin' - Minimum price for price grid boundary

if unspecified, StockSpec values are calculated based on asset distributions at maturity (default) | array
Minimum price for price grid boundary, specified by a 1-by-2 array. The first entry in the array corresponds to the first asset defined by StockSpec1 and the second entry corresponds to the second asset defined by StockSpec2.
For the finite difference method, the composition of the grid affects the quality of the output as well as the execution time. It is highly recommended to use the optional arguments AssetPriceMin, AssetPriceMax, PriceGridSize, and TimeGridSize to control the composition of the grid to ensure the quality of the output and a reasonable execution time.

## Data Types <br> single | double

## 'AssetPriceMax' - Maximum price for price grid boundary <br> if unspecified, StockSpec values are calculated based on asset distributions at maturity (default) | array

Maximum price for price grid boundary, specified by a 1-by-2 array. The first entry in the array corresponds to the first asset defined by StockSpec1 and the second entry corresponds to the second asset defined by StockSpec2.

For the finite difference method, the composition of the grid affects the quality of the output as well as the execution time. It is highly recommended to use the optional arguments AssetPriceMin, AssetPriceMax, PriceGridSize, and TimeGridSize to control the composition of the grid to ensure the quality of the output and a reasonable execution time.

## Data Types <br> single | double

## 'PriceGridSize' - Size for finite difference grid [300,300] (default) | array

Size for finite difference grid, specified by a 1-by-2 array. The first entry corresponds to the first asset defined by StockSpec1 and the second entry corresponds to the second asset defined by StockSpec2.

For the finite difference method, the composition of the grid affects the quality of the output as well as the execution time. It is highly recommended to use the optional argumentsAssetPriceMax, PriceGridSize, and TimeGridSize to control the composition of the grid to ensure the quality of the output and a reasonable execution time.

```
Data Types
single | double
```


## 'TimeGridSize' - Size of the time grid for finite difference grid <br> 100 (default) | scalar | nonnegative integer

Size of the time grid for finite difference grid, specified as a nonnegative integer.

For the finite difference method, the composition of the grid affects the quality of the output as well as the execution time. It is highly recommended to use the optional argumentsAssetPriceMax, PriceGridSize, and TimeGridSize to control the composition of the grid to ensure the quality of the output and a reasonable execution time.
Data Types
single | double

## 'AmericanOpt' - Option type

0 European (default) | scalar | vector of positive integers [0, 1]
Option type, specified as NINST-by-1 positive integer scalar flags with values:

- 0 - European
- 1 - American

Data Types
single | double

## Output <br> Arguments

Price - Expected prices of spread option
vector
Expected prices of the spread option, returned as a NINST-by-1 vector.

## PriceGrid - Grid containing prices calculated by finite difference method

array
Grid containing prices calculated by finite difference method, returned as a 3-D grid with a size of PriceGridSize(1) * PriceGridSize(2) * TimeGridSize. The price for $\mathrm{t}=0$ is contained in PriceGrid(:, :, 1).

## AssetPrice 1 - Prices for first asset defined by StockSpec1

vector
Prices for first asset defined by StockSpec1, corresponding to the first dimension of PriceGrid, returned as a vector.

## AssetPrice2 - Prices for second asset defined by StockSpec2

vector
Prices for second asset defined by StockSpec2, corresponding to the second dimension of PriceGrid, returned as a vector.

## Times - Times corresponding to third dimension of PriceGrid vector

Times corresponding to third dimension of PriceGrid, returned as a vector.

## Examples Compute the Price of a Spread Option Using the Alternate Direction Implicit (ADI) Finite Difference Method

Define the spread option dates.

```
Settle = '01-Jan-2012';
Maturity = '01-April-2012';
```

Define asset 1. Price and volatility of RBOB gasoline

```
Price1gallon = 2.85; % $/gallon
Price1 = Price1gallon * 42; % $/barrel
Vol1 = 0.29;
```

Define asset 2. Price and volatility of WTI crude oil

```
Price2 = 93.20; % $/barrel
Vol2 = 0.36;
```

Define the correlation between the underlying asset prices of asset 1 and asset 2 .

Corr $=0.42$;
Define the spread option.

```
OptSpec = 'call';
```

```
Strike = 20;
```

Define the RateSpec.

```
rates = 0.05;
```

Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,
'EndDates', Maturity, 'Rates', rates, ...
'Compounding', Compounding, 'Basis', Basis)
RateSpec =


Define the StockSpec for the two assets.

```
StockSpec1 = stockspec(Vol1, Price1)
StockSpec2 = stockspec(Vol2, Price2)
```

StockSpec1 =

```
FinObj: 'StockSpec'
    Sigma: 0.2900
```

```
            AssetPrice: 119.7000
            DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
StockSpec2 =
```

```
                    FinObj: 'StockSpec
```

                    FinObj: 'StockSpec
                    Sigma: 0.3600
                    Sigma: 0.3600
        AssetPrice: 93.2000
        AssetPrice: 93.2000
    DividendType: []
    DividendType: []
    DividendAmounts: 0
DividendAmounts: 0
ExDividendDates: []

```
ExDividendDates: []
```

Compute the spread option price based on the Alternate Direction Implicit (ADI) finite difference method.

```
[Price, PriceGrid, AssetPrice1, AssetPrice2, Times] = ...
    spreadbyfd(RateSpec, StockSpec1, StockSpec2, Settle, ...
    Maturity, OptSpec, Strike, Corr);
```

Display the price.
Price

Price =
11.1998

Plot the finite difference grid.

```
mesh(AssetPrice1, AssetPrice2, PriceGrid(:, :, 1)');
    title('Spread Option Prices for Range of Underlying Prices');
```

```
xlabel('Price of underlying asset 1');
ylabel('Price of underlying asset 2');
zlabel('Price of spread option');
```

Spread Option Prices for Range of Underlying Prices

ReferencesCarmona, R., Durrleman, V., "Pricing and Hedging Spread Options,"SIAM Review, Vol. 45, No. 4, pp. 627-685, Society for Industrial andApplied Mathematics, 2003.
Villeneuve, S., Zanette, A., "Parabolic ADI Methods for PricingAmerican Options on Two Stocks," Mathematics of Operations Research,Vol. 27, No. 1, pp. 121-149, INFORMS, 2002.Ikonen, S., Toivanen, J., Efficient Numerical Methods for PricingAmerican Options Under Stochastic Volatility, Wiley InterScience, 2007.
See Also spreadsensbyfd | spreadbybjs | spreadbykirk | spreadbyls
Related- "Pricing European and American Spread Options" on page 3-41
Examples

- "Hedging Strategies Using Spread Options" on page 3-57
Concepts - "Spread Option" on page 3-37


## Purpose

Syntax

## Description

Price European or American spread options using Monte Carlo simulations

```
Price = spreadbyls(RateSpec,StockSpec1,StockSpec2,Settle,Maturity,
OptSpec,Strike,Corr)
Price = spreadbyls(___,Name,Value)
[Price,Paths,Times,Z] = spreadbyls(RateSpec,StockSpec1,StockSpec2,Set
OptSpec,Strike,Corr)
[Price,Paths,Times,Z] = spreadbyls(__ ,Name,Value)
```

Price =
spreadbyls(RateSpec,StockSpec1,StockSpec2,Settle, Maturity, OptSpec, Strike, Corr) returns the price of a European or American call or put spread option using Monte Carlo simulations.

Price = spreadbyls( $\qquad$ , Name, Value) returns the price of a European or American call or put spread option using Monte Carlo simulations using optional name-value pair arguments.
> [Price,Paths,Times,Z] = spreadbyls(RateSpec,StockSpec1,StockSpec2,Settle, Maturity, OptSpec, Strike, Corr) returns the Price, Paths, Times, and Z of a European or American call or put spread option using Monte Carlo simulations.

[Price,Paths,Times,Z] = spreadbyls(__ , Name, Value) returns the Price, Paths, Times, and Z of a European or American call or put spread option using Monte Carlo simulations using optional name-value pair arguments.

## Input Arguments

## RateSpec - Interest-rate term structure <br> structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from intenvset. For information on the interest-rate specification, see intenvset.

## Data Types <br> struct

## StockSpec 1 - Stock specification for underlying asset 1

structure
Stock specification for underlying asset 1. For information on the stock specification, see stockspec.
stockspec can handle other types of underlying assets. For example, for physical commodities the price is represented by StockSpec.Asset, the volatility is represented by StockSpec.Sigma, and the convenience yield is represented by StockSpec.DividendAmounts.

## Data Types

struct

## StockSpec2-Stock specification for underlying asset 2

structure
Stock specification for underlying asset 2. For information on the stock specification, see stockspec.
stockspec can handle other types of underlying assets. For example, for physical commodities the price is represented by StockSpec.Asset, the volatility is represented by StockSpec.Sigma, and the convenience yield is represented by StockSpec.DividendAmounts.

Data Types
struct

## Settle - Settlement date for spread option

date string | nonnegative scalar integer

Settlement date for the spread option specified, as a date string or nonnegative scalar integer.
Data Types
char

## Maturity - Maturity date for spread option <br> date string | nonnegative scalar integer

Maturity date for spread option, specified as a date string or a nonnegative scalar integer.

Data Types
char

## OptSpec - Definition of option

string with values 'call' or 'put'
Definition of option as 'call' or 'put', specified as a string.
Data Types
char

## Strike - Option strike price value

nonnegative scalar integer
Option strike price value, specified, as a nonnegative scalar integer.

## Data Types

single | double

## Corr - Correlation between underlying asset prices <br> scalar integer

Correlation between underlying asset prices, specified as an scalar integer.

Data Types<br>single | double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

Example: Price =
spreadbyls(RateSpec,StockSpec1,StockSpec2,Settle, Maturity, OptSpec, Strike,

## 'AmericanOpt' - Option type

0 European (default) | scalar with value [0,1]
Option type, specified as an integer scalar flag with value:

- 0 - European
- 1 - American

For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium.

Data Types<br>single | double

## 'NumTrials' - Scalar number of independent sample paths <br> 1000 (default) | nonnegative scalar integer

Scalar number of independent sample paths (simulation trials), specified as a nonnegative integer.

Data Types<br>single | double

## 'NumPeriods' - Scalar number of simulation periods per trial 100 (default) | nonnegative scalar integer

Scalar number of simulation periods per trial, specified as a nonnegative integer. NumPeriods is considered only when pricing European basket
options. For American spread options, NumPeriod is equal to the number of exercise days during the life of the option.
Data Types
single | double

## ' $\mathbf{Z}$ ' - Time series array of dependent random variates

 vectorTime series array of dependent random variates specified as a NumPeriods-by-2-by-NumTrials 3-D array. The $Z$ value generates the Brownian motion vector (i.e., Wiener processes) that drives the simulation.

Data Types
single | double

## 'Antithetic' - Indicator for antithetic sampling

false (default) | scalar logical flag with value of true or false
Indicator for antithetic sampling, specified with a value of true or false.

Data Types
logical

## Output Price - Expected price of spread option Arguments <br> scalar

Expected price of the spread option, returned as a 1-by-1 scalar.

## Paths - Simulated paths of correlated state variables vector

Simulated paths of correlated state variables, returned as a NumPeriods + 1-by-2-by-NumTrials 3-D time series array. Each row of Paths is the transpose of the state vector $X(t)$ at time $t$ for a given trial.

Times - Observation times associated with simulated paths vector

## Examples Compute the Price of a Spread Option Using Monte Carlo Simulation

Define the spread option dates.

```
Settle = '01-Jan-2012';
Maturity = '01-April-2012';
```

Define asset 1. Price and volatility of RBOB gasoline

```
Price1gallon = 2.85; % $/gallon
Price1 = Price1gallon * 42; % $/barrel
Vol1 = 0.29;
```

Define asset 2. Price and volatility of WTI crude oil

```
Price2 = 93.20; % $/barrel
Vol2 = 0.36;
```

Define the correlation between the underlying asset prices of asset 1 and asset 2.

Corr $=0.42$;
Define the spread option.

```
OptSpec = 'call';
Strike = 20;
```

Define the RateSpec.

```
rates = 0.05;
```

Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,
'EndDates', Maturity, 'Rates', rates, ...
'Compounding', Compounding, 'Basis', Basis)

RateSpec =
FinObj: 'RateSpec'
Compounding: -1
Disc: 0.9876
Rates: 0.0500
EndTimes: 0.2500
StartTimes: 0
EndDates: 734960
StartDates: 734869
ValuationDate: 734869
Basis: 1
EndMonthRule: 1

Define the StockSpec for the two assets.
StockSpec1 = stockspec(Vol1, Price1)
StockSpec2 = stockspec(Vol2, Price2)

StockSpec1 =
FinObj: 'StockSpec'

```
            Sigma: 0.2900
            AssetPrice: 119.7000
            DividendType: []
DividendAmounts: 0
ExDividendDates: []
```


## StockSpec2 =

```
                    FinObj: 'StockSpec
```

                    FinObj: 'StockSpec
                    Sigma: 0.3600
                    Sigma: 0.3600
            AssetPrice: 93.2000
            AssetPrice: 93.2000
            DividendType: []
            DividendType: []
    DividendAmounts: O
DividendAmounts: O
ExDividendDates: []

```
ExDividendDates: []
```

Compute the spread option price using Monte Carlo simulation based on the Longstaff-Schwartz model.

Price = spreadbyls(RateSpec, StockSpec1, StockSpec2, Settle, ... Maturity, OptSpec, Strike, Corr)

Price =
11.0799

## References

Carmona, R., Durrleman, V., "Pricing and Hedging Spread Options," SIAM Review, Vol. 45, No. 4, pp. 627-685, Society for Industrial and Applied Mathematics, 2003.

See Also
spreadsensbyls | spreadbybjs | spreadbyfd | spreadbykirk

## Related <br> Examples

## Concepts

- "Pricing European and American Spread Options" on page 3-41
- "Hedging Strategies Using Spread Options" on page 3-57
- "Spread Option" on page 3-37

| Purpose | Calculate European spread option prices and sensitivities using Kirk pricing model |
| :---: | :---: |
| Syntax | ```PriceSens = spreadbykirk(RateSpec,StockSpec1,StockSpec2,Settle,Maturity, OptSpec,Strike,Corr) PriceSens = spreadsensbykirk(__ ,Name,Value)``` |
| Description | PriceSens = <br> spreadbykirk(RateSpec,StockSpec1,StockSpec2,Settle, Maturity, OptSpec, Strike, Corr) returns the European spread option prices and sensitivities using the Kirk pricing model. |
|  | PriceSens = spreadsensbykirk( $\qquad$ , Name, Value) returns the European spread option prices and sensitivities using the Kirk pricing model with optional name-value pair arguments. |
| Input <br> Arguments | RateSpec - Interest-rate term structure structure |
|  | Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from intenvset. For information on the interest-rate specification, see intenvset. |
|  | Data Types struct |
|  | StockSpec 1 - Stock specification for underlying asset 1 structure |

## structure

Stock specification for underlying asset 1. For information on the stock specification, see stockspec.
stockspec can handle other types of underlying assets. For example, for physical commodities the price is represented by StockSpec.Asset, the volatility is represented by StockSpec.Sigma, and the convenience yield is represented by StockSpec.DividendAmounts.

## Data Types <br> struct

## StockSpec2-Stock specification for underlying asset 2

structure
Stock specification for underlying asset 2. For information on the stock specification, see stockspec.
stockspec can handle other types of underlying assets. For example, for physical commodities the price is represented by StockSpec.Asset, the volatility is represented by StockSpec.Sigma, and the convenience yield is represented by StockSpec.DividendAmounts.

## Data Types

struct

## Settle - Settlement dates for spread option

## nonnegative integer | vector of nonnegative integers | date string | cell array of date strings

Settlement dates for the spread option, specified as a date string or as nonnegative integers using a NINST-by- 1 vector or cell array of dates.

## Data Types <br> char | cell

## Maturity - Maturity date for spread option

nonnegative integer | vector of nonnegative integers | date string | cell array of date strings

Maturity date for spread option, specified as a date string or as nonnegative integers using a NINST-by-1 vector or cell array of dates.

```
Data Types
char | cell
```


## OptSpec - Definition of option

string with values 'call' or 'put' | cell array of strings

Definition of option as 'call' or 'put', specified as a NINST-by-1 cell array of strings.

## Data Types

char | cell

## Strike - Option strike price values

integer | vector of integers
Option strike price values, specified as an integer using as NINST-by-1 vector of strike price values.

If Strike is equal to zero, this function computes the price and sensitivities of an exchange option.

## Data Types <br> single | double

## Corr - Correlation between underlying asset prices

integer | vector of integers
Correlation between underlying asset prices, specified as an integer using as NINST-by-1 vector.

## Data Types

single | double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

Example: PriceSens =
spreadsensbykirk(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec , §

## 'OutSpec' - Define outputs

\{'Price'\} (default) | string with values 'Price', 'Delta', 'Gamma',
'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | cell array of strings

```
with values 'Price','Delta','Gamma', 'Vega','Lambda', 'Rho',
'Theta', and 'All'
```

Define outputs specifying NOUT- by-1 or 1-by-NOUT cell array of strings with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

OutSpec = \{'All'\} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec to include each sensitivity:

```
Example: OutSpec =
{'delta','gamma','vega','lambda','rho','theta','price'}
```

```
Data Types
char | cell
```


## Output Arguments

PriceSens - Expected price or sensitivities values of spread option vector

Expected price or sensitivities values of the spread option, returned as a NINST-by-1 or NINST-by-2 vector.

## Examples Compute the Price and Sensitivities of a Spread Option Using the Kirk Model

Define the spread option dates.

```
Settle = '01-Jun-2012';
Maturity = '01-Sep-2012';
```

Define asset 1. Price and volatility of RBOB gasoline

```
Price1gallon = 2.85; % $/gallon
Price1 = Price1gallon * 42; % $/barrel
Vol1 = 0.29;
```

Define asset 2. Price and volatility of WTI crude oil

```
Price2 = 93.20;
% $/barrel
```

```
Vol2 = 0.36;
```

Define the correlation between the underlying asset prices of asset 1 and asset 2.

Corr $=0.42$;
Define the spread option.

```
OptSpec = 'call';
Strike = 20;
```

Define the RateSpec.
rates $=0.05$;
Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ... 'EndDates', Maturity, 'Rates', rates, ...
'Compounding', Compounding, 'Basis', Basis)

RateSpec =

```
                    FinObj: 'RateSpec'
    Compounding: -1
                Disc: 0.9876
                    Rates: 0.0500
        EndTimes: 0.2500
        StartTimes: 0
            EndDates: 735113
        StartDates: 735021
    ValuationDate: 735021
            Basis: 1
    EndMonthRule: 1
```

Define the StockSpec for the two assets.

```
StockSpec1 = stockspec(Vol1, Price1)
StockSpec2 = stockspec(Vol2, Price2)
```


## StockSpec1 =

FinObj: 'StockSpec'
Sigma: 0.2900
AssetPrice: 119.7000
DividendType: []
DividendAmounts: 0
ExDividendDates: []

## StockSpec2 =

FinObj: 'StockSpec'
Sigma: 0.3600
AssetPrice: 93.2000
DividendType: []
DividendAmounts: 0
ExDividendDates: []

Compute the spread option price and sensitivities based on the Kirk model.

```
OutSpec = {'Price', 'Delta', 'Gamma'};
[Price, Delta, Gamma] = spreadsensbykirk(RateSpec, StockSpec1, StockS
Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec)
```

Price =
11.1904

```
Delta =
    0.6722 -0.6067
    Gamma =
    0.0191 0.0217
```


## References

Carmona, R., Durrleman, V., "Pricing and Hedging Spread Options," SIAM Review, Vol. 45, No. 4, pp. 627-685, Society for Industrial and Applied Mathematics, 2003.

See Also spreadbykirk \| spreadbybjs | spreadbyfd | spreadbyls
Related - "Pricing European and American Spread Options" on page 3-41
Examples

- "Hedging Strategies Using Spread Options" on page 3-57

Concepts - "Spread Option" on page 3-37

| Purpose | Calculate European spread option prices and sensitivities using <br> Bjerksund-Stensland pricing model |
| :--- | :--- |
| Syntax | PriceSens = spreadbybjs(RateSpec, StockSpec1, StockSpec2, Settle, Maturit) <br> OptSpec, Strike, Corr) |
| PriceSens = spreadsensbybjs (__ , Name, Value) |  |

PriceSens = spreadsensbybjs( __ , Name, Value) returns the European spread option prices and sensitivities using the Bjerksund-Stensland pricing model with optional name-value pair arguments.

## Input Arguments

## RateSpec - Interest-rate term structure <br> structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from intenvset. For information on the interest-rate specification, see intenvset.

Data Types
struct

## StockSpec 1 - Stock specification for underlying asset 1

## structure

Stock specification for underlying asset 1. For information on the stock specification, see stockspec.
stockspec can handle other types of underlying assets. For example, for physical commodities the price is represented by StockSpec.Asset, the volatility is represented by StockSpec.Sigma, and the convenience yield is represented by StockSpec.DividendAmounts.

## Data Types <br> struct <br> StockSpec2-Stock specification for underlying asset 2 <br> structure

Stock specification for underlying asset 2. For information on the stock specification, see stockspec.
stockspec can handle other types of underlying assets. For example, for physical commodities the price is represented by StockSpec.Asset, the volatility is represented by StockSpec.Sigma, and the convenience yield is represented by StockSpec.DividendAmounts.

## Data Types <br> struct

## Settle - Settlement dates for spread option

nonnegative integer | vector of nonnegative integers | date string | cell array of date strings

Settlement dates for the spread option, specified as a date string or as nonnegative integers using a NINST-by-1 vector or cell array of dates.

## Data Types <br> char | cell

## Maturity - Maturity date for spread option

## nonnegative integer | vector of nonnegative integers | date string | cell array of date strings

Maturity date for spread option, specified as a date string or as nonnegative integers using a NINST-by-1 vector or cell array of dates.

```
Data Types
char | cell
```


## OptSpec - Definition of option

string with values 'call' or 'put' | cell array of strings

Definition of option as 'call' or 'put', specified as a NINST-by-1 cell array of strings.

```
Data Types
char | cell
```


## Strike - Option strike price values

integer | vector of integers
Option strike price values, specified as an integer using a NINST-by-1 vector of strike price values.

If Strike is equal to zero the function computes the price and sensitivities of an exchange option.

## Data Types

single | double

## Corr - Correlation between underlying asset prices

integer | vector of integers
Correlation between underlying asset prices, specified as an integer using a NINST-by-1 vector.
Data Types
single | double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: PriceSens = spreadsensbykirk(RateSpec,StockSpec1,StockSpec2, Settle, Maturity, OptSp
'OutSpec' - Define outputs
\{'Price'\} (default) | string with values 'Price', 'Delta', 'Gamma',
'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | cell array of strings

```
with values 'Price','Delta', 'Gamma', 'Vega','Lambda', 'Rho',
'Theta', and 'All'
```

Define outputs specifying NOUT- by-1 or 1-by-NOUT cell array of strings with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

OutSpec = \{'All'\} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec to include each sensitivity:

```
Example: OutSpec =
{'delta','gamma','vega','lambda','rho','theta','price'}
```

```
Data Types
char | cell
```


## Output PriceSens - Expected prices or sensitivities values of spread Arguments <br> option

## vector

Expected prices or sensitivities values of the spread option, returned as a NINST-by-1 or NINST-by-2 vector.

## Examples Compute the Price and Sensitivities of a Spread Option Using the Bjerksund-Stensland Model

Define the spread option dates.

```
Settle = '01-Jun-2012';
Maturity = '01-Sep-2012';
```

Define asset 1. Price and volatility of RBOB gasoline

```
Price1gallon = 2.85; % $/gallon
Price1 = Price1gallon * 42; % $/barrel
Vol1 = 0.29;
```

Define asset 2. Price and volatility of WTI crude oil

```
Price2 = 93.20; % $/barrel
Vol2 = 0.36;
```

Define the correlation between the underlying asset prices of asset 1 and asset 2.

```
Corr = 0.42;
```

Define the spread option.

```
OptSpec = 'call';
Strike = 20;
```

Define the RateSpec.

```
rates = 0.05;
```

Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,
'EndDates', Maturity, 'Rates', rates, ...
'Compounding', Compounding, 'Basis', Basis)

## RateSpec =

FinObj: 'RateSpec'<br>Compounding: -1<br>Disc: 0.9876<br>Rates: 0.0500<br>EndTimes: 0.2500<br>StartTimes: 0<br>EndDates: 735113<br>StartDates: 735021<br>ValuationDate: 735021<br>Basis: 1<br>EndMonthRule: 1

```
Define the StockSpec for the two assets.
StockSpec1 = stockspec(Vol1, Price1)
StockSpec2 = stockspec(Vol2, Price2)
StockSpec1 =
                    FinObj: 'StockSpec'
                    Sigma: 0.2900
        AssetPrice: 119.7000
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```


## StockSpec2 =

```
                    FinObj: 'StockSpec'
```

                    FinObj: 'StockSpec'
                    Sigma: 0.3600
                    Sigma: 0.3600
            AssetPrice: 93.2000
            AssetPrice: 93.2000
    DividendType: []
    DividendType: []
    DividendAmounts: 0
    DividendAmounts: 0
    ExDividendDates: []
    ```
    ExDividendDates: []
```

Compute the spread option price and sensitivities based on the Kirk model.

OutSpec = \{'Price', 'Delta', 'Gamma'\};
[Price, Delta, Gamma] = spreadsensbybjs(RateSpec, StockSpec1, StockSpec2, Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec)

Price =

Delta =
$0.6737-0.6082$

Gamma =
$0.0190 \quad 0.0216$

## References

Carmona, R., Durrleman, V., "Pricing and Hedging Spread Options," SIAM Review, Vol. 45, No. 4, pp. 627-685, Society for Industrial and Applied Mathematics, 2003.

Bjerksund, Petter, Stensland, Gunnar, "Closed form spread option valuation," Department of Finance, NHH, 2006.

## See Also

Related
Examples
Concepts
spreadbykirk | spreadbybjs | spreadbyfd | spreadbyls

- "Pricing European and American Spread Options" on page 3-41
- "Hedging Strategies Using Spread Options" on page 3-57
- "Spread Option" on page 3-37

Purpose

Syntax

## Description

Calculate price and sensitivities for European or American spread options using Monte Carlo simulations

```
PriceSens = spreadsensbyls(RateSpec,StockSpec1,StockSpec2,Settle,Maturity
OptSpec,Strike,Corr)
PriceSens = spreadsensbyls(___ ,Name,Value)
[PriceSens,Paths,Times,Z] = spreadsensbyls(RateSpec,StockSpec1,StockSpec2
OptSpec,Strike,Corr)
[PriceSens,Paths,Times,Z] = spreadsensbyls( __ ,Name,Value)
```

PriceSens =
spreadsensbyls(RateSpec,StockSpec1,StockSpec2,Settle, Maturity, OptSpec, Strike, Corr) returns the price of a European or American call or put spread option using Monte Carlo simulations.

PriceSens = spreadsensbyls( __ , Name, Value) returns the price of a European or American call or put spread option using Monte Carlo simulations with optional name-value pair arguments.
[PriceSens,Paths,Times,Z] = spreadsensbyls(RateSpec,StockSpec1,StockSpec2,Settle, Maturity, OptSpec, Strike, Corr) returns the PriceSens, Paths, Times, and Z of a European or American call or put spread option using Monte Carlo simulations.
[PriceSens, Paths,Times, Z] = spreadsensbyls( __ , Name, Value) returns the PriceSens, Paths, Times, and Z of a European or American call or put spread option using Monte Carlo simulations with optional name-value pair arguments.

## spreadsensbyls

## Input Arguments

## RateSpec - Interest-rate term structure <br> structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from intenvset. For information on the interest-rate specification, see intenvset.

## Data Types

struct

## StockSpec 1 - Stock specification for underlying asset 1

structure
Stock specification for underlying asset 1. For information on the stock specification, see stockspec.
stockspec can handle other types of underlying assets. For example, for physical commodities the price is represented by StockSpec.Asset, the volatility is represented by StockSpec.Sigma, and the convenience yield is represented by StockSpec.DividendAmounts.

## Data Types

struct

## StockSpec2 - Stock specification for underlying asset 2

structure
Stock specification for underlying asset 2. For information on the stock specification, see stockspec.
stockspec can handle other types of underlying assets. For example, for physical commodities the price is represented by StockSpec.Asset, the volatility is represented by StockSpec.Sigma, and the convenience yield is represented by StockSpec.DividendAmounts.

## Data Types

struct

## Settle - Settlement date for spread option

date string | nonnegative scalar integer

Settlement date for the spread option, specified as a date string or as a nonnegative scalar integer.

## Data Types <br> char

## Maturity - Maturity date for spread option

date string | nonnegative scalar integer
Maturity date for spread option, specified as a date string or as a nonnegative scalar integer.

## Data Types <br> char

## OptSpec - Definition of option

string with values 'call' or 'put'
Definition of option as 'call' or 'put', specified as a string.
Data Types
char

## Strike - Option strike price value

## nonnegative scalar integer

Option strike price value, specified as a scalar integer.

## Data Types <br> single | double

## Corr - Correlation between underlying asset prices

scalar integer
Correlation between underlying asset prices, specified as a scalar integer.

Data Types<br>single | double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

Example: PriceSens = spreadbyls(RateSpec, StockSpec1,StockSpec2,Settle, Maturity, OptSpec, Str:

## 'AmericanOpt' - Option type

0 European (default) | scalar with values [0,1]
Option type, specified as a scalar integer flag with values:

- 0 - European
- 1 - American

For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium.

Data Types<br>single | double

'NumTrials' - Number of independent sample paths
1000 (default) | nonnegative scalar integer
Number of independent sample paths (simulation trials) specified as a nonnegative scalar integer.

```
Data Types
single | double
```


## 'NumPeriods' - Number of simulation periods per trial

100 (default) | nonnegative scalar integer
Number of simulation periods per trial, specified as a nonnegative scalar integer. NumPeriods is considered only when pricing European basket options. For American spread options, NumPeriods is equal to the number of exercise days during the life of the option.

Data Types<br>single | double

## 'Z' - Time series array of dependent random variates

## vector

Time series array of dependent random variates, specified as a NumPeriods-by-2-by-NumTrials 3-D array. The $Z$ value generates the Brownian motion vector (i.e., Wiener processes) that drives the simulation.

```
Data Types
single | double
```


## 'Antithetic' - Indicator for antithetic sampling

## false (default) | logical flag with value of true or false

Indicator for antithetic sampling, specified with value of true or false.

## Data Types

logical

## 'OutSpec' - Define outputs

\{'Price'\} (default) | string with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | cell array of strings with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

Define outputs specifying NOUT- by-1 or 1-by-NOUT cell array of strings with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

OutSpec $=$ \{'All'\} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec to include each sensitivity:
Example: OutSpec =
\{'delta','gamma','vega','lambda','rho','theta','price'\}

## Data Types <br> char | cell

## Output Arguments

## Examples

## PriceSens - Expected price and sensitivities of spread option vector

Expected price and sensitivities of the spread option, returned as a NINST-by- 1 vector.

## Paths - Simulated paths of correlated state variables vector

Simulated paths of correlated state variables, returned as a NumPeriods + 1-by-2-by-NumTrials 3-D time series array. Each row of Paths is the transpose of the state vector $X(t)$ at time $t$ for a given trial.

## Times - Observation times associated with simulated paths

 vectorObservation times associated with simulated paths, returned as a NumPeriods + 1-by-1 column vector of observation times associated with the simulated paths. Each element of Times is associated with the corresponding row of Paths.

## Z - Time series array of dependent random variates

## vector

Time series array of dependent random variates, returned as a NumPeriods-by-2-by-NumTrials 3-D array when Z is specified as an input argument. If the $Z$ input argument is not specified, then the $Z$ output argument contains the random variates generated internally.

## Compute the Price and Sensitivities of a Spread Option Using Monte Carlo Simulation

Define the spread option dates.

```
Settle = '01-Jun-2012';
Maturity = '01-Sep-2012';
```

Define asset 1. Price and volatility of RBOB gasoline

```
Price1gallon = 2.85; % $/gallon
Price1 = Price1gallon * 42; % $/barrel
Vol1 = 0.29;
```

Define asset 2. Price and volatility of WTI crude oil

```
Price2 = 93.20; % $/barrel
Vol2 = 0.36;
```

Define the correlation between the underlying asset prices of asset 1 and asset 2 .

Corr $=0.42$;
Define the spread option.

```
OptSpec = 'call';
Strike = 20;
```

Define the RateSpec.
rates = 0.05;
Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
'EndDates', Maturity, 'Rates', rates, ...
'Compounding', Compounding, 'Basis', Basis)

## RateSpec =

```
                    FinObj: 'RateSpec'
Compounding: -1
            Disc: 0.9876
                    Rates: 0.0500
            EndTimes: 0.2500
                StartTimes: 0
            EndDates: 735113
```

```
    StartDates: 735021
ValuationDate: 735021
    Basis: 1
EndMonthRule: 1
```

Define the StockSpec for the two assets.

```
StockSpec1 = stockspec(Vol1, Price1)
StockSpec2 = stockspec(Vol2, Price2)
```

StockSpec1 =

```
            FinObj: 'StockSpec'
                    Sigma: 0.2900
        AssetPrice: 119.7000
        DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```


## StockSpec2 =

```
            FinObj: 'StockSpec'
            Sigma: 0.3600
        AssetPrice: 93.2000
        DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Compute the spread option price and sensitivities using Monte Carlo simulation based on the Longstaff-Schwartz model.

OutSpec = \{'Price', 'Delta', 'Gamma'\};
[Price, Delta, Gamma] = spreadsensbyls(RateSpec, StockSpec1, StockSpec

```
Settle, Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec)
```

Price =
11.0799

Delta =
$0.6626-0.5972$

Gamma =
$0.0209 \quad 0.0240$

## References

Carmona, R., Durrleman, V., "Pricing and Hedging Spread Options," SIAM Review, Vol. 45, No. 4, pp. 627-685, Society for Industrial and Applied Mathematics, 2003.

See Also spreadbyls | spreadbybjs | spreadbyfd | spreadbykirk
Related

- "Pricing European and American Spread Options" on page 3-41

Examples
Concepts

- "Spread Option" on page 3-37


## Purpose

Syntax

## Description

Calculate price and sensitivities of European or American spread options using finite difference method

```
PriceSens = spreadbyfd(RateSpec,StockSpec1,StockSpec2,Settle,Maturity
OptSpec,Strike,Corr)
PriceSens = spreadbyfd( ___ ,Name,Value)
[PriceSens,PriceGrid,AssetPrice1,AssetPrice2,Times] = spreadbyfd(Rate؛
OptSpec,Strike,Corr)
[PriceSens,PriceGrid,AssetPrice1,AssetPrice2,Times]
= spreadbyfd(
```

$\qquad$

``` ,
    Name,Value)
PriceSens =
spreadbyfd(RateSpec,StockSpec1,StockSpec2,Settle,Maturity, OptSpec, Strike, Corr) returns the price and sensitivities of European or American call or put spread options using the Alternate Direction Implicit (ADI) finite difference method. The spread is between the asset defined in StockSpec1 minus the asset defined in StockSpec2.
```

PriceSens = spreadbyfd( $\qquad$ ,Name, Value) returns the price and sensitivities of European or American call or put spread options using the Alternate Direction Implicit (ADI) finite difference method using optional name-value pair arguments. The spread is between the asset defined in StockSpec1 minus the asset defined in StockSpec2.
[PriceSens,PriceGrid,AssetPrice1,AssetPrice2,Times] = spreadbyfd(RateSpec,StockSpec1,StockSpec2,Settle,Maturity, OptSpec,Strike,Corr) returns the PriceSens, PriceGrid, AssetPrice1, AssetPrice2, and Times for European or American call or put spread options using the Alternate Direction Implicit (ADI) finite difference method. The spread is between the asset defined in StockSpec1 minus the asset defined in StockSpec2.

## Input Arguments

[PriceSens,PriceGrid,AssetPrice1,AssetPrice2,Times] = spreadbyfd( __, Name, Value) returns the PriceSens, PriceGrid, AssetPrice1, AssetPrice2, and Times for European or American call or put spread options using the Alternate Direction Implicit (ADI) finite difference method using optional name-value pair arguments. The spread is between the asset defined in StockSpec 1 minus the asset defined in StockSpec2.

## RateSpec - Interest-rate term structure

## structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from intenvset. For information on the interest-rate specification, see intenvset.

## Data Types

struct

## StockSpec 1 - Stock specification for underlying asset 1

structure
Stock specification for underlying asset 1. For information on the stock specification, see stockspec.
stockspec can handle other types of underlying assets. For example, for physical commodities the price is represented by StockSpec.Asset, the volatility is represented by StockSpec.Sigma, and the convenience yield is represented by StockSpec.DividendAmounts.

## Data Types

struct

## StockSpec2-Stock specification for underlying asset 2

structure
Stock specification for underlying asset 2. For information on the stock specification, see stockspec.
stockspec can handle other types of underlying assets. For example, for physical commodities the price is represented by StockSpec.Asset,
the volatility is represented by StockSpec.Sigma, and the convenience yield is represented by StockSpec.DividendAmounts.

## Data Types

struct

## Settle - Settlement dates for spread option

nonnegative integer | vector of nonnegative integers | date string | cell array of date strings

Settlement dates for the spread option, specified as date strings or as nonnegative integers using a NINST-by-1 vector or cell array of dates.

## Data Types <br> char | cell

## Maturity - Maturity date for spread option

nonnegative integer | vector of nonnegative integers | date string | cell array of date strings

Maturity date for spread option, specified as date strings or as nonnegative integers using a NINST-by-1 vector or cell array of dates.

## Data Types <br> char | cell

## OptSpec - Definition of option

string with values 'call' or 'put' | cell array of strings
Definition of option as 'call' or 'put', specified as a NINST-by-1 cell array of strings.

## Data Types

char | cell

## Strike - Option strike price values

integer | vector of integers
Option strike price values, specified as an integer using a NINST-by-1 vector of strike price values.

## Data Types <br> single | double <br> Corr - Correlation between underlying asset prices

integer | vector of integers
Correlation between underlying asset prices, specified as an integer using aNINST-by-1 vector.

```
Data Types
single | double
```


## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example:
[PriceSens,PriceGrid,AssetPrice1,AssetPrice2,Times] = spreadbyfd(RateSpec,StockSpec1,StockSpec2, Settle, Maturity, OptSpec, Strike, 'AssetPriceMin', 'AssetPriceMax', 'PriceGridSize', 'TimeGridSize', 'Americand

## 'OutSpec' - Define outputs

\{'Price'\} (default) | string with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | cell array of strings with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

Define outputs specifying NOUT- by-1 or 1-by-NOUT cell array of strings with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

OutSpec $=$ \{'All' $\}$ specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec to include each sensitivity:

```
Example: OutSpec =
{'delta','gamma','vega','lambda','rho','theta','price'}
```

Data Types<br>char | cell

## 'AssetPriceMin' - Minimum price for price grid boundary <br> if unspecified, StockSpec values are calculated based on asset distributions at maturity (default) | array

Minimum price for price grid boundary, specified by a 1-by-2 array. The first entry in the array corresponds to the first asset defined by StockSpec1 and the second entry corresponds to the second asset defined by StockSpec2.

For the finite difference method, the composition of the grid affects the quality of the output as well as the execution time. It is highly recommended to use the optional arguments AssetPriceMin, AssetPriceMax, PriceGridSize, and TimeGridSize to control the composition of the grid to ensure the quality of the output and a reasonable execution time.

Data Types
single | double

## 'AssetPriceMax' - Maximum price for price grid boundary if unspecified, StockSpec values are calculated based on asset distributions at maturity (default) | array

Maximum price for price grid boundary, specified by a 1-by-2 array. The first entry in the array corresponds to the first asset defined by StockSpec 1 and the second entry corresponds to the second asset defined by StockSpec2.

For the finite difference method, the composition of the grid affects the quality of the output as well as the execution time. It is highly recommended to use the optional arguments AssetPriceMin, AssetPriceMax, PriceGridSize, and TimeGridSize to control the composition of the grid to ensure the quality of the output and a reasonable execution time.

```
Data Types
single | double
```


## 'PriceGridSize' - Size for finite difference grid <br> [300,300] (default) | array

Size for finite difference grid, specified by a 1-by-2 array. The first entry corresponds to the first asset defined by StockSpec1 and the second entry corresponds to the second asset defined by StockSpec2.

For the finite difference method, the composition of the grid affects the quality of the output as well as the execution time. It is highly recommended to use the optional argumentsAssetPriceMax, PriceGridSize, and TimeGridSize to control the composition of the grid to ensure the quality of the output and a reasonable execution time.

## Data Types

single | double

## 'TimeGridSize' - Size of time grid for finite difference grid

100 (default) | scalar | nonnegative integer
Size of time grid for finite difference grid, specified as a nonnegative integer.

For the finite difference method, the composition of the grid affects the quality of the output as well as the execution time. It is highly recommended to use the optional argumentsAssetPriceMax, PriceGridSize, and TimeGridSize to control the composition of the grid to ensure the quality of the output and a reasonable execution time.

## Data Types

single | double

## 'AmericanOpt' - Option type

0 European (default) | scalar | vector of positive integers [0, 1]
Option type, specified as NINST-by-1 positive integer scalar flags with values:

- 0 - European
- 1 - American
Data Types
single | double


## Output PriceSens - Expected prices and sensitivities of spread option Arguments

Examples Compute the Price of a Spread Option Using the Alternate Direction Implicit (ADI) Finite Difference Method
Define the spread option dates.

```
Settle = '01-Jun-2012';
Maturity = '01-Sep-2012';
```

Define asset 1. Price and volatility of RBOB gasoline

```
Price1gallon = 2.85; % $/gallon
Price1 = Price1gallon * 42; % $/barrel
Vol1 = 0.29;
```

Define asset 2. Price and volatility of WTI crude oil

```
Price2 = 93.20; % $/barrel
Vol2 = 0.36;
```

Define the correlation between the underlying asset prices of asset 1 and asset 2 .

Corr $=0.42$;
Define the spread option.

```
OptSpec = 'call';
Strike = 20;
```

Define the RateSpec.

```
rates = 0.05;
```

Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
'EndDates', Maturity, 'Rates', rates, ...
'Compounding', Compounding, 'Basis', Basis)

RateSpec =

FinObj: 'RateSpec'
Compounding: -1
Disc: 0.9876Rates: 0.0500EndTimes: 0.2500
StartTimes: 0EndDates: 735113
StartDates: 735021
ValuationDate: 735021Basis: 1
EndMonthRule: 1
Define the StockSpec for the two assets.
StockSpec1 = stockspec(Vol1, Price1)
StockSpec2 = stockspec(Vol2, Price2)
StockSpec1 =
FinObj: 'StockSpec
Sigma: 0.2900
AssetPrice: 119.7000
DividendType: [
DividendAmounts: 0
ExDividendDates: []
StockSpec2 =
FinObj: 'StockSpec
Sigma: 0.3600AssetPrice: 93.2000
DividendType: [
DividendAmounts: 0
ExDividendDates: []

Compute the spread option price and sensitivities based on the Alternate Direction Implicit (ADI) finite difference method.

OutSpec = \{'Price', 'Delta', 'Gamma'\};
[Price, Delta, Gamma, PriceGrid, AssetPrice1, AssetPrice2, Times] = ... spreadsensbyfd(RateSpec, StockSpec1, StockSpec2, Settle, ... Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec);

Display the price and sensitivities.
Price
Delta
Gamma

Price =
11.1998

Delta =

Gamma =
$0.0190 \quad 0.0214$

Plot the finite difference grid.

```
mesh(AssetPrice1, AssetPrice2, PriceGrid(:, :, 1)');
    title('Spread Option Prices for Range of Underlying Prices');
    xlabel('Price of underlying asset 1');
    ylabel('Price of underlying asset 2');
    zlabel('Price of spread option');
```



## References

Carmona, R., Durrleman, V., "Pricing and Hedging Spread Options," SIAM Review, Vol. 45, No. 4, pp. 627-685, Society for Industrial and Applied Mathematics, 2003.
Villeneuve, S., Zanette, A., "Parabolic ADI Methods for PricingAmerican Options on Two Stocks," Mathematics of Operations Research,Vol. 27, No. 1, pp. 121-149, INFORMS, 2002.Ikonen, S., Toivanen, J., Efficient Numerical Methods for PricingAmerican Options Under Stochastic Volatility, Wiley InterScience, 2007.
See Also spreadsensbyfd | spreadbybjs | spreadbykirk | spreadbyls
Related- "Pricing European and American Spread Options" on page 3-41
Examples

- "Hedging Strategies Using Spread Options" on page 3-57
Concepts - "Spread Option" on page 3-37


## Purpose Specify European stock option structure

```
Syntax
[StockOptSpec] = stockoptspec(OptPrice, Strike, Settle,
Maturity, OptSpec, InterpMethod)
```


## Arguments

## Description

## Examples

[StockOptSpec] = stockoptspec(OptPrice, Strike, Settle, Maturity, OptSpec, InterpMethod) creates a structure encapsulating the properties of a stock option structure.

## Specify a European Stock Option Structure

This example shows how to specify a European stock option structure using the following data quoted from liquid options in the market with varying strikes and maturity.

```
Settle = '01/01/06';
Maturity = ['07/01/06';
    '07/01/06';
```

```
    '07/01/06';
    '07/01/06';
    '01/01/07';
    '01/01/07';
    '01/01/07';
    '01/01/07';
    '07/01/07';
    '07/01/07';
    '07/01/07';
    '07/01/07';
'01/01/08';
'01/01/08';
'01/01/08';
'01/01/08'];
Strike = [113;
    101;
    100;
        88;
        128;
        112;
        100;
        78;
        144;
        112;
        100;
        69;
        162;
        112;
        100;
        61];
OptPrice =[ 0;
    4.807905472659144;
    1.306321897011867;
    0.048039195057173;
        0;
```

```
    2.310953054191461;
    1.421950392866235;
    0.020414826276740;
            0;
    5.091986935627730;
    1.346534812295291;
    0.005101325584140;
            0;
    8.047628153217246;
    1.219653432150932;
    0.001041436654748];
OptSpec = { 'call';
    'call';
    'put';
    'put';
    'call';
    'call';
    'put';
    'put';
    'call';
    'call';
    'put';
    'put';
    'call';
    'call';
    'put';
    'put'};
StockOptSpec = stockoptspec(OptPrice, Strike, Settle, Maturity, OptSp
StockOptSpec =
FinObj: 'StockOptSpec'
OptPrice: [16x1 double]
```


## stockoptspec

> Strike: $[16 \times 1$ double]
> Settle: 732678
> Maturity: $[16 \times 1$ double]
> OptSpec: $\{16 \times 1$ cell\}
> InterpMethod: 'price'

See Also ittprice | itttree | stockspec

## Purpose Create stock structure

Syntax StockSpec = stockspec(Sigma, AssetPrice, DividendType, DividendAmounts, ExDividendDates)

## Arguments

| Sigma | NINST-by-1 decimal annual price volatility of <br> underlying security. |
| :--- | :--- |
| AssetPrice | NINST-by-1 vector of underlying asset price <br> values at time 0. |
| DividendType | (Optional) NINST-by-1 cell array of strings <br> specifying each stock's dividend type. |
| Dividend type must be either cash for actual <br> dollar dividends, constant for constant <br> dividend yield, or continuous for continuous <br> dividend yield. This function does not handle <br> stock option dividends. |  |

Note Dividends are assumed to be paid in cash. Noncash dividends (stock) are not allowed. When combining two or more type of dividends, shorter rows should be padded with the value NaN.

| DividendAmounts | (Optional) NINST-by-NDIV matrix of cash <br> dividends or NINST-by-1 vector representing <br> a constant or continuous annualized dividend <br> yield. |
| :--- | :--- |
| ExDividendDates | (Optional) NINST-by-NDIV matrix of <br> ex-dividend dates for cash type or NINST-by-1 <br> vector of ex-dividend dates for constant <br> dividend type. For continuous dividend <br> type, this argument should be ignored. |

Description StockSpec = stockspec(Sigma, AssetPrice, DividendType, DividendAmounts, ExDividendDates) creates a MATLAB structure containing the properties of a stock.

Note StockSpec handles other types of underliers when pricing instruments other than equities.

## Examples Create a StockSpec for Stocks With Cash Dividends

Consider a stock that provides four cash dividends of $\$ 0.50$ on January 3, 2008, April 1, 2008, July 5, 2008 and October 1, 2008. The stock is trading at $\$ 50$, and has a volatility of $20 \%$ per annum. Using this data, create the structure StockSpec:

```
AssetPrice = 50;
Sigma = 0.20;
DividendType = {'cash'};
DividendAmounts = [0.50, 0.50, 0.50, 0.50];
ExDividendDates = {'03-Jan-2008', '01-Apr-2008', '05-July-2008', '01-Oct-
StockSpec = stockspec(Sigma, AssetPrice, DividendType, DividendAmounts,
```

```
StockSpec =
```

```
            FinObj: 'StockSpec'
```

            FinObj: 'StockSpec'
            Sigma: 0.2000
            Sigma: 0.2000
        AssetPrice: 50
        AssetPrice: 50
    DividendType: {'cash'}
    DividendType: {'cash'}
    DividendAmounts: [0.5000 0.5000 0.5000 0.5000]
DividendAmounts: [0.5000 0.5000 0.5000 0.5000]
ExDividendDates: [733410 733499 733594 733682]
ExDividendDates: [733410 733499 733594 733682]
Examine the StockSpec structure.
datedisp(StockSpec.ExDividendDates)
StockSpec.DividendType
03-Jan-2008 01-Apr-2008 05-Jul-2008 01-Oct-2008
ans =
'cash'

```

The StockSpec structure encapsulates the information of the stock and its four cash dividends.

\section*{Create a StockSpec for Stocks With Cash and Continuous Dividends}

Consider two stocks that are trading at \(\$ 40\) and \(\$ 35\). The first one provides two cash dividends of \(\$ 0.25\) on March 1, 2008 and June 1, 2008. The second stock provides a continuous dividend yield of \(3 \%\). The stocks have a volatility of \(30 \%\) per annum. Using this data, create the structure StockSpec:
```

AssetPrice = [40; 35];
Sigma = .30;

```
```

DividendType = {'cash'; 'continuous'};
DividendAmount = [0.25, 0.25 ; 0.03 NaN];
DividendDate1 = 'March-01-2008';
DividendDate2 = 'Jun-01-2008';
StockSpec = stockspec(Sigma, AssetPrice, DividendType, DividendAmount,...
{ DividendDate1, DividendDate2 ; NaN NaN})
StockSpec =
FinObj:
Sigma:
[2x1 double
AssetPrice:

```

Examine the StockSpec structure.
datedisp(StockSpec.ExDividendDates)
StockSpec.DividendType
```

01-Mar-2008
NaN
01-Jun-2008
NaN
ans =
cash'
'continuous'

```

The StockSpec structure encapsulates the information of the two stocks and their dividends.

See Also
crrprice | crrtree | intenvset | optstockbybjs | optstockbyblk | optstockbyls | optstockbybls | spreadbykirk | spreadbybjs | spreadbyfd | spreadbyls | optstockbyrgw

Purpose Calculate price of supershare digital options using Black-Scholes model
Synfax Price = supersharebybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, StrikeLow, StrikeHigh)

Arguments

\section*{Description Price = supersharebybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, StrikeLow, StrikeHigh) computes supershare digital option prices using the Black-Scholes model. \\ Price is a NINST-by-1 vector of expected option prices. \\ Examples \\ Compute the Price of Supershare Digital Options Using Black-Scholes Model}

The annualized, continuously compounded rate term structure. For information on the interest rate specification, see intenvset.

StockSpec Stock specification. See stockspec.
Settle NINST-by-1 vector of settlement or trade dates.
Maturity NINST-by-1 vector of maturity dates.
StrikeLow NINST-by-1 vector of low strike price values.

StrikeHigh NINST-by-1 vector of high strike price values.

This example shows how to compute the price of supershare digital options using Black-Scholes model. Consider a supershare based on a portfolio of nondividend paying stocks with a lower strike of 350 and an upper strike of 450 . The value of the portfolio on November 1, 2008 is 400 . The risk-free rate is \(4.5 \%\) and the volatility is \(18 \%\). Using this data, calculate the price of the supershare option on February 1, 2009.
```

Settle = 'Nov-1-2008';
Maturity = 'Feb-1-2009';
Rates = 0.045;
Basis = 1;
Compounding = -1;
% create the RateSpec
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,..
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Bas
% define the StockSpec
AssetPrice = 400;
Sigma = .18;
StockSpec = stockspec(Sigma, AssetPrice);
% define the high and low strike points
StrikeLow = 350;
StrikeHigh = 450;
% calculate the price
Pssh = supersharebybls(RateSpec, StockSpec, Settle, Maturity,...
StrikeLow, StrikeHigh)
Pssh =
0.9411

```

See Also
assetbybls | cashbybls | gapbybls | supersharesensbybls

Purpose

Syntax
PriceSens = supersharesensbybls(RateSpec, StockSpec, Settle, Maturity, StrikeLow, StrikeHigh) PriceSens = supersharesensbybls(RateSpec, StockSpec, Settle, Maturity, StrikeLow, StrikeHigh, OutSpec)

\section*{Arguments}
\(\left.\begin{array}{ll}\text { RateSpec } & \begin{array}{l}\text { The annualized, continuously compounded rate } \\ \text { term structure. For information on the interest } \\ \text { rate specification, see intenvset. }\end{array} \\ \text { StockSpec } & \text { Stock specification. See stockspec. } \\ \text { Settle } & \text { NINST-by-1 vector of settlement or trade dates. } \\ \text { Maturity } & \text { NINST-by-1 vector of maturity dates. } \\ \text { StrikeLow } & \text { NINST-by-1 vector of low strike price values. } \\ \text { StrikeHigh } & \begin{array}{l}\text { NINST-by-1 vector of high strike price values. }\end{array} \\ \text { OutSpec } & \begin{array}{l}\text { (Optional) All optional inputs are specified as }\end{array} \\ & \begin{array}{l}\text { matching parameter name/value pairs. The } \\ \text { parameter name is specified as a character string, } \\ \text { followed by the corresponding parameter value. }\end{array} \\ & \begin{array}{l}\text { You can specify parameter name/value pairs } \\ \text { in any order. Names are case-insensitive and }\end{array} \\ \text { partial string matches are allowed provided no } \\ \text { ambiguities exist. Valid parameter names are: }\end{array}\right\}\)
should be Price, Lambda, and Rho, in that order.

To invoke from a function: [Price, Lambda, Rho] = supersharesensbybls(..., 'OutSpec', \{'Price', 'Lambda', 'Rho'\})

OutSpec = \{'All'\} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec as OutSpec = \{'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', 'Price'\};
- Default is OutSpec \(=\{\) 'Price' \(\}\).

\section*{Description}

PriceSens = supersharesensbybls(RateSpec, StockSpec, Settle, Maturity, StrikeLow, StrikeHigh) computes supershare option prices using the Black-Scholes option pricing model.

PriceSens = supersharesensbybls(RateSpec, StockSpec, Settle, Maturity, StrikeLow, StrikeHigh, OutSpec) includes an OutSpec argument defined as parameter/value pairs, and computes supershare option prices and sensitivities using the Black-Scholes option pricing model.

PriceSens is a NINST-by-1 vector of expected option prices and sensitivities.

\section*{Examples Compute Price and Sensitivities of Supershare Digital Options Using Black-Scholes Model}

This example shows how to compute price and sensitivities of supershare digital options using a Black-Scholes model. Consider a supershare based on a portfolio of nondividend paying stocks with a lower strike of 350 and an upper strike of 450 . The value of the
portfolio on November 1, 2008 is 400 . The risk-free rate is \(4.5 \%\) and the volatility is \(18 \%\). Using this data, calculate the price and sensitivity of the supershare option on February 1, 2009.
```

Settle = 'Nov-1-2008';
Maturity = 'Feb-1-2009';
Rates = 0.045;
Basis = 1;
Compounding = -1;
% define the RateSpec
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Basis'
% define the StockSpec
AssetPrice = 400;
Sigma = .18;
StockSpec = stockspec(Sigma, AssetPrice);
% define the high and low strike points
StrikeLow = 350;
StrikeHigh = 450;
% calculate the price
Pssh = supersharebybls(RateSpec, StockSpec, Settle, Maturity,...
StrikeLow, StrikeHigh)
% compute the delta and theta of the supershare option
OutSpec = { 'delta';'theta'};
[Delta, Theta]= supersharesensbybls(RateSpec, StockSpec, Settle,...
Maturity, StrikeLow, StrikeHigh, 'OutSpec', OutSpec)

```
Pssh =
```

Delta =
-0.0010
Theta =
-1.0102

```

See Also
supersharebybls

Purpose
Price swap instrument from Black-Derman-Toy interest-rate tree
```

Syntax

```
[Price, PriceTree, CFTree, SwapRate] = swapbybdt(BDTTree,
```

[Price, PriceTree, CFTree, SwapRate] = swapbybdt(BDTTree,
LegRate, Settle, Maturity)
LegRate, Settle, Maturity)
[Price, PriceTree, CFTree, SwapRate] = swapbybdt(BDTTree,
[Price, PriceTree, CFTree, SwapRate] = swapbybdt(BDTTree,
LegRate, Settle, Maturity, LegReset, Basis, Principal,
LegRate, Settle, Maturity, LegReset, Basis, Principal,
LegType, EndMonthRule)
LegType, EndMonthRule)
[Price, PriceTree, CFTree, SwapRate] = swapbybdt(BDTTree,
[Price, PriceTree, CFTree, SwapRate] = swapbybdt(BDTTree,
LegRate, Settle, Maturity, Name,Value)

```
```

LegRate, Settle, Maturity, Name,Value)

```
```


## Input

Arguments

BDTTree Interest-rate tree structure created by bdttree.
LegRate Number of instruments (NINST)-by-2 matrix, with each row defined as:
[CouponRate Spread] or [Spread CouponRate] CouponRate is the decimal annual rate. Spread is the number of basis points over the reference rate. The first column represents the receiving leg, while the second column represents the paying leg.

Settle Settlement date. NINST-by-1 vector of serial date numbers or date strings representing the settlement date for each swap. Settle must be earlier than Maturity.
Maturity Maturity date. NINST-by-1 vector of dates representing the maturity date for each swap.

The Settle date for every swap is set to the ValuationDate of the BDT tree. The swap argument Settle is ignored.

This function also calculates the SwapRate (fixed rate) so that the value of the swap is initially 0 . To do this, enter CouponRate as NaN.

## Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

## LegReset

NINST-by-2 matrix representing the reset frequency per year for each swap. NINST-by-1 vector representing the frequency of payments per year.

## Default: [11]

## Basis

Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual $/ 365$
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- $9=$ actual/360 (ISMA)
- $10=$ actual/365 (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

## Principal

NINST-by- 1 vector or NINST-by- 1 cell array of the notional principal amounts or principal value schedules. For the latter case, each element of the cell array is a NumDates-by-2 call array where the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

Default: 100

## LegType

NINST-by-2 matrix. Each row represents an instrument. Each column indicates if the corresponding leg is fixed (1) or floating (0). This matrix defines the interpretation of the values entered in LegRate.

Default: [1 0] for each instrument

## Options

Derivatives pricing options structure created with derivset.

## EndMonthRule

End-of-month rule. NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Default: 1

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

## AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. NINST-by-1 of logicals.

Default: false

## BusinessDayConvention

Require payment dates to be business dates. NINST-by- 1 cell array with possible choices of business day convention:

- actual
- follow
- modifiedfollow
- previous
- modifiedprevious

Default: actual

## Holidays

Holidays used for business day convention. NHOLIDAYS-by-1 of MATLAB date numbers.

Default: If no dates are specified, holidays.m is used.

## StartDate

NINST-by-1 vector of dates when the swap actually starts. Use this argument to price forward swaps, i.e., swaps that start in a future date

Default: Settle date

## Description

[Price, PriceTree, CFTree, SwapRate] = swapbybdt(BDTTree, LegRate, Settle, Maturity) computes the price of a swap instrument from a BDT interest-rate tree.
[Price, PriceTree, CFTree, SwapRate] = swapbybdt(BDTTree, LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType, EndMonthRule) computes the price of a swap instrument from a BDT interest-rate tree using optional input arguments.
[Price, PriceTree, CFTree, SwapRate] = swapbybdt(BDTTree, LegRate, Settle, Maturity, Name, Value) computes the price of a swap instrument from a BDT interest-rate tree with additional options specified by one or more Name, Value pair arguments.
Price is number of instruments (NINST)-by-1 expected prices of the swap at time 0.
PriceTree is a tree structure with a vector of the swap values at each node.

CFTree is a tree structure with a vector of the swap cash flows at each node. This structure contains only NaNs because with binomial recombining trees, cash flows cannot be computed accurately at each node of a tree.
SwapRate is a NINST-by- 1 vector of rates applicable to the fixed leg such that the swaps' values are zero at time 0 . This rate is used in calculating the swaps' prices when the rate specified for the fixed leg in LegRate is NaN. SwapRate is padded with NaN for those instruments in which CouponRate is not set to NaN.

## Definitions Amortizing Swap

In an amortizing swap, the notional principal decreases periodically because it is tied to an underlying financial instrument with a declining (amortizing) principal balance, such as a mortgage.

## Forward Swap

Agreement to enter into an interest-rate swap arrangement on a fixed date in future.

## Examples Price an Interest-Rate Swap

Price an interest-rate swap with a fixed receiving leg and a floating paying leg. Payments are made once a year, and the notional principal amount is $\$ 100$. The values for the remaining arguments are:

- Coupon rate for fixed leg: 0.15 ( $15 \%$ )
- Spread for floating leg: 10 basis points
- Swap settlement date: Jan. 01, 2000
- Swap maturity date: Jan. 01, 2003

Based on the information above, set the required arguments and build the LegRate, LegType, and LegReset matrices:

```
Settle = '01-Jan-2000';
Maturity = '01-Jan-2003';
Basis = 0;
Principal = 100;
LegRate = [0.15 10]; % [CouponRate Spread]
LegType = [1 0]; % [Fixed Float]
LegReset = [1 1]; % Payments once per year
```

Price the swap using the BDTTree included in the MAT-file deriv.mat. BDTTree contains the time and forward-rate information needed to price the instrument.

```
load deriv.mat;
```

Use swapbybdt to compute the price of the swap.
Price = swapbybdt(BDTTree, LegRate, Settle, Maturity,... LegReset, Basis, Principal, LegType)

## Price =

7.4222

Using the previous data, calculate the swap rate, the coupon rate for the fixed leg, such that the swap price at time $=0$ is zero.

LegRate = [NaN 20];
[Price, PriceTree, CFTree, SwapRate] = swapbybdt(BDTTree,... LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType)

Price =

$$
-1.4211 \mathrm{e}-14
$$

PriceTree =

```
FinObj: 'BDTPriceTree'
tObs: [0 12234\(]\)
PTree: \{1x5 cell\}
```

CFTree $=$

FinObj: 'BDTCFTree' tObs: [0 12234$]$

## SwapRate =

0.1205

## Price an Amortizing Swap

Price an amortizing swap using the Principal input argument to define the amortization schedule.

Create the RateSpec.
Rates = 0.035;
ValuationDate = '1-Jan-2011';
StartDates = ValuationDate;
EndDates = '1-Jan-2017';
Compounding = 1 ;
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', Star 'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)

## RateSpec =

```
                    FinObj: 'RateSpec'
        Compounding: 1
            Disc: 0.8135
            Rates: 0.0350
            EndTimes: 6
        StartTimes: 0
            EndDates: 736696
        StartDates: 734504
        ValuationDate: 734504
            Basis: 0
```


## EndMonthRule: 1

Create the swap instrument using the following data:

```
Settle ='1-Jan-2011';
Maturity = '1-Jan-2017';
Period = 1;
LegRate = [0.04 10];
```

Define the swap amortizing schedule.

```
Principal ={{'1-Jan-2013' 100;'1-Jan-2014' 80;'1-Jan-2015' 60;'1-Jan-2016
```

Build the BDT tree and assume volatility is $10 \%$.

```
MatDates = {'1-Jan-2012'; '1-Jan-2013';'1-Jan-2014';'1-Jan-2015';'1-Jan-2
BDTTimeSpec = bdttimespec(ValuationDate, MatDates);
Volatility = 0.10;
BDTVolSpec = bdtvolspec(ValuationDate, MatDates, Volatility*ones(1,lengtr
BDTT = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec);
Compute the price of the amortizing swap.
Price = swapbybdt(BDTT, LegRate, Settle, Maturity, 'Principal' , Principa
```

Price =
1.4574

## Price a Forward Swap

Price a forward swap using the StartDate input argument to define the future starting date of the swap.

Create the RateSpec.

```
Rates = 0.0325;
ValuationDate = '1-Jan-2012';
StartDates = ValuationDate;
EndDates = '1-Jan-2018';
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', Star
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
RateSpec =
                    FinObj: 'RateSpec'
        Compounding: 1
            Disc: 0.8254
            Rates: 0.0325
        EndTimes: 6
        StartTimes: 0
        EndDates: 737061
        StartDates: 734869
ValuationDate: }73486
            Basis: 0
        EndMonthRule: 1
```

Build the tree with a volatility of $10 \%$.
MatDates $=$ \{'1-Jan-2013'; '1-Jan-2014';'1-Jan-2015';'1-Jan-2016';'1-J
BDTTimeSpec = bdttimespec(ValuationDate, MatDates);
Volatility = 0.10;
BDTVolSpec $=$ bdtvolspec(ValuationDate, MatDates, Volatility*ones(1,ler
BDTT = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec);

Compute the price of a forward swap that starts in two years (Jan 1, 2014) and matures in three years with a forward swap rate of $3.85 \%$.

```
Settle ='1-Jan-2012';
```

```
Maturity = '1-Jan-2017';
StartDate = '1-Jan-2014';
LegRate = [0.0385 10];
Price = swapbybdt(BDTT, LegRate, Settle, Maturity, 'StartDate', StartDate
Price =
    1.3203
```

Using the previous data, compute the forward swap rate, the coupon rate for the fixed leg, such that the forward swap price at time $=0$ is zero.

LegRate = [NaN 10];
[Price, ~,~, SwapRate] = swapbybdt(BDTT, LegRate, Settle, Maturity, 'Star

Price =

$$
-2.8422 e-14
$$

## SwapRate =

0.0335

See Also
bdttree | capbybdt | cfbybdt | floorbybdt

## Purpose

Price swap instrument from Black-Karasinski interest-rate tree
Syntax
[Price, PriceTree, SwapRate] = swapbybk(BKTree, LegRate, Settle, Maturity) [Price, PriceTree, SwapRate] = swapbybk(BKTree, LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType, EndMonthRule)
[Price, PriceTree, SwapRate] = swapbybk(BKTree, LegRate, Settle, Maturity, Name, Value)

## Input <br> Arguments

$\left.\begin{array}{ll}\text { BKTree } & \text { Interest-rate tree structure created by bktree. } \\ \text { LegRate } & \begin{array}{l}\text { Number of instruments (NINST)-by-2 matrix, with } \\ \text { each row defined as: }\end{array} \\ & \text { [CouponRate Spread] or [Spread CouponRate] } \\ \text { CouponRate is the decimal annual rate. Spread is the } \\ \text { number of basis points over the reference rate. The } \\ \text { first column represents the receiving leg, while the } \\ \text { second column represents the paying leg. }\end{array}\right\}$

The Settle date for every swap is set to the ValuationDate of the BK tree. The swap argument Settle is ignored.

This function also calculates the SwapRate (fixed rate) so that the value of the swap is initially zero. To do this, enter CouponRate as NaN.
swapbybk computes prices of vanilla swaps, amortizing swaps and forward swaps.

## Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

## LegReset

NINST-by-2 matrix representing the reset frequency per year for each swap. NINST-by-1 vector representing the frequency of payments per year.

Default: [11 1 ]

## Basis

Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual $/ 360$
- $3=$ actual $/ 365$
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- $9=$ actual/360 (ISMA)
- $10=\mathrm{actual} / 365$ (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

## Principal

NINST-by- 1 vector or NINST-by- 1 cell array of the notional principal amounts or principal value schedules. For the latter case, each element of the cell array is a NumDates-by- 2 call array where the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

Default: 100

## LegType

NINST-by-2 matrix. Each row represents an instrument. Each column indicates if the corresponding leg is fixed (1) or floating (0). This matrix defines the interpretation of the values entered in LegRate.

Default: [1 0] for each instrument

## Options

Derivatives pricing options structure created with derivset.

## EndMonthRule

End-of-month rule. A NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Default: 1

## StartDate

NINST-by-1 vector of dates when the swap actually starts. Use this argument to price forward swaps, i.e., swaps that start in a future date

Default: Settle date

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

## AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. NINST-by-1 of logicals.

Default: false

## BusinessDayConvention

Require payment dates to be business dates. NINST-by-1 cell array with possible choices of business day convention:

- actual
- follow
- modifiedfollow
- previous
- modifiedprevious

Default: actual

## Holidays

Holidays used for business day convention. NHOLIDAYS-by-1 of MATLAB date numbers.

Default: If no dates are specified, holidays.m is used.

## Description

[Price, PriceTree, SwapRate] = swapbybk(BKTree, LegRate, Settle, Maturity) computes the price of a swap instrument from a Black-Karasinski interest-rate tree.
[Price, PriceTree, SwapRate] = swapbybk(BKTree, LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType, EndMonthRule) computes the price of a swap instrument from a Black-Karasinski interest-rate tree using optional input arguments.
[Price, PriceTree, SwapRate] = swapbybk(BKTree, LegRate, Settle, Maturity, Name, Value) computes the price of a swap instrument from a Black-Karasinski interest-rate tree with additional options specified by one or more Name, Value pair arguments.

Price is the number of instruments (NINST)-by-1 expected prices of the swap at time 0 .

PriceTree is the tree structure with a vector of the swap values at each node:

- PriceTree. PTree contains swap values.
- PriceTree.tObs contains the observation times.
- PriceTree.Connect contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are NumNodes elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicates where the down branch connects to.


## Definitions

## Examples

- PriceTree. Probs contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

SwapRate is a NINST-by-1 vector of rates applicable to the fixed leg such that the swaps' values are zero at time 0 . This rate is used in calculating the swaps' prices when the rate specified for the fixed leg in LegRate is NaN. The SwapRate output is padded with NaN for those instruments in which CouponRate is not set to NaN .

## Amortizing Swap

In an amortizing swap, the notional principal decreases periodically because it is tied to an underlying financial instrument with a declining (amortizing) principal balance, such as a mortgage.

## Forward Swap

Agreement to enter into an interest-rate swap arrangement on a fixed date in future.

## Price an Interest-Rate Swap

Price an interest-rate swap with a fixed receiving leg and a floating paying leg. Payments are made once a year, and the notional principal amount is $\$ 100$. The values for the remaining arguments are:

- Coupon rate for fixed leg: 0.06 (6\%)
- Spread for floating leg: 20 basis points
- Swap settlement date: Jan. 01, 2000
- Swap maturity date: Jan. 01, 2003

Based on the information above, set the required arguments and build the LegRate, LegType, and LegReset matrices:

```
Settle = '01-Jan-2000';
Maturity = '01-Jan-2003';
Basis = 0;
Principal = 100;
```

```
LegRate = [0.06 20]; % [CouponRate Spread]
LegType = [1 0]; % [Fixed Float]
LegReset = [11 1]; % Payments once per year
```

Load the file deriv.mat, which provides ZeroRateSpec, the interest-rate term structure needed to price the bond.

```
load deriv.mat;
```

Use swapbyzero to compute the price of the swap.

```
Price = swapbyzero(ZeroRateSpec, LegRate, Settle, Maturity,...
LegReset, Basis, Principal, LegType)
```


## Price =

3.6923

Using the previous data, calculate the swap rate, which is the coupon rate for the fixed leg, such that the swap price at time $=0$ is zero.

```
LegRate = [NaN 20];
```

[Price, SwapRate] = swapbyzero(ZeroRateSpec, LegRate, Settle,...
Maturity, LegReset, Basis, Principal, LegType)

## Price =

0

SwapRate =
0.0466

Use swapbyzero with name-value pair arguments for LegRate, LegType, LatestFloatingRate, AdjustCashFlowsBasis, and BusinessDayConvention to calculate output for Price, SwapRate, AI, RecCF, RecCFDates, PayCF, and PayCFDates:

```
Settle = datenum('08-Jun-2010');
RateSpec = intenvset('Rates', [.005 .0075 .01 .014 .02 .025 .03]',...
'StartDates',Settle, 'EndDates',{'08-Dec-2010','08-Jun-2011',...
'08-Jun-2012', '08-Jun-2013', '08-Jun-2015', '08-Jun-2017', '08-Jun-2020'}');
Maturity = datenum('15-Sep-2020');
LegRate = [.025 50];
LegType = [1 0]; % fixed/floating
LatestFloatingRate = .005;
[Price, SwapRate, AI, RecCF, RecCFDates, PayCF,PayCFDates] = ...
swapbyzero(RateSpec, LegRate, Settle, Maturity,'LegType',LegType,...
'LatestFloatingRate',LatestFloatingRate,'AdjustCashFlowsBasis',true,...
'BusinessDayConvention','modifiedfollow')
```

Price =
-6.7259
SwapRate =
NaN
AI =
1.4575

```
RecCF =
    Columns 1 through 7
        llllllll}\begin{array}{lllll}{-1.8219 2.5000 2.5000 2.5137 2.4932 2.4932 2.5000}
        Columns 8 through 12
        2.5000 2.5000 2.5137 2.4932 102.4932
RecCFDates =
    Columns 1 through 6
        734297 734396
        734761
        7 3 5 1 2 9
        7 3 5 4 9 3
                            7358
    Columns 7 through 12
        736222 736588
        7 3 6 9 5 3
        7 3 7 3 2 0
        7 3 7 6 8 4
    7 3 8
PayCF =
    Columns 1 through 7
        lllllll
        Columns 8 through 12
        4.1733 4.5164 4.4920 4.7950 104.6608
PayCFDates =
    Columns 1 through 6
```

| 734297 | 734396 | 734761 | 735129 | 735493 | 735857 |
| ---: | ---: | ---: | ---: | ---: | ---: |
| Columns 7 through 12 |  |  |  |  |  |
| 73622 | 736588 | 736953 | 737320 | 737684 | 738049 |

## Price an Amortizing Swap

Price an amortizing swap using the Principal input argument to define the amortization schedule.

Create the RateSpec.
Rates = 0.035;
ValuationDate = '1-Jan-2011';
StartDates = ValuationDate;
EndDates = '1-Jan-2017';
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDa
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)

RateSpec =

```
                    FinObj: 'RateSpec'
        Compounding: 1
            Disc: 0.8135
            Rates: 0.0350
            EndTimes: 6
        StartTimes: 0
            EndDates: 736696
        StartDates: 734504
        ValuationDate: 734504
            Basis: 0
            EndMonthRule: 1
```

Create the swap instrument using the following data:

```
Settle ='1-Jan-2011';
Maturity = '1-Jan-2017';
Period = 1;
LegRate = [0.04 10];
```

Define the swap amortizing schedule.

```
Principal ={{'1-Jan-2013' 100;'1-Jan-2014' 80;'1-Jan-2015' 60;'1-Jan-&
```

Build the BK tree and assume volatility is $10 \%$.

```
MatDates = {'1-Jan-2012'; '1-Jan-2013';'1-Jan-2014';'1-Jan-2015';'1-Ja
BKTimeSpec = bktimespec(ValuationDate, MatDates);
Volatility = 0.10;
AlphaDates = '01-01-2017';
AlphaCurve = 0.1;
BKVolSpec = bkvolspec(ValuationDate, MatDates, Volatility*ones(1,leng
AlphaDates, AlphaCurve);
BKT = bktree(BKVolSpec, RateSpec, BKTimeSpec);
```

Compute the price of the amortizing swap.

```
Price = swapbybk(BKT, LegRate, Settle, Maturity, 'Principal' , Princi
```


## Price =

1.4574

## Price a Forward Swap

Price a forward swap using the StartDate input argument to define the future starting date of the swap.

Create the RateSpec.
Rates = 0.0374;
ValuationDate = '1-Jan-2012';
StartDates = ValuationDate;
EndDates = '1-Jan-2018';
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDa
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)

RateSpec =

> FinObj: 'RateSpec'
> Compounding: 1
> Disc: 0.8023
> Rates: 0.0374
> EndTimes: 6
> StartTimes: 0
> EndDates: 737061
> StartDates: 734869
> ValuationDate: 734869
> Basis: 0
> EndMonthRule: 1

Build a BK tree.
VolDates = \{'1-Jan-2013'; '1-Jan-2014';'1-Jan-2015';'1-Jan-2016';'1-Jan-2
VolCurve = 0.1;
AlphaDates = '01-01-2018';
AlphaCurve = 0.1;
BKVolSpec = bkvolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
BKTimeSpec = bktimespec(RateSpec.ValuationDate, VolDates, Compounding);

```
BKT = bktree(BKVolSpec, RateSpec, BKTimeSpec);
```

Compute the price of a forward swap that starts in a year (Jan 1, 2013) and matures in four years with a forward swap rate of $4.25 \%$.

```
Settle ='1-Jan-2012';
Maturity = '1-Jan-2017';
StartDate = '1-Jan-2013';
LegRate = [0.0425 10];
Price = swapbybk(BKT, LegRate, Settle, Maturity, 'StartDate', StartDa
```

Price =
1.4434

Using the previous data, compute the forward swap rate, the coupon rate for the fixed leg, such that the forward swap price at time $=0$ is zero.

```
LegRate = [NaN 10];
[Price, ~,SwapRate] = swapbybk(BKT, LegRate, Settle, Maturity, 'Start
```

Price =
$1.4211 \mathrm{e}-14$
SwapRate =
0.0384

See Also bktree | bondbybk | capbybk | fixedbybk | floorbybk

Purpose
Price swap instrument from Heath-Jarrow-Morton interest-rate tree

```
Syntax
```

[Price, PriceTree, CFTree, SwapRate] = swapbyhjm(HJMTree,

```
[Price, PriceTree, CFTree, SwapRate] = swapbyhjm(HJMTree,
LegRate, Settle, Maturity)
LegRate, Settle, Maturity)
[Price, PriceTree, CFTree, SwapRate] = swapbyhjm(HJMTree,
[Price, PriceTree, CFTree, SwapRate] = swapbyhjm(HJMTree,
LegRate, Settle, Maturity, LegReset, Basis, Principal,
LegRate, Settle, Maturity, LegReset, Basis, Principal,
LegType, EndMonthRule)
LegType, EndMonthRule)
[Price, PriceTree, CFTree, SwapRate] = swapbyhjm(HJMTree,
[Price, PriceTree, CFTree, SwapRate] = swapbyhjm(HJMTree,
LegRate, Settle, Maturity, Name,Value)
```

```
LegRate, Settle, Maturity, Name,Value)
```

```

\section*{Input}

Arguments
\(\left.\begin{array}{ll}\text { HJMTree } & \text { Forward-rate tree structure created by hjmtree. } \\
\text { LegRate } & \begin{array}{l}\text { Number of instruments (NINST)-by-2 matrix, with } \\
\text { each row defined as: }\end{array} \\
& \text { [CouponRate Spread] or [Spread CouponRate] } \\
\text { CouponRate is the decimal annual rate. Spread is the } \\
\text { number of basis points over the reference rate. The } \\
\text { first column represents the receiving leg, while the } \\
\text { second column represents the paying leg. }\end{array}\right\}\)\begin{tabular}{l} 
Settlement date. NINST-by-1 vector of serial date \\
numbers or date strings representing the settlement \\
Maturity \\
\\
\begin{tabular}{l} 
Maturity. \\
Mater swap. Settle must be earlier than
\end{tabular} \\
\begin{tabular}{l} 
Maturity date. NINST-by-1 vector of dates \\
representing the maturity date for each swap.
\end{tabular}
\end{tabular}

The Settle date for every swap is set to the ValuationDate of the HJM tree. The swap argument Settle is ignored.

This function also calculates the SwapRate (fixed rate) so that the value of the swap is initially zero. To do this, enter CouponRate as NaN.

\section*{Ordered Input or Name-Value Pair Arguments}

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

\section*{LegReset}

NINST-by-2 matrix representing the reset frequency per year for each swap. NINST-by- 1 vector representing the frequency of payments per year.

Default: [11 1]

\section*{Basis}

Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- \(9=\) actual/360 (ISMA)
- \(10=\mathrm{actual} / 365\) (ISMA)
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

\section*{Principal}

NINST-by-1 vector or NINST-by- 1 cell array of the notional principal amounts or principal value schedules. For the latter case, each element of the cell array is a NumDates-by-2 call array where the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

Default: 100

\section*{LegType}

NINST-by-2 matrix. Each row represents an instrument. Each column indicates if the corresponding leg is fixed (1) or floating (0). This matrix defines the interpretation of the values entered in LegRate.

Default: [1 0] for each instrument

\section*{Options}

Derivatives pricing options structure created with derivset.

\section*{EndMonthRule}

End-of-month rule. A NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.
- \(0=\) Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

\section*{Default: 1}

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

\section*{AdjustCashFlowsBasis}

Adjust the cash flows based on the actual period day count. NINST-by-1 of logicals.

Default: false

\section*{BusinessDayConvention}

Require payment dates to be business dates. NINST-by- 1 cell array with possible choices of business day convention:
- actual
- follow
- modifiedfollow
- previous
- modifiedprevious

Default: actual

\section*{Holidays}

Holidays used for business day convention. NHOLIDAYS-by-1 of MATLAB date numbers.

Default: If no dates are specified, holidays.m is used.

\section*{StartDate}

NINST-by- 1 vector of dates when the swap actually starts. Use this argument to price forward swaps, i.e., swaps that start in a future date

Default: Settle date

\section*{Description}

\section*{Definitions}
[Price, PriceTree, CFTree, SwapRate] = swapbyhjm(HJMTree, LegRate, Settle, Maturity) computes the price of a swap instrument from an HJM interest-rate tree.
[Price, PriceTree, CFTree, SwapRate] = swapbyhjm(HJMTree, LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType, EndMonthRule) computes the price of a swap instrument from an HJM interest-rate tree with optional input arguments.
[Price, PriceTree, CFTree, SwapRate] = swapbyhjm(HJMTree, LegRate, Settle, Maturity, Name, Value) computes the price of a swap instrument from an HJM interest-rate tree with additional options specified by one or more Name, Value pair arguments.

Price is the number of instruments (NINST-by-1) expected prices of the swap at time 0.

PriceTree is the tree structure with a vector of the swap values at each node.

CFTree is the tree structure with a vector of the swap cash flows at each node.

SwapRate is a NINST-by-1 vector of rates applicable to the fixed leg such that the swaps' values are zero at time 0 . This rate is used in calculating the swaps' prices when the rate specified for the fixed leg in LegRate is NaN. The SwapRate output is padded with NaN for those instruments in which CouponRate is not set to NaN.

\section*{Amortizing Swap}

In an amortizing swap, the notional principal decreases periodically because it is tied to an underlying financial instrument with a declining (amortizing) principal balance, such as a mortgage.

\section*{Forward Swap}

Agreement to enter into an interest-rate swap arrangement on a fixed date in future.

\section*{Examples Price an Interest-Rate Swap}

Price an interest-rate swap with a fixed receiving leg and a floating paying leg. Payments are made once a year, and the notional principal amount is \(\$ 100\). The values for the remaining arguments are:
- Coupon rate for fixed leg: 0.06 (6\%)
- Spread for floating leg: 20 basis points
- Swap settlement date: Jan. 01, 2000
- Swap maturity date: Jan. 01, 2003

Based on the information above, set the required arguments and build the LegRate, LegType, and LegReset matrices:
```

Settle = '01-Jan-2000';
Maturity = '01-Jan-2003';
Basis = 0;
Principal = 100;
LegRate = [0.06 20]; % [CouponRate Spread]
LegType = [1 0]; % [Fixed Float]
LegReset = [1 1]; % Payments once per year

```

Price the swap using the HJMTree included in the MAT-file deriv.mat. The HJMTree structure contains the time and forward-rate information needed to price the instrument.
```

load deriv.mat;

```

Use swapbyhjm to compute the price of the swap.
[Price, PriceTree, CFTree] = swapbyhjm(HJMTree, LegRate,... Settle, Maturity, LegReset, Basis, Principal, LegType)

Price =
3.6923

PriceTree =
```

FinObj: 'HJMPriceTree'
tObs: [ $\left.0 \begin{array}{lllll}0 & 1 & 2 & 3 & 4\end{array}\right]$ PBush: \{1x5 cell\}

```

CFTree =

FinObj: 'HJMCFTree'
tObs: [ \(\left.\begin{array}{lllll}0 & 1 & 2 & 3 & 4\end{array}\right]\)
CFBush: \{[0] [1x1x2 double] [1x2x2 double] ... [1x8 double]\}
Use treeviewer to examine CFTree graphically and see the cash flows from the swap along both the up and the down branches. A positive cash flow indicates an inflow (income - payments \(>0\) ), while a negative cash flow indicates an outflow (income - payments \(<0\) ).
treeviewer(CFTree)


In this example, you have sold a swap (receive fixed rate and pay floating rate). At time \(t=3\), if interest rates go down, your cash flow is positive (\$2.63), meaning that you will receive this amount. But if interest rates go up, your cash flow is negative ( \(-\$ 1.58\) ), meaning that you owe this amount.
treeviewer price tree diagrams follow the convention that increasing prices appear on the upper branch of a tree and, consequently, decreasing prices appear on the lower branch. Conversely, for interest-rate displays, decreasing interest rates appear on the upper branch (prices are rising) and increasing interest rates on the lower branch (prices are falling).

Using the previous data, calculate the swap rate, which is the coupon rate for the fixed leg, such that the swap price at time \(=0\) is zero.
```

LegRate = [NaN 20];

```
```

[Price, PriceTree, CFTree, SwapRate] = swapbyhjm(HJMTree,...
LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType)
Price =
0
PriceTree =
FinObj: 'HJMPriceTree'
tObs: [00 1 2 3 4 4]
PBush:{[0] [1x1x2 double] [1x2x2 double] ... [1x8 double]}
CFTree =
FinObj: 'HJMCFTree'
tObs: [00 1 1 2 3 4
CFBush:{[0] [1x1x2 double] [1x2x2 double] ... [1x8 double]}
SwapRate =
0.0466

```

\section*{Price an Amortizing Swap}

Price an amortizing swap using the Principal input argument to define the amortization schedule.

Create the RateSpec.
Rates = 0.035;
ValuationDate = '1-Jan-2011';
StartDates = ValuationDate;
EndDates = '1-Jan-2017';
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDa
```

'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
RateSpec =
FinObj: 'RateSpec'
Compounding: 1
Disc: 0.8135
Rates: 0.0350
EndTimes: 6
StartTimes: 0
EndDates: 736696
StartDates: 734504
ValuationDate: 734504
Basis: 0
EndMonthRule: 1

```

Create the swap instrument using the following data:
```

Settle ='1-Jan-2011';
Maturity = '1-Jan-2017';
Period = 1;
LegRate = [0.04 10];

```

Define the swap amortizing schedule.
```

Principal ={{'1-Jan-2013' 100;'1-Jan-2014' 80;'1-Jan-2015' 60;'1-Jan-2

```

Build the HJM tree using the following data:
```

MatDates = {'1-Jan-2012'; '1-Jan-2013';'1-Jan-2014';'1-Jan-2015';'1-Ja
HJMTimeSpec = hjmtimespec(RateSpec.ValuationDate, MatDates);
Volatility = [.10; .08; .06; .04];
CurveTerm = [ 1; 2; 3; 4];
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec,RateSpec,HJMTimeSpec);

```

Compute the price of the amortizing swap.
Price = swapbyhjm(HJMT, LegRate, Settle, Maturity, 'Principal', Principal

Price =
1.4574

\section*{Price a Forward Swap}

Price a forward swap using the StartDate input argument to define the future starting date of the swap.

Create the RateSpec.
Rates = 0.0374;
ValuationDate = '1-Jan-2012';
StartDates = ValuationDate;
EndDates = '1-Jan-2018';
Compounding = 1;

RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDa
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)

RateSpec =
```

            FinObj: 'RateSpec'
    Compounding: 1
            Disc: 0.8023
            Rates: 0.0374
            EndTimes: 6
        StartTimes: 0
        EndDates: 737061
    ```
```

StartDates: 734869 ValuationDate: 734869
Basis: 0
EndMonthRule: 1

```

Build an HJM tree.
```

MatDates = {'1-Jan-2013'; '1-Jan-2014';'1-Jan-2015';'1-Jan-2016';'1-J؛
HJMTimeSpec = hjmtimespec(RateSpec.ValuationDate, MatDates);
Volatility = [.10; .08; .06; .04];
CurveTerm = [ 1; 2; 3; 4];
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec,RateSpec,HJMTimeSpec);

```

Compute the price of a forward swap that starts in a year (Jan 1, 2013) and matures in four years with a forward swap rate of \(4.25 \%\).
```

Settle ='1-Jan-2012';
Maturity = '1-Jan-2017';
StartDate = '1-Jan-2013';
LegRate = [0.0425 10];

```
Price = swapbyhjm(HJMT, LegRate, Settle, Maturity, 'StartDate', Start
Price =
    1.4434

Using the previous data, compute the forward swap rate, the coupon rate for the fixed leg, such that the forward swap price at time \(=0\) is zero.
```

LegRate = [NaN 10];
[Price, ~,~, SwapRate] = swapbyhjm(HJMT, LegRate, Settle, Maturity,

```
```

Price =
0

```
```

SwapRate =

```
SwapRate =
    0.0384
```

    0.0384
    ```

See Also capbyhjm \| cfbyhjm \| floorbyhjm | hjmtree \| treeviewer

\section*{Purpose}

Price swap instrument from Hull-White interest-rate tree
Syntax
[Price, PriceTree, SwapRate] = swapbyhw(HWTree, LegRate, Settle, Maturity) [Price, PriceTree, SwapRate] = swapbyhw(HWTree, LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType, Options, EndMonthRule) [Price, PriceTree, SwapRate] = swapbyhw(HWTree, LegRate, Settle, Maturity, Name, Value)

\section*{Input \\ Arguments}
\begin{tabular}{ll} 
HWTree & Forward-rate tree structure created by hwtree. \\
LegRate & \begin{tabular}{l} 
Number of instruments (NINST)-by-2 matrix, with \\
each row defined as: \\
[CouponRate Spread] or [Spread CouponRate]
\end{tabular} \\
CouponRate is the decimal annual rate. Spread is the \\
Settle & \begin{tabular}{l} 
number of basis points over the reference rate. The \\
first column represents the receiving leg, while the \\
second column represents the paying leg.
\end{tabular} \\
Maturity & \begin{tabular}{l} 
Settlement date. NINST-by-1 vector of serial date \\
numbers or date strings representing the settlement \\
date for each swap. Settle must be earlier than \\
Maturity.
\end{tabular} \\
\begin{tabular}{l} 
Maturity date. NINST-by-1 vector of dates \\
representing the maturity date for each swap.
\end{tabular}
\end{tabular}

The Settle date for every swap is set to the ValuationDate of the HW tree. The swap argument Settle is ignored.

This function also calculates the SwapRate (fixed rate) so that the value of the swap is initially zero. To do this, enter CouponRate as NaN.

\section*{Ordered Input or Name-Value Pair Arguments}

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

\section*{LegReset}

NINST-by-2 matrix representing the reset frequency per year for each swap. NINST-by-1 vector representing the frequency of payments per year.

Default: [11]

\section*{Basis}

Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- \(3=\) actual \(/ 365\)
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- \(9=\) actual/360 (ISMA)
- \(10=\mathrm{actual} / 365\) (ISMA)
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

\section*{Principal}

NINST-by- 1 vector or NINST-by- 1 cell array of the notional principal amounts or principal value schedules. For the latter case, each element of the cell array is a NumDates-by- 2 call array where the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

Default: 100

\section*{LegType}

NINST-by-2 matrix. Each row represents an instrument. Each column indicates if the corresponding leg is fixed (1) or floating (0). This matrix defines the interpretation of the values entered in LegRate.

Default: [1 0] for each instrument

\section*{Options}

Derivatives pricing options structure created with derivset.

\section*{EndMonthRule}

End-of-month rule. NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.
- \(0=\) Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Default: 1

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

\section*{AdjustCashFlowsBasis}

Adjust the cash flows based on the actual period day count. NINST-by-1 of logicals.

Default: false

\section*{BusinessDayConvention}

Require payment dates to be business dates. NINST-by-1 cell array with possible choices of business day convention:
- actual
- follow
- modifiedfollow
- previous
- modifiedprevious

Default: actual

\section*{Holidays}

Holidays used for business day convention. NHOLIDAYS-by-1 of MATLAB date numbers.

Default: If no dates are specified, holidays.m is used.

\section*{StartDate}

NINST-by- 1 vector of dates when the swap actually starts. Use this argument to price forward swaps, i.e., swaps that start in a future date

Default: Settle date

\section*{Description}

\section*{Definitions}
[Price, PriceTree, SwapRate] = swapbyhw(HWTree, LegRate, Settle, Maturity) computes the price of a swap instrument from a Hull-White interest-rate tree.
[Price, PriceTree, SwapRate] = swapbyhw(HWTree, LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType, Options, EndMonthRule) computes the price of a swap instrument from a Hull-White interest-rate tree with optional input arguments.
[Price, PriceTree, SwapRate] = swapbyhw(HWTree, LegRate, Settle, Maturity, Name, Value) computes the price of a swap instrument from a Hull-White interest-rate tree with additional options specified by one or more Name, Value pair arguments.
Price is number of instruments (NINST)-by-1 expected prices of the swap at time 0 .

PriceTree is the tree structure with a vector of the swap values at each node.

SwapRate is a NINST-by- 1 vector of rates applicable to the fixed leg such that the swaps' values are zero at time 0 . This rate is used in calculating the swaps' prices when the rate specified for the fixed leg in LegRate is NaN. The SwapRate output is padded with NaNs for those instruments in which CouponRate is not set to NaN.

\section*{Amortizing Swap}

In an amortizing swap, the notional principal decreases periodically because it is tied to an underlying financial instrument with a declining (amortizing) principal balance, such as a mortgage.

\section*{Forward Swap}

Agreement to enter into an interest-rate swap arrangement on a fixed date in future.

\section*{Examples}

\section*{Price an Interest-Rate Swap}

Price an interest-rate swap with a fixed receiving leg and a floating paying leg. Payments are made once a year, and the notional principal amount is \(\$ 100\). The values for the remaining arguments are:
- Coupon rate for fixed leg: 0.06 (6\%)
- Spread for floating leg: 20 basis points
- Swap settlement date: Jan. 01, 2005
- Swap maturity date: Jan. 01, 2008

Based on the information above, set the required arguments and build the LegRate, LegType, and LegReset matrices:
```

Settle = '01-Jan-2005';
Maturity = '01-Jan-2008';
Basis = 0;
Principal = 100;
LegRate = [0.06 20]; % [CouponRate Spread]
LegType = [1 0]; % [Fixed Float]
LegReset = [1 1]; % Payments once per year

```

Price the swap using the HWTree included in the MAT-file deriv.mat. The HWTree structure contains the time and forward-rate information needed to price the instrument.
load deriv.mat;
Use swapbyhw to compute the price of the swap.
[Price, PriceTree, SwapRate] = swapbyhw(HWTree, LegRate, ... Settle, Maturity, LegReset, Basis, Principal, LegType)
```

Warning: Swaps are valued at Tree ValuationDate rather than Settle
Price =
5.9109
PriceTree =
FinObj: 'HWPriceTree'
PTree: {1x5 cell}
tObs: [[0 1 1 2 3 4 4]
Connect: {[2] [2 3 4] [2 2 2 3 4 4 4]}
Probs: {[3x1 double] [3x3 double] [3x5 double]}
SwapRate =
NaN
Using the previous data, calculate the swap rate, which is the coupon rate for the fixed leg, such that the swap price at time $=0$ is zero.
LegRate = [NaN 20];
[Price, PriceTree, SwapRate] = swapbyhw(HWTree, LegRate, ... Settle, Maturity, LegReset, Basis, Principal, LegType)
Warning: Swaps are valued at Tree ValuationDate rather than Settle
Price =
$1.4211 \mathrm{e}-14$
PriceTree =

```
```

FinObj: 'HWPriceTree'
PTree: {1x5 cell}
tObs: [0 1 2 3 4]
Connect: {[2] [2 3 4] [2 2 3 4 4]}
Probs: {[3x1 double] [3x3 double] [3\times5 double]}

```
SwapRate =
0.0438

\section*{Price an Amortizing Swap}

Price an amortizing swap using the Principal input argument to define the amortization schedule.

Create the RateSpec.
Rates = 0.035;
ValuationDate = '1-Jan-2011';
StartDates = ValuationDate;
EndDates = '1-Jan-2017';
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDa 'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)

RateSpec =

\author{
FinObj: 'RateSpec' \\ Compounding: 1 \\ Disc: 0.8135 \\ Rates: 0.0350 \\ EndTimes: 6
}
```

    StartTimes: 0
    EndDates: 736696
    StartDates: 734504
    ValuationDate: 734504
Basis:
EndMonthRule: 1

```

Create the swap instrument using the following data:
```

Settle ='1-Jan-2011';
Maturity = '1-Jan-2017';
Period = 1;
LegRate = [0.04 10];

```

Define the swap amortizing schedule.
```

Principal ={{'1-Jan-2013' 100;'1-Jan-2014' 80;'1-Jan-2015' 60;'1-Jan-2

```

Build the HW tree using the following data:
```

VolDates = ['1-Jan-2012'; '1-Jan-2013';'1-Jan-2014';'1-Jan-2015';'1-J؛
VolCurve = 0.1;
AlphaDates = '01-01-2017';
AlphaCurve = 0.1;

```
HWVolSpec = hwvolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTimeSpec \(=\) hwtimespec(RateSpec.ValuationDate, VolDates, Compounding)
HWT = hwtree(HWVolSpec, RateSpec, HWTimeSpec);

Compute the price of the amortizing swap.
```

Price = swapbyhw(HWT, LegRate, Settle, Maturity, 'Principal', Princip

```

\section*{Price a Forward Swap}

Price a forward swap using the StartDate input argument to define the future starting date of the swap.

Create the RateSpec.
Rates \(=0.0374\);
ValuationDate = '1-Jan-2012';
StartDates = ValuationDate;
EndDates = '1-Jan-2018';
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDa
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)

RateSpec =
```

                    FinObj: 'RateSpec'
        Compounding: 1
            Disc: 0.8023
            Rates: 0.0374
        EndTimes: 6
        StartTimes: 0
            EndDates: 737061
        StartDates: 734869
    ValuationDate: 734869
            Basis: 0
    EndMonthRule: 1
    ```

Build an HW tree.
```

VolDates = {'1-Jan-2013'; '1-Jan-2014';'1-Jan-2015';'1-Jan-2016';'1-J؛
VolCurve = 0.1;
AlphaDates = '01-01-2018';
AlphaCurve = 0.1;
HWVolSpec = hwvolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RateSpec.ValuationDate, VolDates, Compounding
HWT = hwtree(HWVolSpec, RateSpec, HWTimeSpec);

```

Compute the price of a forward swap that starts in a year (Jan 1, 2013) and matures in four years with a forward swap rate of \(4.25 \%\).
```

Settle ='1-Jan-2012';
Maturity = '1-Jan-2017';
StartDate = '1-Jan-2013';
LegRate = [0.0425 10];

```
Price = swapbyhw(HWT, LegRate, Settle, Maturity, 'StartDate', StartDat
Price =
    1.4434

Using the previous data, compute the forward swap rate, the coupon rate for the fixed leg, such that the forward swap price at time \(=0\) is zero.
```

LegRate = [NaN 10];
[Price, ~,SwapRate] = swapbyhw(HWT, LegRate, Settle, Maturity, 'Start

```
Price =
    0
```

SwapRate =
0.0384

```

See Also
bondbyhw | capbyhw | cfbyhw | floorbyhw | fixedbyhw | hwtree

\section*{Purpose}

Price swap instrument from set of zero curves

\section*{Description}
[Price, SwapRate AI, RecCF, RecCFDates, PayCF, PayCFDates] =
swapbyzero(RateSpec, LegRate, Settle, Maturity) prices a swap instrument from a set of zero coupon bond rates. All inputs are either scalars or NINST-by- 1 vectors unless otherwise specified. Any date can be a serial date number or date string. An optional argument can be passed as an empty matrix [].
[Price, SwapRate AI, RecCF, RecCFDates, PayCF, PayCFDates] =
swapbyzero(RateSpec, LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType, EndMonthRule) prices a swap instrument from a set of zero coupon bond rates with optional input arguments. All inputs are either scalars or NINST-by-1 vectors unless otherwise specified. Any date can be a serial date number or date string. An optional argument can be passed as an empty matrix [].
```

[Price, SwapRate, AI, RecCF, RecCFDates, PayCF, PayCFDates]

``` =
swapbyzero(RateSpec, LegRate, Settle, Maturity, Name, Value) prices a swap instrument from a set of zero coupon bond rates with additional options specified by one or more Name, Value pair arguments.

\section*{Input \\ Arguments}

\section*{RateSpec}

Structure containing the properties of an interest-rate structure. See intenvset for information on creating RateSpec.

RateSpec can also be a 1-by-2 input variable of RateSpecs, with the second RateSpec structure containing the discount curve(s) for the paying leg. If only one RateSpec structure is specified, then this RateSpec is used to discount both legs.

\section*{LegRate}

Number of instruments (NINST)-by-2 matrix, with each row defined as:

\section*{[CouponRate Spread] or [Spread CouponRate]}

CouponRate is the decimal annual rate. Spread is the number of basis points over the reference rate. The first column represents the receiving leg, while the second column represents the paying leg.

\section*{Settle}

Settlement date. Settle must be either a scalar or NINST-by-1 vector of serial date numbers or date strings of the same value which represent the settlement date for each swap. Settle must be earlier than Maturity.

\section*{Maturity}

Maturity date. NINST-by-1 vector of dates representing the maturity date for each swap.

\section*{Ordered Input or Name-Value Pair Arguments}

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

\section*{LegReset}

NINST-by-2 matrix representing the reset frequency per year for each swap. NINST-by-1 vector representing the frequency of payments per year.

Default: [11 1]

\section*{Basis}

Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- \(3=\) actual \(/ 365\)
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\mathrm{actual} / 365\) (Japanese)
- \(8=\) actual/actual (ISMA)
- \(9=\) actual/360 (ISMA)
- \(10=\mathrm{actual} / 365\) (ISMA)
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

\section*{Principal}

NINST-by- 1 vector or NINST-by- 1 cell array of the notional principal amounts or principal value schedules. For the latter case, each element
of the cell array is a NumDates-by-2 call array where the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

Default: 100

\section*{LegType}

NINST-by-2 matrix. Each row represents an instrument. Each column indicates if the corresponding leg is fixed (1) or floating (0). This matrix defines the interpretation of the values entered in LegRate.

Default: [1 0] for each instrument

\section*{EndMonthRule}

End-of-month rule. NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.
- \(0=\) Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Default: 1

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

\section*{AdjustCashFlowsBasis}

Adjust the cash flows based on the actual period day count. NINST-by-1 of logicals.

\section*{Default: false}

\section*{BusinessDayConvention}

Require payment dates to be business dates. NINST-by-1 cell array with possible choices of business day convention:
- actual
- follow
- modifiedfollow
- previous
- modifiedprevious

\section*{Default: actual}

\section*{Holidays}

Holidays used for business day convention. NHOLIDAYS-by-1 of MATLAB date numbers.

Default: If none specified, holidays.m is used.

\section*{LatestFloatingRate}

Rate for the next floating payment, set at the last reset date. NINST-by-1 of scalars.

Default: If not specified, then the RateSpec must contain this information.

\section*{ProjectionCurve}

The rate curve to be used in generating cash flows for the floating leg of the swap. This structure must be created using intenvset.

Default: If ProjectionCurve is not specified, then the RateSpec is used both for discounting and generating cash flows for the floating leg.

\section*{StartDate}

NINST-by-1 vector of dates when the swap actually starts. Use this argument to price forward swaps, i.e., swaps that start in a future date

Default: Settle date

\section*{Output}

\section*{Price}

Number of instruments (NINST) by number of curves (NUMCURVES) matrix of swap prices. Each column arises from one of the zero curves.

\section*{SwapRate}

NINST-by-NUMCURVES matrix of rates applicable to the fixed leg such that the swap's values are zero at time 0 . This rate is used in calculating the swaps' prices when the rate specified for the fixed leg in LegRate is NaN. The SwapRate output is padded with NaN for those instruments in which CouponRate is not set to NaN .

Output cash flows, cash flow dates, and accrued interest.

\section*{Al}

NINST-by-NUMCURVES matrix of accrued interest.

\section*{RecCF}

NINST-by-NUMCURVES matrix of cash flows for the receiving leg.

Note If there is more than one curve specified in the RateSpec input, then the first NCURVES row corresponds to the first swap, the second NCURVES row correspond to the second swap, and so on.

\section*{RecCFDates}

NINST-by-NUMCURVES matrix of payment dates for the receiving leg.

\section*{PayCF}

NINST-by-NUMCURVES matrix of cash flows for the paying leg.

\section*{PayCFDates}

NINST-by-NUMCURVES matrix of payment dates for the paying leg.

\section*{Definitions Amortizing Swap}

In an amortizing swap, the notional principal decreases periodically because it is tied to an underlying financial instrument with a declining (amortizing) principal balance, such as a mortgage.

\section*{Forward Swap}

Agreement to enter into an interest-rate swap arrangement on a fixed date in future.

\section*{Examples Price an Interest-Rate Swap}

Price an interest-rate swap with a fixed receiving leg and a floating paying leg. Payments are made once a year, and the notional principal amount is \(\$ 100\). The values for the remaining arguments are:
- Coupon rate for fixed leg: 0.06 (6\%)
- Spread for floating leg: 20 basis points
- Swap settlement date: Jan. 01, 2000
- Swap maturity date: Jan. 01, 2003

Based on the information above, set the required arguments and build the LegRate, LegType, and LegReset matrices:
```

Settle = '01-Jan-2000';

```
```

Maturity = '01-Jan-2003';
Basis = 0;
Principal = 100;
LegRate = [0.06 20]; % [CouponRate Spread]
LegType = [1 0]; % [Fixed Float]
LegReset = [1 1]; % Payments once per year

```

Load the file deriv.mat, which provides ZeroRateSpec, the interest-rate term structure needed to price the bond.
```

load deriv.mat;

```

Use swapbyzero to compute the price of the swap.
Price = swapbyzero(ZeroRateSpec, LegRate, Settle, Maturity,...
LegReset, Basis, Principal, LegType)

\section*{Price =}
3.6923

Using the previous data, calculate the swap rate, which is the coupon rate for the fixed leg, such that the swap price at time \(=0\) is zero.

LegRate = [NaN 20];
[Price, SwapRate] = swapbyzero(ZeroRateSpec, LegRate, Settle,... Maturity, LegReset, Basis, Principal, LegType)

Price =
0

\section*{SwapRate =}
0.0466

Use swapbyzero with name-value pair arguments for LegRate, LegType, LatestFloatingRate, AdjustCashFlowsBasis, and BusinessDayConvention to calculate output for Price, SwapRate, AI, RecCF, RecCFDates, PayCF, and PayCFDates:

Settle = datenum('08-Jun-2010');
RateSpec \(=\) intenvset('Rates', \([.005\). 0075 . 01 . 014 . 02 . 025 . 03]',...
'StartDates',Settle, 'EndDates', \{'08-Dec-2010', '08-Jun-2011',...
'08-Jun-2012' , '08-Jun-2013', '08-Jun-2015', '08-Jun-2017' , '08-Jun-2020' Maturity \(=\) datenum('15-Sep-2020');
LegRate = [. 025 50];
LegType = [1 0]; \% fixed/floating
LatestFloatingRate = .005;
[Price, SwapRate, AI, RecCF, RecCFDates, PayCF,PayCFDates] = ... swapbyzero(RateSpec, LegRate, Settle, Maturity,'LegType',LegType,... 'LatestFloatingRate', LatestFloatingRate, 'AdjustCashFlowsBasis', true, . 'BusinessDayConvention', 'modifiedfollow')

Price =
-6.7259

SwapRate =
NaN

AI =
```

    1.4575
    RecCF =
Columns 1 through 7
llllllll}\begin{array}{lllll}{-1.8219 2.5000 2.5000 2.5137 2.4932 2.4932 2.5000}
Columns 8 through 12
2.5000 2.5000 2.5137 2.4932 102.4932
RecCFDates =
Columns 1 through 6
7 3 4 2 9 7
734396
734761
7 3 5 1 2 9
7 3 5 4 9 3
7 3 5 8 5 7
Columns 7 through 12
736222 736588
7 3 6 9 5 3
7 3 7 3 2 0737684738049
PayCF =
Columns 1 through 7
-lllllll
Columns 8 through 12
4.1733 4.5164 4.4920 4.7950 104.6608
PayCFDates =

```

Columns 1 through 6
\(\begin{array}{llllll}734297 & 734396 & 734761 & 735129 & 735493 & 7358\end{array}\)

\section*{Columns 7 through 12}

736222
736588
736953
737320
737684

\section*{Price Swaps By Specifying Multiple Term Structures Using RateSpec}

Price three swaps using two interest-rate curves. First, define the data for the interest-rate term structure:

StartDates = '01-May-2012';
EndDates = \{'01-May-2013'; '01-May-2014';'01-May-2015';'01-May-2016'\} Rates \(=[[0.0356 ; 0.041185 ; 0.04489 ; 0.047741],[0.0366 ; 0.04218 ; 0.04589 ; 0\)

Create the RateSpec using intenvset.
RateSpec = intenvset('Rates', Rates, 'StartDates',StartDates,... 'EndDates', EndDates, 'Compounding', 1)

RateSpec =
```

                    FinObj: 'RateSpec'
    Compounding: 1
            Disc: [4x2 double]
            Rates: [4x2 double]
            EndTimes: [4x1 double]
        StartTimes: [4x1 double]
            EndDates: [4x1 double]
        StartDates: 734990
    ValuationDate: 734990

```

\section*{Basis: 0 \\ EndMonthRule: 1}

Look at the Rates for the two interest-rate curves.
```

RateSpec.Rates

```
ans =
\[
\begin{array}{ll}
0.0356 & 0.0366 \\
0.0412 & 0.0422 \\
0.0449 & 0.0459 \\
0.0477 & 0.0497
\end{array}
\]

Define the swap instruments.
```

Settle = '01-May-2012';
Maturity = '01-May-2015';
LegRate = [0.06 10];
Principal = [100;50;100]; % Three notional amounts

```

Price three swaps using two curves.
Price = swapbyzero(RateSpec, LegRate, Settle, Maturity, 'Principal', Prir
```

Price =
3.9688 3.6869
1.9844 1.8434
3.9688 3.6869

```

\section*{Price Swap By Specifying Multiple Term Structures Using a 1-by-2 RateSpec}

Price a swap using two interest-rate curves. First, define data for the two interest-rate term structures:

StartDates = '01-May-2012';
EndDates = \{'01-May-2013'; '01-May-2014';'01-May-2015';'01-May-2016'\} Rates1 = [0.0356;0.041185;0.04489;0.047741]; Rates2 = [0.0366;0.04218;0.04589;0.04974];

Create the RateSpec using intenvset.
```

RateSpecReceiving = intenvset('Rates', Rates1, 'StartDates',StartDate
'EndDates', EndDates, 'Compounding', 1);
RateSpecPaying= intenvset('Rates', Rates2, 'StartDates',StartDates,..
'EndDates', EndDates, 'Compounding', 1);
RateSpec=[RateSpecReceiving RateSpecPaying]

```
RateSpec =
1x2 struct array with fields:

Fin0bj
Compounding
Disc
Rates
EndTimes
StartTimes
EndDates
StartDates
ValuationDate
Basis
EndMonthRule

Define the swap instruments.
```

Settle = '01-May-2012';
Maturity = '01-May-2015';
LegRate = [0.06 10];
Principal = [100;50;100];

```

Price three swaps using the two curves.
```

Price = swapbyzero(RateSpec, LegRate, Settle, Maturity, 'Principal', Prir

```

\section*{Price =}
3.9693
1.9846
3.9693

\section*{Price an Amortizing Swap}

Price an amortizing swap using the Principal input argument to define the amortization schedule.

Create the RateSpec.
Rates = 0.035;
ValuationDate = '1-Jan-2011';
StartDates = ValuationDate;
EndDates = '1-Jan-2017';
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDa
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
Create the swap instrument using the following data:
```

Settle ='1-Jan-2011';
Maturity = '1-Jan-2017';
Period = 1;

```
```

LegRate = [0.04 10];

```

Define the swap amortizing schedule.
```

Principal ={{'1-Jan-2013' 100;'1-Jan-2014' 80;'1-Jan-2015' 60;'1-Jan-2

```

Compute the price of the amortizing swap.
```

Price = swapbyzero(RateSpec, LegRate, Settle, Maturity, 'Principal'

```

\section*{Price =}
1.4574

\section*{Price a Forward Swap}

Price a forward swap using the StartDate input argument to define the future starting date of the swap.

Create the RateSpec.
```

Rates = 0.0325;
ValuationDate = '1-Jan-2012';
StartDates = ValuationDate;
EndDates = '1-Jan-2018';
Compounding = 1;

```
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', Star
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)

RateSpec =
FinObj: 'RateSpec'
Compounding: 1
Disc: 0.8254
```

            Rates: 0.0325
        EndTimes: 6
        StartTimes: 0
        EndDates: 737061
        StartDates: 734869
    ValuationDate: 734869
Basis: 0
EndMonthRule: 1

```

Compute the price of a forward swap that starts in a year (Jan 1, 2013) and matures in three years with a forward swap rate of \(4.27 \%\).
```

Settle ='1-Jan-2012';
StartDate = '1-Jan-2013';
Maturity = '1-Jan-2016';
LegRate = [0.0427 10];

```
Price = swapbyzero(RateSpec, LegRate, Settle, Maturity, 'StartDate' , Sta

Price =
2.5083

Using the previous data, compute the forward swap rate, the coupon rate for the fixed leg, such that the forward swap price at time \(=0\) is zero.
```

LegRate = [NaN 10];
[Price, SwapRate] = swapbyzero(RateSpec, LegRate, Settle, Maturity,...
'StartDate' , StartDate)

```
Price =

\section*{Specify the Rate at the Instrument's Starting Date When it Cannot be Obtained From the RateSpec}

If Settle is not on a reset date of a floating-rate note, swapbyzero attempts to obtain the latest floating rate before Settle from RateSpec or the LatestFloatingRate parameter. When the reset date for this rate is out of the range of RateSpec (and LatestFloatingRate is not specified), swapbyzero fails to obtain the rate for that date and generates an error. This example shows how to use the LatestFloatingRate input parameter to avoid the error.

Create the error condition when a swap instrument's StartDate cannot be determined from the RateSpec.
```

Settle = '01-Jan-2000';
Maturity = '01-Dec-2003';
Basis = 0;
Principal = 100;
LegRate = [0.06 20]; % [CouponRate Spread]
LegType = [1 0]; % [Fixed Float]
LegReset = [1 1]; % Payments once per year
load deriv.mat;
Price = swapbyzero(ZeroRateSpec, LegRate, Settle, Maturity,...
'LegReset', LegReset, 'Basis', Basis, 'Principal', Principal, ...
'LegType', LegType)

```
```

The rate at the instrument starting date cannot be obtained from RateSpec.
Its reset date (01-Dec-1999) is out of the range of dates contained in RateSpec.
This rate is required to calculate cash flows at the instrument starting date.
Consider specifying this rate with the 'LatestFloatingRate' input parameter.
Error in swapbyzero (line 289)

```
[FloatFullPrice, FloatPrice,FloatCF,FloatCFDates] = floatbyzero(FloatRateSpec, Spreads, Settle, ...

Here, the reset date for the rate at Settle was 01-Dec-1999, which was earlier than the valuation date of ZeroRateSpec (01-Jan-2000). This error can be avoided by specifying the rate at the swap instrument's starting date using the LatestFloatingRate input parameter.

Define LatestFloatingRate and calculate the floating-rate price.
```

Price = swapbyzero(ZeroRateSpec, LegRate, Settle, Maturity,...
'LegReset', LegReset, 'Basis', Basis, 'Principal', Principal, ...
'LegType', LegType, 'LatestFloatingRate', 0.03)
Price =
4.7594

```

\section*{Price a Swap Using a Different Curve to Generate the Cash Flows of the Floating Leg}

Define the OIS and Libor rates.
```

Settle = datenum('15-Mar-2013');
CurveDates = daysadd(Settle,360*[1/12 2/12 3/12 6/12 1 2 3 4 5 7 10],1);
OISRates = [ . 0018 .0019 .0021 .0023 .0031 .006 .011 .017 .021 . 026 .03]'
LiborRates = [.0045 . 0047 . 005 . 0055 .0075 .011 . 016 . 022 . 026 . 030 . 0348
Plot the dual curves.
figure, plot(CurveDates,OISRates, 'r');hold on;plot(CurveDates,LiborRates,' datetick
legend(\{'OIS Curve', 'Libor Curve'\})

```


Create an associated RateSpec for the OIS and Libor curves.
OISCurve = intenvset('Rates',OISRates,'StartDate',Settle,'EndDates',C LiborCurve = intenvset('Rates',LiborRates,'StartDate',Settle,'EndDates

Define the swap.
Maturity = datenum('15-Mar-2018'); \% Five year swap
FloatSpread = 0;
FixedRate = .025;
```

LegRate = [FixedRate FloatSpread];

```

Compute the price of the swap instrument. The LiborCurve term structure will be used to generate the cash flows of the floating leg. The OISCurve term structure will be used for discounting the cash flows.

Price = swapbyzero(OISCurve, LegRate, Settle,... Maturity,'ProjectionCurve',LiborCurve)

\section*{Price =}
\(-0.3697\)

Compare results when the term structure OISCurve is used both for discounting and also generating the cash flows of the floating leg.

PriceSwap = swapbyzero(OISCurve, LegRate, Settle, Maturity)

PriceSwap =
2.0517

See Also | bondbyzero | intenvset | cfbyzero | fixedbyzero | floatbyzero

\section*{Purpose}

Price swaption from Black-Derman-Toy interest-rate tree

\author{
Syntax \\ [Price, PriceTree] = swaptionbybdt(BDTTree, OptSpec, Strike, ExerciseDates, Spread, Settle, Maturity, 'Name1', Value1, 'Name2', Value2)
}

\section*{Arguments}

Maturity NINST-by-1 vector of dates representing the maturity date for the underlying swap for each swap.

Note All optional inputs that follow are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. Parameter name/value pairs may be specified in any order; names are case-insensitive and partial string matches are allowed provided no ambiguities exist.

AmericanOpt (Optional) NINST-by-1 flags options:
- 0 for European options
- 1 for American options

SwapReset

Basis
(Optional) NINST-by-1 vector representing the reset frequency per year for the underlying swap. Default is 1 .
(Optional) Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- 3 = actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
\begin{tabular}{ll} 
& - \(9=\) actual/360 (ICMA) \\
& - \(10=\) actual/365 (ICMA) \\
& - \(11=30 / 360 \mathrm{E}\) (ICMA) \\
& - \(12=\) actual/actual (ISDA) \\
- \(13=\mathrm{BUS} / 252\) \\
Principal & For more information, see basis on page \\
Glossary-1. \\
Options & \begin{tabular}{l} 
(Optional) NINST-by- 1 vector of the notional \\
\\
\end{tabular}\(\quad\)\begin{tabular}{l} 
principal amounts. Default is 100. \\
\\
\\
\end{tabular} (Optional) Derivatives pricing options \\
structure created with derivset.
\end{tabular}
\begin{tabular}{ll} 
Description & [Price, PriceTree] = swaptionbybdt(BDTTree, OptSpec, \\
& Strike, \\
& ExerciseDates, Spread, Settle, Maturity, \\
& 'Name1', Value1, 'Name2', Value2) computes the price of a \\
& swaption from a BDT interest-rate tree. The swaption may be a call \\
& swaption or a put swaption.
\end{tabular}

Note The Settle date for every swaption is set to the ValuationDate of the BDT tree. The swap argument Settle is ignored. The underlying swap starts at the maturity of the swaption.

A call swaption or payer swaption allows the option buyer to enter into an interest rate swap in which the buyer of the option pays the fixed rate and receives the floating rate.

A put swaption or receiver swaption allows the option buyer to enter into an interest rate swap in which the buyer of the option receives the fixed rate and pays the floating rate.

Price is a NINST-by-1 vector of expected swaption prices at time 0 . PriceTree is a MATLAB structure of trees containing vectors of swaption instrument prices and a vector of observation times for each node. Within PriceTree:
- PriceTree. PTree contains the clean prices.
- PriceTree.tObs contains the observation times.

\section*{Examples}

\section*{Price a 5-Year Call Swaption Using a BDT Interest-Rate Tree}

This example shows how to price a 5 -year call swaption using a BDT interest-rate tree. Assume that interest rate and volatility are fixed at \(6 \%\) and \(20 \%\) annually between the valuation date of the tree until its maturity. Build a tree with the following data.
```

Rates = 0.06 * ones (10,1);
StartDates = ['jan-1-2007';'jan-1-2008';'jan-1-2009';'jan-1-2010';'jan-1-
'jan-1-2012';'jan-1-2013';'jan-1-2014';'jan-1-2015';'jan-1-2016'];
EndDates =['jan-1-2008';'jan-1-2009';'jan-1-2010';'jan-1-2011';'jan-1-201
'jan-1-2013';'jan-1-2014';'jan-1-2015';'jan-1-2016';'jan-1-2017'];
ValuationDate = 'jan-1-2007';
Compounding = 1;
% define the RateSpec
RateSpec = intenvset('Rates', Rates, 'StartDates', StartDates, 'EndDates'
'Compounding', Compounding);
% use VolSpec to compute interest-rate volatility
Volatility = 0.20 * ones (10,1); VolSpec = bdtvolspec(ValuationDate,...
EndDates, Volatility);
% use TimeSpec to specify the structure of the time layout for a BDT tre\epsilon
TimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);
% build the BDT tree
BDTTree = bdttree(VolSpec, RateSpec, TimeSpec);

```
\% use the following swaption arguments
ExerciseDates = 'jan-1-2012';
SwapSettlement = ExerciseDates;
SwapMaturity = 'jan-1-2015';
Spread = 0;
SwapReset = 1;
Principal = 100;
OptSpec = 'call';
Strike=.062;
Basis=1;
\% price the swaption
[Price, PriceTree] = swaptionbybdt(BDTTree, OptSpec, Strike, Exercise Spread, SwapSettlement, SwapMaturity, 'SwapReset', SwapReset, ... 'Basis', Basis, 'Principal', Principal)

Price =
2.0592

PriceTree =
```

FinObj: 'BDTPriceTree'
tObs: [00 1 2 2 3 4 5 6 7 8 9 10]
PTree: {1x11 cell}

```

See Also bdttree | instswaption | swapbybdt

Purpose Price swaption from Black-Karasinski interest-rate tree

\author{
Syntax [Price, PriceTree] = swaptionbybk(BKTTree, OptSpec, Strike, ExerciseDates, Spread, Settle, Maturity, 'Name1', Value1, 'Name2', Value2)
}

\section*{Arguments}
\begin{tabular}{ll} 
BKTree & \begin{tabular}{l} 
Interest-rate tree structure created by \\
bktree.
\end{tabular} \\
OptSpec & \begin{tabular}{l} 
NINST-by-1 cell array of strings ' call' or \\
'put'. A call swaption entitles the buyer to \\
pay the fixed rate. A put swaption entitles \\
the buyer to receive the fixed rate.
\end{tabular} \\
Strike & \begin{tabular}{l} 
NINST-by-1 vector for strike swap rate values.
\end{tabular} \\
ExerciseDates & \begin{tabular}{l} 
For a European option: NINST-by-1 vector of \\
exercise dates. Each row is the schedule for \\
one option. For a European option, there is \\
only one ExerciseDate on the option expiry \\
date.
\end{tabular} \\
& \begin{tabular}{l} 
For an American option: NINST-by-2 vector \\
of exercise date boundaries. For each \\
instrument, the option can be exercised \\
on any coupon date between or including \\
the pair of dates on that row. If only one \\
non-NaN date is listed, or if ExerciseDates \\
is NINST-by-1, the option can be exercised \\
between the ValuationDate of the tree and \\
the single listed ExerciseDate.
\end{tabular} \\
Spread & \begin{tabular}{l} 
NINST-by-1 vector representing the number \\
of basis points over the reference rate.
\end{tabular} \\
Settle & \begin{tabular}{l} 
NINST-by-1 vector of dates representing the \\
settle date for each swap.
\end{tabular}
\end{tabular}

Maturity
NINST-by- 1 vector of dates representing the maturity date for the underlying swap for each swap.

Note All optional inputs that follow are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. Parameter name/value pairs may be specified in any order; names are case-insensitive and partial string matches are allowed provided no ambiguities exist.
\begin{tabular}{ll} 
AmericanOpt & (Optional) NINST-by-1 flags options: \\
& - 0 for European options \\
- 1 for American options \\
SwapReset & \begin{tabular}{l} 
(Optional) NINST-by-1 vector representing the \\
\\
reset frequency per year for the underlying \\
swap. Default is 1.
\end{tabular} \\
(Optional) Day-count basis of the instrument. \\
Basis & A vector of integers. \\
& - \(0=\) actual/actual (default) \\
& - \(1=30 / 360\) (SIA) \\
& - \(2=\) actual/360 \\
& - \(3=\) actual/365 \\
& - \(4=30 / 360\) (BMA) \\
& - \(5=30 / 360\) (ISDA) \\
& - \(6=30 / 360\) (European) \\
& - \(7=\) actual/365 (Japanese) \\
& - \(8=\) actual/actual (ICMA)
\end{tabular}
- \(9=\) actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- \(12=\) actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Principal (Optional) NINST-by-1 vector of the notional principal amounts. Default is 100 .
Options
(Optional) Derivatives pricing options structure created with derivset.
[Price, PriceTree] = swaptionbybk(BKTTree, OptSpec, Strike, ExerciseDates, Spread, Settle, Maturity, 'Name1', Value1, 'Name2', Value2) computes the price of a swaption from a BK interest-rate tree.

Note The Settle date for every swaption is set to the ValuationDate of the BK tree. The swap argument Settle is ignored. The underlying swap starts at the maturity of the swaption.

The swaption may be a call swaption or a put swaption.
A call swaption or payer swaption allows the option buyer to enter into an interest rate swap in which the buyer of the option pays the fixed rate and receives the floating rate.

A put swaption or receiver swaption allows the option buyer to enter into an interest rate swap in which the buyer of the option receives the fixed rate and pays the floating rate.

Price is a NINST-by-1 vector of expected swaption prices at time 0.
PriceTree is a MATLAB structure of trees containing vectors of swaption instrument prices and a vector of observation times for each node. Within PriceTree:
- PriceTree. PTree contains the clean prices.
- PriceTree.tObs contains the observation times.

\section*{Examples \\ Price a 4-Year Call and Put Swaption Using a BK Interest-Rate Tree}

This example shows how to price a 4 -year call and put swaption using a BK interest-rate tree, assuming the interest rate is fixed at 7\% annually.
```

Rates =0.07 * ones (10,1);
Compounding = 2;
StartDates = ['jan-1-2007';'jul-1-2007';'jan-1-2008';'jul-1-2008';'jar
'jul-1-2009'; 'jan-1-2010'; 'jul-1-2010';'jan-1-2011';'jul-1-2011'];
EndDates =[ 'jul-1-2007';'jan-1-2008';'jul-1-2008';'jan-1-2009';'jul-1.
'jan-1-2010'; 'jul-1-2010';'jan-1-2011';'jul-1-2011';'jan-1-2012'];
ValuationDate = 'jan-1-2007';
% define the RateSpec
RateSpec = intenvset('Rates', Rates, 'StartDates', StartDates, 'EndDa\
'Compounding', Compounding);
% use BKVolSpec to compute the interest-rate volatility
Volatility = 0.10*ones(10,1);
AlphaCurve = 0.05*ones(10,1);
AlphaDates = EndDates;
BKVolSpec = bkvolspec(ValuationDate, EndDates, Volatility, AlphaDates
% use BKTimeSpec to specify the structure of the time layout for the
BKTimeSpec = bktimespec(ValuationDate, EndDates, Compounding);
% build the BK tree

```
```

BKTree = bktree(BKVolSpec, RateSpec, BKTimeSpec);
% use the following arguments for a 1-year swap and 4-year swaption
ExerciseDates = 'jan-1-2011';
SwapSettlement = ExerciseDates;
SwapMaturity = 'jan-1-2012';
Spread = 0;
SwapReset = 2 ;
Principal = 100;
OptSpec = {'call' ;'put'};
Strike= [ 0.07 ; 0.0725];
Basis=1;
% price the swaption
PriceSwaption = swaptionbybk(BKTree, OptSpec, Strike, ExerciseDates,
Spread, SwapSettlement, SwapMaturity, 'SwapReset', SwapReset, 'Basis', Ba
'Principal', Principal)
PriceSwaption =
0.3593
0.4756

```

See Also
bktree | instswaption | swapbybk

\section*{Purpose}

Price European swaption instrument using Black model
Syntax
```

Price = swaptionbyblk(RateSpec, OptSpec, Strike,
Settle, ExerciseDates, Maturity, Volatility)
Price = swaptionbyblk(RateSpec, OptSpec, Strike, Settle,
ExerciseDates, Maturity, Volatility, Name, Value)

```

\section*{Input} Arguments

\section*{RateSpec}

Structure containing the properties of an interest-rate structure. See intenvset for information on creating RateSpec.

RateSpec can be a NINST-by-2 input variable of RateSpecs, with the second input being the discount curve for the paying leg if different than the receiving leg. If only one curve is specified, than it is used to discount both legs.

\section*{OptSpec}

NINST-by-1 cell array of strings 'call' or 'put'. A call swaption entitles the buyer to pay the fixed rate. A put swaption entitles the buyer to receive the fixed rate.

\section*{Strike}

NINST-by-1 vector of strike swap rate values.

\section*{Settle}

Settlement date. NINST-by-1 vector of serial date numbers or date strings representing the settlement date for each swaption. Settle must not be later than ExerciseDate.

The Settle date input for swaptionbyblk is the valuation date on which the swaption (an option to enter into a swap) is priced. The swaption buyer pays this price on this date to hold the swaption.

\section*{ExerciseDate}

For a European option: NINST-by-1 vector of exercise dates. Each row is the schedule for one option. When using an European option, there is only one ExerciseDate on the option expiry date.

The ExerciseDate is the date on which the swaption expires and the underlying swap starts. The swaption holder can choose to enter into the swap on this date if the situation is favorable.

\section*{Maturity}

Maturity date. NINST-by- 1 vector of dates representing the maturity date for the underlying swap for each swaption.

\section*{Volatility}

Volatilities values. NINST-by-1 vector of volatilities.

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

\section*{Basis}

Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- \(3=\) actual \(/ 365\)
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- \(9=\) actual/360 (ISMA)
- \(10=\mathrm{actual} / 365\) (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

\section*{Principal}

NINST-by-1 vector of the notional principal amount.
Default: 100

\section*{SwapReset}

NINST-by- 1 vector representing the reset frequency per year for the underlying forward swap.

Default: 1

\section*{ProjectionCurve}

The rate curve to be used in generating the future forward rates. This structure must be created using intenvset. Use this optional input if the forward curve is different from the discount curve.

Default: If ProjectionCurve is not specified, then RateSpec is used both for discounting cash flows and projecting future forward rates.

\section*{Output \\ Arguments}

\section*{Price}

NINST-by- 1 vector of prices for the swaptions at time 0.

\section*{Definitions}

\section*{Forward Swap}

A forward swap is a swap that starts at a future date.

\section*{Examples Price a European Swaption Using the Black Model Where the Yield Curve is Flat at 6\%}

Price a European swaption that gives the holder the right to enter in five years into a three-year paying swap where a fixed-rate of \(6.2 \%\) is paid and floating is received. Assume that the yield curve is flat at \(6 \%\) per annum with continuous compounding, the volatility of the swap rate is \(20 \%\), the principal is \(\$ 100\), and payments are exchanged semiannually.

Create the RateSpec.
```

Rate = 0.06;
Compounding = -1;
ValuationDate = 'Jan-1-2010';
EndDates = 'Jan-1-2020';
Basis = 1;

```
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', Valuati
'EndDates', EndDates, 'Rates', Rate, 'Compounding', Compounding, 'Basis',

Price the swaption using the Black model.
```

Settle = 'Jan-1-2011';
ExerciseDates = 'Jan-1-2016';
Maturity = 'Jan-1-2019';
Reset = 2;
Principal = 100;
Strike = 0.062;
Volatility = 0.2;
OptSpec = 'call';
Price= swaptionbyblk(RateSpec, OptSpec, Strike, Settle, ExerciseDates,
Volatility, 'Reset', Reset, 'Principal', Principal, 'Basis', Basis)

```
Price =
    2.0710

\section*{Price a European Swaption Using the Black Model Where the Yield Curve Is Incrementally Increasing}

Price a European swaption that gives the holder the right to enter into a 5 -year receiving swap in a year, where a fixed rate of \(3 \%\) is received and floating is paid. Assume that the 1 -year, 2 -year, 3 -year, 4 -year and 5 - year zero rates are \(3 \%, 3.4 \%, 3.7 \%, 3.9 \%\) and \(4 \%\) with continuous compounding. The swap rate volatility is \(21 \%\), the principal is \(\$ 1000\), and payments are exchanged semiannually.

Create the RateSpec.
```

ValuationDate = 'Jan-1-2010';
EndDates = {'Jan-1-2011';'Jan-1-2012';'Jan-1-2013';'Jan-1-2014';'Jan-
Rates = [0.03; 0.034 ; 0.037; 0.039; 0.04;];
Compounding = -1;
Basis = 1;

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', Va:
```

'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding,'Basis',
RateSpec =
FinObj: 'RateSpec'
Compounding: -1
Disc: [5x1 double]
Rates: [5x1 double]
EndTimes: [5x1 double]
StartTimes: [5x1 double]
EndDates: [5x1 double]
StartDates: 734139
ValuationDate: 734139
Basis: 1
EndMonthRule: 1

```

Price the swaption using the Black model.
```

Settle = 'Jan-1-2011';
ExerciseDates = 'Jan-1-2012';
Maturity = 'Jan-1-2017';
Strike = 0.03;
Volatility = 0.21;
Principal =1000;
Reset = 2;
OptSpec = 'put';
Price = swaptionbyblk(RateSpec, OptSpec, Strike, Settle, ExerciseDates,
Maturity, Volatility,'Basis', Basis, 'Reset', Reset,'Principal', Principa

```
Price =
    0.5771

\section*{Price a Swaption Using a Different Curve to Generate the Future Forward Rates}

Define the OIS and Libor curves.
```

Settle = datenum('15-Mar-2013');
CurveDates = daysadd(Settle,360*[1/12 2/12 3/12 6/12 1 2 3 4 5 7 10],
OISRates = [.0018 .0019 .0021 .0023 . 0031 .006 . 011 . 017 . 021 . 026
LiborRates = [.0045 .0047 .005 .0055 .0075 .0109 . 0162 . 0216 . 0262

```

Create an associated RateSpec for the OIS and Libor curves.
```

OISCurve = intenvset('Rates',OISRates,'StartDate',Settle,'EndDates',C
LiborCurve = intenvset('Rates',LiborRates,'StartDate',Settle,'EndDates

```

Define the swaption instruments.
```

ExerciseDate = '15-Mar-2018';
Maturity = {'15-Mar-2020';'15-Mar-2023'};
OptSpec = 'call';
Strike = 0.04;
BlackVol = 0.2;

```

Price the swaption instruments using the term structure OISCurve both for discounting the cash flows and generating the future forward rates.

Price = swaptionbyblk(OISCurve, OptSpec, Strike, Settle, ExerciseDate

\section*{Price =}
1.0956
2.6944

Price the swaption instruments using the term structure LiborCurve to generate the future forward rates. The term structure OISCurve is used for discounting the cash flows.

PriceLC = swaptionbyblk(OISCurve, OptSpec, Strike, Settle, ExerciseDate,
```

PriceLC =

```
1.5346
3.8142

\section*{See Also bondbyzero | cfbyzero | fixedbyzero | floatbyzero | blackvolbysabr | intenvset}

\author{
Related \\ - "Price a Swaption Using the SABR Model" on page 2-26
}

\section*{Purpose}

Price swaption from Heath-Jarrow-Morton interest-rate tree
```

Syntax
[Price, PriceTree] = swaptionbyhjm(HJMTree, OptSpec, Strike,
ExerciseDates, Spread, Settle, Maturity,
'Name1', Value1, 'Name2', Value2)

```

\section*{Arguments}
\begin{tabular}{ll} 
HJMTree & \begin{tabular}{l} 
Interest-rate tree structure created by \\
hjmtree.
\end{tabular} \\
OptSpec & \begin{tabular}{l} 
NINST-by-1 cell array of strings ' call' or \\
'put '. A call swaption entitles the buyer to \\
pay the fixed rate. A put swaption entitles \\
the buyer to receive the fixed rate.
\end{tabular} \\
Strike & \begin{tabular}{l} 
NINST-by-1 vector of strike swap rate values.
\end{tabular} \\
ExerciseDates & \begin{tabular}{l} 
For a European option: NINST-by-1 vector of \\
exercise dates. Each row is the schedule for \\
one option. For a European option, there is \\
only one ExerciseDate on the option expiry \\
date.
\end{tabular} \\
\begin{tabular}{l} 
For an American option: NINST-by-2 vector \\
of exercise date boundaries. For each \\
instrument, the option can be exercised \\
on any coupon date between or including \\
the pair of dates on that row. If only one \\
non-NaN date is listed, or if ExerciseDates \\
is NINST-by-1, the option can be exercised \\
between the ValuationDate of the tree and \\
the single listed ExerciseDate.
\end{tabular} \\
Spread & \begin{tabular}{l} 
NINST-by-1 vector representing the number \\
of basis points over the reference rate.
\end{tabular} \\
Settle & \begin{tabular}{l} 
NINST-by-1 vector of dates representing the \\
settle date for each swap.
\end{tabular}
\end{tabular}

Maturity
NINST-by-1 vector of dates representing the maturity date for the underlying swap for each swap.

Note All optional inputs that follow are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. Parameter name/value pairs may be specified in any order; names are case-insensitive and partial string matches are allowed provided no ambiguities exist.
AmericanOpt (Optional) NINST-by-1 flags options:
- 0 for European options
- 1 for American options

SwapReset
(Optional) NINST-by-1 vector representing the reset frequency per year for the underlying swap. Default is 1 .
Basis
(Optional) Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- \(3=\) actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
\begin{tabular}{ll} 
& - \(9=\) actual/360 (ICMA) \\
& - \(10=\) actual/365 (ICMA) \\
& - \(11=30 / 360 \mathrm{E}\) (ICMA) \\
& - \(12=\) actual/actual (ISDA) \\
- \(13=\mathrm{BUS} / 252\) \\
Principal & For more information, see basis on page \\
Glossary-1. \\
Options & (Optional) NINST-by-1 vector of the notional \\
& \begin{tabular}{l} 
principal amounts. Default is 100. \\
\\
\end{tabular}\(\quad\)\begin{tabular}{l} 
(Optional) Derivatives pricing options \\
structure created with derivset.
\end{tabular}
\end{tabular}

Description [Price, PriceTree] = swaptionbyhjm(HJMTree, OptSpec, Strike, ExerciseDates, Spread, Settle, Maturity, 'Name1', Value1, 'Name2', Value2) computes the price of a swaption from a HJM interest-rate tree.

Note The Settle date for every swaption is set to the ValuationDate of the HJM tree. The swap argument Settle is ignored. The underlying swap starts at the maturity of the swaption.

The swaption may be a call swaption or a put swaption.
A call swaption or payer swaption allows the option buyer to enter into an interest rate swap in which the buyer of the option pays the fixed rate and receives the floating rate.

A put swaption or receiver swaption allows the option buyer to enter into an interest rate swap in which the buyer of the option receives the fixed rate and pays the floating rate.

Price is a (NINST-by-1 vector of expected swaption prices at time 0 .
PriceTree is a MATLAB structure of trees containing vectors of swaption instrument prices and a vector of observation times for each node. Within PriceTree:
- PriceTree. PTree contains the clean prices.
- PriceTree.tObs contains the observation times.

\section*{Examples}

\section*{Price a 1-Year Call Swaption Using an HJM Interest-Rate Tree}

This example shows how to price a 1-year call swaption using an HJM interest-rate tree. Assume that interest rate is fixed at \(5 \%\) annually between the valuation date of the tree until its maturity. Build a tree with the following data.
```

Rates = [ 0.05;0.05;0.05;0.05];
StartDates = 'jan-1-2007';
EndDates =['jan-1-2008';'jan-1-2009';'jan-1-2010';'jan-1-2011'];
ValuationDate = StartDates;
Compounding = 1;
% define the RateSpec
RateSpec = intenvset('Rates', Rates, 'StartDates', StartDates, 'EndDates'
EndDates, 'Compounding', Compounding);
% use VolSpec to compute the interest-rate volatility
VolSpec=hjmvolspec('Constant',0.01);
% use TimeSpec to specify the structure of the time layout for the HJM ir
TimeSpec = hjmtimespec(ValuationDate, EndDates, Compounding);
% build the HJM tree
HJMTree = hjmtree(VolSpec, RateSpec, TimeSpec);
% use the following swaption arguments
ExerciseDates = '01-Jan-2008';
SwapSettlement = ExerciseDates;

```
```

SwapMaturity = 'jan-1-2010';
Spread = [0];
SwapReset = 1;
Basis = 1;
Principal = 100;
OptSpec = 'call';
Strike=0.05;
% price the swaption
[Price, PriceTree] = swaptionbyhjm(HJMTree, OptSpec, Strike, Exercise
Spread, SwapSettlement, SwapMaturity,'SwapReset', SwapReset, ...
'Basis', Basis, 'Principal', Principal)
Price =
0.9296
PriceTree =
FinObj: 'HJMPriceTree'
tObs: [5x1 double]
PBush: {1x5 cell}

```
hjmtree | instswaption | swapbyhjm

Purpose Price swaption from Hull-White interest-rate tree

\author{
Syntax [Price, PriceTree] = swaptionbyhw(HWTree, OptSpec, Strike, ExerciseDates, Spread, Settle, Maturity, 'Name1', Value1, 'Name2', Value2)
}

\section*{Arguments}
\begin{tabular}{ll} 
HWTree & \begin{tabular}{l} 
Interest-rate tree structure created by \\
hwtree.
\end{tabular} \\
OptSpec & \begin{tabular}{l} 
NINST-by-1 cell array of strings ' call' or \\
'put'. A call swaption entitles the buyer to \\
pay the fixed rate. A put swaption entitles \\
the buyer to receive the fixed rate.
\end{tabular} \\
Strike & NINST-by-1 vector for strike swap rate values. \\
ExerciseDates & \begin{tabular}{l} 
For a European option: NINST-by-1 vector of \\
exercise dates. Each row is the schedule for \\
one option. For a European option, there is \\
only one ExerciseDate on the option expiry \\
date.
\end{tabular} \\
& \begin{tabular}{l} 
For an American option: NINST-by-2 vector \\
of exercise date boundaries. For each \\
instrument, the option can be exercised \\
on any coupon date between or including \\
the pair of dates on that row. If only one \\
non-NaN date is listed, or if ExerciseDates \\
is NINST-by-1, the option can be exercised \\
between the ValuationDate of the tree and \\
the single listed ExerciseDate.
\end{tabular} \\
Spread & \begin{tabular}{l} 
NINST-by-1 vector representing the number \\
of basis points over the reference rate.
\end{tabular} \\
Settle & \begin{tabular}{l} 
NINST-by-1 vector of dates representing the \\
settle date for each swap.
\end{tabular}
\end{tabular}

Maturity
NINST-by- 1 vector of dates representing the maturity date for the underlying swap for each swap.

Note All optional inputs that follow are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. Parameter name/value pairs may be specified in any order; names are case-insensitive and partial string matches are allowed provided no ambiguities exist.
\begin{tabular}{ll} 
AmericanOpt & (Optional) NINST-by-1 flags options: \\
& - 0 for European options \\
- 1 for American options \\
SwapReset & \begin{tabular}{l} 
(Optional) NINST-by-1 vector representing the \\
\\
reset frequency per year for the underlying \\
swap. Default is 1.
\end{tabular} \\
(Optional) Day-count basis of the instrument. \\
Basis & A vector of integers. \\
& - \(0=\) actual/actual (default) \\
& - \(1=30 / 360\) (SIA) \\
& - \(2=\) actual/360 \\
& - \(3=\) actual/365 \\
& - \(4=30 / 360\) (BMA) \\
& - \(5=30 / 360\) (ISDA) \\
& - \(6=30 / 360\) (European) \\
& - \(7=\) actual/365 (Japanese) \\
& - \(8=\) actual/actual (ICMA)
\end{tabular}
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- \(12=\) actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Principal (Optional) NINST-by-1 vector of the notional principal amounts. Default is 100 .
Options
(Optional) Derivatives pricing options structure created with derivset.

\section*{Description}
[Price, PriceTree] = swaptionbyhw(HWTree, OptSpec, Strike, ExerciseDates, Spread, Settle, Maturity, 'Name1', Value1, 'Name2', Value2) computes the price of a swaption from a HW interest-rate tree.

> Note The Settle date for every swaption is set to the ValuationDate of the HW tree. The swap argument Settle is ignored. The underlying swap starts at the maturity of the swaption.

The swaption may be a call swaption or a put swaption.
A call swaption or payer swaption allows the option buyer to enter into an interest rate swap in which the buyer of the option pays the fixed rate and receives the floating rate.

A put swaption or receiver swaption allows the option buyer to enter into an interest rate swap in which the buyer of the option receives the fixed rate and pays the floating rate.

Price is a NINST-by- 1 vector of expected swaption prices at time 0 .
PriceTree is a MATLAB structure of trees containing vectors of swaption instrument prices and a vector of observation times for each node. Within PriceTree:
- PriceTree. PTree contains the clean prices.
- PriceTree.tObs contains the observation times.

\section*{Examples}

\section*{Price a 3-Year Put Swaption Using an HW Interest-Rate Tree}

This example shows how to price a 3 -year put swaption using an HW interest-rate tree with the following data.
```

Rates =0.075 * ones (10,1);
Compounding = 2;
StartDates = ['jan-1-2007';'jul-1-2007';'jan-1-2008';'jul-1-2008';'jar
'jul-1-2009';'jan-1-2010'; 'jul-1-2010';'jan-1-2011';'jul-1-2011'];
EndDates =[ 'jul-1-2007';'jan-1-2008';'jul-1-2008';'jan-1-2009';'jul-1.
'jan-1-2010';'jul-1-2010';'jan-1-2011';'jul-1-2011';'jan-1-2012'];
ValuationDate = 'jan-1-2007';
% define the RatesSpec
RateSpec = intenvset('Rates', Rates, 'StartDates', StartDates, 'EndDat
EndDates, 'Compounding', Compounding);
% use HWVolSpec to compute the interest-rate volatility
Volatility = 0.05*ones(10,1);
AlphaCurve = 0.01*ones(10,1);
AlphaDates = EndDates;
HWVolSpec = hwvolspec(ValuationDate, EndDates, Volatility, AlphaDates
% use HWTimeSpec to specify the structure of the time layout for an HV
HWTimeSpec = hwtimespec(ValuationDate, EndDates, Compounding);
% build the HW tree
HWTree = hwtree(HWVolSpec, RateSpec, HWTimeSpec);

```
```

% use the following arguments for a 1-year swap and 3-year swaption
ExerciseDates = 'jan-1-2010';
SwapSettlement = ExerciseDates;
SwapMaturity = 'jan-1-2012';
Spread = 0;
SwapReset = 2 ;
Principal = 100;
OptSpec = 'put';
Strike= 0.04;
Basis=1;
% price the swaption
PriceSwaption = swaptionbyhw(HWTree, OptSpec, Strike, ExerciseDates, ...
Spread, SwapSettlement, SwapMaturity,'SwapReset', SwapReset, ...
'Basis', Basis,'Principal', Principal)
PriceSwaption =
2.9081

```

See Also hwtree | instswaption | swapbyhw

Purpose
Price European swaption using Linear Gaussian two-factor model
Syntax
```

Price = swaptionbylg2f(ZeroCurve,a,b,sigma,eta,rho,
Strike,ExerciseDate,Maturity)
Price = swaptionbylg2f( ___, Name,Value)

```

Description

Input Arguments

Price = swaptionbylg2f(ZeroCurve, a, b, sigma, eta, rho, Strike, ExerciseDate, Maturity) returns the European swaption price for a two-factor additive Gaussian interest-rate model.

Price = swaptionbylg2f( __ , Name, Value) returns the Eurpoean swaption price for two-factor additive Gaussian interest-rate model using optional name-value pairs.

\section*{ZeroCurve - Zero-curve for Linear Gaussian two-factor model structure}

Zero-curve for the Linear Gaussian two-factor model, specified using IRDataCurve or RateSpec.

\section*{Data Types}
struct

\section*{a-Mean reversion for first factor for Linear Gaussian two-factor model}
scalar
Mean reversion for first factor for the Linear Gaussian two-factor model, specified as a scalar.

\author{
Data Types \\ single | double
}

\section*{b-Mean reversion for second factor for Linear Gaussian two-factor model}
scalar

Mean reversion for second factor for the Linear Gaussian two-factor model, specified as a scalar.

Data Types
single | double

\section*{sigma - Volatility for first factor for Linear Gaussian two-factor model}
scalar
Volatility for first factor for the Linear Gaussian two-factor model, specified as a scalar.
```

Data Types
single | double
eta - Volatility for second factor for Linear Gaussian two-factor
model

```
scalar

Volatility for second factor for the Linear Gaussian two-factor model, specified as a scalar.
```

Data Types
single | double

```
rho-Scalar correlation of the factors
scalar

Scalar correlation of the factors, specified as a scalar.
```

Data Types
single | double

```

\section*{Strike - Swaption strike price}
nonnegative integer | vector of nonnegative integers
Swaption strike price, specified as a nonnegative integer using a NumSwaptions-by-1 vector.

Data Types
single | double

\section*{ExerciseDate - Swaption exercise dates}
nonnegative integer | vector of nonnegative integers
Swaption exercise dates, specified as a NumSwaptions-by-1 vector of serial date numbers or date strings.

Data Types
single | double

\section*{Maturity - Underlying swap maturity date}
nonnegative integer | vector of nonnegative integers | string of dates
Underlying swap maturity date, specified using a NumSwaptions-by-1 vector of serial date numbers or date strings.

\section*{Data Types}
single | double | cell

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.
Example: Price =
swaptionbylg2f(irdc,a,b, sigma, eta, rho,Strike, ExerciseDate, Maturity, 'R

\section*{'Reset' - Frequency of swaption payments per year}

2 (default) | positive integer from the set[1,2,3,4,6,12] | vector of positive integers from the set [1, 2, 3, 4, 6, 12]

Frequency of swaption payments per year, specified as positive integers for the values \(1,2,4,6,12\) ] in a NumSwaptions-by-1 vector.

\author{
Data Types \\ single | double
}

\section*{'Notional' - Notional value of swaption}

100 (default) | nonnegative integer | vector of nonnegative integers
Notional value of swaption, specified as a nonnegative integer using a NumSwaptions-by-1 vector of notional amounts.

\author{
Data Types \\ single | double
}

\section*{'OptSpec' - Option specification for the swaption}
'call' (default) | string with value of 'call' or 'put ' | cell array of string with values of 'call' or 'put'

Option specification for the swaption, specified as a string or a NumSwaptions-by- 1 cell array of strings with a value of 'call' or 'put'. A 'call' swaption entitles the buyer to pay the fixed rate. A 'put' swaption entitles the buyer to receive the fixed rate.

\section*{Data Types \\ char | cell}

\section*{Output Arguments}

\section*{Algorithms}

\section*{Price - Swaption price}
scalar | vector
Swaption price, returned as a scalar or an NumSwaptions-by-1 vector.
The following defines the swaption price for a two-factor additive Gaussian interest-rate model, given the ZeroCurve, a, b, sigma, eta, and rho parameters:
\[
\begin{aligned}
& r(t)=x(t)+y(t)+\phi(t) \\
& d x(t)=-a(x)(t) d t+\sigma\left(d W_{1}(t), x(0)=0\right. \\
& d y(t)=-b(y)(t) d t+\eta\left(d W_{2}(t), y(0)=0\right.
\end{aligned}
\]
where \(d W_{1}(t) d W_{2}(t)=\rho d t\) is a two-dimensional Brownian motion with correlation \(\rho\) and \(\varphi\) is a function chosen to match the initial zero curve.

\section*{Examples Price a European Swaption Using a Linear Gaussian Two-Factor Model}

Define the ZeroCurve, a, b, sigma, eta, and rho parameters to compute the price of the swaption.
```

Settle = datenum('15-Dec-2007');
ZeroTimes = [3/12 6/12 1 5 7 10 20 30]';
ZeroRates = [0.033 0.034 0.035 0.040 0.042 0.044 0.048 0.0475]';
CurveDates = daysadd(Settle,360*ZeroTimes,1);
irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);
a = .07;
b = .5;
sigma = .01;
eta = .006;
rho = -.7;
Reset = 1;
ExerciseDate = daysadd(Settle,360*5,1);
Maturity = daysadd(ExerciseDate,360*[3;4],1);
Strike = .05;
Price = swaptionbylg2f(irdc,a,b,sigma,eta,rho,Strike,ExerciseDate,Matl

```
Price =
    1.1870
    1.5633

\section*{References}
[1] Brigo, D. and F. Mercurio, Interest Rate Models - Theory and Practice, Springer Finance, 2006.

\author{
See Also capbylg2f | floorbylg2f | LinearGaussian2F \\ Relafed - "Price Swaptions With Interest-Rate Models Using Simulation" on Examples page 2-105
}

\section*{Purpose Dates from time and frequency}

\author{
Syntax Dates = time2date(Settle, Times, Compounding, Basis, EndMonthRule)
}

\section*{Arguments}
\(\left.\begin{array}{ll}\text { Settle } & \begin{array}{l}\text { Settlement date. A vector of serial date numbers } \\ \text { or date strings. }\end{array} \\ \text { Times } & \begin{array}{l}\text { Vector of times corresponding to the compounding } \\ \text { value. Times must be equal to or greater than } 0 .\end{array} \\ \text { (Optional) Scalar value representing the rate } \\ \text { at which the input zero rates were compounded } \\ \text { when annualized. Default }=2 \text {. This argument } \\ \text { determines the formula for the discount factors: }\end{array}\right\}\)


\section*{Description}

\section*{Examples}

Dates = time2date(Settle, Times, Compounding, Basis, EndMonthRule) computes dates corresponding to compounded rate quotes between Settle and time factors.

Note To obtain accurate results from this function, the Basis and Dates arguments must be consistent. If the Dates argument contains months that have 31 days, Basis must be one of the values that allow months to contain more than 30 days; for example, Basis \(=0\), 3 , or 7 .

The time2date function is the inverse of date2time.

Show that date2time and time2date are the inverse of each other. First compute the time factors using date2time.
```

Settle = '1-Sep-2002';
Dates = datenum(['31-Aug-2005'; '28-Feb-2006'; '15-Jun-2006';
'31-Dec-2006']);
Compounding = 2;
Basis = 0;
EndMonthRule = 1;
Times = date2time(Settle, Dates, Compounding, Basis,...
EndMonthRule)

```

Times =
5.9945
6.9945
7.5738
8.6576

Now use the calculated Times in time2date and compare the calculated dates with the original set.
```

Dates_calc = time2date(Settle, Times, Compounding, Basis,...
EndMonthRule)

```

\section*{time2date}
Dates calc =732555732736
732843733042
datestr(Dates_calc)
ans =
31-Aug-2005
28-Feb-2006
15-Jun-2006
31-Dec-2006
See Also cftimes | date2time | cfamounts | disc2rate | rate2disc
Purpose Entries from node of recombining binomial tree
Syntax Values = treepath(Tree, BranchList)
Arguments
Description Values = treepath(Tree, BranchList) extracts entries of a node of a recombining binomial tree. The node path is described by the sequence of branchings taken, starting at the root. The top branch is number one, the second-to-top is two, and so on. Set the branch sequence to zero to obtain the entries at the root node.
Values is a number of values (NUMVALS)-by-NUMPATHS matrix containing the retrieved entries of a recombining tree.
Examples Create a BDT tree by loading the example file.
load deriv.mat;
Then

returns the rates at the tree nodes located by taking the up branch,then the down branch, and finally the up branch again.
FwdRates =
1.1000
1.0979
1.1377

\section*{treepath}
1.1183

You can visualize this with the treeviewer function.
treeviewer (BDTTree)


\section*{See Also}
mktree | treeshape

Purpose Shape of recombining binomial tree
Syntax [NumLevels, NumPos, IsPriceTree] = treeshape(Tree)
Arguments
Tree Recombining binomial tree.

\section*{Description}
[NumLevels, NumPos, IsPriceTree] = treeshape(Tree) returns information on a recombining binomial tree's shape.

NumLevels is the number of time levels of the tree.
NumPos is a 1 -by-NUMLEVELS vector containing the length of the state vectors in each level.

IsPriceTree is a Boolean determining if a final horizontal branch is present in the tree.

Examples Create a BDT tree by loading the example file.
load deriv.mat;
With treeviewer you can see the general shape of the BDT interest-rate tree.
treeviewer (BDTTree)

\section*{treeshape}


With this tree
[NumLevels, NumPos, IsPriceTree] = treeshape(BDTTree.FwdTree)
returns

NumLevels =
4

NumPos =
\(\begin{array}{llll}1 & 1 & 1 & 1\end{array}\)

IsPriceTree =
0
\begin{tabular}{ll} 
Purpose & Tree information \\
Syntax & \begin{tabular}{l} 
treeviewer(Tree) \\
treeviewer(PriceTree, InstSet) \\
\\
\\
\end{tabular}
\end{tabular}

\section*{Arguments}
Tree \(\quad\) Tree can be any of the following types of trees.
Interest-rate trees:
- Black-Derman-Toy (BDTTree)
- Black-Karasinski (BKTree)
- Heath-Jarrow-Morton (HJMTree)
- Hull-White (HWTree)

For information on creating interest-rate trees, see:
- bktree for information on creating BKTree.
- bdttree for information on creating BDTTree.
- hjmtree for information on creating HJMTree.
- hwtree for information on creating HWTree.

Money market trees:
- Money market tree (MMktTree)

For information on creating money-market trees, see:
- mmktbybdt for information on creating a money-market tree from a BDT interest-rate tree.
- mmktbyhjm for information on creating a money-market tree from an HJM interest-rate tree.

\section*{treeviewer}

> Note Money market trees cannot be created from BK or HW interest-rate trees.

Stock price trees:
- Cox-Ross-Rubinstein (CRRTree)
- Implied Trinomial tree (ITTTree)
- Leisen-Reimer stock tree (LRTree)
- Equal probabilities (EQPTree)

For information on creating stock price trees, see:
- crrtree for information on creating CRRTree.
- eqptree for information on creating EQPTree.
- itttree for information on creating ITTTree.
- Irtree for information on creating LRTree.

Cash flow trees:
- Black-Derman-Toy (BDTCFTree)
- Heath-Jarrow-Morton (HJMCFTree)

Cash flow trees are created as outputs from the swap functions swapbyhjm and swapbybdt.

Note For the function swapbybdt, which uses a recombining binomial tree, this structure contains only NaNs because cash flows cannot be accurately calculated at every tree node for floating-rate notes.
\begin{tabular}{ll} 
PriceTree & \begin{tabular}{l} 
PriceTree is a Black-Derman-Toy \\
(BDTPriceTree), Black-Karasinski (BKPriceTree), \\
Heath-Jarrow-Morton (HJMPriceTree), Hull-White \\
(HWPriceTree), Cox-Ross-Rubinstein (crrprice),
\end{tabular} \\
CFTree & \begin{tabular}{l} 
Equal probabilities (eqprice), or Implied Trinomial \\
tree (ittprice) tree of instrument prices.
\end{tabular} \\
& \begin{tabular}{l} 
CFTree is a tree of swap cash flows. You create cash \\
flow trees when executing the Black-Derman-Toy \\
and Heath-Jarrow-Morton swap functions. \\
(Black-Derman-Toy cash flow trees contain only
\end{tabular} \\
InstSet & \begin{tabular}{l} 
NaNs.) \\
\\
\\
(Optional) Variable containing a collection of \\
instruments whose prices or cash flows are contained \\
in a tree. The collection can be created with the \\
function instadd or as a cell array containing the
\end{tabular} \\
& \begin{tabular}{l} 
names of the instruments. To display the names \\
of the instruments, the field Name should exist in \\
InstSet. If InstSet is not passed, treeviewer
\end{tabular} \\
uses default instruments names (numbers) when \\
displaying prices or cash flows.
\end{tabular}

\section*{Description}
treeviewer(Tree) displays an interest rate, stock price, or money-market tree.
treeviewer(PriceTree, InstSet) displays a tree of instrument prices. If you provide the name of an instrument set (InstSet) and you have named the instruments using the field Name, the treeviewer display identifies the instrument being displayed with its name. (See Example 3 for a description.) If you do not provide the optional InstSet argument, the instruments are identified by their sequence number in the instrument set. (See Example 6 for a description.)
treeviewer(CFTree, InstSet) displays a cash flow tree that has been created with swapbybdt or swapbyhjm. If you provide the name of an instrument set (InstSet) containing cash flow names, the treeviewer

\section*{treeviewer}
display identifies the instrument being displayed with its name. (See Example 3 for a description.) If the optional InstSet argument is not present, the instruments are identified by their sequence number in the instrument set. See Example 6 for a description.)
treeviewer price tree diagrams follow the convention that increasing prices appear on the upper branch of a tree and, consequently, decreasing prices appear on the lower branch. Conversely, for interest rate displays, decreasing interest rates appear on the upper branch (prices are rising) and increasing interest rates on the lower branch (prices are falling).
treeviewer provides an interactive display of prices or interest rates. The display is activated by clicking the nodes along the price or interest rate path shown in the left pane when the function is called. For HJM trees you select the endpoints of the path, and treeviewer displays all data from beginning to end. With recombining trees, such as BDT, BK and HW, you must click each node in succession from the beginning \((\mathrm{t}=1)\) to the last node \((\mathrm{t}=\mathrm{n})\). Do not include the root node, the node at \(\mathrm{t}=0\). If you do not click the nodes in the proper order, you are reminded with the message

Parent of selected node must be selected.

\section*{Examples Example 1. Display an HJM Interest-Rate Tree.}
load deriv.mat
treeviewer(HJMTree)
The treeviewer function displays the structure of an HJM tree in the left pane. The tree visualization in the right pane is blank.


To visualize the actual interest-rate tree, go to the Tree Visualization pane and click onPath(the default) and Diagram. Now, select the first path by clicking on the last node \((t=3)\) of the upper branch.

\section*{treeviewer}


Note that the entire upper path is highlighted in red.
To complete the process, select a second path by clicking on the last node ( \(t=3\) ) of another branch. The second path is highlighted in purple. The final display looks like this.


\section*{Alternative Forms of Display}

The Tree Visualization pane allows you to select alternative ways to display tree data. For example, if you select Path and Table as your visualization choices, the final display above instead appears in tabular form.

\section*{treeviewer}


To see a plot of interest rates along the chosen branches, click Path and Plot in the Tree Visualization pane.


Note that with Plot selected, rising interest rates are shown on the upper branch and declining interest rates on the lower.

Finally, if you clicked Node and Children under Tree Visualization, you restrict the data displayed to just the selected parent node and its children.

\section*{treeviewer}
\begin{tabular}{|c|c|}
\hline 4 Tree Viewer & - - \(\square\) ] x \\
\hline Eile Edit yiew Insert Iools window Help & \\
\hline & \(\left[\begin{array}{ll}\text { Tree Visualization } & \\ \text { Selection } & \text { Visualization } \\ \text { C Path } & \text { C Table } \\ \text { C Node and Children } & 6 \\ & \text { C Pliagiam }\end{array}\right.\) \\
\hline  &  \\
\hline \(\begin{array}{llll}0 & 1 & 2 & 3\end{array}\) & Help Close \\
\hline
\end{tabular}

With Node and Children selected, the choices under Visualization are unavailable.

\section*{Example 2. Display a BDT Interest-Rate Tree.}
load deriv.mat
treeviewer(BDTTree)
The treeviewer function displays the structure of a BDT tree in the left pane. The tree visualization in the right pane is blank.


To visualize the actual interest-rate tree, go to the Tree Visualization pane and click Path (the default) and Diagram. Now, select the first path by clicking on the first node of the up branch ( \(\mathrm{t}=1\) ). Continue by clicking the down branch at the next node \((\mathrm{t}=2)\). The two figures below show the treeviewer path diagrams for these selections.

\section*{treeviewer}


Continue clicking all nodes in succession until you reach the end of the branch. Note that the entire path you have selected is highlighted in red.

Select a second path by clicking the first node of the lower branch ( \(\mathrm{t}=1\) ). Continue clicking lower nodes as you did on the first branch.
Note that the second branch is highlighted in purple. The final display looks like this.


\footnotetext{
Example 3. Display an HJM Price Tree for Named Instruments.
load deriv.mat
[Price, PriceTree] = hjmprice(HJMTree, HJMInstSet); treeviewer(PriceTree, HJMInstSet)
}

\section*{treeviewer}


\section*{Example 4. Display a BDT Price Tree for Named Instruments.}
load deriv.mat
[Price, PriceTree] = bdtprice(BDTTree, BDTInstSet); treeviewer(PriceTree, BDTInstSet)


\section*{Example 5. Display an HJM Price Tree with Renamed Instruments.}
```

load deriv.mat
[Price, PriceTree] = hjmprice(HJMTree, HJMInstSet);
Names = {'Bond1', 'Bond2', 'Option', 'Fixed','Float', 'Cap',...
'Floor', 'Swap'};
treeviewer(PriceTree, Names)

```

\section*{treeviewer}


\section*{Example 6. Display an HJM Price Tree Using Default Instrument Names (Numbers).}
load deriv.mat
[Price, PriceTree] = hjmprice(HJMTree, HJMInstSet); treeviewer(PriceTree)


See Also
bdttree | bktree | crrtree | eqptree | hjmtree | hwtree | instadd | itttree | lrtree | mmktbybdt | mmktbyhjm | swapbybdt | swapbyhjm

\section*{trintreepath}

Purpose Entries from node of recombining trinomial tree
```

Syntax Values = trintreepath(TrinTree, BranchList)

```

\section*{Arguments}

TrinTree Recombining price or interest-rate trinomial tree.

BranchList Number of paths (NUMPATHS) by path length (PATHLENGTH) matrix containing the sequence of branchings.

\section*{Description \\ Values = trintreepath(TrinTree, BranchList) extracts entries of a node of a recombining trinomial tree. The node path is described by the sequence of branchings taken, starting at the root. The top branch is number 1 , the middle branch is 2 , and the bottom branch is 3 . Set the branch sequence to 0 to obtain the entries at the root node. \\ Values is a number of values (NUMVALS)-by-NUMPATHS matrix containing the retrieved entries of a recombining tree.}

\section*{Examples Create a Hull-White tree by loading the example file.}
load deriv.mat;

Then, for example
FwdRates = trintreepath(HWTree, [1 2 3])
returns the rates at the tree nodes located by starting at 0 , taking the up branch at the first node, the middle branch at the second node, and finally the bottom branch at the third node.

FwdRates =
1.0279
\[
\begin{aligned}
& 1.0528 \\
& 1.0652 \\
& 1.0591
\end{aligned}
\]

You can visualize this with the treeviewer function.
```

treeviewer(HWTree)

```


See Also
mktrintree | trintreeshape

\section*{trintreeshape}

Purpose Shape of recombining trinomial tree

\author{
Syntax [NumLevels, NumPos, NumStates] = trintreeshape(TrinTree)
}

Arguments
TrinTree Recombining price or interest-rate trinomial tree.

Description [NumLevels, NumPos, NumStates] = trintreeshape(TrinTree) returns information on a recombining trinomial tree's shape.
NumLevels is the number of time levels of the tree.
NumPos is a 1 -by-NUMLEVELS vector containing the length of the state vectors in each level.

NumStates is a 1-by-NUMLEVELS vector containing the number of state vectors in each level.

\section*{Examples Create a Hull-White tree by loading the example file.}
load deriv.mat;

With treeviewer you can see the general shape of the HW interest-rate tree.
treeviewer(HWTree)


With this tree
[NumLevels, NumPos, NumStates] = trintreeshape(HWTree)
returns
NumLevels =
4
\begin{tabular}{cccc} 
NumPos & \(=\) & & \\
1 & 1 & 1 & 1 \\
NumStates & & & \\
1 & 3 & 5 & 5
\end{tabular}

\section*{See Also}
mktrintree | trintreepath

Purpose Compute credit exposures from contract values
```

Syntax [exposures,exposurecpty] =
creditexposures(values,counterparties)
[exposures,exposurecpty] = creditexposures( __ ,Name,Value)

```

\section*{Description}

\section*{Input Arguments}
[exposures, exposurecpty] =
creditexposures(values, counterparties) computes the counterparty credit exposures from an array of mark-to-market OTC contract values. These exposures are used when calculating the CVA (credit value adjustment) for a portfolio.
[exposures,exposurecpty] = creditexposures( _ _ ,Name, Value) computes the counterparty credit exposures from an array of mark-to-market OTC contract values using optional name-value pair arguments. These exposures are used when calculating the CVA (credit value adjustment) for a portfolio.

\section*{values - 3-D array of simulated mark-to-market values of portfolio of contracts \\ array}

3-D array of simulated mark-to-market values of a portfolio of contracts simulated over a series of simulation dates and across many scenarios, specified as a NumDates-by-NumContracts-by-NumScenarios "cube" of contract values. Each row represents a different simulation date, each column a different contract, and each "page" is a different scenario from a Monte-Carlo simulation.

\section*{Data Types \\ double}

\section*{counterparties - Counterparties corresponding to each contract \\ vector | cell array}

Counterparties corresponding to each contract in values, specified as a NumContracts-element vector of counterparties. Counterparties
can be a vector of numeric IDs or a cell array of counterparty names. By default, each counterparty is assumed to have one netting set that covers all of its contracts. If counterparties are covered by multiple netting sets, then use the NettingID parameter. A value of NaN (or in a cell array) indicates that a contract is not included in any netting set unless otherwise specified by NettingID. counterparties is case insensitive and leading or trailing white spaces are removed.

\section*{Data Types \\ double | cell}

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: [exposures,exposurecpty] =
creditexposures(values, counterparties, 'NettingID', '10', 'ExposureType'

\section*{'NettingID' - Netting set IDs indicate which netting set each contract belongs}
(default) | vector | cell array
Netting set IDs to indicate to which netting set each contract in values belongs, specified by a NumContracts-element vector of netting set IDs. NettingID can be a vector of numeric IDs or else a cell array of string identifiers. The creditexposures function uses counterparties and NettingID to define each unique netting set (all contracts in a netting set must be with the same counterparty). By default, each counterparty has a single netting set which covers all of their contracts. A value of NaN (or ' ' in a cell array) indicates that a contract is not included in any netting set. NettingID is case insensitive and leading or trailing white spaces are removed.
```

Data Types
double | cell

```

\title{
'ExposureType' - Calculation method for exposures \\ 'counterparty' (default) | string with value of 'counterparty' or 'additive'
}

Calculation method for exposures, specified with values:
- 'Counterparty' - Compute exposures per counterparty.
- 'Additive' - Compute additive exposures at the contract level. Exposures are computed per contract and sum to the total counterparty exposure.

\section*{Data Types \\ char}

\section*{Output Arguments}

\section*{exposures - 3-D array of credit exposures}
array
3-D array of credit exposures representing the potential losses from each counterparty or contract at each date and over all scenarios. The size of exposures depends on the ExposureType input argument:
- When ExposureType is 'Counterparty', exposures returns a NumDates-by-NumCounterparties-by-NumScenarios "cube" of credit exposures representing potential losses that could be incur over all dates, counterparties, and scenarios, if a counterparty defaulted (ignoring any post-default recovery).
- When ExposureType is 'Additive', exposures returns a NumDates-by-NumContracts-by-NumScenarios "cube," where each element is the additive exposure of each contract (over all dates and scenarios). Additive exposures sum to the counterparty-level exposure.

\footnotetext{
exposurecpty - Counterparties that correspond to columns of exposures array
vector
}

Counterparties that correspond to columns of the exposures array, returned as NumCounterparties or NumContracts elements depending on the ExposureType.

\section*{Examples View Contract Values and Exposures Over Time for a Particular Counterparty}

After computing the mark-to-market contract values for a portfolio of swaps over many scenarios, compute the credit exposure for a particular counterparty. View the contract values and credit exposure over time. First, load data containing the mark-to-market contract values for a portfolio of swaps over many scenarios.
```

load ccr.mat
% Look at one counterparty.
cpID = 4;
cpValues = squeeze(sum(values(:,swaps.Counterparty == cpID,:),2));
subplot(2,1,1)
plot(simulationDates,cpValues);
title(sprintf('Mark-to-Market Contract Values for Counterparty: %d',cp
datetick('x','mmmyy')
ylabel('Portfolio Value ($)')
% Compute the exposure by counterparty.
[exposures, expcpty] = creditexposures(values,swaps.Counterparty,...
'NettingID',swaps.NettingID);
% View the credit exposure over time for the counterparty.
subplot(2,1,2)
cpIdx = find(expcpty == cpID);
plot(simulationDates,squeeze(exposures(:,cpIdx,:)));
title(sprintf('Exposure for counterparty: %d',cpIdx));
datetick('x','mmmyy')
ylabel('Exposure ($)')
xlabel('Simulation Dates')

```


\section*{Compute the Credit Exposure and Determine the Incremental Exposure for a New Trade}

Load data containing the mark-to-market contract values for a portfolio of swaps over many scenarios.
load cor.mat
Look at one counterparty.
```

cpID = 4;
cpIdx = swaps.Counterparty == cpID;
cpValues = values(:,cpIdx,:);
plot(simulationDates,squeeze(sum(cpValues,2)));
grid on;
title(sprintf('Potential Mark-to-Market Portfolio Values for Counterp
datetick('x','mmmyy')
ylabel('Portfolio Value (\$)')

```


Compute the exposures.
```

netting = swaps.NettingID(cpIdx);
exposures = creditexposures(cpValues,cpID,'NettingID',netting);

```

View the credit exposure over time for the counterparty.
```

figure;
plot(simulationDates,squeeze(exposures));
grid on
title(sprintf('Exposure for counterparty: %d',cpID));
datetick('x','mmmyy')
ylabel('Exposure (\$)')
xlabel('Simulation Dates')

```


Compute the credit exposure profiles.
profilesBefore = exposureprofiles(simulationDates,exposures)
profilesBefore =
\[
\begin{array}{r}
\text { Dates: }[37 \times 1 \text { double }] \\
\text { EE: }[37 \times 1 \text { double }] \\
\text { PFE: }[37 \times 1 \text { double }]
\end{array}
\]
```

    MPFE: 2.1580e+05
    EffEE: [37x1 double]
EPE: 2.8602e+04
EffEPE: 4.9579e+04

```

Consider a new trade with a counterparty. For this example, take another trade from the original swap portfolio and "copy" it for a new counterparty. This example is only for illustrative purposes.
```

newTradeIdx = 3;

```
newTradeValues = values(:,newTradeIdx,:);
\% Append a new trade to your existing portfolio.
cpValues = [cpValues newTradeValues];
netting = [netting; cpID];
exposures = creditexposures(cpValues,cpID,'NettingID',netting);

Compute the new credit exposure profiles.
```

profilesAfter = exposureprofiles(simulationDates,exposures)

```
profilesAfter =
```

    Dates: [37x1 double]
            EE: [37x1 double]
            PFE: [37x1 double]
        MPFE: 2.4689e+05
    EffEE: [37x1 double]
        EPE: 3.1609e+04
    EffEPE: 5.6178e+04

```

Visualize the expected exposures and the new trade's incremental exposure. The incremental exposure is used to compute the incremental credit value adjustment (CVA) charge.
```

figure;
subplot(2,1,1)
plot(simulationDates,profilesBefore.EE,...
simulationDates,profilesAfter.EE);
grid on;
legend({'EE before','EE with trade'})
datetick('x','mmmyy','keeplimits')
title('Expected Exposure before and after new trade');
ylabel('Exposure ($)')
subplot(2,1,2)
incrementalEE = profilesAfter.EE - profilesBefore.EE;
plot(simulationDates,incrementalEE);
grid on;
legend('incremental EE')
datetick('x','mmmyy','keeplimits')
ylabel('Exposure ($)')
xlabel('Simulation Dates')

```


\section*{References}

Pykhtin M., and S. Zhu. A Guide to Modeling Counterparty Credit Risk, GARP, July/August 2007, issue 37.

Pykhtin, Michael., and Dan Rosen. Pricing Counterparty Risk at the Trade Level and CVA Allocations, FEDS Working Paper No. 10., February 1, 2010.

See Also exposureprofiles
\begin{tabular}{|c|c|}
\hline Purpose & Determine option-adjusted spread of callable bond using Agency OAS model \\
\hline Syntax & ```
OAS = agencyoas(ZeroData, Price, CouponRate, Settle, Maturity,
Vol, CallDate)
OAS = agencyoas(ZeroData, Price, CouponRate, Settle, Maturity,
Vol, CallDate, Name,Value)
``` \\
\hline Description & \begin{tabular}{l}
OAS = agencyoas(ZeroData, Price, CouponRate, Settle, Maturity, \\
Vol, CallDate) computes OAS of a callable bond given price using the Agency OAS model. \\
OAS = agencyoas(ZeroData, Price, CouponRate, Settle, Maturity, \\
Vol, CallDate, Name, Value) computes OAS of a callable bond given price using the Agency OAS model with additional options specified by one or more Name, Value pair arguments.
\end{tabular} \\
\hline Input & ZeroData \\
\hline Arguments & Zero curve represented as a numRates-by-2 matrix where the first column is zero dates and the second column is the accompanying zero rates. \\
\hline & Price \\
\hline & \\
\hline & CouponRate \\
\hline
\end{tabular}
numBonds-by- 1 vector of coupon rates in decimal form.

\section*{Settle}

Scalar MATLAB date number for the settlement date for all bonds and the zero data.

Note The Settle date must be an identical settlement date for all the bonds and the zero curve.

\section*{Maturity}
numBonds-by- 1 vector of maturity dates.

\section*{Vol}
numBonds-by- 1 vector of volatilities in decimal form. This is the volatility of interest rates corresponding to the time of the CallDate.

\section*{CallDate}
numBonds-by-1 vector of call dates.

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

\section*{'Basis'}

N -by-1 vector of day-count basis:
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- 3 = actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

\section*{'CurveBasis'}

Basis of the zero curve, where the choices are identical to Basis.
Default: 0 (actual/actual)

\section*{'CurveCompounding'}

Compounding frequency of the zero curve. Possible values include: -1 , \(0,1,2,3,4,6,12\).

Default: 2 (Semi-annual)

\section*{'EndMonthRule'}

End-of-month rule; 1, indicating in effect, and 0, indicating rule not in effect for the bond(s). When 1 , the rule is in effect for the bond(s), this means that a security that pays coupon interest on the last day of the month will always make payment on the last day of the month.

Default: 1 - Indicates in effect

\section*{'Face'}

Face value of the bond.
Default: 100

\section*{'FirstCouponDate'}

Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.

Default: If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

\section*{'InterpMethod'}

Interpolation method used to obtain points from the zero curve. Values are:
- linear - linear interpolation
- cubic - piecewise cubic spline interpolation
- pchip - piecewise cubic Hermite interpolation

Default: linear

\section*{'IssueDate'}

Bond issue date.
Default: If you do not specify an IssueDate, the cash flow payment dates are determined from other inputs.

\section*{'LastCouponDate'}

Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at
the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date.

Default: If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

\section*{'Period'}

Number of coupon payments per year. Possible values include: \(0,1,2\), \(3,4,6,12\).

Default: 2

\section*{'StartDate'}

Forward starting date of payments.
Default: If you do not specify a StartDate, the effective start date is the Settle date.

\section*{Output}

Arguments

\section*{Definitions}

\section*{Examples}

\section*{OAS}
numBonds-by- 1 matrix of option-adjusted spreads.

\section*{Agency OAS Model}

The BMA European Callable Securities Formula provides a standard methodology for computing price and option-adjusted spread for European Callable Securities (ECS).

\section*{Compute the Agency oAS Value}

This example shows how to compute the agency OAS value.
```

Settle = datenum('20-Jan-2010');
ZeroRates = [.07 .164 . 253 1.002 1.732 2.226 2.605 3.316 ...
3.474 4.188 4.902]'/100;
ZeroDates = daysadd(Settle,360*[.25 .5 1 2 3 4 5 7 10 20 30],1);
ZeroData = [ZeroDates ZeroRates];

```
```

Maturity = datenum('30-Dec-2013');
CouponRate = .022;
Price = 99.155;
Vol = .5117;
CallDate = datenum('30-Dec-2010');
OAS = agencyoas(ZeroData, Price, CouponRate, Settle, Maturity, Vol, Ca
OAS =
8.5837

```
References

SIFMA, The BMA European Callable Securities Formula,
 http://www.sifma.org.
See Also agencyprice
Tutorials - "Agency Option-Adjusted Spreads" on page 6-2

\section*{agencyprice}

Purpose Price callable bond using Agency OAS model
```

Syntax
Price = agencyprice(ZeroData, OAS, CouponRate, Settle, Maturity, Vol, CallDate)
Price = agencyprice(ZeroData, OAS, CouponRate, Settle, Maturity, Vol, CallDate, Name,Value)

```

\section*{Description}

\section*{Input Arguments}

Price = agencyprice(ZeroData, OAS, CouponRate, Settle, Maturity, Vol, CallDate) computes the price for a callable bond, given OAS, using the Agency OAS model.

Price = agencyprice(ZeroData, OAS, CouponRate, Settle, Maturity, Vol, CallDate, Name, Value) computes the price for a callable bond, given OAS, using the Agency OAS model with additional options specified by one or more Name, Value pair arguments.

\section*{ZeroData}

Zero curve represented as a numRates-by-2 matrix where the first column is zero dates and the second column is the accompanying zero rates.

\section*{OAS}
numBonds-by- 1 vector of option-adjusted spreads, expressed as a decimal (i.e., 50 basis points is entered as .005).

\section*{CouponRate}
numBonds-by- 1 vector of coupon rates in decimal form.

\section*{Settle}

Scalar MATLAB date number for the settlement date for all the bonds and the zero data.

Note The Settle date must be an identical settlement date for all bonds and the zero curve.

\section*{Maturity}
numBonds-by-1 vector of maturity dates.

\section*{Vol}
numBonds-by- 1 vector of volatilities in decimal form. This is the volatility of interest rates corresponding to the time of the CallDate.

\section*{CallDate}
numBonds-by- 1 vector of call dates.

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

\section*{'Basis'}

N -by-1 vector of day-count basis:
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- 3 = actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

\section*{'CurveBasis'}

Basis of the zero curve, where the choices are identical to Basis.
Default: 0 (actual/actual)

\section*{'CurveCompounding'}

Compounding frequency of the curve. Possible values include: -1 , \(0,1,2,3,4,6,12\).

Default: 2 (Semi-annual)

\section*{'EndMonthRule'}

End-of-month rule; 1, indicating in effect, and 0, indicating rule not in effect for the bond(s). When 1, the rule is in effect for the bond(s). This means that a security that pays coupon interest on the last day of the month will always make payment on the last day of the month.

Default: 1 - Indicates in effect

\section*{'Face'}

Face value of the bond.
Default: 100

\section*{'FirstCouponDate'}

Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.

Default: If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

\section*{'InterpMethod'}

Interpolation method used to obtain points from the zero curve. Values are:
- linear - linear interpolation
- cubic - piecewise cubic spline interpolation
- pchip - piecewise cubic Hermite interpolation

Default: linear

\section*{'IssueDate'}

Bond issue date.
Default: If you do not specify an IssueDate, the cash flow payment dates are determined from other inputs.

\section*{'LastCouponDate'}

Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at

\section*{agencyprice}
the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date.

Default: If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

\section*{'Period'}

Number of coupon payments per year. Possible values include: 0, 1, 2 , \(3,4,6,12\).

Default: 2

\section*{'StartDate'}

Forward starting date of payments.
Default: If you do not specify a StartDate, the effective start date is the Settle date.

\section*{Output Arguments}

\section*{Price}
numBonds-by- 1 matrix of the price.

\section*{Definitions}

\section*{Agency OAS Model}

The BMA European Callable Securities Formula provides a standard methodology for computing price and option-adjusted spread for European Callable Securities (ECS).

\section*{Examples Compute the Agency Price}

This example shows how to compute the agency Price.
```

Settle = datenum('20-Jan-2010');
ZeroRates = [.07 .164 .253 1.002 1.732 2.226 2.605 3.316 ...
3.474 4.188 4.902]'/100;
ZeroDates = daysadd(Settle,360*[.25 .5 1 2 3 4 5 7 10 20 30],1);
ZeroData = [ZeroDates ZeroRates];

```
```

Maturity = datenum('30-Dec-2013');
CouponRate = .022;
OAS = 6.53/10000;
Vol = .5117;
CallDate = datenum('30-Dec-2010');
Price = agencyprice(ZeroData, OAS, CouponRate, Settle, Maturity, Vol,
Price =
99.4212

```
References

SIFMA, The BMA European Callable Securities Formula,
 http://www.sifma.org.
See Also agencyoas
Tutorials - "Agency Option-Adjusted Spreads" on page 6-2

\section*{bkcall}

\section*{Purpose}

Price European call option on bonds using Black model

\author{
Syntax CallPrice = bkcall(Strike, ZeroData, Sigma, BondData, Settle, Expiry, Period, Basis, EndMonthRule, InterpMethod, StrikeConvention)
}
\begin{tabular}{|c|c|c|}
\hline Arguments & Strike & Scalar or number of options (NOPT)-by-1 vector of strike prices. \\
\hline & ZeroData & Two-column (optionally three-column) matrix containing zero (spot) rate information used to discount future cash flows. \\
\hline & & - Column 1: Serial maturity date associated with the zero rate in the second column. \\
\hline & & - Column 2: Annualized zero rates, in decimal form, appropriate for discounting cash flows occurring on the date specified in the first column. All dates must occur after Settle (dates must correspond to future investment horizons) and must be in ascending order. \\
\hline & & - Column 3 (optional): Annual compounding frequency. Values are 1 (annual), 2 (semiannual, default), 3 (three times per year), 4 (quarterly), 6 (bimonthly), 12 (monthly), and -1 (continuous). \\
\hline & Sigma & Scalar or NOPT-by-1 vector of annualized price volatilities required by Black's model. \\
\hline
\end{tabular}
\(\left.\begin{array}{ll}\text { BondData } & \begin{array}{l}\text { Row vector with three (optionally four) } \\
\text { columns or NOPT-by-3 (optionally NOPT-by-4) } \\
\text { matrix specifying characteristics of underlying } \\
\text { bonds in the form: }\end{array} \\
\text { [CleanPrice CouponRate Maturity Face] } \\
\text { CleanPrice is the price excluding accrued } \\
\text { interest. }\end{array}\right\}\)\begin{tabular}{l} 
CouponRate is the decimal coupon rate.
\end{tabular}

\section*{bkcall}

\begin{tabular}{ll} 
InterpMethod & \begin{tabular}{l} 
(Optional) Scalar integer zero curve \\
interpolation method. For cash flows that \\
do not fall on a date found in the ZeroData \\
spot curve, indicates the method used to \\
interpolate the appropriate zero discount rate.
\end{tabular} \\
& Available methods are (0) nearest, (1) linear, \\
and (2) cubic. Default = 1. See interp1 for \\
more information.
\end{tabular}

StrikeConvention = 0 (default) defines the strike price as the cash (dirty) price paid for the underlying bond.

StrikeConvention = 1 defines the strike price as the quoted (clean) price paid for the underlying bond. When evaluating Black's model, the accrued interest of the bond at option expiration is added to the input strike price.

\section*{Description}

CallPrice = bkcall(Strike, ZeroData, Sigma, BondData, Settle, Expiry, Period, Basis, EndMonthRule, InterpMethod, StrikeConvention) using Black's model, derives an NOPT-by-1 vector of prices of European call options on bonds.
If cash flows occur beyond the dates spanned by ZeroData, the input zero curve, the appropriate zero rate for discounting such cash flows is obtained by extrapolating the nearest rate on the curve (that is, if a cash flow occurs before the first or after the last date on the input zero curve, a flat curve is assumed).

In addition, you can use the Financial Instruments Toolbox method getZeroRates for an IRDataCurve object with a Dates property to create a vector of dates and data acceptable for bkcall. For more information, see "Converting an IRDataCurve or IRFunctionCurve Object" on page 9-40.

\section*{bkcall}

\section*{Examples Price a European Call Option On Bonds Using the Black Model}

This example shows how to price a European call option on bonds using the Black model. Consider a European call option on a bond maturing in 9.75 years. The underlying bond has a clean price of \(\$ 935\), a face value of \(\$ 1000\), and pays \(10 \%\) semiannual coupons. Since the bond matures in 9.75 years, a \(\$ 50\) coupon will be paid in 3 months and again in 9 months. Also, assume that the annualized volatility of the forward bond price is \(9 \%\). Furthermore, suppose the option expires in 10 months and has a strike price of \(\$ 1000\), and that the annualized continuously compounded risk-free discount rates for maturities of 3,9 , and 10 months are \(9 \%, 9.5 \%\), and \(10 \%\), respectively.
```

% specify the option information
Settle = '15-Mar-2004';
Expiry = '15-Jan-2005'; % 10 months from settlement
Strike = 1000;
Sigma = 0.09;
Convention = [0 1]';
% specify the interest-rate environment

```
\% specify the bond information
CleanPrice \(=935\);
CouponRate \(=0.1\);
Maturity = '15-Dec-2013'; \% 9.75 years from settlement
Face \(=1000\);
BondData \(=\) [CleanPrice CouponRate datenum(Maturity) Face];
Period \(=2\);
Basis \(=1\);
\% call Black's model
CallPrices = bkcall(Strike, ZeroData, Sigma, BondData, Settle,...
Expiry, Period, Basis, [], [], Convention)
```

CallPrices =
9.4873
7.9686

```

When the strike price is the dirty price (Convention \(=0\) ), the call option value is \(\$ 9.49\). When the strike price is the clean price (Convention \(=\) 1 ), the call option value is \(\$ 7.97\).

\author{
References \\ [1] Hull, John C., Options, Futures, and Other Derivatives, Prentice Hall, 5th edition, 2003, pp. 287-288, 508-515.
}

\section*{See Also \\ bkput}

\section*{bkcaplet}

Purpose Price interest-rate caplet using Black model
```

Syntax CapPrices = bkcaplet(CapData, FwdRates, ZeroPrice, Settle,
StartDate, EndDate, Sigma)

```

\section*{Arguments}

CapData Number of caps (NCAP)-by-2 matrix containing cap rates and bases: [CapRates Basis].
Values for bases may be:
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- 3 = actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- \(10=\mathrm{actual} / 365\) (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- \(12=\) actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
FwdRates Scalar or NCAP-by-1 vector containing forward rates in decimal. FwdRates accrue on the same basis as CapRates.

ZeroPrice Scalar or NCAP-by-1 vector containing zero coupon prices with maturities corresponding to those of each cap in CapData, per \(\$ 100\) nominal value.
Settle Scalar or NCAP-by-1 vector of identical elements containing settlement date of caplets.

StartDate Scalar or NCAP-by-1 vector containing start dates of the caplets.

EndDate \(\quad\) Scalar or NCAP-by-1 vector containing maturity dates of caplets.

Sigma Scalar or NCAP-by-1 vector containing volatility of forward rates in decimal, corresponding to each caplet.

\section*{Description CapPrices = bkcaplet(CapData, FwdRates, ZeroPrice, Settle, StartDate, EndDate, Sigma) computes the prices of interest-rate caplets for every \(\$ 100\) face value of principal.}

\section*{Examples Compute the Price of Interest-Rate Caplets for Every \(\mathbf{\$ 1 0 0}\) Face Value of Principal}

This example shows how to compute the price of interest-rate caplets for every \(\$ 100\) face value of principal. Given a notional amount of \(\$ 1,000,000\), compute the value of a caplet on October 15, 2002 that starts on October 15, 2003 and ends on January 15, 2004.
```

CapData = [0.08, 1];
FwdRates = 0.07;
ZeroPrice = 100*exp(-0.065*1.25);
Settle = datenum('15-0ct-2002');
BeginDates = datenum('15-Oct-2003');
EndDates = datenum('15-Jan-2004');
Sigma = 0.20;
% because the caplet is \$100 notional, divide \$1,000,000 by \$100
Notional = 1000000/100;

```

\section*{bkcaplet}
```

CapPrice = Notional*bkcaplet(CapData, FwdRates, ZeroPrice, ...
Settle, BeginDates, EndDates, Sigma)
CapPrice =
519.0046

```

\section*{See Also}
bkfloorlet

\section*{Purpose}

Price interest-rate floorlet using Black model
Syntax FloorPrices = bkfloorlet(FloorData, FwdRates, ZeroPrice, Settle, StartDate, EndDate, Sigma)

\section*{Arguments}

FloorData Number of floors (NFLR)-by-2 matrix containing floor rates and bases: [FloorRate Basis].
Values for bases may be:
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- \(3=\) actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- \(10=\mathrm{actual} / 365\) (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
FwdRates Scalar or NFLR-by-1 vector containing forward rates in decimal. FwdRates accrue on the same basis as FloorRates.

\section*{bkfloorlet}
\begin{tabular}{|c|c|c|}
\hline & ZeroPrice & Scalar or NFLR-by-1 vector containing zero coupon prices with maturities corresponding to those of each floor in FloorData, per \(\$ 100\) nominal value. \\
\hline & Settle & Scalar or NFLR-by-1 vector of identical elements containing settlement date of floorlets. \\
\hline & StartDate & Scalar or NFLR-by-1 vector containing start dates of the floorlets. \\
\hline & EndDate & Scalar or NFLR-by-1 vector containing maturity dates of floorlets. \\
\hline & Sigma & Scalar or NFLR-by-1 vector containing volatility of forward rates in decimal, corresponding to each floorlet. \\
\hline Description & FloorPrice Settle, St interest-rate & = bkfloorlet(FloorData, FwdRates, ZeroPrice, tDate, EndDate, Sigma) computes the prices of floorlets for every \(\$ 100\) of notional value. \\
\hline Examples & \begin{tabular}{l}
Price an In \\
Value Usin
\end{tabular} & erest-Rate Floorlet For Every \$100 of Notional the Black Model \\
\hline & This exampl of notional \(\$ 1,000,000\), starts on Oc & shows how to price an interest-rate floorlet for every \(\$ 100\) lue using the Black model. Given a notional amount of mpute the value of a floorlet on October 15, 2002 that ber 15, 2003 and ends on January 15, 2004. \\
\hline & FloorData \(=\) & [0.08, 1]; \\
\hline & FwdRates = & .07; \\
\hline & ZeroPrice = & 100*exp(-0.065*1.25) ; \\
\hline & Settle = da & enum ('15-Oct-2002'); \\
\hline & BeginDates & = datenum('15-0ct-2003'); \\
\hline & EndDates = & datenum('15-Jan-2004'); \\
\hline & Sigma \(=0.2\) & \\
\hline & \begin{tabular}{l}
\% because \\
Notional =
\end{tabular} & loorlet is \(\$ 100\) notional, divide \(\$ 1,000,000\) by \(\$ 100\) 1000000/100; \\
\hline
\end{tabular}

\section*{bkfloorlet}
```

FloorPrice = Notional*bkfloorlet(FloorData, FwdRates, ...
ZeroPrice, Settle, BeginDates, EndDates, Sigma)
FloorPrice =
2.8239e+03

```

\section*{See Also \\ bkcaplet}

\section*{bkput}

Purpose Price European put option on bonds using Black model

\author{
Syntax PutPrice = bkput(Strike, ZeroData, Sigma, BondData, Settle, Expiry, Period, Basis, EndMonthRule, InterpMethod, StrikeConvention)
}

Arguments
Strike

ZeroData

Sigma

Scalar or number of options (NOPT)-by-1 vector of strike prices.

Two-column (optionally three-column) matrix containing zero (spot) rate information used to discount future cash flows.
- Column 1: Serial maturity date associated with the zero rate in the second column.
- Column 2: Annualized zero rates, in decimal form, appropriate for discounting cash flows occurring on the date specified in the first column. All dates must occur after Settle (dates must correspond to future investment horizons) and must be in ascending order.
- Column 3 (optional): Annual compounding frequency. Values are 1 (annual), 2 (semiannual, default), 3 (three times per year), 4 (quarterly), 6 (bimonthly), 12 (monthly), and -1 (continuous).

Scalar or NOPT-by-1 vector of annualized price volatilities required by Black's model.
\(\left.\begin{array}{ll}\text { BondData } & \begin{array}{l}\text { Row vector with three (optionally four) columns } \\ \text { or NOPT-by-3 (optionally NOPT-by-4) matrix } \\ \text { specifying characteristics of underlying bonds in } \\ \text { the form [CleanPrice CouponRate Maturity }\end{array} \\ \text { Face] where: } \\ \text { - CleanPrice is the price excluding accrued } \\ \text { interest. }\end{array}\right\}\) - CouponRate is the decimal coupon rate.

\section*{bkput}

Basis (Optional) Day-count basis of the bond. A vector of integers.
- \(0=\) actual/actual (default)
- 1 = 30/360 (SIA)
- 2 = actual/360
- \(3=\) actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
EndMonthRule
(Optional) End-of-month rule. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. \(0=\) ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
\begin{tabular}{ll} 
InterpMethod & \begin{tabular}{l} 
(Optional) Scalar integer zero curve \\
interpolation method. For cash flows that do \\
not fall on a date found in the ZeroData spot \\
curve, indicates the method used to interpolate \\
the appropriate zero discount rate. Available \\
methods are (0) nearest, (1) linear, and (2) cubic. \\
Default = 1. See interp1 for more information.
\end{tabular} \\
StrikeConvention & \begin{tabular}{l} 
(Optional) Scalar or NOPT-by-1 vector of option \\
contract strike price conventions.
\end{tabular} \\
& \begin{tabular}{l} 
StrikeConvention = 0 (default) defines the \\
strike price as the cash (dirty) price paid for the
\end{tabular} \\
underlying bond.
\end{tabular}\(\quad\)\begin{tabular}{l} 
StrikeConvention = 1 defines the strike \\
price as the quoted (clean) price paid for the \\
underlying bond. The accrued interest of the \\
bond at option expiration is added to the input \\
strike price when evaluating Black's model.
\end{tabular}

\section*{Description}

PutPrice = bkput(Strike, ZeroData, Sigma, BondData, Settle, Expiry, Period, Basis, EndMonthRule, InterpMethod, StrikeConvention) using Black's model, derives an NOPT-by-1 vector of prices of European put options on bonds.

If cash flows occur beyond the dates spanned by ZeroData, the input zero curve, the appropriate zero rate for discounting such cash flows is obtained by extrapolating the nearest rate on the curve (that is, if a cash flow occurs before the first or after the last date on the input zero curve, a flat curve is assumed).

In addition, you can use the Financial Instruments Toolbox method getZeroRates for an IRDataCurve object with a Dates property to create a vector of dates and data acceptable for bkput. For more information, see "Converting an IRDataCurve or IRFunctionCurve Object" on page 9-40.

\section*{Examples Price European Put Options On Bonds Using the Black Model}

This example shows how to price European put options on bonds using the Black model. Consider a European put option on a bond maturing in 10 years. The underlying bond has a clean price of \(\$ 122.82\), a face value of \(\$ 100\), and pays \(8 \%\) semiannual coupons. Also, assume that the annualized volatility of the forward bond yield is \(20 \%\). Furthermore, suppose the option expires in 2.25 years and has a strike price of \(\$ 115\), and that the annualized continuously compounded risk free zero (spot) curve is flat at \(5 \%\). For a hypothetical settlement date of March 15, 2004, the following code illustrates the use of Black's model to duplicate the put prices in Example 22.2 of the Hull reference. In particular, it illustrates how to convert a broker's yield volatility to a price volatility suitable for Black's model.
```

% Specify the option information.
Settle = '15-Mar-2004';
Expiry = '15-Jun-2006'; % 2.25 years from settlement
Strike = 115;
YieldSigma = 0.2;
Convention = [0; 1];
% Specify the interest-rate environment. Since the
% zero curve is flat, interpolation into the curve always returns
% 0.05. Thus, the following curve is not unique to the solution.
ZeroData = [datenum('15-Jun-2004') 0.05 -1;
datenum('15-Dec-2004') 0.05 -1;
datenum(Expiry) 0.05 -1];
% Specify the bond information.
CleanPrice = 122.82;
CouponRate = 0.08;
Maturity = '15-Mar-2014'; % 10 years from settlement
Face = 100;
BondData = [CleanPrice CouponRate datenum(Maturity) Face];
Period = 2; % semiannual coupons
Basis = 1; % 30/360 day-count basis

```
\% Convert a broker's yield volatility quote to a price volatility \% required by Black's model. To duplicate Example 22.2 in Hull,
\% first compute the periodic (semiannual) yield to maturity from \% the clean bond price.
Yield = bndyield(CleanPrice, CouponRate, Settle, Maturity,... Period, Basis);
\% Compute the duration of the bond at option expiration. Most
\% fixed-income sensitivity analyses use the modified duration
\% statistic to examine the impact of small changes in periodic
\% yields on bond prices. However, Hull's example operates in
\% continuous time (annualized instantaneous volatilities and
\% continuously compounded zero yields for discounting coupons).
\% To duplicate Hull's results, use the second output of BNDDURY,
\% the Macaulay duration.
[Modified, Macaulay] = bnddury(Yield, CouponRate, Expiry,...
Maturity, Period, Basis);
\% Convert the yield-to-maturity from a periodic to a
\% continuous yield.
Yield = Period .* log(1 + Yield./Period);
\% Convert the yield volatility to a price volatility via
\% Hull's Equation 22.6 (page 514).
PriceSigma = Macaulay .* Yield .* YieldSigma;
\% Finally, call Black's model.
PutPrices = bkput(Strike, ZeroData, PriceSigma, BondData,... Settle, Expiry, Period, Basis, [], [], Convention)

PutPrices =
1.7838
2.4071

\section*{bkput}

When the strike price is the dirty price (Convention \(=0\) ), the call option value is \(\$ 1.78\). When the strike price is the clean price (Convention \(=\) 1 ), the call option value is \(\$ 2.41\).

\author{
References [1] Hull, John C., Options, Futures, and Other Derivatives, Prentice Hall, 5th edition, 2003, pp. 287-288, 508-515.
}

\section*{See Also \\ bkcall}

Purpose
Implied repo rates for bond future given price

\author{
Syntax \\ \section*{Description}
}

ImpRepo = bndfutimprepo(Price, FutPrice, FutSettle, Delivery, ConvFactor, CouponRate, Maturity) ImpRepo = bndfutimprepo(Price, FutPrice, FutSettle, Delivery, ConvFactor, CouponRate, Maturity, 'ParameterName', 'ParameterValue ...)

\section*{Input Arguments}

ImpRepo = bndfutimprepo(Price, FutPrice, FutSettle, Delivery, ConvFactor, CouponRate, Maturity) computes the implied repo rate for a bond future given the price of a bond, the bond properties, the price of the bond future, and the bond conversion factor.

\begin{abstract}
ImpRepo = bndfutimprepo(Price, FutPrice, FutSettle, Delivery, ConvFactor, CouponRate, Maturity, 'ParameterName', 'ParameterValue ...) accepts optional inputs as one or more comma-separated parameter/value pairs. 'ParameterName' is the name of the parameter inside single quotes. Parametervalue is the value corresponding to 'ParameterName'. Specify parameter-value pairs in any order. Names are case-insensitive.
\end{abstract}

\section*{Price}
numBonds-by-1 vector of bond prices.

\section*{FutPrice}
numBonds-by- 1 vector of future prices

\section*{FutSettle}
numBonds-by-1 vector of future settle dates.

\section*{Delivery}
numBonds-by-1 vector of future delivery dates.

\section*{bndfutimprepo}

\section*{ConvFactor}
numBonds-by-1 vector of bond conversion factors. For more information, see convfactor.

\section*{CouponRate}
numBonds-by-1 vector of coupon rates in decimal form.

\section*{Maturity}
numBonds-by-1 vector of coupon rates in decimal form.

\section*{Parameter-Value Pairs}

\section*{Basis}

Day-count basis. Possible values include
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- 3 = actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- \(12=\) actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0

\section*{EndMonthRule}

End-of-month rule. Values are:
- 0 - Rule is not in effect for the bond.
- 1 - Rule is in effect for the bond. This means that a security that pays coupon interest on the last day of the month always makes payment on the last day of the month.

Default: 1

\section*{Face}

Face value of the bond. Face has no impact on key rate duration. This calling sequence is preserved for consistency.

Default: 100

\section*{FirstCouponDate}

Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.

Default: If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

\section*{IssueDate}

Issue date for a bond.

\section*{LastCouponDate}

\section*{bndfutimprepo}

Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date.

Default: If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

\section*{Period}

Number of coupons payments per year. Possible values include:
- 0
- 1
- 2
- 3
- 4
- 6
- 12

\section*{Default: 2}

\section*{ReinvestBasis}

Day count basis for resinvestment rate.
Default: Identical to RepoBasis.

\section*{ReinvestRate}

Rate for reinvesting intermediate coupons from the bond.
Default: Identical to ImpRepo.

\section*{RepoBasis}

Day count basis for ImpRepo.
Default: 2

\section*{StartDate}

Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify StartDate, the effective start date is the Settle date.

\section*{Output \\ Arguments}

\section*{Definitions}

\section*{ImpRepo}

Implied repo rate, or the repo rate that would produce the price input.
bndfutimprepo computes the implied repo rate for a bond future given:
- Price of a bond
- Bond properties
- Price of the bond future
- Bond conversion factor

The default behavior is that the coupon reinvestment rate matches the repo rate. However, you can specify a separate reinvestment rate using optional inputs.

\section*{Examples Compute the Repro Rate For a Bond Future}

This example shows how to compute the repro rate for a bond future using the following data.
bndfutimprepo(129,98, '9/21/2000', '12/29/2000' , 1.3136, . 0875, ' \(8 / 15 / 2020\)
ans \(=\)

\section*{bndfutimprepo}

\subsection*{0.0584}

\author{
References Burghardt, G., T. Belton, M. Lane, and J. Papa, The Treasury Bond Basis, McGraw-Hill, 2005. \\ Krgin, Dragomir, Handbook of Global Fixed Income Calculations, John Wiley \& Sons, 2002. \\ See Also bndfutprice | convfactor \\ How To - "Bond Futures" on page 7-12
}

Purpose
Syntax
Price bond future given repo rates
[FutPrice,AccrInt] = bndfutprice(RepoRate, Price, FutSettle, Delivery, ConvFactor, CouponRate, Maturity) FutPrice,AccrInt] = bndfutprice(RepoRate, FutPrice, FutSettle, Delivery, ConvFactor, CouponRate, Maturity, 'ParameterName','ParameterValue ...)

\section*{Description}

\section*{Input \\ Arguments}
[FutPrice,AccrInt] = bndfutprice(RepoRate, Price, FutSettle, Delivery, ConvFactor, CouponRate, Maturity) computes the price of a bond futures contract for one or more bonds given a repo rate, and bond properties, including the bond conversion factor.
FutPrice,AccrInt] = bndfutprice(RepoRate, FutPrice, FutSettle, Delivery, ConvFactor, CouponRate, Maturity, 'ParameterName', 'ParameterValue ...) accepts optional inputs as one or more comma-separated parameter/value pairs. 'ParameterName' is the name of the parameter inside single quotes. Parametervalue is the value corresponding to 'ParameterName'. Specify parameter-value pairs in any order. Names are case-insensitive.

\section*{RepoRate}
numBonds-by-1 vector of repo rates.

\section*{Price}
numBonds-by-1 vector of bond prices

\section*{FutSettle}
numbonds-by-1 vector of future settle dates.

\section*{Delivery}

\section*{bndfutprice}
numBonds-by- 1 vector of future delivery dates.

\section*{ConvFactor}
numBonds-by-1 vector of bond conversion factors. For more information, see convfactor.

\section*{CouponRate}
numBonds-by-1 vector of coupon rates in decimal form.

\section*{Maturity}
numBonds-by-1 vector of coupon rates in decimal form.

\section*{Parameter-Value Pairs}

\section*{Basis}

Day-count basis. Possible values include
- 0 = actual/actual (default)
- 1 = 30/360 (SIA)
- 2 = actual \(/ 360\)
- \(3=\) actual \(/ 365\)
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- \(10=\) actual \(/ 365\) (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0

\section*{EndMonthRule}

End-of-month rule. Values are:
- 0 - Rule is not in effect for the bond.
- 1 - Rule is in effect for the bond. This means that a security that pays coupon interest on the last day of the month always makes payment on the last day of the month.

Default: 1

\section*{IssueDate}

Issue date for a bond.

\section*{Face}

Face value of the bond. Face has no impact on key rate duration. This calling sequence is preserved for consistency.

Default: 100

\section*{FirstCouponDate}

Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.

Default: If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

\section*{LastCouponDate}

\section*{bndfutprice}

Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date.

Default: If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

\section*{Period}

Number of coupons payments per year. Possible values include:
- 0
- 1
- 2
- 3
- 4
- 6
- 12

Default: 2

\section*{ReinvestBasis}

Day count basis for reinvestment rate.
Default: Identical to RepoBasis.

\section*{ReinvestRate}

Compounding convention for reinvestment rate.
Default: Identical to RepoRate.

\section*{RepoBasis}

Day count basis for RepoRate.
Default: 2

\section*{StartDate}

Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify StartDate, the effective start date is the Settle date.

\section*{Output \\ Arguments}

Definitions

\section*{Examples}

\section*{FutPrice}

Quoted futures price, per \(\$ 100\) notional.

\section*{Accrint}

Accrued interest due at delivery date, per \(\$ 100\) notional.
bndfutprice computes the price of a bond futures contract for one or more bonds, given a repo rate, and bond properties, including the bond conversion factor. The default behavior is that the coupon reinvestment rate matches the repo rate. However, you can specify a separate reinvestment rate using optional inputs.

\section*{Compute the Price For a Bond Future}

This example shows how to compute the price for a bond future using the following data.
```

bndfutprice(.064, 129, '9/21/2000','12/29/2000', 1.3136, .0875, '8/15

```

\section*{bndfutprice}
References Burghardt, G., T. Belton, M. Lane, and J. Papa, The Treasury Bond Basis, McGraw-Hill, 2005.
Krgin, Dragomir, Handbook of Global Fixed Income Calculations, John Wiley \& Sons, 2002.
See Also bndfutimprepo | convfactor
How To - "Bond Futures" on page 7-12


\section*{bootstrap}

Note When using fra, the first column of N -by- 3 data matrix for Instruments is the start dates for the forward rate agreement (FRA).

\author{
Compounding
}
(Optional) Scalar that sets the compounding frequency per year for an IRDataCurve object:
- \(-1=\) Continuous compounding
- 1 = Annual compounding
- \(2=\) Semiannual compounding (default)
- \(3=\) Compounding three times per year
- \(4=\) Quarterly compounding
- \(6=\) Bimonthly compounding
- \(12=\) Monthly compounding

Note Simple interest can be specified for an instrument by specifying the Compounding value as 0 .
\begin{tabular}{|c|c|}
\hline Basis & (Optional) Day-count basis of the \\
\hline & - \(0=\) actual/actual (default) \\
\hline & - \(1=30 / 360\) (SIA) \\
\hline & - \(2=\) actual/360 \\
\hline & - 3 = actual/365 \\
\hline & - \(4=30 / 360\) (BMA) \\
\hline & - \(5=30 / 360\) (ISDA) \\
\hline & - \(6=30 / 360\) (European) \\
\hline & - \(7=\) actual/365 (Japanese) \\
\hline & - \(8=\) actual/actual (ICMA) \\
\hline & - \(9=\) actual/360 (ICMA) \\
\hline & - \(10=\mathrm{actual} / 365\) (ICMA) \\
\hline & - \(11=30 / 360 \mathrm{E}\) (ICMA) \\
\hline & - \(12=\) actual/actual (ISDA) \\
\hline & - 13 = BUS/252 \\
\hline & For more information, see basis on page Glossary-1. \\
\hline InterpMethod & (Optional) Values are: \\
\hline & - 'linear' - Linear interpolation (default). \\
\hline & - 'constant' - Piecewise constant interpolation. \\
\hline & - 'pchip' - Piecewise cubic Hermite interpolation. \\
\hline & - 'spline' - Cubic spline interpolation \\
\hline
\end{tabular}

\section*{bootstrap}
\begin{tabular}{ll} 
IRBootstrapOptionsObj & \begin{tabular}{l} 
(Optional) An IRBootstrapOptions \\
object.
\end{tabular} \\
DiscountCurve & \begin{tabular}{l} 
(Optional) RateSpec for a curve used to \\
discount the cash flows.
\end{tabular}
\end{tabular}

\section*{Instrument Parameters}

For each bond Instrument, you can specify the following additional instrument parameters as parameter/value pairs. For example, InstrumentBasis distinguishes a bond instrument's Basis value from the curve's Basis value. For instruments of type deposit, futures, or swap the Basis and Compounding values must be identical for each instance of the instrument.

InstrumentCouponRat(Optional) Decimal number indicating the annual percentage rate used to determine the coupons payable on an instrument.

InstrumentPeriod (Optional) Coupons per year of the instrument. A vector of integers. Allowed values are 0, 1, 2 (default), 3, 4, 6, and 12.

InstrumentBasis (Optional) Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- \(3=\) actual \(/ 365\)
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- \(10=\) actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- \(12=\) actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
InstrumentEndMonthR(CHyetional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. \(0=\) ignore rule, meaning that an instrument's coupon payment date is always the same numerical day of the month. \(1=\) set rule on (default), meaning that an instrument's coupon payment date is always the last actual day of the month.

InstrumentIssueData(Optional) Date when an instrument was issued.

InstrumentFirstCoup(ondàdeal) Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

InstrumentLastCoupdortenal) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.
InstrumentFace (Optional) Face or par value. Default \(=100\).

Note When using Instrument parameter/value pairs, you can specify simple interest for an Instrument by specifying the InstrumentPeriod value as 0 . If InstrumentBasis and InstrumentPeriod are not specified for an Instrument, the following default values are used:
- deposit instrument uses Basis as 2 (act/360) and Compounding is 0 (simple interest).
- futures instrument uses Basis as 2 (act/360) and Compounding is 4 (quarterly).
- swap instrument uses Basis as 2 (act/360) and Compounding is 2.
- bond instrument uses Basis as 0 (act/act) and Compounding is 2.
- FRA instrument uses Basis as 2 (act/360) and Compounding is 4 (quarterly).

\section*{Description Dcurve = IRDataCurve.bootstrap(Type, Settle,} InstrumentTypes, Instruments, 'Parameter1', Value1, 'Parameter2', Value2, ...) bootstraps an interest-rate curve from market data. The dates of the bootstrapped curve correspond to the maturity dates of the input instruments. You must enter
the optional arguments for Basis, Compounding, Interpmethod, IRBootstrapOptionsObj, and DiscountCurve as parameter/value pairs.

\section*{Examples}

\section*{Use the bootstrap Method to Create an IRDataCurve Object}

In this bootstrapping example, InstrumentTypes, Instruments, and a Settle date are defined:
```

InstrumentTypes = {'Deposit';'Deposit';...
'Futures';'Futures';'Futures';'Futures';'Futures';'Futures';...
'Swap';'Swap';'Swap';'Swap';};

```
Instruments = [datenum('08/10/2007'), datenum('09/17/2007'),.0532000;
datenum('08/10/2007'), datenum('11/17/2007'),.0535866; ...
datenum('08/08/2007'), datenum('19-Dec-2007'),9485; ...
datenum('08/08/2007'), datenum('19-Mar-2008'), 9502; ...
datenum('08/08/2007'), datenum('18-Jun-2008'), 9509.5; ...
datenum('08/08/2007'), datenum('17-Sep-2008'), 9509; ...
datenum('08/08/2007'), datenum('17-Dec-2008'), 9505.5; ...
datenum('08/08/2007'), datenum('18-Mar-2009'), 9501; ...
datenum('08/08/2007'), datenum('08/08/2014'),.0530; ...
datenum('08/08/2007'), datenum('08/08/2019'),.0551; ...
datenum('08/08/2007'), datenum('08/08/2027'),.0565; ...
datenum('08/08/2007'), datenum('08/08/2037'),.0566];
CurveSettle = datenum('08/10/2007');

Use the bootstrap method to create an IRDataCurve object.
bootModel = IRDataCurve.bootstrap('Forward', CurveSettle, ... InstrumentTypes, Instruments,'InterpMethod','pchip');

To create the plot for the bootstrapped market data:
```

PlottingDates = (datenum('08/11/2007'):30:CurveSettle+365*25)';
plot(PlottingDates,bootModel.getParYields(PlottingDates),'r')
set(gca,'ylim',[0 .06])

```

\section*{bootstrap}
datetick


\section*{Use the bootstrap Method to Create an IRDataCurve Object That Includes Bonds}

In this bootstrapping example, InstrumentTypes, Instruments, and a Settle date are defined:

CurveSettle = datenum('8-Mar-2010');
```

InstrumentTypes = \{'Deposit';'Deposit';'Deposit';'Deposit';...
'Futures'; 'Futures'; 'Futures';'Futures';'Swap';'Swap';'Bond' ;'Bonc
Instruments $=$ [datenum('8-Mar-2010'), datenum('8-Apr-2010'),.003; ...
datenum ('8-Mar-2010'), datenum ('8-Jun-2010'), .005; ...
datenum ('8-Mar-2010'), datenum ('8-Sep-2010'), 007; ...
datenum ('8-Mar-2010'), datenum('8-Mar-2011'), .009; ...
datenum ('8-Mar-2010'), datenum('18-Jun-2011'), 9840; ...
datenum ('8-Mar-2010'), datenum('17-Sep-2011'), 9820; ...
datenum ('8-Mar-2010'), datenum('17-Dec-2011'), 9810; ...
datenum ('8-Mar-2010'), datenum('18-Mar-2012'), 9800; ...
datenum ('8-Mar-2010'), datenum('8-Mar-2015'), 025; ...
datenum ('8-Mar-2010'), datenum('8-Mar-2020'),.035; ...
datenum ('8-Mar-2010') , datenum ('8-Mar-2030'), 99; ...
datenum('8-Mar-2010'), datenum('8-Mar-2040'), 101];

```

When bonds are used, InstrumentCouponRate must be specified:
```

InstrumentCouponRate = [zeros(10,1);.045;.05];

```

Note, for parameters that are only applicable to bonds (e.g.: InstumentFirstCouponDate, InstrumentLastCouponDate, InstumentIssueDate, InstumentFace) the entries for non-bond instruments (deposits and futures) are ignored.
Use the bootstrap method to create an IRDataCurve object.
bootModel = IRDataCurve.bootstrap('Forward', CurveSettle, ... InstrumentTypes, Instruments,'InterpMethod','pchip',... 'InstrumentCouponRate', InstrumentCouponRate);

To create the plot for the bootstrapped market data:
```

PlottingDates = daysadd(CurveSettle,30:30:30*360,1);
plot(PlottingDates,bootModel.getParYields(PlottingDates),'r')
set(gca,'ylim',[0 .06])
datetick

```

\section*{bootstrap}

\begin{tabular}{ll} 
See Also & \begin{tabular}{l} 
"@IRDataCurve" on page A-7 | "@IRBootstrapOptions" on page A-2 \\
toRateSpec
\end{tabular} \\
Related & \begin{tabular}{l} 
- "IRDataCurve Bootstrapping Based on Market Instruments" on \\
Examples
\end{tabular} \\
& page 9-7 \\
& - "Dual Curve Bootstrapping" on page 9-16
\end{tabular}

\section*{Purpose}

Price convertible bond
[CbMatrix, UndMatrix, DebtMatrix, EqtyMatrix] = cbprice(RiskFreeRate, StaticSpread, Sigma, Price, ConvRatio, NumSteps, IssueDate, Settle, Maturity, CouponRate)
[CbMatrix, UndMatrix, DebtMatrix, EqtyMatrix] = cbprice(RiskFreeRate, StaticSpread, Sigma, Price, ConvRatio, NumSteps, IssueDate, Settle, Maturity, CouponRate, Name, Value)

\section*{Description}

Input
Arguments
[CbMatrix, UndMatrix, DebtMatrix, EqtyMatrix] = cbprice(RiskFreeRate, StaticSpread, Sigma, Price, ConvRatio, NumSteps, IssueDate, Settle, Maturity, CouponRate) price a convertible bond with a one-factor lattice method.
[CbMatrix, UndMatrix, DebtMatrix, EqtyMatrix] = cbprice(RiskFreeRate, StaticSpread, Sigma, Price, ConvRatio, NumSteps, IssueDate, Settle, Maturity, CouponRate, Name, Value) price a convertible bond with a one-factor lattice method with additional options specified by one or more Name, Value pair arguments.

\section*{RiskFreeRate}

Annual yield of the risk-free bond with the same maturity as the convertible, compounded continuously. Scalar value of risk-free rates is in decimal. (Recommended value is the yield of a risk-free bond with the same maturity as the convertible.)

\section*{StaticSpread}

Scalar value of the constant spread to risk-free rate. Adding StaticSpread to the RiskFreeRate produces the issuer's yield, which reflects the credit risk.

\section*{Sigma}

\section*{cbprice}

Scalar value of the annual volatility environment in decimal.

\section*{Price}

Scalar value of the price of the asset at the settlement or valuation date.

\section*{ConvRatio}

Scalar value of the number of assets convertible to one bond. cbprice expects the ConvRatio to be in terms of a \(\$ 100\) face or par value.

\section*{NumSteps}

Scalar value of the number of steps within the binomial tree.

\section*{IssueDate}

Scalar value of the issue date of the convertible bond.

\section*{Settle}

Scalar value of the settlement date of the convertible bond.

\section*{Maturity}

Scalar value of the maturity date of the convertible bond.

\section*{CouponRate}

Scalar value of the coupon rate in decimal form or a C-by-2 vector of dates and associated coupon rates.

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

\section*{'Basis'}

Day-count basis of the bond. A vector of integers.
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- \(3=\) actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0 (actual/actual)

\section*{'Calliype'}

Scalar value for the call type. Values are 0 for a call on cash price, or 1 for a call on clean price.

Default: 0 (cash price)

\section*{'Callinfo'}

Two-column matrix containing the call information. The first column is the call dates and the second column is the call prices for every \(\$ 100\)

\section*{cbprice}
face of the bond. The call, in the amount of call prices, is activated after the corresponding call date.

Default: No call feature

\section*{'Convinfo'}

Two-column matrix containing convertible information. The first column is the convertible dates and the second column is whether the issue is convertible or not.

Default: Bond is always convertible

\section*{'DividendInfo'}

Two-column matrix of dividend information. The first column is the ex-dividend date and the second column is the corresponding amount. Enter any amount known at any time; only the amounts that are within the lifespan of the option are used. If the DividendType is 2, DividendInfo is a 1-by-2 matrix where the first entry is the Settle date and the second entry is the continuous dividend yield.

Default: No dividend

\section*{'DividendType'}

Scalar value for dividend type. Values are:
- 0 - Dollar dividend
- 1 - Dividend yield
- 2 - Continuous dividend yield

Default: 0 (Dollar dividend)

\section*{'EndMonthRule'}

NINST-by-1 vector for end-of-month rule. Values are 1 (on, in effect) and 0 (off, not in effect).

Default: 1 (on, in effect)

\section*{'Period'}

Scalar value for number of coupon payments. Values are:
- 1 - One coupon per year
- 2 -Semiannual
- 3 - Three times a year
- 4 - Quarterly
- 6 - Bimonthly compounding
- 12 - Monthly

Default: 2 (Semiannual)

\section*{'Putlnfo'}

Two-column matrix containing put information. The first column is the put dates and the second column is the put prices for every \(\$ 100\) face of the bond. The put, in the amount of put prices, is activated after the corresponding put date.

Default: No put feature

\section*{'Pułtype'}

Scalar value for put type. Value are 0 for a put on cash price or 1 for a put on clean price.

Default: 0 (put on cash price)

\section*{'TreeType'}

Scalar value for tree type. Values are 0 for binomial lattice or 1 for trinomial lattice.

Default: 0 (binomial lattice)

\section*{cbprice}

\section*{Output \\ Arguments}

\section*{Definitions}

\section*{Examples}

\section*{Price a Convertible Bond}

This example shows how to price a convertible bond that matures in 2035 and is callable after 2025.

Define the convertible bond characteristics.
```

Settle = datenum('20-Sep-2013');
IssueDate = datenum('15-Dec-2005');
Maturity = datenum('15-Dec-2035');
CouponRate = .03;
ConvRatio = 3.4; % Per 100 face value
CallDate = datenum('15-Dec-2025');
CallPrice = 100;
RiskFreeRate = 0.03;

```
```

StockPrice = 24;
Volatility = 0.2;
CreditSpread = 0.005;
DivType = 2; % Continous
DivYield = 0.04;

```

Calculate the price for the convertible bond.
```

NumberSteps = 100;

```
ConvPrice = cbprice(RiskFreeRate,CreditSpread,Volatility,StockPrice,C
NumberSteps, IssueDate, Settle, Maturity, CouponRate, 'DividendType', DivTyp
'DividendInfo',[Settle DivYield],'CallInfo',[CallDate CallPrice]);

\section*{Price a Convertible Bond With a One-Factor Lattice Method}

This example shows how to perform a spread effect analysis of a 4\% coupon convertible bond callable at 110 at the end of the second year, maturing at par in 5 years, with yield to maturity of \(5 \%\), and spread (of yield to maturity versus 5 -year treasury) of \(0,50,100\), and 150 basis points. The underlying stock pays no dividend.
```

RiskFreeRate = 0.05;
Sigma = 0.3;
ConvRatio = 1;
NumSteps = 200;
IssueDate = '2-Jan-2002';
Settle = '2-Jan-2002';
Maturity = '2-Jan-2007';
CouponRate = 0.04;
Period = 2;
Basis = 1;
EndMonthRule = 1;
DividendType = 0;
DividendInfo = [];
CallInfo = [datenum('2-Jan-2004') , 110];
CallType = 1;

```

\section*{cbprice}
```

TreeType = 1;
Spreads = 0:0.005:0.015;
Prices = 40:10:140;
stock = repmat(Prices',1,length(Spreads));
convprice = zeros(length(Prices),length(Spreads));
for spreadidx = 1:length(Spreads)
for priceidx = 1:length(Prices)
[CbMatrix, UndMatrix, DebtMatrix, EqtyMatrix] = ...
cbprice(RiskFreeRate, Spreads(spreadidx), Sigma, Prices(pricei
ConvRatio, NumSteps, IssueDate, Settle,
Maturity, CouponRate, Period, Basis, EndMonthRule, ...
DividendType, DividendInfo, CallType, CallInfo, TreeType);
convprice(priceidx,spreadidx) = CbMatrix(1,1);
end
end
plot(stock,convprice);
legend({'+0 bp'; '+50 bp'; '+100 bp'; '+150 bp'});
title ('Effect of Spread using Trinomial tree - 200 steps')
xlabel('Stock Price');
ylabel('Convertible Price');
text(50, 150, ['Coupon 4 semiannual,', sprintf('\n'), ...
'110 Call-on-clean after 2 years,' sprintf('\n'), ...
'maturing par in 5 years'],'fontweight','Bold')

```


References Andersen, L. and D. Buffum, "Calibration and implementation of convertible bonds models," Working paper, Banc of America Securities, 2003.

Ayache, E., P.A. Forsyth, and K.R. Vetzal, "Valuation of Convertible Bonds with Credit Risk," Journal of Derivatives, 11 (Fall 2003), 9-29.

Tsiveriotis, K. and C. Fernandes, "Valuing Convertible Bonds with Credit Risk," Journal of Fixed Income 8, 95-102, 1998

Zabolotnyuk, Yuriy, Jones, Robert A. and Veld, Chris H., "An Empirical Comparison of Convertible Bond Valuation Models," (October 15, 2009). Available at SSRN: http://ssrn.com/abstract=994805.

\section*{Tutorials}
- "Convertible Bond Valuation" on page 7-10

\section*{Purpose}

\author{
Syntax \\ Description
}

\section*{Input \\ Arguments}

Bootstrap default probability curve from credit default swap market quotes
```

[ProbData, HazData] = cdsbootstrap(ZeroData, MarketData,
Settle)
[ProbData, HazData] = cdsbootstrap(ZeroData, MarketData,
Settle, Name,Value)

```
[ProbData, HazData] = cdsbootstrap(ZeroData, MarketData, Settle) bootstraps the default probability curve using credit default swap (CDS) market quotes. The market quotes can be expressed as a list of maturity dates and corresponding CDS market spreads, or as a list of maturities and corresponding upfronts and standard spreads for standard CDS contracts. The estimation uses the standard model of the survival probability.
[ProbData, HazData] = cdsbootstrap(ZeroData, MarketData, Settle, Name, Value) bootstraps the default probability curve using CDS market quotes with additional options specified by one or more Name, Value pair arguments. The market quotes can be expressed as a list of maturity dates and corresponding CDS market spreads, or as a list of maturities and corresponding upfronts and standard spreads for standard CDS contracts. The estimation uses the standard model of the survival probability.

\section*{ZeroData}

M-by-2 vector of dates and zero rates or an IRDataCurve object of zero rates.

\section*{MarketData}

N -by- 2 matrix of dates and corresponding market spreads or N -by- 3 matrix of dates, upfronts, and standard spreads of CDS contracts.

\section*{Settle}

Settlement date is a serial date number or date string. This must be earlier than or equal to the dates in MarketData.

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

Note Any optional input of size N -by- 1 is also acceptable as an array of size 1-by-N, or as a single value applicable to all contracts. Single values are internally expanded to an array of size N -by- 1 .

\section*{'Basis'}

N -by- 1 vector of day-count basis of the CDS:
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- \(3=\) actual \(/ 365\)
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- \(10=\) actual \(/ 365\) (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 2 (actual/360)

\section*{'BusDayConvention'}

String or N-by-1 cell array of strings of business day conventions. Values are:
- actual
- follow
- modifiedfollow
- previous
- modifiedprevious

Default: actual

\section*{'PayAccruedPremium'}

N -by- 1 vector of Boolean flags, True (default), if accrued premiums are paid upon default, False otherwise.

Default: True

\section*{'Period'}

N -by- 1 vector of the number of premiums per year of the CDS. Allowed values are \(1,2,3,4,6\), and 12 .

Default: 4

\section*{'ProbDates'}

P-by-1 vector of dates for ProbData.

Default: Column of dates in MarketData

\section*{'RecoveryRate'}

N -by-1 vector of recovery rates, expressed as a decimal from 0 to 1.
Default: 0.4

\section*{'TimeStep'}

Positive integer indicating the number of days to take as time step for the numerical integration.

Default: 10 (days)

\section*{'ZeroBasis'}

Basis of the zero curve. Choices are identical to Basis.
Default: 0 (actual/actual)

\section*{'ZeroCompounding'}

Compounding frequency of the zero curve. Allowed values are:
- 1 - Annual compounding
- 2 - Semiannual compounding
- 3 - Compounding three times per year
- 4 - Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding
- - 1 - Continuous compounding

> Note When ZeroData is an IRDataCurve object, the arguments ZeroCompounding and ZeroBasis are implicit in ZeroData and are redundant inside this function. In that case, specify these optional arguments when constructing the IRDataCurve object before calling this function.

Default: 2 (Semiannual compounding)

\section*{Output Arguments}

\section*{ProbData}

P-by-2 matrix with dates and corresponding cumulative default probability values. The dates match those in MarketData, unless the optional input parameter ProbDates is provided.

\section*{HazData}

N-by-2 matrix with dates and corresponding hazard rate values for the standard survival probability model. The dates match those in MarketData.

Note A warning is displayed when non-monotone default probabilities (i.e., negative hazard rates) are found.

\section*{Examples Bootstrap Default Probability Curve from Credit Default Swap Market Quotes}

This example shows how to use cdsbootstrap with market quotes for CDS contracts to generate ProbData and HazData values.
```

Settle = '17-Jul-2009'; % valuation date for the CDS
Spread_Time = [1 2 3 5 7]';
Spread = [140 175 210 265 310]';
Market_Dates = daysadd(datenum(Settle),360*Spread_Time,1);
MarketD

```
```

Zero_Time = [l.5 1 2 3 4 5]';
Zero_Rate = [ll.35 1.43 1.9 2.47 2.936 3.311]'/100;
Zero_Dates = daysadd(datenum(Settle),360*Zero_Time,1);
ZeroData = [Zero_Dates Zero_Rate];
[ProbData,HazData] = cdsbootstrap(ZeroData,MarketData,Settle)
ProbData =
1.0e+05 *
7.3434 0.0000
7.3470 0.0000
7.3507 0.0000
7.3580 0.0000
7.3653 0.0000
HazData =
1.0e+05 *
7.3434 0.0000
7.3470 0.0000
7.3507 0.0000
7.3580 0.0000
7.3653 0.0000

```

Algorithms If the time to default is denoted by \(\tau\), the default probability curve, or function, \(P D(t)\), and its complement, the survival function \(Q(t)\), are given by:
\[
P D(t)=P[\tau \leq t]=1-P[\tau>t]=1-Q(t)
\]

In the standard model, the survival probability is defined in terms of a piecewise constant hazard rate \(h(t)\). For example, if \(h(t)=\)
\(\lambda_{1}\), for \(0 \leq t \leq t_{1}\)
\(\lambda_{2}\), for \(t_{1}<t \leq t_{2}\)
\(\lambda_{3}\), for \(t_{2}<t\)
then the survival function is given by \(Q(t)=\)
\(e^{-\lambda_{1} t}\), for \(0 \leq t \leq t_{1}\)
\(e^{-\lambda_{1} t-\lambda_{2}\left(t-t_{1}\right)}\), for \(t_{1}<t \leq t_{2}\)
\(e^{-\lambda_{1} t_{1}-\lambda_{2}\left(t_{2}-t_{1}\right)-\lambda_{3}\left(t-t_{2}\right)}\), for \(t_{2}<t\)
Given \(n\) market dates \(t_{1}, \ldots, t_{n}\) and corresponding market CDS spreads \(S_{1}, \ldots, S_{n}\), cdsbootstrap calibrates the parameters \(\lambda_{1}, \ldots, \lambda_{n}\) and evaluates \(P D(t)\) on the market dates, or an optional user-defined set of dates.

\section*{References}

Beumee, J., D. Brigo, D. Schiemert, and G. Stoyle. "Charting a Course Through the CDS Big Bang," Fitch Solutions, Quantitative Research, Global Special Report. April 7, 2009.
Hull, J., and A. White, "Valuing Credit Default Swaps I: No Counterparty Default Risk," Journal of Derivatives 8, 29-40.

O'Kane, D. and S. Turnbull, "Valuation of Credit Default Swaps." Lehman Brothers, Fixed Income Quantitative Credit Research, April, 2003.

See Also
cdsspread | cdsprice | cdsrpv01 | "@IRDataCurve" on page A-7

\section*{Related}
- "Credit Default Swap (CDS)" on page 8-2

Purpose Price payer and receiver credit default swap options
\begin{tabular}{|c|c|}
\hline Syntax & \begin{tabular}{l}
[Payer, Receiver] = cdsoptprice(ZeroData, ProbData, Settle, OptionMaturity, CDSMaturity, Strike, SpreadVol) \\
[Payer, Receiver] = cdsoptprice(ZeroData, ProbData, Settle, OptionMaturity, CDSMaturity, Strike, SpreadVol, Name, Value)
\end{tabular} \\
\hline
\end{tabular}

\section*{Description}

\section*{Input Arguments}
[Payer, Receiver] = cdsoptprice(ZeroData, ProbData, Settle, OptionMaturity, CDSMaturity, Strike, SpreadVol) computes the price of payer and receiver credit default swap options.
[Payer, Receiver] = cdsoptprice(ZeroData, ProbData, Settle, OptionMaturity, CDSMaturity, Strike, SpreadVol, Name, Value) computes the price of payer and receiver credit default swap options with additional options specified by one or more Name, Value pair arguments.

\section*{ZeroData}

M-by-2 vector of dates and zero rates or an IRDataCurve object of zero rates.

\section*{ProbData}

P-by-2 array of dates and default probabilities.

\section*{Settle}

Settlement date is a serial date number or date string. Settle must be earlier than the maturity date.

\section*{OptionMaturity}

N -by- 1 vector of serial date numbers or date strings containing the option maturity dates.

\section*{CDSMaturity}

N -by-1 vector of serial date numbers or date strings containing the CDS maturity dates.

\section*{Strike}

N -by-1 vector of option strikes expressed in basis points.

\section*{SpreadVol}

N -by-1 vector of annualized credit spread volatilities expressed as a positive decimal number.

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

Note Any optional input of size N-by-1 is also acceptable as an array of size 1-by-N, or as a single value applicable to all contracts. Single values are internally expanded to an array of size N -by- 1 .

\section*{'AdjustedForwardSpread'}

N -by-1 vector of adjusted forward spreads (in basis points) to be used when pricing CDS index options.

Default: unadjusted forward spread normally used for single-name CDS options

\section*{'Basis'}

N -by-1 vector of contract day-count basis:
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual \(/ 360\)
- \(3=\) actual \(/ 365\)
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- \(10=\mathrm{actual} / 365\) (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 2 (actual/360)

\section*{'BusDayConvention'}

String or N -by- 1 cell array of strings of business day conventions. Values are:
- actual
- follow
- modifiedfollow
- previous
- modifiedprevious

Default: actual

\section*{'Knockout'}

N -by- 1 vector of Boolean flags. If the credit default swaptions is a knockout, the flag is True, otherwise it is False.

Default: False

\section*{'PayAccruedPremium'}

N-by-1 vector of Boolean flags. If accrued premiums are paid upon default, the flag is True, otherwise it is False.

Default: True

\section*{'Period'}

N -by- 1 vector of the number of premiums per year of the CDS. Allowed values are \(1,2,3,4,6\), and 12 .

Default: 4

\section*{'RecoveryRate'}

N -by-1 vector of recovery rates, expressed as a decimal from 0 to 1.
Default: 0.4

\section*{'ZeroBasis'}

Basis of the zero curve. Choices are identical to Basis.
Default: 0 (actual/actual)

\section*{'ZeroCompounding'}

Compounding frequency of the zero curve. Allowed values are:
- 1 - Annual compounding
- 2 - Semiannual compounding
- 3 - Compounding three times per year

\section*{cdsoptprice}
- 4 - Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding
- -1 - Continuous compounding

Note When ZeroData is an IRDataCurve object, the arguments ZeroCompounding and ZeroBasis are implicit in ZeroData and are redundant inside this function. In that case, specify these optional arguments when constructing the IRDataCurve object before calling this function.

Default: 2 (Semiannual compounding)

\section*{Output Arguments}

\section*{Definitions}

\section*{Payer}

N-by-1 vector of prices for payer swap options in Basis points.

\section*{Receiver}

N -by-1 vector of prices for receiver swap options in Basis points.

\section*{Credit Default Swap Option}

A credit default swap (CDS) option, or credit default swaption, is a contract that provides the option holder with the right, but not the obligation, to enter into a credit default swap in the future. CDS options can either be payer swaptions or receiver swaptions. In a payer swaption, the option holder has the right to enter into a CDS in which they are paying premiums and in a receiver swaptions, the option holder is receiving premiums.

\section*{Examples Obtain Payer and Receiver Values for a Credit Default Swap Option}

Use cdsoptprice to generate Payer and Receiver values for a credit default swap option.
```

Settle = datenum('12-Jun-2012');
OptionMaturity = datenum('20-Sep-2012');
CDSMaturity = datenum('20-Sep-2017');
OptionStrike = 200;
SpreadVolatility = .4;
Zero_Time = [.5 1 1 2 3 4 5]';
Zero_Rate = [.5 .75 1.5 1.7 1.9 2.2]'/100;
Zero_Dates = daysadd(Settle,360*Zero_Time,1);
ZeroData = [Zero_Dates Zero_Rate];
Market_Time = [11 2 3 5 7 10]';
Market_Rate = [l100 120 145 220 245 270]';
Market_Dates = daysadd(Settle,360*Market_Time,1);
MarketData = [Market_Dates Market_Rate];
ProbData = cdsbootstrap(ZeroData, MarketData, Settle);
[Payer,Receiver] = cdsoptprice(ZeroData, ProbData, Settle,...
OptionMaturity, CDSMaturity, OptionStrike, SpreadVolatility)

```
Payer =
    223.5780
Receiver =
22.7460

\section*{Algorithms}

The payer and receiver credit default swap options are computed using the Black's model as described in O'Kane [1]:
\[
\begin{aligned}
& V_{\text {Pay }(\text { Knockout })}=R P V 01\left(t, t_{E}, T\right)\left(F \Phi\left(d_{1}\right)-K \Phi\left(d_{2}\right)\right) \\
& V_{\text {Rec }(\text { Knockout })}=R P V 01\left(t, t_{E}, T\right)\left(K \Phi\left(-d_{2}\right)-F \Phi\left(-d_{1}\right)\right) \\
& d_{1}=\frac{\ln \left(\frac{F}{K}\right)+\frac{1}{2} \sigma^{2}\left(t_{E}-t\right)}{\sigma \sqrt{t_{E}-t}} \\
& d_{2}=d_{1}-\sigma \sqrt{t_{E}-t} \\
& V_{\text {Pay }(\text { Non-Knockout })}=V_{\text {Pay }(\text { Knockout })}+F E P \\
& V_{\text {Pay }(\text { Non-Knockout })}=V_{\text {Rec }(\text { Knockout })}
\end{aligned}
\]
where
RPV01 is the risky present value of a basis point (see cdsrpv01).
\(\Phi\) is the normal cumulative distribution function.
\(\sigma\) is the spread volatility.
\(t\) is the valuation date.
\(t_{E}\) is the option expiry date.
\(T\) is the CDS maturity date.
\(F\) is the forward spread (from option expiry to CDS maturity).
\(K\) is the strike spread.
\(F E P\) is the front end protection (from option initiation to option expiry).
\begin{tabular}{ll} 
References & \begin{tabular}{l} 
[1] O’Kane, D., Modelling Single-name and Multi-name Credit \\
Derivatives, Wiley, 2008, pp. 156-169.
\end{tabular} \\
See Also & \begin{tabular}{l} 
cdsbootstrap | cdsspread | cdsprice \| cdsrpv01 | "@IRDataCurve" \\
on page A-7
\end{tabular} \\
Related & - "Pricing a Single-Name CDS Option" on page 8-40 \\
Examples & - "Pricing a CDS Index Option" on page 8-42 \\
Concepts & - "Credit Default Swap Option" on page 8-40
\end{tabular}

Purpose Determine price for credit default swap
\begin{tabular}{|c|c|}
\hline Syntax & ```
[Price, AccPrem, PaymentDates, PaymentTimes,
PaymentCF] = cdsprice(ZeroData, ProbData, Settle,
Maturity, ContractSpread)
[Price, AccPrem, PaymentDates, PaymentTimes,
PaymentCF] = cdsprice(ZeroData, ProbData,
Settle, Maturity, ContractSpread,
Name,Value)
``` \\
\hline
\end{tabular}

\section*{Description}

Input
Arguments
[Price, AccPrem, PaymentDates, PaymentTimes, PaymentCF] = cdsprice(ZeroData, ProbData, Settle, Maturity, ContractSpread) computes the price, or the mark-to-market value for CDS instruments.
[Price, AccPrem, PaymentDates, PaymentTimes, PaymentCF] = cdsprice(ZeroData, ProbData, Settle, Maturity, ContractSpread, Name, Value) computes the price, or the mark-to-market value for CDS instruments with additional options specified by one or more Name, Value pair arguments.

\section*{ZeroData}

M-by-2 vector of dates and zero rates or an IRDataCurve object of zero rates.

\section*{ProbData}

P-by-2 array of dates and default probabilities.

\section*{Settle}

Settlement date is a serial date number or date string. This must be earlier than or equal to the Maturity dates.

\section*{Maturity}

N -by-1 vector of serial date numbers or date strings containing the maturity dates.

\section*{ContractSpread}

N -by-1 vector of contract spreads, expressed in basis points.

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Note Any optional input of size N-by-1 is also acceptable as an array of size 1-by-N, or as a single value applicable to all contracts. Single values are internally expanded to an array of size N -by- 1 .

\section*{'Basis'}

N -by- 1 vector of day-count basis of the CDS:
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- 3 = actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- \(10=\) actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- \(12=\) actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 2 (actual/360)

\section*{'BusDayConvention'}

String or N -by- 1 cell array of strings of business day conventions. Values are:
- actual
- follow
- modifiedfollow
- previous
- modifiedprevious

Default: actual

\section*{'Notional'}

N -by-1 vector of contract notional values. Use positive values for long positions and negative values for short positions.

Default: 10MM

\section*{'PayAccruedPremium'}

N-by- 1 vector of Boolean flags. True, if accrued premiums are paid upon default, False otherwise.

Default: True

\section*{'Period'}

N -by- 1 vector of number of premiums per year of the CDS. Allowed values are \(1,2,3,4,6\), and 12 .

Default: 4

\section*{'RecoveryRate'}

N -by- 1 vector of recovery rates, expressed as a decimal from 0 to 1.
Default: 0.4

\section*{'TimeStep'}

Positive integer indicating the number of days to take as time step for the numerical integration.

Default: 10 (days)

\section*{'ZeroBasis'}

Basis of the zero curve, where the choices are identical to Basis.
Default: 0 (actual/actual)

\section*{'ZeroCompounding'}

Compounding frequency of the zero curve. Allowed values are:
- 1 - Annual compounding
- 2 - Semiannual compounding
- 3 - Compounding three times per year
- 4 - Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding
- - 1 - Continuous compounding

\title{
Note When ZeroData is an IRDataCurve object, the arguments ZeroCompounding and ZeroBasis are implicit in ZeroData and are redundant inside this function. In that case, specify these optional arguments when constructing the IRDataCurve object before calling this function.
}

Default: 2 (Semiannual compounding)

\section*{Output Arguments}

\section*{Definitions}

\section*{Price}

N -by- 1 vector of CDS clean prices.

\section*{AccPrem}

N -by-1 vector of accrued premiums.

\section*{PaymentDates}

N -by-numCF matrix of payment dates.

\section*{PaymentTimes}

N -by-numCF matrix of accrual fractions.

\section*{PaymentCF}

N -by-numCF matrix of payments.

\section*{CDS Price}

The price or mark-to-market (MtM) value of an existing CDS contract is computed using the following formula:
```

CDS price = Notional * (Current Spread - Contract Spread)

```
* RPVO1

Current Spread is the current breakeven spread for a similar contract, according to current market conditions. RPV01 is the 'risky present
value of a basis point,' the present value of the premium payments, taking into consideration the default probability. This formula assumes a long position, and the right side is multiplied by -1 for short positions.

\section*{Examples}

Price =
\(1.5461 e+04\)

AccPrem =

10500

\section*{Algorithms}

The premium leg is computed as the product of a spread \(S\) and the risky present value of a basis point (RPV01). The RPV01 is given by:
\[
R P V 01=\sum_{j=1}^{N} Z\left(t_{j}\right) \Delta\left(t_{j-1}, t_{j}, B\right) Q\left(t_{j}\right)
\]
when no accrued premiums are paid upon default, and it can be approximated by
\[
R P V 01 \approx \frac{1}{2} \sum_{j=1}^{N} Z\left(t_{j}\right) \Delta\left(t_{j-1}, t_{j}, B\right)\left(Q\left(t_{j-1}\right)+Q\left(t_{j}\right)\right)
\]
when accrued premiums are paid upon default. Here, \(t_{0}=0\) is the valuation date, and \(t_{1}, \ldots, t_{n}=T\) are the premium payment dates over the life of the contract, \(T\) is the maturity of the contract, \(Z(t)\) is the discount factor for a payment received at time \(t\), and \(\Delta\left(t_{j-1}, t_{j}, B\right)\) is a day count between dates \(t_{j-1}\) and \(t_{j}\) corresponding to a basis \(B\).
The protection leg of a CDS contract is given by the following formula:
\[
\begin{aligned}
& \text { ProtectionLeg }=\int_{0}^{T} Z(\tau)(1-R) d P D(\tau) \\
& \approx(1-R) \sum_{i=1}^{M} Z\left(\tau_{i}\right)\left(P D\left(\tau_{i}\right)-P D\left(\tau_{i-1}\right)\right) \\
& =(1-R) \sum_{i=1}^{M} Z\left(\tau_{i}\right)\left(Q\left(\tau_{i-1}\right)-Q\left(\tau_{i}\right)\right)
\end{aligned}
\]
where the integral is approximated with a finite sum over the discretization \(\tau_{0}=0, \tau_{1}, \ldots, \tau_{M}=T\).
If the spread of an existing CDS contract is \(S_{C}\), and the current breakeven spread for a comparable contract is \(S_{0}\), the current price, or mark-to-market value of the contract is given by:
MtM \(=\) Notional \(\left(S_{0}-S_{C}\right)\) RPV01

This assumes a long position from the protection standpoint (protection was bought). For short positions, the sign is reversed.
\begin{tabular}{ll} 
References & \begin{tabular}{l} 
Beumee, J., D. Brigo, D. Schiemert, and G. Stoyle. "Charting a Course \\
Through the CDS Big Bang," Fitch Solutions, Quantitative Research, \\
Global Special Report. April 7, 2009.
\end{tabular} \\
& \begin{tabular}{l} 
Hull, J., and A. White, "Valuing Credit Default Swaps I: No \\
Counterparty Default Risk," Journal of Derivatives 8, 29-40. \\
O'Kane, D. and S. Turnbull, "Valuation of Credit Default Swaps." \\
Lehman Brothers, Fixed Income Quantitative Credit Research, April, \\
\\
2003.
\end{tabular} \\
See Also & \begin{tabular}{l} 
cdsbootstrap I cdsspread I cdsoptprice I "@IRDataCurve" on page \\
A-7
\end{tabular} \\
\begin{tabular}{ll} 
Related & - "Credit Default Swap (CDS)" on page 8-2 \\
Examples &
\end{tabular}
\end{tabular}

Purpose
Compute risky present value of a basis point for credit default swap
Syntax

\section*{Description}

\section*{Input Arguments}

RPV01 = cdsrpv01(ZeroData, ProbData, Settle, Maturity) RPV01 = cdsrpv01 (__ , Name, Value)
[RPV01,PaymentDates,PaymentTimes] = cdsrpv01(ZeroData,ProbData,Settle, Maturity)
[RPV01,PaymentDates,PaymentTimes] = cdsrpv01 (__ , Name, Value)

RPV01 = cdsrpv01(ZeroData,ProbData,Settle, Maturity) computes the risky present value of a basis point (RPV01) for a credit default swap (CDS).

RPV01 = cdsrpv01( \(\qquad\) , Name, Value) computes the risky present value of a basis point (RPV01) for a credit default swap (CDS) using optional name-value pair arguments.
[RPV01,PaymentDates,PaymentTimes] = cdsrpv01(ZeroData, ProbData,Settle, Maturity) computes the risky present value of a basis point (RPV01), PaymentDates, and PaymentTimes for a credit default swap (CDS).
[RPV01,PaymentDates,PaymentTimes] = cdsrpv01 ( _ _ , Name, Value) computes the risky present value of a basis point (RPV01), PaymentDates, and PaymentTimes for a credit default swap (CDS) using optional name-value pair arguments.

\section*{ZeroData - Dates and zero rates}
object from IRDataCurve or vector of dates and zero rates
Dates and zero rates, specified by an M-by-2 vector of dates and zero rates or the object IRDataCurve for zero rates.

\section*{Data Types \\ struct | double \\ ProbData - Dates and default probabilities}
vector of dates and default probabilities
Dates and default probabilities, specified by a P-by-2 array.
Data Types
double

\section*{Settle - Settlement date}
string or cell array of dates or numeric dates
Settlement date, specified by a serial date number or date string. This must be earlier than or equal to the dates in Maturity.

\section*{Data Types}
char | cell | double

\section*{Maturity - CDS maturity date}
string or cell array of dates or numeric dates
CDS maturity date, specified by an \(N\)-by- 1 vector of serial date numbers or date strings containing the maturity dates. The CDS premium payment dates occur at regular intervals, and the last payment occurs on these maturity dates.

\section*{Data Types}
char | cell | double

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.
```

Example: RPV01 =
cdsrpv01(ZeroData,ProbData,Settle,Maturity,'Period',1,'StartDate','20-Sep
'BusDayConvention', actual, 'CleanRPV01', true, 'PayAccruedPremium' , true, 'Zer

```

\section*{'Period' - Number of premium payments per year \\ 4 (default)| positive integer from the set [1,2,3,4,6,12] | vector of positive integers from the set [1,2,3,4,6,12]}

Number of premium payments per year, specified by an N -by- 1 vector. Values are \(1,2,3,4,6\), and 12.

\section*{Data Types \\ double}

\section*{'StartDate' - Dates the CDS premium leg starts}

Settle date (default) | string | cell array
Dates when the CDS premium leg actually starts, specified by an N-by-1 vector of serial date numbers or date strings. Must be on or between the Settle and Maturity dates. For a forward-starting CDS, specify this date as a future date after Settle.

\section*{Data Types double}

\section*{'Basis' - Day-count basis of contract}
```

2 (actual/360) (default) | positive integers of the set [1...13] | vector
of positive integers of the set [1...13]

```

Day-count basis of the contract, specified as a positive integer using a NINST-by-1 vector.
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- 3 = actual/365
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- \(10=\mathrm{actual} / 365\) (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

\section*{Data Types}
double

\section*{'BusDayConvention' - Business day conventions}
actual (default) | string | cell array
Business day conventions, specified by a string or N -by- 1 cell array of strings of business day conventions. Values are:
- actual
- follow
- modifiedfollow
- previous
- modifiedprevious

Data Types
char | cell
'CleanRPVO1' - Flag for premium accrual
true (default) | boolean flag with value true or false

Flag for premium accrual, specified as an N -by-1 vector of Boolean flags, which is true if the premium accrued at StartDate is excluded in the RPV01, and false otherwise.

\section*{Data Types}
logical

\section*{'PayAccruedPremium' - Flag for accrued premium payment true (default) | boolean flag with value true or false}

Flag for accrued premium payment, specified as a N -by- 1 vector of Boolean flags, true if accrued premiums are paid upon default, false otherwise.

\section*{Data Types}
logical

\section*{'ZeroCompounding' - Compounding frequency of zero curve}

2 semiannual compounding (default) | integer with acceptable value [1,2,3,4,6,12, -1]
Compounding frequency of the zero curve, specified with integer values:
- 1 - Annual compounding
- 2 - Semiannual compounding
- 3 - Compounding three times per year
- 4 - Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding
- -1 - Continuous compounding

> Note When ZeroData is an IRDataCurve object, the arguments ZeroCompounding and ZeroBasis are implicit in ZeroData and are redundant inside this function. In that case, specify these optional arguments when constructing the IRDataCurve object before calling this function.

\section*{Data Types}
double

\section*{'ZeroBasis' - Basis of zero curve}

0 (actual/actual) (default) | positive integers of the set [1...13] | vector of positive integers of the set [1...13]

Basis of the zero curve, specified as a positive integer using a NINST-by-1 vector.
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual \(/ 360\)
- 3 = actual \(/ 365\)
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- \(10=\) actual/365 (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

\section*{Data Types \\ double}

\section*{Output \\ Arguments}

\section*{Definitions}

RPV01
RPV01, associated with a CDS, is the value of a stream of 1-basis-point premiums according to the payment structure of the CDS contract, and taking into consideration the default probability over time.

For more information, see [3] and [4] for details.

\section*{Examples Calculate the RPvo1 Value for a CDS}

Calculate the RPV01 value, given the following specification for a CDS.
```

Settle = '17-Jul-2009'; % valuation date for the CDS
Zero_Time = [.5 1 2 3 4 5]';
Zero_Rate = [1.35 1.43 1.9 2.47 2.936 3.311]'/100;
Zero_Dates = daysadd(Settle,360*Zero_Time,1);
ZeroData = [Zero_Dates Zero_Rate];
ProbData = [daysadd(datenum(Settle),360,1), 0.0247];
Maturity = '20-Sep-2010';

```
```

RPV01 = cdsrpv01(ZeroData,ProbData,Settle,Maturity)

```

\section*{RPV01 =}
1.1651

\section*{References}
[1] Beumee, J., D. Brigo, D. Schiemert, and G. Stoyle. "Charting a Course Through the CDS Big Bang," Fitch Solutions, Quantitative Research, Global Special Report. April 7, 2009.
[2] Hull, J., and A. White, "Valuing Credit Default Swaps I: No Counterparty Default Risk," Journal of Derivatives 8, 29-40.
[3] O’Kane, D. and S. Turnbull, "Valuation of Credit Default Swaps," Lehman Brothers, Fixed Income Quantitative Credit Research, April, 2003.
[4] O'Kane, D., Modelling Single-name and Multi-name Credit Derivatives, Wiley Finance, 2008.
See Also \(\quad\)\begin{tabular}{l} 
cdsbootstrap | cdsspread | cdsprice | cdsoptprice |
\end{tabular}

Related
- "Pricing a CDS Index Option" on page 8-42

Examples
Concepts - basis on page Glossary-1
External • www.cdsmodel.com
Web Sites

Purpose Determine spread of credit default swap
```

Syntax [Spread, PaymentDates, PaymentTimes] = cdsspread(ZeroData,
ProbData, Settle, Maturity)
[Spread, PaymentDates, PaymentTimes] = cdsspread(ZeroData,
ProbData, Settle, Maturity,
Name,Value)

```

\section*{Description}

\section*{Input \\ Arguments}
[Spread, PaymentDates, PaymentTimes] = cdsspread(ZeroData, ProbData, Settle, Maturity) computes the spread of the CDS.
[Spread, PaymentDates, PaymentTimes] = cdsspread(ZeroData, ProbData, Settle, Maturity,
Name, Value) computes the spread of the CDS with additional options specified by one or more Name, Value pair arguments.

\section*{ZeroData}

M-by-2 vector of dates and zero rates or IRCurve of zero rates.

\section*{ProbData}

P-by-2 array of dates and default probabilities.

\section*{Settle}

Settlement date is a serial date number or date string. This must be earlier than or equal to the dates in MarketData.

\section*{Maturity}

N -by-1 vector of serial date numbers or date strings containing the maturity dates.

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can
specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

Note Any optional input of size N-by-1 is also acceptable as an array of size 1-by-N, or as a single value applicable to all contracts. Single values are internally expanded to an array of size N -by- 1 .

\section*{'Basis'}

N -by- 1 vector of day-count basis of the CDS:
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- 3 = actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- \(10=\mathrm{actual} / 365\) (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- \(12=\mathrm{actual} /\) actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 2 (actual/360)

\section*{'BusDayConvention'}

String or N -by-1 cell array of strings of business day conventions. Values are:
- actual
- follow
- modifiedfollow
- previous
- modifiedprevious

Default: actual

\section*{'PayAccruedPremium'}

N-by- 1 vector of Boolean flags, True, if accrued premiums are paid upon default, False otherwise.

Default: True

\section*{'Period'}

N -by-1 vector of number of premiums per year of the CDS. Allowed values are \(1,2,3,4,6\), and 12 .

Default: 4

\section*{'RecoveryRate'}

N -by-1 vector of recovery rates, expressed as a decimal from 0 to 1.
Default: 0.4

\section*{'TimeStep'}

Positive integer indicating the number of days to take as time step for the numerical integration.

Default: 10 (days)

\section*{'ZeroBasis'}

Basis of the zero curve, where the choices are identical to Basis.
Default: 0 (actual/actual)

\section*{'ZeroCompounding'}

Compounding frequency of the zero curve. Allowed values are:
- 1 - Annual compounding
- 2 - Semiannual compounding
- 3 - Compounding three times per year
- 4 - Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding
- - 1 - Continuous compounding

Note When ZeroData is an IRCurve object, the arguments ZeroCompounding and ZeroBasis are implicit in ZeroData and are redundant inside this function. In that case, specify these optional arguments when constructing the IRCurve object before calling this function.

Default: 2 (semiannual compounding )
\begin{tabular}{ll} 
Output & Spread \\
Arguments & N -by-1 vector of spreads (in basis points). \\
& PaymentDates
\end{tabular}

N -by-numCF matrix of payment dates.

\section*{PaymentTimes}

N -by-numCF matrix of accrual fractions.

\section*{Definitions}

\section*{Examples}

\section*{CDS Spread}

The market, or breakeven, spread value of a CDS can be computed by equating the value of the protection leg with the value of the premium leg:
```

Market Spread * RPV01 = Value of Protection Leg

```

The left side corresponds to the value of the premium leg, and this has been decomposed as the product of the market or breakeven spread times the RPV01 or 'risky present value of a basis point' of the contract. The latter is the present value of the premium payments, taking into consideration the default probability. The Market Spread can be computed as the ratio of the value of the protection leg, to the RPV01 of the contract. cdsspread returns the resulting spread in basis points.

\section*{Determine the Spread of a Credit Default Swap}

This example shows how to use cdsspread to compute the clean price for a CDS contract with the following data.
```

Settle = '17-Jul-2009'; % valuation date for the CDS
Zero_Time = [. .5 1 2 2 3 4 5]';
Zero_Rate = [ll.35 1.43 1.9 2.47 2.936 3.311]'/100;
Zero_Dates = daysadd(Settle,360*Zero_Time,1);
ZeroData = [Zero_Dates Zero_Rate];
ProbData = [daysadd(datenum(Settle),360,1), 0.0247];
Maturity = '20-Sep-2010';
Spread = cdsspread(ZeroData,ProbData,Settle,Maturity)

```

\section*{Algorithms}

The premium leg is computed as the product of a spread \(S\) and the risky present value of a basis point (RPV01). The RPV01 is given by:
\[
R P V 01=\sum_{j=1}^{N} Z\left(t_{j}\right) \Delta\left(t_{j-1}, t_{j}, B\right) Q\left(t_{j}\right)
\]
when no accrued premium are paid upon default, and it can be approximated by
\[
R P V 01 \approx \frac{1}{2} \sum_{j=1}^{N} Z\left(t_{j}\right) \Delta\left(t_{j-1}, t_{j}, B\right)\left(Q\left(t_{j-1}\right)+Q\left(t_{j}\right)\right)
\]
when accrued premiums are paid upon default. Here, \(t_{0}=0\) is the valuation date, and \(t_{1}, \ldots, t_{n}=T\) are the premium payment dates over the life of the contract, \(T\) is the maturity of the contract, \(Z(t)\) is the discount factor for a payment received at time \(t\), and \(\Delta\left(t_{j-1}, t_{j}, B\right)\) is a day count between dates \(t_{j-1}\) and \(t_{j}\) corresponding to a basis \(B\).
The protection leg of a CDS contract is given by the following formula:
\[
\begin{aligned}
& \text { ProtectionLeg }=\int_{0}^{T} Z(\tau)(1-R) d P D(\tau) \\
& \approx(1-R) \sum_{i=1}^{M} Z\left(\tau_{i}\right)\left(P D\left(\tau_{i}\right)-P D\left(\tau_{i-1}\right)\right) \\
& =(1-R) \sum_{i=1}^{M} Z\left(\tau_{i}\right)\left(Q\left(\tau_{i-1}\right)-Q\left(\tau_{i}\right)\right)
\end{aligned}
\]
where the integral is approximated with a finite sum over the discretization \(\tau_{0}=0, \tau_{1}, \ldots, \tau_{M}=T\).
A breakeven spread \(S_{0}\) makes the value of the premium and protection legs equal. It follows that:
\[
S_{0}=\frac{\text { ProtectionLeg }}{R P V 01}
\]
References Beumee, J., D. Brigo, D. Schiemert, and G. Stoyle. "Charting a CourseThrough the CDS Big Bang," Fitch Solutions, Quantitative Research,Global Special Report. April 7, 2009.Hull, J., and A. White, "Valuing Credit Default Swaps I: NoCounterparty Default Risk," Journal of Derivatives 8, 29-40.O'Kane, D. and S. Turnbull, "Valuation of Credit Default Swaps."Lehman Brothers, Fixed Income Quantitative Credit Research, April,2003.
See Also | cdsprice | cdsbootstrap
Tutorials - "Credit Default Swap (CDS)" on page 8-2
Purpose Generate principal balance schedule for planned amortization class (PAC) or targeted amortization class (TAC) bond
Syntax \begin{tabular}{l} 
[BalanceSchedule, InitialBalance] = cmosched(Principal, \\
Coupon, OriginalTerm, TermRemaining, PrepaySpeed) \\
[BalanceSchedule, InitialBalance] = cmosched(Principal, \\
Coupon, OriginalTerm, TermRemaining, PrepaySpeed, \\
TranchePrincipal)
\end{tabular}

\section*{Principal}
Principal of the underlying mortgage pool.

\section*{Coupon}
Coupon of the underlying mortgage pool.

\section*{OriginalTerm}
Original term in months of the underlying mortgage pool.

\section*{TermRemaining}
Terms remaining in months of the underlying mortgage pool.

\section*{PrepaySpeed}

PSA speed. For a PAC, the speed is a 1-by-2 matrix where the first element is the lower band and the second element is the upper band. For a TAC, the speed is a scalar.

\section*{TranchePrincipal}
(Optional) Principal of the scheduled tranche. If it is unspecified or empty [], the principal of the scheduled tranche is assumed to be the sum of the payment schedule calculated from the PSA prepayment speeds.

\section*{Output}

Arguments

\section*{Definitions}

\section*{BalanceSchedule}

Matrix of size 1-by-NUMTERMS, where NUMTERMS is the number of terms remaining. Each column contains the scheduled principal balance for the time period corr-by-esponding to the column number.

\section*{InitialBalance}

Scalar containing the initial principal balance of the scheduled tranche.

\section*{Planned Amortization Class (PAC) Bond}

PAC bonds are a type of CMO bond. They are designed to largely eliminate prepayment risk for investors. They do this by transferring essentially all prepayment risk to other bonds in the CMO that are called support bonds.

\section*{Targeted Amortization Class (TAC) Bond}

TAC bonds are analogous to PAC bonds, but are structured differently. TAC bonds offer one-sided protection, shielding investors from high prepayment rates up to a specified PSA and do not protect against low prepayment rates.

\section*{Examples}

\section*{Calculate the Principal Balance Schedule for a CMO PAC Bond}

Define the mortgage pool under consideration and generate a principal balance schedule for planned amortization class (PAC) bonds using two bands of PSA speeds.

Principal = 128687000;
GrossRate = 0.0648;
OriginalTerm = 360;
TermRemaining = 325;
PrepaySpeed = [300 525];
PacPrincipal = 100250000;
```

[BalanceSchedule, InitialBalance] ...
= cmosched(Principal, GrossRate, OriginalTerm, TermRemaining, ...
PrepaySpeed, PacPrincipal)

```

\section*{BalanceSchedule =}
```

    1.0e+07 *
    ```
Columns 1 through 7
\(9.7996 \quad 9.5780\) ..... 9.3602
9.1461 8.9357 8.7289
Columns 8 through ..... 14
8.3259 8.1296 7.9366 7.7469 7.5605 ..... 7.3773
Columns 15 through ..... 21
7.0202 6.8463 ..... 6.6754
6.5073 ..... 6.3422
6.1799

Columns 29 through 35
4.8394
4.7030
4.5689
4.4372
4.3077
4.1804
4.0554

Columns 36 through 42
3.9325
3.8118
3.6931
3.5765
3.4619
3.3494
3.2406

Columns 43 through 49
3.1353
3.0334
2.9348
2.8394
2.7470
2.6576
2.5711

Columns 50 through 56
2.4873
2.4063
2.3279
2.2520
2.1786
2.1075
2.0387

Columns 57 through 63
\begin{tabular}{lllllll}
1.9722 & 1.9078 & 1.8455 & 1.7852 & 1.7268 & 1.6703 & 1.6157
\end{tabular}

Columns 64 through 70
\(\begin{array}{lllllll}1.5628 & 1.5117 & 1.4622 & 1.4142 & 1.3679 & 1.3231 & 1.2797\end{array}\)

Columns 71 through 77
\begin{tabular}{lllllll}
1.2377 & 1.1970 & 1.1577 & 1.1197 & 1.0829 & 1.0473 & 1.0129
\end{tabular}

Columns 78 through 84
0.9795
0.9473
0.9161
0.8859
0.8567
0.8285
0.8011

Columns 85 through 91
0.7747
0.7491
0.7244
0.7004
0.6773
0.6549
0.6332

Columns 92 through 98
0.6122
0.5920
0.5723
0.5534
0.5350
0.5172
0.500

Columns 99 through 105
0.4835
0.4674
0.4518
0.4368
0.4223
0.4082
0.394

Columns 106 through 112
\(\begin{array}{lllllll}0.3814 & 0.3687 & 0.3564 & 0.3445 & 0.3330 & 0.3219 & 0.311\end{array}\)
Columns 113 through 119
\(\begin{array}{lllllll}0.3007 & 0.2906 & 0.2809 & 0.2715 & 0.2623 & 0.2535 & 0.2450\end{array}\)
Columns 120 through 126
\(\begin{array}{lllllll}0.2368 & 0.2288 & 0.2211 & 0.2137 & 0.2065 & 0.1995 & 0.1928\end{array}\)
Columns 127 through 133
0.1863
0.1800
0.1739
0.1680
0.1623
0.1568
0.151

Columns 134 through 140
0.1464
0.1414
0.1366
0.1319
0.1275
0.1231
0.118

Columns 141 through 147
\(\begin{array}{lllllll}0.1149 & 0.1109 & 0.1072 & 0.1035 & 0.0999 & 0.0965 & 0.093\end{array}\)

Columns 148 through 154
\(\begin{array}{lllllll}0.0900 & 0.0869 & 0.0839 & 0.0811 & 0.0783 & 0.0756 & 0.073\end{array}\)
Columns 155 through 161
0.0704
0.0680
0.0657
0.0634
0.0612
0.0591
0.0570

Columns 162 through 168
0.0550
0.0531
0.0513
0.0495
0.0478
0.0461
0.0445

Columns 169 through 175
0.0429
0.0414
0.0400
0.0386
0.0372
0.0359
0.0346

Columns 176 through 182
0.0334
0.0322
0.0311
0.0300
0.0289
0.0279
0.0269

Columns 183 through 189
0.0260
0.0250
0.0241
0.0233
0.0224
0.0216
0.0209

Columns 190 through 196
0.0201
0.0194
0.0187
0.0180
0.0174
0.0167
0.0161

Columns 197 through 203
0.0155
0.0150
0.0144
0.0139
0.0134
0.0129
0.0124

Columns 204 through 210
0.0120
0.0115
0.0111
0.0107
0.0103
0.0099
0.0096

Columns 211 through 217
0.0092
0.0089
0.0085
0.0082
0.0079
0.0076
0.0073

Columns 218 through 224
\begin{tabular}{lllllll}
0.0070 & 0.0068 & 0.0065 & 0.0063 & 0.0060 & 0.0058 & 0.0056
\end{tabular}

Columns 225 through 231
0.0054
0.0052
0.0050
0.0048
0.0046
0.0044
0.004

Columns 232 through 238
0.0041
0.0039
0.0037
0.0036
0.0035
0.0033
0.003

Columns 239 through 245
0.0031
0.0029
0.0028
0.0027
0.0026
0.0025
0.002

Columns 246 through 252
0.0023
0.0022
0.0021
0.0020
0.0019
0.0018
0.001

Columns 253 through 259
0.0017
0.0016
0.0016
0.0015
0.0014
0.0014
0.001

Columns 260 through 266
0.0012
0.0012
0.0011
0.0011
0.0010
0.0010
0.000

Columns 267 through 273
\begin{tabular}{lllllll}
0.0009 & 0.0009 & 0.0008 & 0.0008 & 0.0007 & 0.0007 & 0.000
\end{tabular}

Columns 274 through 280
\begin{tabular}{llllll}
0.0006 & 0.0006 & 0.0006 & 0.0005 & 0.0005 & 0.0005
\end{tabular}

Columns 281 through 287
\begin{tabular}{lllllll}
0.0004 & 0.0004 & 0.0004 & 0.0004 & 0.0004 & 0.0003 & 0.0003
\end{tabular}
```

    Columns 288 through 294
    0.0003 0.0003 0.0003 0.0002 0.0002 0.0002 0.0002
    Columns 295 through 301
    0.0002 0.0002 0.0002 0.0002 0.0001 0.0001 0.0001
    Columns 302 through 308
    0.0001 0.0001 0.0001 0.0001 0.0001 0.0.0001 0.0001
    Columns 309 through 315
    0.0001 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
    Columns 316 through 322
    0.0000 0.0000 0.0000 0 0 0
    Columns 323 through 325
        0 0 0
    InitialBalance =
1 0 0 2 5 0 0 0 0

```

\section*{References Hayre, Lakhbir, ed., Salomon Smith Barney Guide to Mortgage-Backed} and Asset-Backed Securities, John Wiley and Sons, New York, 2001.

Lyuu, Yuh-Dah, Financial Engineering and Computation, Cambridge University Press, 2004.

\section*{See Also | cmoschedcf}

\author{
Tutorials \\ - "Using Collateralized Mortgage Obligations (CMOs)" on page 5-38 \\ How To \\ - "Create PAC and Sequential CMO" on page 5-53
}

Purpose Generate cash flows for scheduled collateralized mortgage obligation (CMO) using PAC or TAC model
\begin{tabular}{ll} 
Syntax & \begin{tabular}{l} 
[Balances, Principal, Interest] = \\
cmoschedcf(PrincipalPayments, TranchePrincipals, \\
TrancheCoupons, BalanceSchedule)
\end{tabular} \\
Description \(\quad\)\begin{tabular}{l} 
[Balances, Principal, Interest] = \\
cmoschedcf(PrincipalPayments, TranchePrincipals, \\
TrancheCoupons, BalanceSchedule) generate cash flows for a \\
scheduled CMO such as the planned amortization class (PAC) or \\
targeted amortization class (TAC), given the underlying mortgage \\
pool payments (or payments from another CMO tranche). The output \\
Balance, Principal, and Interest from this function can be used as \\
input into cmoseqcf to further divide the PAC, TAC, or support tranche \\
into sequential tranches.
\end{tabular} \\
Input & \begin{tabular}{l} 
PrincipalPayments
\end{tabular} \\
Arguments & \begin{tabular}{l} 
Matrix of size 1-by-NUMTERMS, where NUMTERMS is the number of terms \\
remaining. Each column contains the underlying principal payment for \\
the time period corresponding to the row number. Calculate underlying
\end{tabular} \\
principal payments using mbscfamounts or mbspassthrough. The \\
underlying principal payments can also be outputs from other CMO
\end{tabular}

\section*{TranchePrincipals}

Matrix of size 2-by-1 specifying the initial principal for the scheduled and the support tranche.

\section*{TrancheCoupons}

Matrix of size 2-by-1 specifying the coupons for the schedule tranche and the support tranche. The weighted average coupon for the CMO should not exceed the coupon of the underlying mortgage.

\section*{BalanceSchedule}

Matrix of size 1 -by-NUMTERMS, where NUMTERMS is the number of terms remaining. Each element represents the targeted balance schedule for the time period corresponding to that column.

\section*{Output Arguments}

\section*{Balance}

Matrix of size 2-by-NUMTERMS, where NUMTERMS is the number of terms remaining. The first row is the principal balances of the scheduled tranche, and the second row is the principal balances of the support tranche at the time period corresponding to the column.

\section*{Principal}

Matrix of size 2-by-NUMTERMS, where NUMTERMS is the number of terms remaining. The first row is the principal payments of the scheduled tranche, and the second row is the principal payments of the support tranche at the time period corresponding to the column.

\section*{Interest}

Matrix of size 2-by-NUMTERMS, where NUMTERMS is the number of terms remaining. The first row is the interest payments of the schedule tranche, and the second row is the interest payments of the support tranche at the time period corresponding to the column.

\section*{Definitions}

\section*{Planned Amortization Class (PAC) Tranches}

In a PAC CMO, there is a main tranche, known as the schedule tranche, and a support tranche. The main purpose of a schedule tranche is to give investors in the PAC tranche a more certain cash flow.

\section*{Targeted Amortization Class (TAC) Tranches}

TACs are like PACs, but principal payment is specified for only one prepayment rate. If prepayment rates are higher or lower, then the principal payment to TAC holders will be higher or lower accordingly.

\section*{Schedule and Support Tranche}

The main purpose of a PAC tranche is to give investors in the PAC tranche a more certain cash flow. The PAC tranche receives priority
for receiving payments of principal and interest that gives investors in the PAC tranche a steadier income. If prepayments differ from what was expected, then the support tranche gets the variable portion of the payments. While income to the support tranche is more variable, it is also higher yielding. Estimates of the yield, average life, and lockout periods of the PAC tranche is more certain.

\section*{Examples Calculare Cash Flows for Each PAC Tranche}

Define the mortgage pool under consideration for CMO structuring using mbscfamounts or mbspassthrough. Calculate the underlying mortgage cash flow, define the PAC schedule and CMO tranches, and calculate the cash flows for each tranche.
```

MortgagePrincipal = 1000000; % underlying mortgage
Coupon = 0.12;
Terms = 6; % months
[PrincipalBalance, MonthlyPayments, SchedPrincipalPayments, ...
InterestPayments, Prepayments] = ...
mbspassthrough(MortgagePrincipal, Coupon, Terms, Terms, 0, []);
PrincipalPayments = SchedPrincipalPayments.' + Prepayments.'
PrincipalPayments =
1.0e+05 *
1.6255 1.6417 1.6582 1.6747 1.6915 1.7084

```

Calculate the PAC schedule for CMO using cmosched.
PrepaySpeed = [100 300];
[BalanceSchedule, InitialBalance] ...
= cmosched(MortgagePrincipal, Coupon, Terms, Terms, PrepaySpeed, [])
```

BalanceSchedule =
1.0e+05 *
8.3617 6.7180 5.0581 3.3828 1.6955 0
InitialBalance =
9.9886e+05
Define CMO tranches.
TranchePrincipals = ...
[InitialBalance; MortgagePrincipal-InitialBalance];
TrancheCoupons = [0.12; 0.12];
Calculate cash flows for each tranche.
[Balance, Principal, Interest] = ... cmoschedcf(PrincipalPayments, TranchePrincipals, ... TrancheCoupons, BalanceSchedule)
Balance =
$1.0 \mathrm{e}+05$ *

```
8.3631
6.7213
5.0632
3.3885
1.6970
0
0.0114
0.0114
0.0114
0.0114
0.0114
0.0000
```

Principal =
$1.0 \mathrm{e}+05$ *

```
```

    1.6255 1.6417 1.6582 1.6747 1.6915 1.6970
    0 0 0 0 0
    Interest =
1.0e+03 *

| 9.9886 | 8.3631 | 6.7213 | 5.0632 | 3.3885 | 1.6970 |
| :--- | :--- | :--- | :--- | :--- | :--- |


| 0.0114 | 0.0114 | 0.0114 | 0.0114 | 0.0114 | 0.0114 |
| :--- | :--- | :--- | :--- | :--- | :--- |

```
References Hayre, Lakhbir, ed., Salomon Smith Barney Guide to Mortgage-Backedand Asset-Backed Securities, John Wiley and Sons, New York, 2001.Lyuu, Yuh-Dah, Financial Engineering and Computation, CambridgeUniversity Press, 2004.
See Also | cmoseqcf | cmosched | mbscfamounts | mbspassthrough
Tułorials - "Using Collateralized Mortgage Obligations (CMOs)" on page 5-38
How To - "Create PAC and Sequential CMO" on page 5-53

\section*{Purpose}

\author{
Syntax
}

\section*{Description}

\section*{Input \\ Arguments}

Generate cash flows for sequential collateralized mortgage obligation (CMO)
```

[balances, principals, interests] =

```
[balances, principals, interests] =
cmoseqcf(PrincipalPayments, TranchePrincipals,
cmoseqcf(PrincipalPayments, TranchePrincipals,
TrancheCoupons)
TrancheCoupons)
[balances, principals, interests] =
[balances, principals, interests] =
cmoseqcf(PrincipalPayments, TranchePrincipals,
cmoseqcf(PrincipalPayments, TranchePrincipals,
TrancheCoupons, HasZ)
```

TrancheCoupons, HasZ)

```
[balances, principals, interests] = cmoseqcf(PrincipalPayments, TranchePrincipals, TrancheCoupons) generates cash flows for a sequential CMO without a Z-bond, given the underlying mortgage pool payments.
[balances, principals, interests] = cmoseqcf(PrincipalPayments, TranchePrincipals, TrancheCoupons, HasZ) generates cash flows for a sequential CMO with a Z-bond, given the underlying mortgage pool payments.

\section*{PrincipalPayments}

Matrix of size 1-by-NUMTERMS, where NUMTERMS is the number of terms remaining. Each row contains the underlying principal payment for the time period corresponding to the row number. The underlying principal payments can be calculated using mbscfamounts or mbspassthrough. The underlying principal payments can also be outputs from other CMO cash flow functions

\section*{TranchePrincipals}

Matrix of size NUMTRANCHES-by-1, where NUMTRANCHES is the number of tranches in the sequential CMO. Each element of the matrix represents the initial principal for each tranche. If the sequential CMO includes a Z-bond (HasZ is true), the last element of this matrix is the principal of the Z-bond.

\section*{TrancheCoupons}

Matrix of size NUMTRANCHES-by-1, where NUMTRANCHES is the number of tranches in the sequential CMO. Each element of the matrix represents the coupon for each tranche. If the sequential CMO includes a Z-bond (HasZ is true), the last element of this matrix is the coupon of the Z-bond. The weighted average coupon for the CMO should not exceed the coupon of the underlying mortgage.

\section*{HasZ}
(Optional) Boolean (true or false). A value of true indicates that the sequential CMO contains a Z-bond, and the last element of TranchePrincipals and TrancheCoupons will be treated as that of the Z-bond. A value of false indicates that there is no Z-bond in the sequential CMO, and the last element of TranchePrincipals and TrancheCoupons will be treated as an ordinary tranche.

Default: false

\section*{Output Arguments}

\section*{Balance}

Matrix of size NUMTRANCHES-by-NUMTERMS, where NUMTRANCHES is the number of terms remaining and NUMTRANCHES is the number of tranches. Each element represents the principal balance at the time period corresponding to the column, and for the tranche corresponding to the row.

\section*{Principal}

Matrix of size NUMTRANCHES-by-NUMTERMS, where NUMTRANCHES is the number of terms remaining and NUMTRANCHES is the number of tranches. Each element represents the principal payments made at the time period corresponding to the column, and to the tranche corresponding to the row.

\section*{Interest}

Matrix of size NUMTRANCHES-by-NUMTERMS, where NUMTRANCHES is the number of terms remaining and NUMTRANCHES is the number of tranches. Each element represents the interest payments made at the time period
corresponding to the column, and to the tranche corresponding to the row.

\section*{Definitions Sequential Pay CMO}

A sequential pay CMO involves tranches that pay off principal sequentially. For example, consider the following case, where all principal from the underlying mortgage pool is repaid on tranche A first, then tranche B, then tranche C. Note that interest is paid on each tranche as long as the principal for the tranche has not been retired.

\section*{CMO Tranche}

Tranche is a term often used to describe a specific class of bonds within an offering wherein each tranche offers varying degrees of risk to the investor.

\section*{Examples Calculate Cash Flows for a Sequential Collateralized Mortgage Obligation (CMO)}

Define the mortgage pool under consideration for CMO structuring using mbscfamounts or mbspassthrough and calculate the cash flows with an A and B tranche for a sequential CMO.
```

MortgagePrincipal = 1000000;
Coupon = 0.12;
Terms = 6; % months
% Calculate underlying mortgage cash flows
[PrincipalBalance, MonthlyPayments, SchedPrincipalPayments, ...
InterestPayments, Prepayments] = ...
mbspassthrough(MortgagePrincipal, Coupon, Terms, Terms, 0, []);
PrincipalPayments = SchedPrincipalPayments.' + Prepayments.'

```

PrincipalPayments =
1.0e+05 *
```

Define CMO tranches, A and B.
TranchePrincipals = [500000; 500000];
TrancheCoupons = [0.12; 0.12];

```

Calculate cash flows for each tranche.
[Balance, Principal, Interest] = ...
cmoseqcf(PrincipalPayments, TranchePrincipals, TrancheCoupons, false)

Balance =
\(1.0 \mathrm{e}+05\) *
\begin{tabular}{rrrrrr}
3.3745 & 1.7328 & 0.0746 & 0 & 0 & 0 \\
5.0000 & 5.0000 & 5.0000 & 3.3999 & 1.7084 & 0.0000
\end{tabular}

Principal =
\(1.0 \mathrm{e}+05\) *
\(\begin{array}{llllll}1.6255 & 1.6417 & 1.6582 & 0.0746 & 0 & 0\end{array}\)
\(\begin{array}{llllll}0 & 0 & 0 & 1.6001 & 1.6915 & 1.7084\end{array}\)

Interest =
\(1.0 \mathrm{e}+03\) *
\begin{tabular}{rrrrrr}
5.0000 & 3.3745 & 1.7328 & 0.0746 & 0 & 0 \\
5.0000 & 5.0000 & 5.0000 & 5.0000 & 3.3999 & 1.7084
\end{tabular}
References Hayre, Lakhbir, ed., Salomon Smith Barney Guide to Mortgage-Backed and Asset-Backed Securities, John Wiley and Sons, New York, 2001.
Lyuu, Yuh-Dah, Financial Engineering and Computation, Cambridge University Press, 2004.
See Also | cmoschedcf | mbscfamounts | mbspassthrough | cmosched
Tutorials - "Using Collateralized Mortgage Obligations (CMOs)" on page 5-38
How To - "Create PAC and Sequential CMO" on page 5-53

Purpose Bond conversion factors
\begin{tabular}{ll} 
Syntax & \(C F=\) convfactor(RefDate, Maturity, CouponRate) \\
& \(C F=\) convfactor (RefDate, Maturity, CouponRate, \\
& 'ParameterName ', ParameterValue ...)
\end{tabular}

\section*{Description}

\section*{Input \\ Arguments}
\(C F=\) convfactor(RefDate, Maturity, CouponRate) computes a conversion factor for a bond futures contract.

CF = convfactor(RefDate, Maturity, CouponRate, 'ParameterName', ParameterValue ...) accepts optional inputs as one or more comma-separated parameter-value pairs. 'ParameterName' is the name of the parameter inside single quotes. 'ParameterValue is the value corresponding to 'ParameterName'. Specify parameter/value pairs in any order. Names are case-insensitive. convfactor computes a conversion factor for a bond futures contract, given a Convention value for a U.S. Treasury bond, German bond, U.K. Gilt, or Japanese Government Bond.

\section*{RefDate}

Reference dates, for which conversion factor is computed (usually the first day of delivery months).

\section*{Maturity}

Maturity date of the underlying bond.

\section*{CouponRate}

Annual coupon rate of the underlying bond in decimal.
Parameter-Valueter the following inputs only as parameter-value pairs. Pairs

\section*{Convention}

Conversion factor convention. Scalar. Valid values are:
- 1 = U.S. Treasury bond (30-year) and Treasury note (10-year) futures contract
- \(2=\) U.S. 2 -year and 5 -year Treasury note futures contract
- 3 = German Bobl, Bund, Buxl, and Schatz
- 4 = U.K. gilts
- 5 = Japanese Government Bonds (JGBs)

Default: 1

\section*{FirstCouponDate}

Irregular or normal first coupon date.

\section*{RefYield}

Reference semiannual yield.
Default: 0.06 (6\%)

\section*{StartDate}

Forward starting date of payments.

\section*{Output \\ CF}

Arguments
N-by1 vector of conversion factors against the \(6 \%\) yield par-bond.
Definitions
Conversion factors of U.S. Treasury bonds and other government bonds are based on a bond yielding \(6 \%\). Optionally, you can specify other types of bonds and yields using inputs for RefYield and Convention. For U.S. Treasury bonds, verify the output of convfactor by comparing the output against the quotations provided by the Chicago Board of Trade (http://www.cmegroup.com/company/cbot.html).

For German bonds, verify the output of convfactor by comparing the output against the quotations provided by Eurex (http://www.eurexchange.com).

For U.K. Gilts, verify the output of convfactor by comparing the output against the quotations provided by Euronext (http://www.euronext.com).

For Japanese Government Bonds, verify the output of convfactor by comparing the output against the quotations provided by the Tokyo Stock Exchange (http://www.tse.or.jp/english/).

\section*{Examples}

\section*{Compute the Conversion Factors For a Bond Futures Contract}

This example shows how to calculate CF, given the following RefDate, Maturity, and CouponRate.
```

RefDate = {'1-Dec-2002';
'1-Mar-2003';
'1-Jun-2003';
'1-Sep-2003';
'1-Dec-2003';
'1-Sep-2003';
'1-Dec-2002';
'1-Jun-2003'};

```
Maturity = \{'15-Nov-2012';
    '15-Aug-2012';
    '15-Feb-2012';
    '15-Feb-2011';
    '15-Aug-2011';
    '15-Aug-2010';
    '15-Aug-2009';
    '15-Feb-2010'\};
CouponRate \(=[0.04 ; 0.04375 ; 0.04875 ; 0.05 ; 0.05 ; 0.0575 ; 0.06 ; 0.065] ;\)
CF = convfactor(RefDate, Maturity, CouponRate)
0.8539
0.8858
0.9259
0.9418
0.9403
0.9862
1.0000
1.0266

\section*{Compute the Conversion Factor For a German Bond}

This example shows how to calculate cf, given the following RefDate, Maturity, and CouponRate for a German bond.
```

cf = convfactor('3/10/2009','1/04/2018', .04,.06,3)

```
cf \(=\)
0.8659

\section*{References Burghardt, G., T. Belton, M. Lane, and J. Papa, The Treasury Bond Basis, McGraw-Hill, 2005. \\ Krgin, Dragomir, Handbook of Global Fixed Income Calculations, John Wiley \& Sons, 2002.}
See Also \begin{tabular}{l} 
tfutbyprice | tfutbyyield | tfutimprepo | bndfutimprepo | \\
bndfutprice
\end{tabular}

How To . "Bond Futures" on page 7-12

\section*{fitFunction}

\section*{Purpose Custom fit interest-rate curve object to bond market data}

Class
@IRFunctionCurve
Synfax \begin{tabular}{ll} 
CurveObj \(=\) IRFunctionCurve.fitFunction(Type, Settle, \\
& FunctionHandle, Instruments, IRFitOptionsObj) \\
& CurveObj \(=\) IRFunctionCurve.fitFunction(Type, Settle, \\
& FunctionHandle, Instruments, IRFitOptionsObj, 'Parameter1', \\
& Value1, 'Parameter2', Value2, ...)
\end{tabular}

\section*{Arguments}
\begin{tabular}{ll} 
Type & \begin{tabular}{l} 
Type of interest-rate curve for a bond: zero, \\
forward, or discount.
\end{tabular} \\
Settle & \begin{tabular}{l} 
Scalar for the Settle date of the curve.
\end{tabular} \\
Instruments & \begin{tabular}{l} 
Function handle that defines the interest-rate \\
curve. The function handle takes two numeric \\
vectors (time-to-maturity and a vector of function \\
coefficients) and returns one numeric output \\
(interest rate or discount factor). For more \\
information on defining a function handle, see \\
the MATLAB Programming Fundamentals \\
documentation.
\end{tabular} \\
\begin{tabular}{l} 
N-by-4 data matrix for Instruments where the \\
first column is Settle date, the second column \\
is Maturity, the third column is the clean price, \\
and the fourth column is a CouponRate for the
\end{tabular} \\
bond.
\end{tabular}

Compounding (Optional) Scalar that sets the compounding frequency per year for the IRFunctionCurve object:
- \(-1=\) Continuous compounding
- \(1=\) Annual compounding
- \(2=\) Semiannual compounding (default)
- 3 = Compounding three times per year
- \(4=\) Quarterly compounding
- 6 = Bimonthly compounding
- \(12=\) Monthly compounding

Basis
(Optional) Day-count basis of the bond. A scalar of integers.
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- \(2=\) actual \(/ 360\)
- 3 = actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- \(10=\mathrm{actual} / 365\) (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

\section*{fitFunction}

For more information, see basis on page Glossary-1.

For each bond Instrument, you can specify the following additional instrument parameters as parameter/value pairs. For example, InstrumentBasis distinguishes a bond instrument's Basis value from the curve's Basis value.
\[
\begin{array}{ll}
\text { InstrumentPeriod } & \begin{array}{l}
\text { (Optional) Coupons per year of the bond. A } \\
\text { vector of integers. Allowed values are } 0,1,2 \\
\text { (default), } 3,4,6 \text {, and } 12 .
\end{array} \\
\text { InstrumentBasis } & \begin{array}{l}
\text { (Optional) Day-count basis of the bond. A } \\
\text { vector of integers. }
\end{array} \\
& \text { - } 0=\text { actual/actual (default) } \\
& \text { - } 1=30 / 360 \text { (SIA) } \\
& \text { - } 2=\text { actual/360 } \\
& \text { - } 3=\text { actual/365 } \\
& \text { - } 4=30 / 360 \text { (BMA) } \\
& \text { - } 5=30 / 360 \text { (ISDA) } \\
& \text { - } 6=30 / 360 \text { (European) } \\
& \text { - } 7=\text { actual/365 (Japanese) } \\
& \text { - } 8=\text { actual/actual (ICMA) } \\
& \text { - } 9=\text { actual/360 (ICMA) } \\
& \text { - } 10=\text { actual/365 (ICMA) } \\
& \text { - } 11=30 / 360 \mathrm{E} \text { (ICMA) } \\
& \text { - } 12=\text { actual/actual (ISDA) } \\
& \text { - } 13=\mathrm{BUS} / 252
\end{array}
\]

For more information, see basis on page Glossary-1.

InstrumentEndMonthP(Whetional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. \(0=\) ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. \(1=\) set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
InstrumentIssueDate(Optional) Date when an instrument was issued.

InstrumentFirstCoup(Ontảderal) Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

InstrumentLastCoupqumenal) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

InstrumentFace (Optional) Face or par value. Default \(=100\).

\section*{fitFunction}

Note When using Instrument parameter/value pairs, you can specify simple interest for a bond by specifying the InstrumentPeriod value as 0 . If InstrumentBasis and InstrumentPeriod are not specified for a bond, the following default values are used: Basis is 0 (act/act) and Period is 2.

\section*{Description}
```

CurveObj = IRFunctionCurve.fitFunction(Type, Settle, FunctionHandle, Instruments, IRFitOptionsObj, 'Parameter1', Value1, 'Parameter2', Value2, ...) fits a bond to a custom fitting function. You must enter the optional arguments for Basis and Compounding as parameter/value pairs.

```

\section*{Examples}
```

Settle = repmat(datenum('30-Apr-2008'),[[6 1]);
Maturity = [datenum('07-Mar-2009');datenum('07-Mar-2011');....
datenum('07-Mar-2013'); datenum('07-Sep-2016'); ...
datenum('07-Mar-2025'); datenum('07-Mar-2036')];
CleanPrice = [100.1;100.1;100.8;96.6;103.3;96.3];
CouponRate = [0.0400;0.0425;0.0450;0.0400;0.0500;0.0425];
Instruments = [Settle Maturity CleanPrice CouponRate];
CurveSettle = datenum('30-Apr-2008');
OptOptions = optimoptions('lsqnonlin','display','iter');
functionHandle = @(t,theta) polyval(theta,t);
CustomModel = IRFunctionCurve.fitFunction('Zero', CurveSettle, ...
functionHandle,Instruments, ...
IRFitOptions([.05 . 05 .05],'FitType','price',...
'OptOptions',OptOptions));

```
\begin{tabular}{cccccc} 
& & \multicolumn{3}{c}{ Norm of } & \multicolumn{1}{c}{ First-order } \\
Iteration & Func-count & \(\mathrm{f}(\mathrm{x})\) & step & optimality & CG-iterations \\
0 & 4 & 38036.7 & & \(4.92 \mathrm{e}+04\) & \\
1 & 8 & 38036.7 & 10 & \(4.92 \mathrm{e}+04\) & 0 \\
2 & 12 & 38036.7 & 2.5 & \(4.92 \mathrm{e}+04\) & 0 \\
3 & 16 & 38036.7 & 0.625 & \(4.92 \mathrm{e}+04\) & 0 \\
4 & 20 & 38036.7 & 0.15625 & \(4.92 \mathrm{e}+04\) & 0
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|}
\hline 5 & 24 & 30741.5 & 0.0390625 & \(1.72 \mathrm{e}+05\) & 0 \\
\hline 6 & 28 & 30741.5 & 0.078125 & \(1.72 \mathrm{e}+05\) & 0 \\
\hline 7 & 32 & 30741.5 & 0.0195312 & \(1.72 \mathrm{e}+05\) & 0 \\
\hline 8 & 36 & 28713.6 & 0.00488281 & \(2.33 e+05\) & 0 \\
\hline 9 & 40 & 20323.3 & 0.00976562 & \(9.47 \mathrm{e}+05\) & 0 \\
\hline 10 & 44 & 20323.3 & 0.0195312 & \(9.47 \mathrm{e}+05\) & 0 \\
\hline 11 & 48 & 20323.3 & 0.00488281 & \(9.47 \mathrm{e}+05\) & 0 \\
\hline 12 & 52 & 20323.3 & 0.0012207 & \(9.47 \mathrm{e}+05\) & 0 \\
\hline 13 & 56 & 19698.8 & 0.000305176 & \(1.08 \mathrm{e}+06\) & 0 \\
\hline 14 & 60 & 17493 & 0.000610352 & \(7 \mathrm{e}+06\) & 0 \\
\hline 15 & 64 & 17493 & 0.0012207 & \(7 \mathrm{e}+06\) & 0 \\
\hline 16 & 68 & 17493 & 0.000305176 & \(7 \mathrm{e}+06\) & 0 \\
\hline 17 & 72 & 15455.1 & 7.62939e-05 & \(2.25 \mathrm{e}+07\) & 0 \\
\hline 18 & 76 & 15455.1 & 0.000177499 & \(2.25 \mathrm{e}+07\) & 0 \\
\hline 19 & 80 & 13317.1 & 3.8147e-05 & \(3.18 \mathrm{e}+07\) & 0 \\
\hline 20 & 84 & 12865.3 & 7.62939e-05 & \(7.83 \mathrm{e}+07\) & 0 \\
\hline 21 & 88 & 11779.8 & 7.62939e-05 & \(7.58 \mathrm{e}+06\) & 0 \\
\hline 22 & 92 & 11747.6 & 0.000152588 & \(1.45 \mathrm{e}+05\) & 0 \\
\hline 23 & 96 & 11720.9 & 0.000305176 & \(2.33 \mathrm{e}+05\) & 0 \\
\hline 24 & 100 & 11667.2 & 0.000610352 & \(1.48 \mathrm{e}+05\) & 0 \\
\hline 25 & 104 & 11558.6 & 0.0012207 & \(3.55 \mathrm{e}+05\) & 0 \\
\hline 26 & 108 & 11335.5 & 0.00244141 & \(1.57 \mathrm{e}+05\) & 0 \\
\hline 27 & 112 & 10863.8 & 0.00488281 & \(6.36 \mathrm{e}+05\) & 0 \\
\hline 28 & 116 & 9797.14 & 0.00976562 & \(2.53 \mathrm{e}+05\) & 0 \\
\hline 29 & 120 & 6882.83 & 0.0195312 & \(9.18 \mathrm{e}+05\) & 0 \\
\hline 30 & 124 & 6882.83 & 0.0373993 & \(9.18 \mathrm{e}+05\) & 0 \\
\hline 31 & 128 & 3218.45 & 0.00934981 & \(1.96 \mathrm{e}+06\) & 0 \\
\hline 32 & 132 & 612.703 & 0.0186996 & \(3.01 \mathrm{e}+06\) & 0 \\
\hline 33 & 136 & 13.0998 & 0.0253882 & \(3.05 \mathrm{e}+06\) & 0 \\
\hline 34 & 140 & 0.0762922 & 0.00154002 & \(5.05 \mathrm{e}+04\) & 0 \\
\hline 35 & 144 & 0.0731652 & 3.61102e-06 & 29.9 & 0 \\
\hline 36 & 148 & 0.0731652 & 6.32335e-08 & 0.063 & 0 \\
\hline
\end{tabular}

Local minimum possible.
lsqnonlin stopped because the final change in the sum of squares relative to its initial value is less than the default value of the function tolerance.

\section*{fitFunction}

\section*{How To}
- "@IRFitOptions" on page A-10
- "@IRFunctionCurve" on page A-12

\section*{fitNelsonSiegel}


\section*{fitNelsonSiegel}

> - \(7=\) actual/365 (Japanese)
> - \(8=\) actual/actual (ICMA)
> - \(9=\) actual/360 (ICMA)
> - \(10=\) actual/365 (ICMA)
> - \(11=30 / 360 \mathrm{E}\) (ICMA)
> - \(12=\) actual/actual (ISDA)
> - \(13=\) BUS/252

For more information, see basis on page Glossary-1.

IRFitOptionsObj (Optional) Object constructed from IRFitOptions.

\section*{Instrument Parameters}

For each bond Instrument, you can specify the following additional instrument parameters as parameter/value pairs. For example, InstrumentBasis distinguishes a bond instrument's Basis value from the curve's Basis value.

InstrumentPeriod (Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2 (default), \(3,4,6\), and 12 .

InstrumentBasis
(Optional) Day-count basis of the bond. A vector of integers.
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- 3 = actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.
InstrumentEndMonthP(O)Hectional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. \(0=\) ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. \(1=\) set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.

InstrumentIssueDate(Optional) Date when an instrument was issued.

InstrumentFirstCoup(Ondàbeal) Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

\section*{fitNelsonSiegel}

InstrumentLastCoupquatenal) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

InstrumentFace (Optional) Face or par value. Default = 100.

Note When using Instrument parameter/value pairs, you can specify simple for a bond by specifying the InstrumentPeriod value as 0 . If InstrumentBasis and InstrumentPeriod are not specified for a bond, the following default values are used: Basis is 0 (act/act) and Period is 2 .
\begin{tabular}{ll} 
Description \(\quad\) CurveObj = IRFunctionCurve.fitNelsonSiegel(Type, Settle, \\
& Instruments, 'Parameter1', Value1, 'Parameter2', Value2, \\
\(\ldots\) fou fits a Nelson-Siegel function to market data for a bond. \\
& You must enter the optional arguments for Basis, Compounding, \\
& and IRFitOptionsObj as parameter/value pairs. After creating a \\
& Nelson-Siegel model, you can view the model parameters using: \\
& CurveObj. Parameters
\end{tabular}

\section*{Examples \\ Use the Nelson-Siegel Function to Fit Bond Market Data}

This example shows how to use the Nelson-Siegel function to fit bond market data.
```

Settle = repmat(datenum('30-Apr-2008'),[6 1]);
Maturity = [datenum('07-Mar-2009');datenum('07-Mar-2011');...

```
```

datenum('07-Mar-2013'); datenum('07-Sep-2016'); ...
datenum('07-Mar-2025');datenum('07-Mar-2036')];
CleanPrice = [100.1;100.1;100.8;96.6;103.3;96.3];
CouponRate = [0.0400;0.0425;0.0450;0.0400;0.0500;0.0425];
Instruments = [Settle Maturity CleanPrice CouponRate];
PlottingPoints = datenum('07-Mar-2009'):180:datenum('07-Mar-2036');
Yield = bndyield(CleanPrice,CouponRate,Settle,Maturity);
NSModel = IRFunctionCurve.fitNelsonSiegel('Zero',datenum('30-Apr-2008
NSModel.Parameters
% create the plot
plot(PlottingPoints,NSModel.getParYields(PlottingPoints), 'r')
hold on
scatter(Maturity,Yield,'black')
datetick('x')
ans =
4.6617 -1.0227 -0.3484 1.2386

```

\section*{fitNelsonSiegel}


Algorithms
The Nelson-Siegel model proposes that the instantaneous forward curve can be modeled with the following:
\[
f=\beta_{0}+\beta_{1} e^{\frac{-m}{\tau}}+\beta_{2} \frac{m}{\tau} e^{\frac{-m}{\tau}}
\]

This can be integrated to derive an equation for the zero curve (see [6] for more information on the equations and the derivation):

\section*{fitNelsonSiegel}
\[
s=\beta_{0}+\left(\beta_{1}+\beta_{2}\right) \frac{\tau}{m}\left(1-e^{\frac{-m}{\tau}}\right)-\beta_{2} e^{\frac{-m}{\tau}}
\]

See [1] for more information.

\section*{References}

See Also
Related Examples
[1] Nelson, C.R., Siegel, A.F., (1987), "Parsimonious modelling of yield curves", Journal of Business, 60, pp 473-89
[2] Svensson, L.E.O. (1994), "Estimating and interpreting forward interest rates: Sweden 1992-4", International Monetary Fund, IMF Working Paper, 1994/114
[3] Fisher, M., Nychka, D., Zervos, D. (1995), "Fitting the term structure of interest rates with smoothing splines", Board of Governors of the Federal Reserve System, Federal Reserve Board Working Paper 95-1
[4] Anderson, N., Sleath, J. (1999), "New estimates of the UK real and nominal yield curves", Bank of England Quarterly Bulletin, November, pp 384-92
[5] Waggoner, D. (1997), "Spline Methods for Extracting Interest Rate Curves from Coupon Bond Prices", Federal Reserve Board Working Paper 97-10
[6] "Zero-coupon yield curves: technical documentation", BIS Papers No. 25, October 2005
[7] Bolder, D.J., Gusba,S (2002), "Exponentials, Polynomials, and Fourier Series: More Yield Curve Modelling at the Bank of Canada," Working Papers 02-29, Bank of Canada
[8] Bolder, D.J., Streliski, D (1999), "Yield Curve Modelling at the Bank of Canada," Technical Reports 84, Bank of Canada
"@IRFitOptions" on page A-10 |"@IRFunctionCurve" on page A-12
- "Fitting Interest Rate Curve Functions" on page 9-34

\section*{fitSmoothingSpline}
\begin{tabular}{ll} 
Purpose & Fit smoothing spline to bond market data \\
Class & @IRFunctionCurve \\
Syntax & \begin{tabular}{l} 
CurveObj \(=\) IRFunctionCurve.fitSmoothingSpline(Type, Settle, \\
\\
Instruments, Lambdafun)
\end{tabular} \\
& \begin{tabular}{l} 
CurveObj \(=\) IRFunctionCurve.fitSmoothingSpline(Type, Settle, \\
\\
Instruments, Lambdafun, 'Parameter1', Value1, 'Parameter2', \\
\\
\\
\\
\end{tabular}\(\quad\).
\end{tabular}

\section*{Arguments}

Note You must have a license for Curve Fitting Toolbox software to use the fitSmoothingSpline method.
\(\left.\begin{array}{ll}\text { Type } & \begin{array}{l}\text { Type of interest-rate curve for a bond: Forward, Zero, } \\ \text { or Discount. }\end{array} \\ \text { Settle } & \begin{array}{l}\text { Scalar for the Settle date of the curve. }\end{array} \\ \text { Lnstruments } & \begin{array}{l}\text { N-by-4 data matrix for Instruments where the } \\ \text { first column is Settle date, the second column is } \\ \text { Maturity, the third column is the clean price, and the } \\ \text { fourth column is a CouponRate for the bond. }\end{array} \\ \text { land } & \begin{array}{l}\text { Penalty function that takes as its input time and } \\ \text { returns a penalty value. Use a function handle to } \\ \text { support the penalty function. The function handle for }\end{array} \\ \text { the penalty function which takes one numeric input } \\ \text { (time-to-maturity) and returns one numeric output } \\ \text { (penalty to be applied to the curvature of the spline). }\end{array}\right\}\)

\section*{fitSmoothingSpline}

Note The smoothing spline represents the forward curve. The spline is penalized for curvature by specifying a penalty function. This fit may only be done with a FitType of DurationWeightedPrice.

Knots (Optional) Vector of knot locations (times-to-maturity); by default, knots is set to be a vector comprised of 0 and the time to maturity of all input instruments. The default is for the spline type to be cubic but you can specify any spline type by explicitly specifying the knots. User-defined knots can be specified using the following command, wherek is the order: augknt (knots,k).

Compounding (Optional) Scalar that sets the compounding frequency per year for the IRFunctionCurve object:
- \(-1=\) Continuous compounding (default)
- \(1=\) Annual compounding
- \(2=\) Semiannual compounding
- 3 = Compounding three times per year
- 4 = Quarterly compounding
- 6 = Bimonthly compounding
- \(12=\) Monthly compounding

Basis (Optional) Day-count basis of the interest-rate curve. A scalar of integers.
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- 3 = actual/365

\section*{fitSmoothingSpline}
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- \(10=\) actual \(/ 365\) (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- \(12=\) actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Instrument Parameters

For each bond Instrument, you can specify the following additional instrument parameters as parameter/value pairs. For example, InstrumentBasis distinguishes a bond instrument's Basis value from the curve's Basis value.
\begin{tabular}{ll} 
InstrumentPeriod & \begin{tabular}{l} 
(Optional) Coupons per year of the bond. A \\
vector of integers. Allowed values are \(0,1,2\) \\
(default), \(3,4,6\), and 12.
\end{tabular} \\
InstrumentBasis & \begin{tabular}{l} 
(Optional) Day-count basis of the bond. A \\
vector of integers.
\end{tabular} \\
& - \(0=\) actual/actual (default) \\
& - \(1=30 / 360\) (SIA) \\
& - \(2=\) actual/360 \\
& - \(3=\) actual/365 \\
& - \(4=30 / 360\) (BMA)
\end{tabular}

\section*{fitSmoothingSpline}
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- \(10=\mathrm{actual} / 365\) (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- \(12=\) actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.

InstrumentEndMonthP(4)4etional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. \(0=\) ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. \(1=\) set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.

InstrumentIssueData(Optional) Date when an instrument was issued.

\section*{fitSmoothingSpline}

> InstrumentFirstCoup(Obtadeal) Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstcouponDate, the cash flow payment dates are determined from other inputs. InstrumentLastCoupq(01attenal) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Note When using Instrument parameter/value pairs, you can specify simple interest for a bond by specifying the InstrumentPeriod value as 0 . If InstrumentBasis and InstrumentPeriod are not specified for a bond, the following default values are used: Basis is 0 (act/act) and Period is 2.

Description Fcurve = IRFunctionCurve.fitSmoothingSpline (Type, Settle, Instruments, Lambdafun, 'Parameter1', Value1, 'Parameter2', Value2, ...) fits a smoothing spline to market data for a bond. You must enter the optional arguments for Basis, Compounding, and Knots as parameter/value pairs.

\section*{fitSmoothingSpline}

\section*{Examples Use a Smoothing Spline Function to Fit Market Data For a Bond}

This example shows how to use a smoothing spline function to fit market data for a bond.
```

Settle = repmat(datenum('30-Apr-2008'),[6 1]);
Maturity = [datenum('07-Mar-2009');datenum('07-Mar-2011');...
datenum('07-Mar-2013');datenum('07-Sep-2016');...
datenum('07-Mar-2025');datenum('07-Mar-2036')];
CleanPrice = [100.1;100.1;100.8;96.6;103.3;96.3];
CouponRate = [0.0400;0.0425;0.0450;0.0400;0.0500;0.0425];
Instruments = [Settle Maturity CleanPrice CouponRate];
PlottingPoints = datenum('07-Mar-2009'):180:datenum('07-Mar-2036');
Yield = bndyield(CleanPrice,CouponRate,Settle,Maturity);
% use the AUGKNT function to construct the knots for a cubic spline a
CustomKnots = augknt(0:5:30,4);
SmoothingModel = IRFunctionCurve.fitSmoothingSpline('Zero',datenum('3(
Instruments,@(t) 1000,'knots', CustomKnots);
% create the plot
plot(PlottingPoints,SmoothingModel.getParYields(PlottingPoints),'b')
hold on
scatter(Maturity,Yield,'black')
datetick('x')

```

\section*{fitSmoothingSpline}


Algorithms
The term structure can be modeled with a spline - specifically, one way to model the term structure is by representing the forward curve with a cubic spline. To ensure that the spline is sufficiently smooth, a penalty is imposed relating to the curvature (second derivative) of the spline:
\[
\sum_{i=1}^{N}\left[\frac{P_{i}-\hat{P}_{i}(f)}{D_{i}}\right]^{2}+\int_{0}^{M} \lambda_{t}(m)\left[f^{\prime \prime}(m)\right]^{2} d m
\]

\section*{fitSmoothingSpline}
where the first term is the difference between the observed price \(P\) and the predicted price, \(\breve{P}\), (weighted by the bond's duration, \(D\) ) summed over all bonds in our data set and the second term is the penalty term (where \(\lambda\) is a penalty function and \(f\) is the spline).

See [3], [4], [5] below.
There have been different proposals for the specification of the penalty function \(\lambda\). One approach, advocated by [4], and currently used by the UK Debt Management Office, is a penalty function of the following form:
\[
\log (\lambda(m))=L-(L-S) e^{\frac{-m}{7}}
\]

\section*{References}
[1] Nelson, C.R., Siegel, A.F., (1987), "Parsimonious modelling of yield curves", Journal of Business, 60, pp 473-89
[2] Svensson, L.E.O. (1994), "Estimating and interpreting forward interest rates: Sweden 1992-4", International Monetary Fund, IMF Working Paper, 1994/114
[3] Fisher, M., Nychka, D., Zervos, D. (1995), "Fitting the term structure of interest rates with smoothing splines", Board of Governors of the Federal Reserve System, Federal Reserve Board Working Paper 95-1
[4] Anderson, N., Sleath, J. (1999), "New estimates of the UK real and nominal yield curves", Bank of England Quarterly Bulletin, November, pp 384-92
[5] Waggoner, D. (1997), "Spline Methods for Extracting Interest Rate Curves from Coupon Bond Prices", Federal Reserve Board Working Paper 97-10
[6] "Zero-coupon yield curves: technical documentation", BIS Papers No. 25, October 2005
[7] Bolder, D.J., Gusba,S (2002), "Exponentials, Polynomials, and Fourier Series: More Yield Curve Modelling at the Bank of Canada," Working Papers 02-29, Bank of Canada

\section*{fitSmoothingSpline}
[8] Bolder, D.J., Streliski, D (1999), "Yield Curve Modelling at the Bank of Canada," Technical Reports 84, Bank of Canada

See Also "@IRFunctionCurve" on page A-12
Related
- "Fitting Interest Rate Curve Functions" on page 9-34 Examples
\begin{tabular}{|c|c|}
\hline Purpose & Fit Svensson function to bond market data \\
\hline Class & @IRFunctionCurve \\
\hline Syntax & ```
CurveObj = IRFunctionCurve.fitSvensson(Type, Settle,
Instruments)
CurveObj = IRFunctionCurve.fitSvensson(Type, Settle,
Instruments, 'Parameter1', Value1, 'Parameter2',
Value2, ...)
``` \\
\hline \multirow[t]{4}{*}{Arguments} & \begin{tabular}{ll} 
Type & Type of interest-rate curve for a bond: zero or \\
forward.
\end{tabular} \\
\hline & Settle Scalar for the Settle date of the curve. \\
\hline & Instruments \(\quad N\)-by- 4 data matrix for Instruments where the first column is Settle date, the second column is Maturity, the third column is the clean price, and the fourth column is a CouponRate for the bond. \\
\hline & \begin{tabular}{l}
Compounding \\
(Optional) Scalar that sets the compounding frequency per year for the IRFunctionCurve object:
\end{tabular} \\
\hline
\end{tabular}

\section*{fitSvensson}
\begin{tabular}{ll} 
Basis & \begin{tabular}{l} 
(Optional) Day-count basis of the interest-rate \\
curve. A scalar of integers.
\end{tabular} \\
- \(0=\) actual/actual (default) \\
- \(1=30 / 360\) (SIA) \\
& - \(2=\mathrm{actual} / 360\) \\
& - \(3=\mathrm{actual} / 365\) \\
& - \(4=30 / 360\) (BMA) \\
& - \(5=30 / 360\) (ISDA) \\
& - \(6=30 / 360\) (European) \\
& - \(7=\) actual/365 (Japanese) \\
& - \(8=\) actual/actual (ICMA) \\
& - \(9=\) actual/360 (ICMA) \\
& - \(10=\) actual/365 (ICMA) \\
& - \(11=30 / 360 \mathrm{E}\) (ICMA) \\
& - \(12=\) actual/actual (ISDA) \\
& - \(13=\) BUS/252 \\
& For more information, see basis on page \\
& Glossary-1. \\
(Optional) Object constructed from \\
IRFitOptions.
\end{tabular}

\section*{Instrument Parameters}

For each bond Instrument, you can specify the following additional instrument parameters as parameter/value pairs. For example, InstrumentBasis distinguishes a bond instrument's Basis value from the curve's Basis value.
\begin{tabular}{ll} 
InstrumentPeriod & \begin{tabular}{l} 
(Optional) Coupons per year of the bond. A \\
vector of integers. Allowed values are \(0,1,2\) \\
(default), \(3,4,6\), and 12.
\end{tabular} \\
InstrumentBasis & (Optional) Day-count basis of the bond. A \\
vector of integers. \\
& - \(0=\) actual/actual (default) \\
& - \(1=30 / 360\) (SIA) \\
& - \(2=\) actual/360 \\
& - \(3=\) actual/365 \\
& - \(4=30 / 360\) (BMA) \\
& - \(5=30 / 360\) (ISDA) \\
& - \(6=30 / 360\) (European) \\
& - \(7=\) actual/365 (Japanese) \\
& - \(8=\) actual/actual (ICMA) \\
& - \(9=\) actual/360 (ICMA) \\
& - \(10=\) actual/365 (ICMA) \\
& - \(11=30 / 360 \mathrm{E}\) (ICMA) \\
- \(12=\) actual/actual (ISDA) \\
- \(13=\mathrm{BUS} / 252\)
\end{tabular}

For more information, see basis.

\section*{fitSvensson}

InstrumentEndMonthR(whetional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. \(0=\) ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. \(1=\) set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
InstrumentIssueDate(Optional) Date when an instrument was issued.

InstrumentFirstCoup(Obtàあal) Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.
InstrumentLastCoupqualtenal) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.
InstrumentFace (Optional) Face or par value. Default = 100 .

> Note When using Instrument parameter/value pairs, you can specify simple interest for a bond by specifying the InstrumentPeriod value as 0 . If InstrumentBasis and InstrumentPeriod are not specified for a bond, the following default values are used: Basis is 0 (act/act) and Period is 2.

\section*{Description}

\section*{Examples}

CurveObj = IRFunctionCurve.fitSvensson(Type, Settle, Instruments, 'Parameter1', Value1, 'Parameter2', Value2, ...) fits the Svensson function to bond market data. You must enter the optional arguments for Basis, Compounding, and IRFitOptions as parameter/value pairs. After creating a Svensson model, you can view the model parameters using:

CurveObj. Parameters

\section*{Use a Svensson Function to Fit Bond Market Data}

This example shows how to use a Svensson function to fit bond market data.
```

Settle = repmat(datenum('30-Apr-2008'),[6 1]);
Maturity = [datenum('07-Mar-2009');datenum('07-Mar-2011');...
datenum('07-Mar-2013');datenum('07-Sep-2016');...
datenum('07-Mar-2025');datenum('07-Mar-2036')];
CleanPrice = [100.1;100.1;100.8;96.6;103.3;96.3];
CouponRate = [0.0400;0.0425;0.0450;0.0400;0.0500;0.0425];
Instruments = [Settle Maturity CleanPrice CouponRate];
PlottingPoints = datenum('07-Mar-2009'):180:datenum('07-Mar-2036');
Yield = bndyield(CleanPrice,CouponRate,Settle,Maturity);
SvenssonModel = IRFunctionCurve.fitSvensson('Zero',datenum('30-Apr-20(
SvenssonModel.Parameters
% create the plot

```
```

plot(PlottingPoints,SvenssonModel.getParYields(PlottingPoints), 'g')
hold on
scatter(Maturity,Yield,'black')
datetick('x')
ans =
0.0002 3.7616 -24.5733 34.3408 10.8871 11.9277

```


Algorithms
A very similar model to the Nelson-Siegel is the Svensson model, which adds two additional parameters to account for greater flexibility in the term structure. This model proposes that the forward rate can be modeled with the following form:
\[
f=\beta_{0}+\beta_{1} e^{\frac{-m}{1}}+\beta_{2} e^{\frac{-m}{2}} \frac{m}{\tau_{1}}+\beta_{3} e^{\frac{-m}{2}} \frac{m}{\tau_{2}}
\]

As above, this can be integrated to derive an equation for the zero curve:

\section*{fitSvensson}
\[
s=\beta_{0}+\beta_{1}\left(1-e^{\frac{-m}{\tau_{1}}}\right)\left(-\frac{\tau_{1}}{m}\right)+\beta_{2}\left(\left(1-e^{\frac{-m}{\tau_{1}}}\right) \frac{\tau_{1}}{m}-e^{\frac{m}{\tau_{1}}}\right)+\beta_{3}\left(\left(1-e^{\frac{-m}{\tau_{2}}}\right) \frac{\tau_{2}}{m}-e^{\frac{m}{r_{2}}}\right)
\]

\section*{References}

See Also "@IRFunctionCurve" on page A-12 |"@IRFitOptions" on page A-10
Related
Examples "Fitting Interest Rate Curve Functions" on page 9-34
[1] Nelson, C.R., Siegel, A.F., (1987), "Parsimonious modelling of yield curves", Journal of Business, 60, pp 473-89
[2] Svensson, L.E.O. (1994), "Estimating and interpreting forward interest rates: Sweden 1992-4", International Monetary Fund, IMF Working Paper, 1994/114
[3] Fisher, M., Nychka, D., Zervos, D. (1995), "Fitting the term structure of interest rates with smoothing splines", Board of Governors of the Federal Reserve System, Federal Reserve Board Working Paper 95-1
[4] Anderson, N., Sleath, J. (1999), "New estimates of the UK real and nominal yield curves", Bank of England Quarterly Bulletin, November, pp 384-92
[5] Waggoner, D. (1997), "Spline Methods for Extracting Interest Rate Curves from Coupon Bond Prices", Federal Reserve Board Working Paper 97-10
[6] "Zero-coupon yield curves: technical documentation", BIS Papers No. 25, October 2005
[7] Bolder, D.J., Gusba,S (2002), "Exponentials, Polynomials, and Fourier Series: More Yield Curve Modelling at the Bank of Canada," Working Papers 02-29, Bank of Canada
[8] Bolder, D.J., Streliski, D (1999), "Yield Curve Modelling at the Bank of Canada," Technical Reports 84, Bank of Canada

\section*{getDiscountFactors}
\begin{tabular}{|c|c|}
\hline Purpose & Get discount factors for input dates for IRDataCurve \\
\hline Class & @IRDataCurve \\
\hline Syntax & \(F=\) getDiscountFactors(CurveObj, InpDates) \\
\hline \multirow[t]{2}{*}{Arguments} & \(\begin{array}{ll}\text { CurveObj } & \text { Interest-rate curve object that is constructed using } \\ & \text { IRDataCurve. }\end{array}\) \\
\hline & InpDates \(\quad\) Vector of input dates using MATLAB date format.
The input dates must be after the settle date. \\
\hline Description & F = getDiscountFactors(CurveObj, InpDates) returns discount factors for the input dates. \\
\hline \multirow[t]{11}{*}{Examples} & Get Discount Factors For Input Dates for an IRDataCurve \\
\hline & This example shows how to get discount factors for input dates for an IRDataCurve.
\[
\begin{aligned}
& \text { Data }=\left[\begin{array}{llllll}
2.09 & 2.47 & 2.71 & 3.12 & 3.43 & 3.85
\end{array} 4.574 .58\right] / 100 ; \\
& \text { Dates }=\text { daysadd(today,[360 } 2 * 3603 * 3605 * 3607 * 360 \quad 10 * 36020 * 36030 * 36 \\
& \text { irdc }=\text { IRDataCurve('Zero',today, Dates, Data); } \\
& \text { irdc.getDiscountFactors(today }+30: 30: \text { today }+720)
\end{aligned}
\] \\
\hline & ans = \\
\hline & 0.9986 \\
\hline & 0.9971 \\
\hline & 0.9956 \\
\hline & 0.9940 \\
\hline & 0.9924 \\
\hline & 0.9907 \\
\hline & 0.9890 \\
\hline & 0.9873 \\
\hline
\end{tabular}

\section*{getDiscountFactors}
\[
\begin{aligned}
& 0.9855 \\
& 0.9836 \\
& 0.9817 \\
& 0.9798 \\
& 0.9778 \\
& 0.9757 \\
& 0.9736 \\
& 0.9715 \\
& 0.9693 \\
& 0.9671 \\
& 0.9649 \\
& 0.9626 \\
& 0.9602 \\
& 0.9578 \\
& 0.9554 \\
& 0.9529
\end{aligned}
\]

\section*{How To}
- "@IRDataCurve" on page A-7

\section*{getDiscountFactors}


\section*{getDiscountFactors}
\[
\begin{aligned}
& 0.9810 \\
& 0.9792 \\
& 0.9773 \\
& 0.9755 \\
& 0.9736 \\
& 0.9717 \\
& 0.9698 \\
& 0.9679 \\
& 0.9660 \\
& 0.9641 \\
& 0.9621 \\
& 0.9602 \\
& 0.9582 \\
& 0.9562
\end{aligned}
\]

\footnotetext{
How To
- "@IRFunctionCurve" on page A-12
}

- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- \(12=\) actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.

\section*{Description}

F = getForwardRates(CurveObj, InpDates, 'Parameter1', Value1, 'Parameter2', Value2, ...) returns forward rates for the input dates. You must enter the optional arguments for Basis and Compounding as parameter/value pairs. The getForwardRates method returns forward rates corresponding to the periodicity of the dates input to getForwardRates. For example, where the dates are monthly, monthly forward rates are returned. The first element of the output is the forward rate from the Settle to 1 month, the second element is the forward rate from 1 month to 2 months, etc.

\section*{Examples}

\section*{Get Forward Rates For Input Dates for an IRDataCurve}

This example shows how to get forward rates for input dates for an IRDataCurve.
```

Data = [2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100;
Dates = daysadd(today,[360 2*360 3*360 5*360 7*360 10*360 20*360 30*360],
irdc = IRDataCurve('Zero',today,Dates,Data);
irdc.getForwardRates(today+30:30:today+720)

```
```

ans =
0.0174
0.0180
0.0187
0.0193
0.0199
0.0205
0.0212
0.0218
0.0224
0.0230
0.0237
0.0243
0.0249
0.0255
0.0262
0.0268
0.0274
0.0280
0.0287
0.0293
0.0299
0.0305
0.0312
0.0318

```

How To
- "@IRDataCurve" on page A-7

- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- \(12=\) actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.

\section*{Description}

F = getForwardRates(CurveObj, InpDates, 'Parameter1', Value1, 'Parameter2', Value2, ...) returns forward rates for the input dates. You must enter the optional arguments for Basis and Compounding as parameter/value pairs.

\section*{Examples \\ Get Forward Rates For Input Dates For an IRFunctionCurve}

This example shows how to get forward rates for input dates for an IRFunctionCurve.
```

irfc = IRFunctionCurve('Forward',today,@(t) polyval([-0.0001 0.003 0.(
irfc.getForwardRates(today+30:30:today+720)

```
ans =
0.0202
0.0205
0.0207
0.0210
0.0212
0.0215
\[
\begin{aligned}
& 0.0217 \\
& 0.0219 \\
& 0.0222 \\
& 0.0224 \\
& 0.0226 \\
& 0.0229 \\
& 0.0231 \\
& 0.0233 \\
& 0.0235 \\
& 0.0238 \\
& 0.0240 \\
& 0.0242 \\
& 0.0244 \\
& 0.0247 \\
& 0.0249 \\
& 0.0251 \\
& 0.0253 \\
& 0.0255
\end{aligned}
\]

How To . "@IRFunctionCurve" on page A-12
\begin{tabular}{|c|c|}
\hline Purpose & Get par yields for input dates for IRDataCurve \\
\hline Class & @IRDataCurve \\
\hline Syntax & \begin{tabular}{l}
\(F=\) getParYields(CurveObj, InpDates) \\
F = getParYields(CurveObj, InpDates, 'Parameter1', Value1, 'Parameter2', Value2, ...)
\end{tabular} \\
\hline \multirow[t]{9}{*}{Arguments} & \(\begin{array}{ll}\text { CurveObj } & \text { Interest-rate curve object that is constructed using } \\ & \text { IRDataCurve. }\end{array}\) \\
\hline & InpDates \(\quad\) Vector of input dates using MATLAB date format.
The input dates must be after the settle date. \\
\hline & \begin{tabular}{l}
(Optional) Scalar that sets the compounding frequency per year for the par yield rates are: \\
- \(-1=\) Continuous compounding \\
- \(1=\) Annual compounding \\
- \(2=\) Semiannual compounding (default) \\
- 3 = Compounding three times per year \\
- \(4=\) Quarterly compounding \\
- \(6=\) Bimonthly compounding \\
- \(12=\) Monthly compounding
\end{tabular} \\
\hline & Basis (Optional) Day-count basis values for the par yield rates: \\
\hline & - \(0=\) actual/actual (default) \\
\hline & - 1 = 30/360 (SIA) \\
\hline & - \(2=\) actual/360 \\
\hline & - \(3=\) actual/365 \\
\hline & \\
\hline
\end{tabular}
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- \(12=\) actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.

\section*{Description}

F = getParYields(CurveObj, InpDates, 'Parameter1', Value1, 'Parameter2', Value2, ...) returns par yields for the input dates. You must enter the optional arguments for Basis and Compounding as parameter/value pairs.

\section*{Examples Get Par Yields For Input Dates For an IRDataCurve}

This example shows how to get par yields for input dates for an IRDataCurve.
```

Data = [2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100;
Dates = daysadd(today,[360 2*360 3*360 5*360 7*360 10*360 20*360 30*360],
irdc = IRDataCurve('Zero',today,Dates,Data);
irdc.getParYields(today+30:30:today+720)

```
ans =
    0.0176
    0.0176
\[
\begin{aligned}
& 0.0180 \\
& 0.0182 \\
& 0.0186 \\
& 0.0188 \\
& 0.0192 \\
& 0.0197 \\
& 0.0199 \\
& 0.0203 \\
& 0.0205 \\
& 0.0208 \\
& 0.0212 \\
& 0.0214 \\
& 0.0218 \\
& 0.0220 \\
& 0.0224 \\
& 0.0226 \\
& 0.0230 \\
& 0.0234 \\
& 0.0236 \\
& 0.0240 \\
& 0.0242 \\
& 0.0245
\end{aligned}
\]

How To
- "@IRDataCurve" on page A-7
\begin{tabular}{|c|c|}
\hline Purpose & Get par yields for input dates for IRFunctionCurve \\
\hline Class & @IRFunctionCurve \\
\hline Syntax & \[
\begin{aligned}
& F=\text { getParYields(CurveObj, InpDates) } \\
& F=\text { getParYields(CurveObj, InpDates, 'Parameter1', } \\
& \text { Value1, 'Parameter2', Value2, ...) }
\end{aligned}
\] \\
\hline \multirow[t]{8}{*}{Arguments} & \begin{tabular}{ll} 
Curve0bj & Interest-rate curve object that is constructed using \\
& IRFunctionCurve.
\end{tabular} \\
\hline & \(\begin{array}{ll}\text { InpDates } & \text { Vector of input dates using MATLAB date format. } \\ & \text { The input dates must be after the settle date. }\end{array}\) \\
\hline & \begin{tabular}{l}
Compounding (Optional) Scalar that sets the compounding frequency per year for par yield rates are: \\
- \(-1=\) Continuous compounding \\
- \(1=\) Annual compounding \\
- \(2=\) Semiannual compounding (default) \\
- 3 = Compounding three times per year \\
- \(4=\) Quarterly compounding \\
- \(6=\) Bimonthly compounding \\
- \(12=\) Monthly compounding
\end{tabular} \\
\hline & Basis (Optional) Day-count basis values for par yield rates: \\
\hline & \begin{tabular}{l}
- \(0=\) actual/actual (default) \\
- \(1=30 / 360\) (SIA)
\end{tabular} \\
\hline & - 2 = actual/360 \\
\hline & - 3 = actual/365 \\
\hline & - \(4=30 / 360\) (BMA) \\
\hline
\end{tabular}
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.

\section*{Description}

F = getParYields(CurveObj, InpDates, 'Parameter1', Value1, 'Parameter2', Value2, ...) returns par yields for the input dates. You must enter the optional arguments for Basis and Compounding as parameter/value pairs.

\section*{Examples Get Par Yields For Input Dates For an IRFunctionCurve}

This example shows how to get par yields for input dates for an IRFunctionCurve.
irfc = IRFunctionCurve('Forward',today,@(t) polyval([-0.0001 0.003 0. irfc.getParYields(today+30:30:today+720)
ans \(=\)
0.0204
0.0201
0.0204
0.0204
\[
\begin{aligned}
& 0.0206 \\
& 0.0206 \\
& 0.0207 \\
& 0.0211 \\
& 0.0211 \\
& 0.0213 \\
& 0.0213 \\
& 0.0214 \\
& 0.0216 \\
& 0.0217 \\
& 0.0218 \\
& 0.0219 \\
& 0.0220 \\
& 0.0221 \\
& 0.0222 \\
& 0.0224 \\
& 0.0225 \\
& 0.0226 \\
& 0.0227 \\
& 0.0228
\end{aligned}
\]

How To . "@IRFunctionCurve" on page A-12

- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- \(12=\) actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.

Description
F = getZeroRates(CurveObj, InpDates, 'Parameter1', Value1, 'Parameter2', Value2, ...) returns zero rates for the input dates. You must enter the optional arguments for Basis and Compounding as parameter/value pairs.

\section*{Examples Get Zero Rates For Input Dates For an IRDataCurve}

This example shows how to get zero rates for input dates for an IRDataCurve.

Data \(=\left[\begin{array}{llll} & .09 & 2.47 & 2.71 \\ 3.12 & 3.43 & 3.854 .574 .58] / 100 ;\end{array}\right.\)
Dates = daysadd(today,[360 2*360 3*360 5*360 7*360 10*360 20*360 30*360],
irdc = IRDataCurve('Zero',today, Dates, Data);
irdc.getZeroRates(today+30:30:today+720)
ans \(=\)
0.0174
0.0177
0.0180
0.0183
\[
\begin{aligned}
& 0.0187 \\
& 0.0190 \\
& 0.0193 \\
& 0.0196 \\
& 0.0199 \\
& 0.0202 \\
& 0.0205 \\
& 0.0208 \\
& 0.0212 \\
& 0.0215 \\
& 0.0218 \\
& 0.0221 \\
& 0.0224 \\
& 0.0227 \\
& 0.0230 \\
& 0.0233 \\
& 0.0237 \\
& 0.0240 \\
& 0.0243 \\
& 0.0246
\end{aligned}
\]

How To . "@IRDataCurve" on page A-7

\section*{getZeroRates}

- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- \(12=\) actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.

\section*{Description}

F = getZeroRates(CurveObj, InpDates, 'Parameter1', Value1, 'Parameter2', Value2, ...) returns zero rates for the input dates. You must enter the optional arguments for Basis and Compounding as parameter/value pairs.

\section*{Examples \\ Get Zero Rates For Input Dates For an IRFunctionCurve}

This example shows how to get zero rates for input dates for an IRFunctionCurve.
```

irfc = IRFunctionCurve('Forward',today,@(t) polyval([-0.0001 0.003 0.(
irfc.getZeroRates(today+30:30:today+720)

```
ans =
0.0201
0.0202
0.0204
0.0205
0.0206
0.0207

\section*{getZeroRates}
\[
\begin{aligned}
& 0.0209 \\
& 0.0210 \\
& 0.0211 \\
& 0.0212 \\
& 0.0213 \\
& 0.0214 \\
& 0.0216 \\
& 0.0217 \\
& 0.0218 \\
& 0.0219 \\
& 0.0220 \\
& 0.0221 \\
& 0.0223 \\
& 0.0224 \\
& 0.0225 \\
& 0.0226 \\
& 0.0227 \\
& 0.0228
\end{aligned}
\]
- "@IRFunctionCurve" on page A-12

\section*{IRBootstrapOptions}


\section*{IRBootstrapOptions}

\section*{ConvexityAdjustment: [10x1 double]}

\author{
How To \\ - "@IRDataCurve" on page A-7
}
\begin{tabular}{ll} 
Purpose & Construct interest-rate curve object from dates and data \\
Class & @IRDataCurve
\end{tabular}

\section*{IRDataCurve}

Basis (Optional) Day-count basis of the interest-rate curve. A scalar of integers.
- \(0=\) actual/actual (default)
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- \(10=\) actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
InterpMethod (Optional) Values are:
- 'linear' - Linear interpolation (default).
- 'constant' - Piecewise constant interpolation.
- 'pchip' - Piecewise cubic Hermite interpolation.
- 'spline' - Cubic spline interpolation.

\section*{Description}

\section*{Examples}

CurveObj = IRDataCurve(Type, Settle, Dates, Data, 'Parameter1', Value1, 'Parameter2', Value2, ...) constructs an interest-rate curve with the specified Dates and Data. You must enter the optional arguments for Basis, Compounding, and InterpMethod as parameter/value pairs.

Alternatively, an IRDataCurve object can be bootstrapped from market data using the bootstrap method.

After an IRDataCurve curve object is constructed, you can use the following methods to determine the forward rates, zero rates, and discount factors. In addition, you can use the toRateSpec method to convert the interest-rate curve object to a RateSpec structure.
\begin{tabular}{l|l}
\hline Method & Description \\
\hline getForwardRates & Returns forward rates for input dates. \\
\hline getZeroRates & Returns zero rates for input dates. \\
\hline getDiscountFactors & Returns discount factors for input dates. \\
\hline getParYields & Returns par yields for input dates. \\
\hline toRateSpec & \begin{tabular}{l} 
Converts to be a RateSpec object; this \\
structure is identical to the RateSpec
\end{tabular} \\
\hline bootstrap & \begin{tabular}{l} 
Bootstraps an interest rate curve from \\
market data.
\end{tabular} \\
\hline
\end{tabular}
```

Data = [l2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100;
Dates = daysadd(today,[360 2*360 3*360 5*360 7*360 10*360 20*360 30*360],1);
irdc = IRDataCurve('Zero',today,Dates,Data)
irdc =

```

Properties:
Dates: [8x1 double]

\section*{IRDataCurve}

\author{
Data: [8x1 double] \\ InterpMethod: 'linear' \\ Type: 'Zero' \\ Settle: 733599 \\ Compounding: 2 \\ Basis: 0
}

How To - "@IRCurve" on page A-4

\section*{IRFitOptions}
Purpose Construct specific options for fitting interest-rate curve object
Class@IRFitOptions
Syntax myfitoptions = IRFitOptions(InitialGuess) ..... myfitoptions = IRFitOptions(InitialGuess, 'Parameter1', Value1)
ArgumentsInitialGuess Initial guess for the parameters of the curvefunction. Vector of values for the starting pointof the optimization.FitType (Optional) Price, Yield, orDurationWeightedPrice determines which isminimized in the curve fitting process. The defaultis DurationWeightedPrice.UpperBound (Optional) Lower bound for the parameters of thecurve function.LowerBound (Optional) Upper bound for the parameters of thecurve function.OptOptions (Optional) Optimization solver options constructedwith optimoptions from the Optimization Toolbox(optimset is also supported).
Description myfitoptions = IRFitOptions('Param1', Value1) constructs the IRFitOptions structure with an initial guess or with an initial guess and bounds. You must enter the optional arguments for FitType, UpperBound, LowerBound, and OptOptions as parameter/value pairs.

Note IRFitOptions constructor must be used with fitFunction method when building a custom fitting function.

\section*{IRFitOptions}
```

Examples myfitoptions = IRFitOptions([lllllo,'FitType','yield')
myfitoptions =
Properties:
FitType: 'yield'
InitialGuess: [7 2 1 0]
UpperBound: []
LowerBound: []
OptOptions: []

```
How To
- "@IRFunctionCurve" on page A-12

\section*{IRFunctionCurve}
\begin{tabular}{ll} 
Purpose & \begin{tabular}{l} 
Construct interest-rate curve object from function handle or function \\
and fit to market data
\end{tabular} \\
Class & @IRFunctionCurve
\end{tabular}

\section*{IRFunctionCurve}
\begin{tabular}{|c|c|}
\hline Basis & (Optional) Day-count basis of the bond. A scalar \\
\hline & - \(0=\mathrm{actual} / \mathrm{actual}\) (default) \\
\hline & - \(1=30 / 360\) (SIA) \\
\hline & - 2 = actual/360 \\
\hline & - 3 = actual/365 \\
\hline & - \(4=30 / 360\) (BMA) \\
\hline & - \(5=30 / 360\) (ISDA) \\
\hline & - \(6=30 / 360\) (European) \\
\hline & - \(7=\) actual/365 (Japanese) \\
\hline & - \(8=\) actual/actual (ICMA) \\
\hline & - \(9=\) actual/360 (ICMA) \\
\hline & - \(10=\mathrm{actual} / 365\) (ICMA) \\
\hline & - \(11=30 / 360 \mathrm{E}\) (ICMA) \\
\hline & - \(12=\) actual/actual (ISDA) \\
\hline & - 13 = BUS/252 \\
\hline & For more information, see basis on page Glossary-1. \\
\hline FunctionHandle & Function handle that defines the interest-rate curve. The function handle requires one numeric input (time-to-maturity) and returns one numeric output (interest rate or discount factor). For more information on defining a function handle, see the MATLAB Programming Fundamentals documentation. \\
\hline Parameters & Fitted parameters for function. \\
\hline
\end{tabular}

\section*{IRFunctionCurve}

\section*{Description}

CurveObj = IRFunctionCurve(Type, Settle, FunctionHandle, 'Parameter1', Value1, 'Parameter2', Value2, ...) constructs an interest-rate curve object directly by specifying a function handle. You must enter the optional arguments for Basis and Compounding as parameter/value pairs.

After you use the IRFunctionCurve constructor to create an IRFunctionCurve object, you can fit the bond using the following methods.
\begin{tabular}{l|l}
\hline Method & Description \\
\hline getForwardRates & Returns forward rates for input dates. \\
\hline getZeroRates & Returns zero rates for input dates. \\
\hline getDiscountFactors & \begin{tabular}{l} 
Returns discount factors for input \\
dates.
\end{tabular} \\
\hline getParYields & Returns par yields for input dates. \\
\hline toRateSpec & \begin{tabular}{l} 
Converts to be a RateSpec object. \\
This RateSpec structure is identical \\
to the RateSpec produced by the \\
Financial Instruments Toolbox \\
function intenvset.
\end{tabular} \\
\hline
\end{tabular}

Alternatively, you can construct an IRFunctionCurve object using the following static methods.
\begin{tabular}{l|l}
\hline Static Method & Description \\
\hline fitNelsonSiegel & \begin{tabular}{l} 
Fits a Nelson-Siegel function to \\
market data.
\end{tabular} \\
\hline fitSvensson & \begin{tabular}{l} 
Fits a Svensson function to market \\
data.
\end{tabular} \\
\hline fitSmoothingSpline & \begin{tabular}{l} 
Fits a smoothing spline function to \\
market data.
\end{tabular} \\
\hline fitFunction & Fits a custom function to market data. \\
\hline
\end{tabular}

\section*{IRFunctionCurve}
```

Examples irfc = IRFunctionCurve('Forward',today,@(t) polyval([-0.0001 0.003 0.02],t))
irfc =
Properties:
FunctionHandle: @(t)polyval([-0.0001,0.003,0.02],t)
Type: 'Forward'
Settle: 733599
Compounding: 2
Basis: 0
How To
-"@IRCurve" on page A-4

```

\section*{liborduration}

\section*{Purpose Duration of LIBOR-based interest-rate swap}
```

Syntax
[PayFixDuration GetFixDuration] = liborduration(SwapFixRate,
Tenor, Settle)

```

\section*{Arguments}

\section*{Description}

Examples
[PayFixDuration GetFixDuration] = liborduration(SwapFixRate, Tenor, Settle) computes the duration of LIBOR-based interest-rate swaps.

PayFixDuration is the modified duration, in years, realized when entering pay-fix side of the swap.

GetFixDuration is the modified duration, in years, realized when entering receive-fix side of the swap.

\section*{Compute the Duration of LIBOR-Based Interest-Rate Swaps}

This example shows how to compute the duration of LIBOR-based interest-rate swaps using the following data.
```

SwapFixRate = 0.0383;
Tenor = 7;
Settle = datenum('11-Oct-2002');

```
[PayFixDuration GetFixDuration] = liborduration(SwapFixRate,...
Tenor, Settle)

PayFixDuration =

\section*{liborduration}
\(-4.7567\)

\section*{GetFixDuration =}
4.7567

\author{
See Also \\ liborfloat2fixed | liborprice
}

\section*{liborfloat2fixed}
\begin{tabular}{|c|c|c|}
\hline Purpose & \multicolumn{2}{|l|}{Compute par fixed-rate of swap given 3-month LIBOR data} \\
\hline Syntax & \multicolumn{2}{|l|}{```
[FixedSpec, ForwardDates, ForwardRates] =
liborfloat2fixed(ThreeMonthRates, Settle, Tenor, StartDate,
Interpolation, ConvexAdj, RateParam, InArrears, Sigma,
FixedCompound, FixedBasis)
```} \\
\hline \multirow[t]{6}{*}{Arguments} & ThreeMonthRates & Three-month Eurodollar futures data or forward rate agreement data. (A forward rate agreement stipulates that a certain interest rate applies to a certain principal amount for a given future time period.) An n-by-3 matrix in the form of [month year IMMQuote]. The floating rate is assumed to compound quarterly and to accrue on an actual/360 basis. \\
\hline & Settle & Settlement date of the swap. Scalar. \\
\hline & Tenor & Life of the swap. Scalar. \\
\hline & StartDate & (Optional) Scalar value to denote reference date for valuation of (forward) swap. This in effect allows forward swap valuation. Default = Settle. \\
\hline & Interpolation & (Optional) Interpolation method to determine applicable forward rate for months when no Eurodollar data is available. Default is 'linear' or 1. Other possible values are 'Nearest' or 0 , and 'Cubic' or 2. \\
\hline & ConvexAdj & (Optional) Default = 0 (off). \(1=\) on. Denotes whether futures/forward convexity adjustment is required. Pertains to forward rate adjustments when those rates are taken from Eurodollar futures data. \\
\hline
\end{tabular}
\(\left.\begin{array}{ll}\text { RateParam } & \left.\begin{array}{l}\text { (Optional) Short-rate model's parameters } \\ \text { (Hull-White) [a S], where the short-rate } \\ \text { process is: } \\ d r\end{array}\right][\theta(t)-a r] d t+S d z . \\ & \text { Default }=[0.050 .015] . \\ \text { (Optional) Default }=0 \text { (off). Set to } 1 \text { for on. If } \\ \text { on, the routine does an automatic a convexity } \\ \text { adjustment to forward rates. }\end{array}\right\}\)

For more information, see basis on page Glossary-1.

\section*{Description}
[FixedSpec, ForwardDates, ForwardRates] = liborfloat2fixed(ThreeMonthRates, Settle, Tenor, StartDate, Interpolation, ConvexAdj, RateParam, InArrears, Sigma, FixedCompound, FixedBasis) computes forward rates, dates, and the swap fixed rate.

FixedSpec specifies the structure of the fixed-rate side of the swap:
- Coupon: Par-swap rate
- Settle: Start date
- Maturity: End date
- Period: Frequency of payment
- Basis: Accrual basis

ForwardDates are dates corresponding to ForwardRates (all third Wednesdays of the month, spread 3 months apart). The first element is the third Wednesday immediately after Settle.

ForwardRates are forward rates corresponding to the forward dates, quarterly compounded, and on the actual/360 basis.

Note To preserve input integrity, Tenor is rounded upward to the closest integer. Currently traded tenors are 2,5 , and 10 years.

The function assumes that floating-rate observations occur quarterly on the third Wednesday of a delivery month. The first delivery month is the month of the first third Wednesday after Settle. Floating-side payments occur on the third-month anniversaries of observation dates.

\section*{liborfloat2fixed}

\section*{Examples Compute the Par Fixed-Rate of a Swap Given 3-Month LIBOR Data}

This example shows how to compute the par fixed-rate of a swap given 3 -month LIBOR data. Use the supplied EDdata.xls file as input to a liborfloat2fixed computation.
```

[EDFutData, textdata] = xlsread('EDdata.xls');
Settle = datenum('15-Oct-2002');
Tenor = 2;

```
[FixedSpec, ForwardDates, ForwardRates] =...
liborfloat2fixed(EDFutData(:,1:3), Settle, Tenor)

FixedSpec =
Coupon: 0.0222
Settle: '16-Oct-2002'
Maturity: '16-Oct-2004'
Period: 4
Basis: 1

ForwardDates =
731505
731596
731687
731778
731869
731967
732058
732149

ForwardRates =
\[
\begin{aligned}
& 0.0177 \\
& 0.0166 \\
& 0.0170 \\
& 0.0188 \\
& 0.0214 \\
& 0.0248 \\
& 0.0279 \\
& 0.0305
\end{aligned}
\]

\section*{See Also \\ liborduration | liborprice}

\section*{liborprice}

\section*{Purpose Price swap given swap rate}

\author{
Syntax Price = liborprice(ThreeMonthRates, Settle, Tenor, SwapRate, StartDate, Interpolation, ConvexAdj, RateParam, InArrears, Sigma, FixedCompound, FixedBasis)
}

\section*{Arguments}

Settle Settlement date of swap. Scalar.
Tenor
SwapRate
StartDate

Interpolation (Optional) Interpolation method to determine applicable forward rate for months when no Eurodollar data is available. Default is 'linear' or 1. Other possible values are 'Nearest' or 0 , and 'Cubic' or 2.
ConvexAdj (Optional) Default = 0 (off). \(1=\) on. Denotes whether futures/forward convexity adjustment is required. Pertains to forward rate adjustments when those rates are taken from Eurodollar futures data.
\begin{tabular}{|c|c|}
\hline RateParam & (Optional) Short-rate model's parameters (Hull-White) [a S], where the short-rate process is:
\[
d r=[\theta(t)-a r] d t+S d z
\]
\[
\text { Default }=\left[\begin{array}{ll}
0.05 & 0.015
\end{array}\right] .
\] \\
\hline InArrears & (Optional) Default \(=0\) (off). Set to 1 for on. If on, the routine does an automatic convexity adjustment to forward rates. \\
\hline Sigma & (Optional) Overall annual volatility of caplets. \\
\hline FixedCompound & (Optional) Scalar value. Compounding or frequency of payment on the fixed side. Also, the reset frequency. Default \(=4\) (quarterly). Other values are 1,2 , and 12 . \\
\hline \multirow[t]{14}{*}{FixedBasis} & (Optional) \\
\hline & - \(0=\) actual/actual (default) \\
\hline & - 1 = 30/360 (SIA) \\
\hline & - 2 = actual/360 \\
\hline & - \(3=\) actual/365 \\
\hline & - \(4=30 / 360\) (BMA) \\
\hline & - \(5=30 / 360\) (ISDA) \\
\hline & - \(6=30 / 360\) (European) \\
\hline & - \(7=\) actual/365 (Japanese) \\
\hline & - \(8=\) actual/actual (ICMA) \\
\hline & - \(9=\) actual/360 (ICMA) \\
\hline & - 10 = actual/365 (ICMA) \\
\hline & - \(11=30 / 360 \mathrm{E}\) (ICMA) \\
\hline & - \(12=\) actual/actual (ISDA) \\
\hline
\end{tabular}

\section*{liborprice}

For more information, see basis on page Glossary-1.

\section*{Description \\ Price = liborprice(ThreeMonthRates, Settle, Tenor, SwapRate, StartDate, Interpolation, ConvexAdj, RateParam, InArrears, Sigma, FixedCompound, FixedBasis) computes the price per \(\$ 100\) notional value of a swap given the swap rate. A positive result indicates that fixed side is more valuable than the floating side. \\ Price is the present value of the difference between floating and fixed-rate sides of the swap per \(\$ 100\) notional. \\ Examples Compute the Price Per \(\mathbf{\$ 1 0 0}\) Notional Value of a Swap Given the Swap Rate}

This example shows that a swap paying the par swap rate has a value of 0 .
\% load the input data
[EDFutData, textdata] = xlsread('EDdata.xls');
Settle = datenum('15-Oct-2002');
Tenor = 2;
\% compute the fixed rate from the Eurodollar data
FixedSpec = liborfloat2fixed(EDFutData(:,1:3), Settle, Tenor)
\% compute the price of a par swap
Price = liborprice(EDFutData(:,1:3), Settle, Tenor, FixedSpec.Coupon)

FixedSpec =
Coupon: 0.0222
Settle: '16-Oct-2002'
Maturity: '16-Oct-2004'
Period: 4
Basis: 1
```

Price =
2.7756e-15

```

Price is effectively equal to 0 .

\section*{See Also \\ liborduration | liborfloat2fixed}

Purpose Cash flow and time mapping for mortgage pool
```

Syntax
[CFlowAmounts, CFlowDates, TFactors, Factors, Payment, Principal,
Interest, Prepayment] = mbscfamounts(Settle, Maturity, IssueDate, GrossRa
CouponRate, Delay, PrepaySpeed, PrepayMatrix)

```

\section*{Arguments}
\begin{tabular}{ll} 
Settle & \begin{tabular}{l} 
Settlement date. A serial date number or date \\
string. Settle must be earlier than Maturity. \\
Maturity date. A serial date number or date \\
string.
\end{tabular} \\
Maturity & \begin{tabular}{l} 
Issue date. A serial date number or date string.
\end{tabular} \\
IssueDate & GrossRate \\
CouponRate & \begin{tabular}{l} 
Gross coupon rate (including fees), in decimal. \\
(Optional) Net coupon rate, in decimal. Default \\
= GrossRate.
\end{tabular} \\
Delay & \begin{tabular}{l} 
(Optional) Delay in days. Default is 0 (no \\
delay).
\end{tabular} \\
PrepaySpeed & \begin{tabular}{l} 
(Optional) Relation of the conditional payment \\
rate (CPR) to the benchmark model. Default \\
= 0. If you input a customized prepayment \\
matrix, set PrepaySpeed to [].
\end{tabular} \\
PrepayMatrix & \begin{tabular}{l} 
(Optional) Used only when PrepaySpeed is \\
unspecified. Customized prepayment vector. A
\end{tabular} \\
& \begin{tabular}{l} 
NaN-padded matrix of size \\
max(TermRemaining) -by-NMBS. Each column
\end{tabular} \\
corresponds to each mortgage-backed security,
\end{tabular}

All inputs (except PrepayMatrix) are number of mortgage-backed securities (NMBS)-by-1 vectors.

\section*{Description}

\section*{Examples Calculate Cash Flow Amounts and Dates, Time Factors, and Mortgage Factors for a Single Mortgage}

Given a mortgage with the following characteristics, compute the cash flow amounts and dates, the time factors, and the mortgage factors.

Define the mortgage characteristics.
```

Settle = datenum('17-April-2002');
Maturity = datenum('1-Jan-2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
CouponRate = 0.075;
Delay = 14;
PrepaySpeed = 100;

```

Use mbscfamonts to evaluate the mortgage.
[CFlowAmounts, CFLowDates, TFactors, Factors] = ... mbscfamounts(Settle, Maturity, IssueDate, GrossRate, ... CouponRate, Delay, PrepaySpeed)

CFlowAmounts =
Columns 1 through 7
\(-0.0033\)
0.0118
0.0120
0.0121
0.0120
0.0119
0.0119

Columns 8 through 14
\(\begin{array}{lllllll}0.0118 & 0.0117 & 0.0117 & 0.0116 & 0.0115 & 0.0115 & 0.0114\end{array}\)

Columns 15 through 21
\begin{tabular}{lllllll}
0.0114 & 0.0113 & 0.0112 & 0.0112 & 0.0111 & 0.0110 & 0.0110
\end{tabular}

Columns 22 through 28
0.0109
0.0109
0.0108
0.0107
0.0107
0.0106
0.0106

Columns 29 through 35
\begin{tabular}{lllllll}
0.0105 & 0.0105 & 0.0104 & 0.0103 & 0.0103 & 0.0102 & 0.0102
\end{tabular}

Columns 36 through 42
\[
\begin{array}{lllllll}
0.0101 & 0.0101 & 0.0100 & 0.0099 & 0.0099 & 0.0098 & 0.0098
\end{array}
\]

Columns 43 through 49
\begin{tabular}{lllllll}
0.0097 & 0.0097 & 0.0096 & 0.0096 & 0.0095 & 0.0095 & 0.0094
\end{tabular}

Columns 50 through 56
0.0094
0.0093
0.0093
0.0092
0.0092
0.0091
0.009

Columns 57 through 63
\(\begin{array}{lllllll}0.0090 & 0.0089 & 0.0089 & 0.0088 & 0.0088 & 0.0087 & 0.008\end{array}\)
Columns 64 through 70
0.0087
0.0086
0.0086
0.0085
0.0085
0.0084
0.008

Columns 71 through 77
\(\begin{array}{lllllll}0.0083 & 0.0083 & 0.0082 & 0.0082 & 0.0081 & 0.0081 & 0.008\end{array}\)

Columns 78 through 84
\(\begin{array}{lllllll}0.0080 & 0.0079 & 0.0079 & 0.0079 & 0.0078 & 0.0078 & 0.0077\end{array}\)

Columns 85 through 91
0.0077
0.0076
0.0076
0.0075
0.0075
0.0075
0.007

Columns 92 through 98
\begin{tabular}{lllllll}
0.0074 & 0.0073 & 0.0073 & 0.0073 & 0.0072 & 0.0072 & 0.007
\end{tabular}

Columns 99 through 105
\begin{tabular}{lllllll}
0.0071 & 0.0070 & 0.0070 & 0.0070 & 0.0069 & 0.0069 & 0.0068
\end{tabular}

Columns 106 through 112
\begin{tabular}{lllllll}
0.0068 & 0.0068 & 0.0067 & 0.0067 & 0.0066 & 0.0066 & 0.0066
\end{tabular}

11-1483

Columns 113 through 119
0.0065
0.0065
0.0065
0.0064
0.0064
0.0063
0.0063

Columns 120 through 126
0.0063
0.0062
0.0062
0.0062
0.0061
0.0061
0.0061

Columns 127 through 133
\[
\begin{array}{lllllll}
0.0060 & 0.0060 & 0.0059 & 0.0059 & 0.0059 & 0.0058 & 0.0058
\end{array}
\]

Columns 134 through 140
0.0058
0.0057
0.0057
0.0057
0.0056
0.0056
0.0056

Columns 141 through 147
\(\begin{array}{lllllll}0.0055 & 0.0055 & 0.0055 & 0.0054 & 0.0054 & 0.0054 & 0.0053\end{array}\)
Columns 148 through 154
0.0053
0.0053
0.0052
0.0052
0.0052
0.0052
0.0051

Columns 155 through 161
\(\begin{array}{lllllll}0.0051 & 0.0051 & 0.0050 & 0.0050 & 0.0050 & 0.0049 & 0.0049\end{array}\)

Columns 162 through 168
0.0049
0.0048
0.0048
0.0048
0.0048
0.0047
0.0047

Columns 169 through 175
0.0047
0.0046
0.0046
0.0046
0.0046
0.0045
0.0045

Columns 176 through 182
0.0045 0.0044 0.0044 0.0044 0.0044 0.0043 0.004
Columns 183 through ..... 189
0.0043 0.0043 0.0042 0.0042 0.0042 0.0041 0.004
Columns 190 through ..... 196
0.0041 0.0041 0.0040 0.0040 0.0040 0.0040 0.003
Columns 197 through ..... 203
0.0039 0.0039 0.0039 0.0038 0.0038 0.0038 0.0038
Columns 204 through 210
0.0037 0.0037 0.0037 0.0037 0.0036 0.0036 0.003
Columns 211 through ..... 217
0.0036 0.0035 0.0035 0.0035 0.0035 0.0035 0.003
Columns 218 through ..... 224
0.0034 0.0034 0.0034 0.0033 0.0033 0.0033 0.003
Columns 225 through ..... 231
0.0033 0.0032 0.0032 0.0032 0.0032 0.0031 0.003
Columns 232 through ..... 238
0.0031 0.0031 0.0031 0.0030 0.0030 0.0030 0.003
Columns 239 through ..... 245
0.0030
0.0029
0.0029
0.0029
0.0029
0.0029
0.0028

Columns 246 through 252
0.0028
0.0028
0.0028
0.0028
0.0027
0.0027
0.0027

Columns 253 through 259
0.0027
0.0027
0.0026
0.0026
0.0026
0.0026
0.0026

Columns 260 through 266
0.0025
0.0025
0.0025
0.0025
0.0025
0.0024
0.0024

Columns 267 through 273
0.0024
0.0024
0.0024
0.0024
0.0023
0.0023
0.0023

Columns 274 through 280
\[
0.0023
\]
0.0023
0.0023
0.0022
0.0022
0.0022
0.0022

Columns 281 through 287
0.0022
0.0021
0.0021
0.0021
0.0021
0.0021
0.0021

Columns 288 through 294
0.0020
0.0020
0.0020
0.0020
0.0020
0.0020
0.0020

Columns 295 through 301
0.0019
0.0019
0.0019
0.0019
0.0019
0.0019
0.0018

Columns 302 through 308
0.0018
0.0018
0.0018
0.0018
0.0018
0.0017
0.0017
Columns 309 through ..... 315
\(0.0017 \quad 0.0017\) 0.0017 0.0017 0.0017 0.0016 ..... 0.0016
Columns 316 through ..... 322
0.0016 0.0016 0.0016 0.0016 0.0016 0.0015 0.0015
Columns 323 through ..... 329
0.0015 0.0015 0.0015 0.0015 0.0015 0.0014 0.001
Columns 330 through ..... 334
0.0014 0.0014 0.0014 0.0014 0.0014
CFLowDates =
Columns 1 through 6
\(731323 \quad 731337\) 731368 ..... 731398
731429 ..... 731
Columns 7 through ..... 12
731490 ..... 731521
731551 ..... 731582
731613 ..... 7316
Columns 13 through ..... 18
7316727317027317337317637317947318
Columns 19 through ..... 24
731855 731886731916731947731978732
Columns 25 through ..... 30

Columns 49 through 54
\(732768 \quad 732798\)
732829
732859
732890

Columns 55 through 60

732951732982

Columns 61 through 66
\(733133 \quad 733163\)
733194
733224
733255
733286

Columns 67 through 72
\(733316 \quad 733347\)
733377
733408
733439
733468

Columns 73 through 78

733499
733529
733560
733590
733621
733652
\begin{tabular}{|c|c|c|c|c|c|}
\hline 733682 & 733713 & 733743 & 733774 & 733805 & 733 \\
\hline \multicolumn{6}{|l|}{Columns 85 through 90} \\
\hline 733864 & 733894 & 733925 & 733955 & 733986 & 734 \\
\hline \multicolumn{6}{|l|}{Columns 91 through 96} \\
\hline 734047 & 734078 & 734108 & 734139 & 734170 & 734 \\
\hline \multicolumn{6}{|l|}{Columns 97 through 102} \\
\hline 734229 & 734259 & 734290 & 734320 & 734351 & 734 \\
\hline \multicolumn{6}{|l|}{Columns 103 through 108} \\
\hline 734412 & 734443 & 734473 & 734504 & 734535 & 734 \\
\hline \multicolumn{6}{|l|}{Columns 109 through 114} \\
\hline 734594 & 734624 & 734655 & 734685 & 734716 & 734 \\
\hline \multicolumn{6}{|l|}{Columns 115 through 120} \\
\hline 734777 & 734808 & 734838 & 734869 & 734900 & 734 \\
\hline \multicolumn{6}{|l|}{Columns 121 through 126} \\
\hline 734960 & 734990 & 735021 & 735051 & 735082 & 735 \\
\hline \multicolumn{6}{|l|}{Columns 127 through 132} \\
\hline 735143 & 735174 & 735204 & 735235 & 735266 & 735 \\
\hline \multicolumn{6}{|l|}{Columns 133 through 138} \\
\hline 735325 & 735355 & 735386 & 735416 & 735447 & 735 \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multicolumn{6}{|l|}{Columns 139 through 144} \\
\hline 735508 & 735539 & 735569 & 735600 & 735631 & 735659 \\
\hline \multicolumn{6}{|l|}{Columns 145 through 150} \\
\hline 735690 & 735720 & 735751 & 735781 & 735812 & 735843 \\
\hline \multicolumn{6}{|l|}{Columns 151 through 156} \\
\hline 735873 & 735904 & 735934 & 735965 & 735996 & 736024 \\
\hline \multicolumn{6}{|l|}{Columns 157 through 162} \\
\hline 736055 & 736085 & 736116 & 736146 & 736177 & 736208 \\
\hline \multicolumn{6}{|l|}{Columns 163 through 168} \\
\hline 736238 & 736269 & 736299 & 736330 & 736361 & 736390 \\
\hline \multicolumn{6}{|l|}{Columns 169 through 174} \\
\hline 736421 & 736451 & 736482 & 736512 & 736543 & 736574 \\
\hline \multicolumn{6}{|l|}{Columns 175 through 180} \\
\hline 736604 & 736635 & 736665 & 736696 & 736727 & 736755 \\
\hline \multicolumn{6}{|l|}{Columns 181 through 186} \\
\hline 736786 & 736816 & 736847 & 736877 & 736908 & 736939 \\
\hline \multicolumn{6}{|l|}{Columns 187 through 192} \\
\hline 736969 & 737000 & 737030 & 737061 & 737092 & 737120 \\
\hline
\end{tabular}


738795
738826

Columns 253 through 258
\(738977 \quad 739007\)

Columns 259 through 264

739160
739191
739221
739252
739283
739312

Columns 265 through 270

739343
739373

Columns 271 through 276

739526
739557
739587
739618
739649
739677

Columns 277 through 282
\(739708 \quad 739738\)

Columns 283 through 288
\(739891 \quad 739922\)

Columns 289 through 294

740073
740103
74013
740164
740195
740226

Columns 295 through 300

740256740287
740317
740348
740379
740407
740438
740468
Columns 307 through 312
740621 - 740652
Columns 313 through 318
740804740834
740865
740895
740926
Columns 319 through 324
Columns 325 through 330
\(741169 \quad 741199-741230\)
Columns 331 through 334
\(741352 \quad 741383\)
741383
741413
741444
TFactors =
Columns 1 through 7
\(0 \quad 0.9333\)
1.9333
2.9333
3.9333
4.9333
5.933
Columns 8 through 14
\(\begin{array}{lllllll}6.9333 & 7.9333 & 8.9333 & 9.9333 & 10.9333 & 11.9333 & 12.933\end{array}\)
Columns 15 through 21
\(13.9333 \quad 14.9333 \quad 15.9333 \quad 16.9333 \quad 17.9333 \quad 18.933319 .933\)

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Columns 22 through ..... 28
20.9333 21.9333 ..... 22.9333
23.9333 24.9333 25.9333 26.9333
Columns 29 through ..... 35
27.9333 28.9333
Columns 36 through ..... 4229.933330.933331.933332.933333.9333
34.9333 35.9333 36.9333 37.9333 ..... 38.9333
39.9333 ..... 40.9333
Columns 43 through ..... 49
41.9333 42.9333 43.9333 44.9333 ..... 45.9333
46.9333 ..... 47.9333
Columns 50 through ..... 56
48.9333 49.9333 ..... 50.9333
51.9333 ..... 52.9333
53.9333 ..... 54.9333
Columns 57 through ..... 63
55.9333 56.9333 57.9333 58.9333 ..... 59.9333
60.9333 61.9333
Columns 64 through ..... 70
62.9333 63.9333 ..... 64.9333
65.9333 66.9333 67.9333 ..... 68.9333
Columns 71 through ..... 77
69.9333 70.9333 71.9333 72.9333 ..... 73.9333
74.9333 75.9333
Columns 78 through 84
76.9333 77.9333 78.9333 79.9333 80.9333 81.9333 82.9333
Columns 85 through ..... 91
83.9333 84.9333 85.9333 86.9333 ..... 87.9333
88.9333 89.933
Columns 92 through ..... 98
90.9333 91.9333 ..... 92.9333
93.9333 94.9333 95.9333 96.933
Columns 99 through ..... 105
\(97.9333 \quad 98.9333 \quad 99.9333 \quad 100.9333 \quad 101.9333 \quad 102.9333 \quad 103.933\)
Columns 106 through ..... 112
\(104.9333 \quad 105.9333 \quad 106.9333 \quad 107.9333 \quad 108.9333 \quad 109.9333 \quad 110.933\)
Columns 113 through ..... 119
111.9333112 .9333113 .9333 114.9333 115.9333 116.9333 ..... 117.933
Columns 120 through ..... 126
118.9333119 .9333120 .9333121 .9333 ..... 122.9333
123.9333 ..... 124.933
Columns 127 through ..... 133
\(125.9333 \quad 126.9333 \quad 127.9333 \quad 128.9333 \quad 129.9333 \quad 130.9333131 .933\)
Columns 134 through ..... 140
\(132.9333 \quad 133.9333 \quad 134.9333 \quad 135.9333 \quad 136.9333 \quad 137.9333138 .933\)
Columns 141 through ..... 147
\(139.9333 \quad 140.9333 \quad 141.9333 \quad 142.9333143 .9333144 .9333145 .933\)
Columns 148 through ..... 154

\section*{mbscfamounts}
```

146.9333 147.9333 148.9333 149.9333 150.9333 151.9333 152.9333
Columns 155 through 161
153.9333 154.9333 155.9333 156.9333 157.9333 158.9333 159.9333
Columns 162 through 168
160.9333 161.9333 162.9333 163.9333 164.9333 165.9333 166.9333
Columns 169 through 175
167.9333 168.9333 169.9333 170.9333 171.9333 172.9333 173.9333
Columns 176 through 182
174.9333 175.9333 176.9333 177.9333 178.9333 179.9333 180.9333
Columns 183 through 189
181.9333 182.9333 183.9333 184.9333 185.9333 186.9333 187.9333
Columns }190\mathrm{ through 196
188.9333 189.9333 190.9333 191.9333 192.9333 193.9333 194.9333
Columns 197 through 203
195.9333 196.9333 197.9333 198.9333 199.9333 200.9333 201.9333
Columns 204 through 210
202.9333 203.9333 204.9333 205.9333 206.9333 207.9333 208.9333
Columns 211 through 217
209.9333 210.9333 211.9333 212.9333 213.9333 214.9333 215.9333

```
```

Columns 218 through 224
216.9333 217.9333 218.9333 219.9333 220.9333 221.9333 222.933)
Columns 225 through 231
223.9333 224.9333 225.9333 226.9333 227.9333 228.9333 229.933'
Columns 232 through 238
230.9333 231.9333 232.9333 233.9333 234.9333 235.9333 236.9333
Columns 239 through 245
237.9333 238.9333 239.9333 240.9333 241.9333 242.9333 243.933'
Columns 246 through 252
244.9333 245.9333 246.9333 247.9333 248.9333 249.9333 250.9333
Columns 253 through 259
251.9333 252.9333 253.9333 254.9333 255.9333 256.9333 257.933'
Columns 260 through 266
258.9333 259.9333 260.9333 261.9333 262.9333 263.9333 264.9333
Columns 267 through 273
265.9333 266.9333 267.9333 268.9333 269.9333 270.9333 271.933'
Columns 274 through 280
272.9333 273.9333 274.9333 275.9333 276.9333 277.9333 278.933`

```

\section*{mbscfamounts}
```Columns 281 through 287
```

$279.9333 \quad 280.9333 \quad 281.9333 \quad 282.9333 \quad 283.9333 \quad 284.9333 \quad 285.9333$

```Columns 288 through 294
```

$286.9333 \quad 287.9333 \quad 288.9333 \quad 289.9333 \quad 290.9333 \quad 291.9333 \quad 292.9333$

```Columns 295 through 301
```

$293.9333 \quad 294.9333 \quad 295.9333 \quad 296.9333 \quad 297.9333 \quad 298.9333 \quad 299.9333$

```Columns 302 through 308
```

$300.9333 \quad 301.9333 \quad 302.9333 \quad 303.9333 \quad 304.9333 \quad 305.9333 \quad 306.9333$
Columns 309 through 315
$307.9333 \quad 308.9333 \quad 309.9333 \quad 310.9333$ 311.9333 ..... 312.9333313 .9333
Columns 316 through ..... 322
$314.9333 \quad 315.9333 \quad 316.9333 \quad 317.9333 \quad 318.9333 \quad 319.9333 \quad 320.9333$

```Columns 323 through 329
```

$321.9333 \quad 322.9333 \quad 323.9333 \quad 324.9333$ 325.9333 326.9333 ..... 327.9333
Columns 330 through ..... 334
$328.9333 \quad 329.9333 \quad 330.9333 \quad 331.9333$ ..... 332.9333
Factors =
Columns 1 through 7
1.0000
0.9944
0.9887
0.9828
0.9769
0.9711
0.965

Columns 8 through 14
0.9595
0.9538
0.9481
0.9424
0.9368
0.9311
0.925

Columns 15 through 21
0.9199
0.9144
0.9089
0.9034
0.8979
0.8925
0.887

Columns 22 through 28
0.8817
0.8763
0.8710
0.8657
0.8604
0.8552
0.849

Columns 29 through 35
0.8447
0.8396
0.8344
0.8293
0.8242
0.8191
0.814

Columns 36 through 42
0.8090
0.8040
0.7990
0.7941
0.7892
0.7842
0.779

Columns 43 through 49
0.7745
0.7697
0.7649
0.7601
0.7553
0.7506
0.745

Columns 50 through 56
0.7411
0.7365
0.7318
0.7272
0.7226
0.7180
0.713

Columns 57 through 63
0.7089
0.7044
0.6999
0.6954
0.6910
0.6865
0.682

Columns 64 through 70
0.6777
0.6734
0.6690
0.6647
0.6604
0.6561
0.651

Columns 71 through 77
0.6476
0.6434
0.6392
0.6350
0.6309
0.6267
0.6226

Columns 78 through 84
0.6185
0.6144
0.6104
0.6063
0.6023
0.5983
0.5943

Columns 85 through 91
0.5903
0.5864
0.5825
0.5785
0.5747
0.5708
0.5669

Columns 92 through 98
$0.5631 \quad 0.5593$
0.5555
0.5517
0.5479
0.5442
0.5405

Columns 99 through 105
0.5368
0.5331
0.5294
0.5257
0.5221
0.5185
0.5149

Columns 106 through 112
0.5113
0.5077
0.5042
0.5006
0.4971
0.4936
0.4901

Columns 113 through 119
$0.4866 \quad 0.4832 \quad 0.4797$
0.4763
0.4729
0.4695
0.4661

Columns 120 through 126
0.4628
0.4594
0.4561
0.4528
0.4495
0.4462
0.4430

Columns 127 through 133
0.4397
0.4365
0.4333
0.4301
0.4269
0.4237
0.4205

## Columns 134 through 140

0.4174
0.4143
0.4111
0.4080
0.4049
0.4019
0.398

Columns 141 through 147
0.3958
0.3927
0.3897
0.3867
0.3837
0.3808
0.3778

Columns 148 through 154
0.3748
0.3719
0.3690
0.3661
0.3632
0.3603
0.357

Columns 155 through 161
0.3546
0.3517
0.3489
0.3461
0.3433
0.3405
0.337

Columns 162 through 168
$\begin{array}{lllllll}0.3350 & 0.3322 & 0.3295 & 0.3267 & 0.3240 & 0.3213 & 0.318\end{array}$
Columns 169 through 175
$\begin{array}{lllllll}0.3160 & 0.3133 & 0.3106 & 0.3080 & 0.3054 & 0.3027 & 0.300\end{array}$
Columns 176 through 182
$0.2975 \quad 0.2950 \quad 0.2898 \quad 0.2873 \quad 0.2847 \quad 0.282$
Columns 183 through 189
$\begin{array}{lllllll}0.2797 & 0.2772 & 0.2747 & 0.2722 & 0.2698 & 0.2673 & 0.2649\end{array}$
Columns 190 through 196
$\begin{array}{lllllll}0.2624 & 0.2600 & 0.2576 & 0.2552 & 0.2528 & 0.2504 & 0.248(1)\end{array}$

Columns 197 through 203
0.2457
0.2433
0.2410
0.2386
0.2363
0.2340
0.2317

Columns 204 through 210
0.2294
0.2271
0.2249
0.2226
0.2204
0.2181
0.2159

Columns 211 through 217
0.2137
0.2115
0.2093
0.2071
0.2049
0.2027
0.2005

Columns 218 through 224
0.1984
0.1962
0.1941
0.1920
0.1899
0.1877
0.1856

Columns 225 through 231
0.1836
0.1815
0.1794
0.1773
0.1753
0.1732
0.1712

Columns 232 through 238
0.1692
0.1671
0.1651
0.1631
0.1611
0.1591
0.1572

Columns 239 through 245
0.1552
0.1532
0.1513
0.1493
0.1474
0.1455
0.1436

Columns 246 through 252
0.1416
0.1397
0.1378
0.1359
0.1341
0.1322
0.1303

Columns 253 through 259
0.1285
0.1266
0.1248
0.1229
0.1211
0.1193
0.1175

Columns 260 through 266
0.1157
0.1139
0.1121
0.1103
0.1085
0.1068
0.105

Columns 267 through 273
0.1032
0.1015
0.0998
0.0980
0.0963
0.0946
0.092

Columns 274 through 280

| 0.0912 | 0.0895 | 0.0878 | 0.0861 | 0.0844 | 0.0827 | 0.081 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Columns 281 through 287
0.0794
0.0778
0.0761
0.0745
0.0728
0.0712
0.069

Columns 288 through 294
0.0680
0.0664
0.0648
0.0632
0.0616
0.0600
0.058

Columns 295 through 301
0.0568
0.0553
0.0537
0.0522
0.0506
0.0491
0.047

Columns 302 through 308
0.0460
0.0445
0.0430
0.0415
0.0400
0.0385
0.037

Columns 309 through 315
0.0355
0.0340
0.0325
0.0311
0.0296
0.0281
0.026

Columns 316 through 322
0.0252
0.0238
0.0223
0.0209
0.0195
0.0180
0.016

Columns 323 through 329

| 0.0152 | 0.0138 | 0.0124 | 0.0110 | 0.0096 | 0.0082 | 0.0068 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Columns 330 through 334
0.0055
0.0041
0.0027
0.0014
0

The result is contained in four 334 -element row vectors.

## Compute Cash Flow Amounts and Dates, Time Factors, and Mortgage Factors for a Mortgage Portfolio

Given a portfolio of mortgage-backed securities, use mbscfamounts to compute the cash flows and other factors from the portfolio.

Define characteristics for a mortgage portfolio.

```
Settle = datenum(['13-Jan-2000';'17-Apr-2002';'17-May-2002']);
Maturity = datenum('1-Jan-2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
CouponRate = [0.075; 0.07875; 0.0775];
Delay = 14;
PrepaySpeed = 100;
```

Use mbscfamonts to evaluate the mortgage.
[CFlowAmounts, CFlowDates, TFactors, Factors] = ... mbscfamounts(Settle, Maturity, IssueDate, GrossRate, ... CouponRate, Delay, PrepaySpeed)

CFlowAmounts =

Columns 1 through 7

| -0.0025 | 0.0071 | 0.0072 | 0.0074 | 0.0076 | 0.0077 | 0.0079 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| -0.0035 | 0.0121 | 0.0123 | 0.0124 | 0.0123 | 0.0122 | 0.0122 |
| -0.0034 | 0.0122 | 0.0123 | 0.0123 | 0.0122 | 0.0121 | 0.0121 |

Columns 8 through 14

| 0.0080 | 0.0082 | 0.0084 | 0.0085 | 0.0087 | 0.0088 | 0.009 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0121 | 0.0120 | 0.0120 | 0.0119 | 0.0118 | 0.0118 | 0.011 |
| 0.0120 | 0.0119 | 0.0119 | 0.0118 | 0.0117 | 0.0117 | 0.011 |

Columns 15 through 21

| 0.0091 | 0.0093 | 0.0094 | 0.0095 | 0.0097 | 0.0098 | 0.009 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0116 | 0.0116 | 0.0115 | 0.0115 | 0.0114 | 0.0113 | 0.011 |
| 0.0116 | 0.0115 | 0.0114 | 0.0114 | 0.0113 | 0.0112 | 0.011 |

Columns 22 through 28

| 0.0101 | 0.0102 | 0.0103 | 0.0104 | 0.0106 | 0.0107 | 0.0108 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0112 | 0.0111 | 0.0111 | 0.0110 | 0.0110 | 0.0109 | 0.0108 |
| 0.0111 | 0.0111 | 0.0110 | 0.0109 | 0.0109 | 0.0108 | 0.0108 |

Columns 29 through 35
0.0109
0.0110
$0.0108 \quad 0.0107$ 0.01070 .0106
Columns 36 through 42
0.0111
0.0110
0.0110
0.0109
0.010
0.0107
0.0106
0.0105
0.0105
0.010
0.0106
0.0105
0.0105
0.0104
0.010

| 0.0108 | 0.0107 | 0.0107 | 0.0106 | 0.0106 | 0.0105 | 0.010 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0104 | 0.0103 | 0.0103 | 0.0102 | 0.0101 | 0.0101 | 0.010 |
| 0.0103 | 0.0102 | 0.0102 | 0.0101 | 0.0101 | 0.0100 | 0.009 |

Columns 43 through 49

| 0.0104 | 0.0103 | 0.0103 | 0.0102 | 0.0102 | 0.0101 | 0.010 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0100 | 0.0099 | 0.0099 | 0.0098 | 0.0098 | 0.0097 | 0.009 |
| 0.0099 | 0.0098 | 0.0098 | 0.0097 | 0.0097 | 0.0096 | 0.009 |

Columns 50 through 56

| 0.0100 | 0.0099 | 0.0099 | 0.0098 | 0.0098 | 0.0097 | 0.0097 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0096 | 0.0095 | 0.0095 | 0.0094 | 0.0094 | 0.0093 | 0.0093 |
| 0.0095 | 0.0095 | 0.0094 | 0.0094 | 0.0093 | 0.0092 | 0.0092 |

0.0099
0.0098
.0098
0.0093
0.0093
olumns 57 through 63

| 0.0096 | 0.0096 | 0.0095 | 0.0095 | 0.0094 | 0.0094 | 0.0093 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0092 | 0.0092 | 0.0091 | 0.0091 | 0.0090 | 0.0090 | 0.0089 |
| 0.0091 | 0.0091 | 0.0090 | 0.0090 | 0.0089 | 0.0089 | 0.0088 |

Columns 64 through 70

| 0.0093 | 0.0092 | 0.0091 | 0.0091 | 0.0090 | 0.0090 | 0.0089 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0089 | 0.0088 | 0.0088 | 0.0087 | 0.0087 | 0.0086 | 0.0086 |
| 0.0088 | 0.0087 | 0.0087 | 0.0086 | 0.0086 | 0.0085 | 0.0085 |

Columns 71 through 77

| 0.0089 | 0.0088 | 0.0088 | 0.0088 | 0.0087 | 0.0087 | 0.0086 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0085 | 0.0085 | 0.0084 | 0.0084 | 0.0083 | 0.0083 | 0.0082 |
| 0.0085 | 0.0084 | 0.0084 | 0.0083 | 0.0083 | 0.0082 | 0.0082 |

Columns 78 through 84

| 0.0086 | 0.0085 | 0.0085 | 0.0084 | 0.0084 | 0.0083 | 0.0083 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0082 | 0.0081 | 0.0081 | 0.0080 | 0.0080 | 0.0080 | 0.0079 |
| 0.0081 | 0.0081 | 0.0080 | 0.0080 | 0.0079 | 0.0079 | 0.0078 |

Columns 85 through 91

| 0.0082 | 0.0082 | 0.0081 | 0.0081 | 0.0080 | 0.0080 | 0.0080 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0079 | 0.0078 | 0.0078 | 0.0077 | 0.0077 | 0.0076 | 0.0076 |
| 0.0078 | 0.0078 | 0.0077 | 0.0077 | 0.0076 | 0.0076 | 0.0075 |

Columns 92 through 98

| 0.0079 | 0.0079 | 0.0078 | 0.0078 | 0.0077 | 0.0077 | 0.007 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0076 | 0.0075 | 0.0075 | 0.0074 | 0.0074 | 0.0073 | 0.007 |
| 0.0075 | 0.0075 | 0.0074 | 0.0074 | 0.0073 | 0.0073 | 0.007 |

Columns 99 through 105

| 0.0076 | 0.0076 | 0.0075 | 0.0075 | 0.0074 | 0.0074 | 0.007 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0073 | 0.0072 | 0.0072 | 0.0071 | 0.0071 | 0.0070 | 0.007 |
| 0.0072 | 0.0072 | 0.0071 | 0.0071 | 0.0070 | 0.0070 | 0.007 |

Columns 106 through 112

| 0.0073 | 0.0073 | 0.0072 | 0.0072 | 0.0071 | 0.0071 | 0.007 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0070 | 0.0069 | 0.0069 | 0.0068 | 0.0068 | 0.0068 | 0.006 |
| 0.0069 | 0.0069 | 0.0068 | 0.0068 | 0.0068 | 0.0067 | 0.006 |

Columns 113 through 119

| 0.0070 | 0.0070 | 0.0069 | 0.0069 | 0.0069 | 0.0068 | 0.006 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0067 | 0.0066 | 0.0066 | 0.0066 | 0.0065 | 0.0065 | 0.006 |
| 0.0066 | 0.0066 | 0.0066 | 0.0065 | 0.0065 | 0.0064 | 0.006 |

Columns 120 through 126

| 0.0067 | 0.0067 | 0.0067 | 0.0066 | 0.0066 | 0.0066 | 0.006 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0064 | 0.0064 | 0.0063 | 0.0063 | 0.0063 | 0.0062 | 0.006 |
| 0.0064 | 0.0063 | 0.0063 | 0.0063 | 0.0062 | 0.0062 | 0.006 |

Columns 127 through 133

| 0.0065 | 0.0064 | 0.0064 | 0.0064 | 0.0063 | 0.0063 | 0.006 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0062 | 0.0061 | 0.0061 | 0.0060 | 0.0060 | 0.0060 | 0.005 |
| 0.0061 | 0.0061 | 0.0060 | 0.0060 | 0.0060 | 0.0059 | 0.005 |

Columns 134 through 140
$\begin{array}{lll}0.0062 & 0.0062 & 0 . \\ 0.0059 & 0.0059 & 0 . \\ 0.0059 & 0.0058 & 0 . \\ & & \\ \text { olumns } 141 & \text { through } & 147\end{array}$

| 0.0060 | 0.0059 | 0.0059 | 0.0059 | 0.0058 | 0.0058 | 0.0058 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0057 | 0.0056 | 0.0056 | 0.0056 | 0.0055 | 0.0055 | 0.0055 |
| 0.0056 | 0.0056 | 0.0056 | 0.0055 | 0.0055 | 0.0055 | 0.0054 |

Columns 148 through 154

| 0.0057 | 0.0057 | 0 |
| :--- | :--- | ---: |
| 0.0054 | 0.0054 | 0 |
| 0.0054 | 0.0054 | 0 |
|  |  |  |
| olumns 155 | through | 161 |


| 0.0055 | 0.0055 | 0. |
| :--- | :--- | :--- |
| 0.0052 | 0.0052 | 0. |
| 0.0052 | 0.0051 | 0. |
|  |  |  |


| 0.0053 | 0.0052 | 0.0052 | 0.0052 | 0.0052 | 0.0051 | 0.0051 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0050 | 0.0050 | 0.0049 | 0.0049 | 0.0049 | 0.0048 | 0.0048 |
| 0.0049 | 0.0049 | 0.0049 | 0.0049 | 0.0048 | 0.0048 | 0.0048 |

Columns 169 through 175
$\begin{array}{ll}0.0051 & 0.0050 \\ 0.0048 & 0.0047 \\ 0.0047 & 0.0047 \\ & \\ \text { olumns } & 176\end{array}$ through
0
0
0
$\begin{array}{lllllll}0.0049 & 0.0048 & 0.0048 & 0.0048 & 0.0047 & 0.0047 & 0.0047\end{array}$

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| 0.0046 | 0.0045 | 0.0045 | 0.0045 | 0.0045 | 0.0044 | 0.004 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0045 | 0.0045 | 0.0045 | 0.0044 | 0.0044 | 0.0044 | 0.004 |

Columns 183 through 189

| 0.0047 | 0.0046 | 0.0046 | 0.0046 | 0.0045 | 0.0045 | 0.004 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0044 | 0.0043 | 0.0043 | 0.0043 | 0.0043 | 0.0042 | 0.004 |
| 0.0043 | 0.0043 | 0.0043 | 0.0043 | 0.0042 | 0.0042 | 0.004 |

Columns 190 through 196

| 0.0045 | 0.0044 | 0.0044 | 0.0044 | 0.0043 | 0.0043 | 0.004 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0042 | 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0040 | 0.004 |
| 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0040 | 0.0040 | 0.004 |

Columns 197 through 203

| 0.0043 | 0.0042 | 0.0042 | 0.0042 | 0.0042 | 0.0041 | 0.004 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0040 | 0.0040 | 0.0039 | 0.0039 | 0.0039 | 0.0039 | 0.003 |
| 0.0040 | 0.0039 | 0.0039 | 0.0039 | 0.0039 | 0.0038 | 0.003 |

Columns 204 through 210

| 0.0041 | 0.0041 | 0.0040 | 0.0040 | 0.0040 | 0.0040 | 0.003 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0038 | 0.0038 | 0.0038 | 0.0037 | 0.0037 | 0.0037 | 0.003 |
| 0.0038 | 0.0038 | 0.0037 | 0.0037 | 0.0037 | 0.0037 | 0.003 |

Columns 211 through 217

| 0.0039 | 0.0039 | 0.0039 | 0.0038 | 0.0038 | 0.0038 | 0.003 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0036 | 0.0036 | 0.0036 | 0.0036 | 0.0035 | 0.0035 | 0.003 |
| 0.0036 | 0.0036 | 0.0036 | 0.0035 | 0.0035 | 0.0035 | 0.003 |

Columns 218 through 224

| 0.0037 | 0.0037 | 0.0037 | 0.0037 | 0.0036 | 0.0036 | 0.003 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0035 | 0.0034 | 0.0034 | 0.0034 | 0.0034 | 0.0034 | 0.003 |

$$
\begin{array}{lllllll}
0.0034 & 0.0034 & 0.0034 & 0.0034 & 0.0034 & 0.0033 & 0.0033
\end{array}
$$

Columns 225 through 231

| 0.0036 | 0.0036 | 0.0035 | 0.0035 | 0.0035 | 0.0035 | 0.0034 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0033 | 0.0033 | 0.0033 | 0.0032 | 0.0032 | 0.0032 | 0.0032 |
| 0.0033 | 0.0033 | 0.0032 | 0.0032 | 0.0032 | 0.0032 | 0.0032 |

Columns 232 through 238

| 0.0034 | 0.0034 | 0.0034 | 0.0033 | 0.0033 | 0.0033 | 0.0033 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0032 | 0.0031 | 0.0031 | 0.0031 | 0.0031 | 0.0030 | 0.0030 |
| 0.0031 | 0.0031 | 0.0031 | 0.0031 | 0.0030 | 0.0030 | 0.0030 |

Columns 239 through 245

| 0.0033 | 0.0032 | 0.0032 | 0.0032 | 0.0032 | 0.0032 | 0.0031 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0030 | 0.0030 | 0.0030 | 0.0029 | 0.0029 | 0.0029 | 0.0029 |
| 0.0030 | 0.0030 | 0.0029 | 0.0029 | 0.0029 | 0.0029 | 0.0029 |

Columns 246 through 252

| 0.0031 | 0.0031 | 0.0031 | 0.0031 | 0.0030 | 0.0030 | 0.0030 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0029 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0027 |
| 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0027 | 0.0027 |

Columns 253 through 259

| 0.0030 | 0.0029 | 0.0 |
| :--- | :--- | :--- |
| 0.0027 | 0.0027 | 0.0 |
| 0.0027 | 0.0027 | 0.0 |
|  |  |  |
| Columns 260 | through | 266 |


| 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0027 | 0.0027 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0026 | 0.0026 | 0.0025 | 0.0025 | 0.0025 | 0.0025 | 0.0025 |
| 0.0026 | 0.0025 | 0.0025 | 0.0025 | 0.0025 | 0.0025 | 0.0024 |

Columns 267 through 273

| 0.0027 | 0.0027 | 0.0027 | 0.0026 | 0.0026 | 0.0026 | 0.002 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0024 | 0.0024 | 0.0024 | 0.0024 | 0.0024 | 0.0024 | 0.002 |
| 0.0024 | 0.0024 | 0.0024 | 0.0024 | 0.0024 | 0.0023 | 0.002 |

Columns 274 through 280

| 0.0026 | 0.0025 | 0.0025 | 0.0025 | 0.0025 | 0.0025 | 0.002 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0023 | 0.0023 | 0.0023 | 0.0023 | 0.0022 | 0.0022 | 0.002 |
| 0.0023 | 0.0023 | 0.0023 | 0.0022 | 0.0022 | 0.0022 | 0.002 |

Columns 281 through 287

| 0.0024 | 0.0024 | 0.0024 | 0.0024 | 0.0024 | 0.0024 | 0.002 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0022 | 0.0022 | 0.0022 | 0.0021 | 0.0021 | 0.0021 | 0.002 |
| 0.0022 | 0.0022 | 0.0021 | 0.0021 | 0.0021 | 0.0021 | 0.002 |

Columns 288 through 294

| 0.0023 | 0.0023 | 0.0023 | 0.0023 | 0.0023 | 0.0022 | 0.002 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0021 | 0.0021 | 0.0020 | 0.0020 | 0.0020 | 0.0020 | 0.002 |
| 0.0021 | 0.0020 | 0.0020 | 0.0020 | 0.0020 | 0.0020 | 0.002 |

Columns 295 through 301

| 0.0022 | 0.0022 | 0.0022 | 0.0022 | 0.0021 | 0.0021 | 0.002 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0020 | 0.0019 | 0.0019 | 0.0019 | 0.0019 | 0.0019 | 0.001 |
| 0.0019 | 0.0019 | 0.0019 | 0.0019 | 0.0019 | 0.0019 | 0.001 |

Columns 302 through 308

| 0.0021 | 0.0021 | 0.0021 | 0.0020 | 0.0020 | 0.0020 | 0.002 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0018 | 0.0018 | 0.0018 | 0.0018 | 0.0018 | 0.0018 | 0.001 |
| 0.0018 | 0.0018 | 0.0018 | 0.0018 | 0.0018 | 0.0018 | 0.001 |

11-1511

Columns 309 through 315
0.0020
0.0020
0.0019
0.0019
0.0019
0.0019
0.0019
0.0017
0.0017
0.0017
0.0017
0.0017
0.0017
0.0017
0.0017
0.0017
0.0016
olumns 316 through 322

| 0.0019 | 0.0019 | 0.0018 | 0.0018 | 0.0018 | 0.0018 | 0.0018 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0016 | 0.0016 | 0.0016 | 0.0016 | 0.0016 | 0.0016 | 0.0015 |
| 0.0016 | 0.0016 | 0.0016 | 0.0016 | 0.0016 | 0.0015 | 0.0015 |

Columns 323 through 329
0.0018
0.0018
$0.0015 \quad 0.0015 \quad 0.0015$
0.0015
0.0015
Columns 330 through 336
0.0017
0.0017
0.0017
0.0017
0.0017
0.0015
0.0015
0.0015
0.0014
0.0015
0.0015
0.0015
0.0014
0.0014
0.0017
0.0017
0.0016
0.0016
0.0016
0.0014
0.0014
0.0014
0.0014
0.0014
NaN
0.0016
NaN
NaN
0.0016
0.0014
0.0014
0.0014
0.0014
NaN
NaN

Columns 337 through 343

$$
\begin{array}{rrr}
0.0016 & 0.0016 & 0.0015 \\
\mathrm{NaN} & \mathrm{NaN} & \mathrm{NaN} \\
\mathrm{NaN} & \mathrm{NaN} & \mathrm{NaN}
\end{array}
$$

> 0.0015 NaN
0.0015
NaN

NaN
0.0015
NaN
NaN
0.0015
NaN
NaN

Columns 344 through 350
0.0015
0.0015
0.0014

NaN
NaN
0.0014

NaN NaN
0.0014

NaN
NaN
0.0014

NaN
NaN

Columns 351 through 357

| 0.0014 | 0.0014 | 0.0014 |
| ---: | ---: | ---: |
| NaN | NaN | NaN |

NaN
NaN
NaN
0.0013
NaN
NaN
0.0013
NaN NaN

> 0.0013
> NaN NaN
0.001
NaN

NaI
Columns 358 through 361

$$
\begin{array}{rrrr}
0.0013 & 0.0013 & 0.0013 & 0.0013 \\
\mathrm{NaN} & \mathrm{NaN} & \mathrm{NaN} & \mathrm{NaN} \\
\mathrm{NaN} & \mathrm{NaN} & \mathrm{NaN} & \mathrm{NaN}
\end{array}
$$

## CFlowDates = <br> Columns 1 through 6

| 730498 | 730517 | 730546 | 730577 | 730607 | 730 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 731323 | 731337 | 731368 | 731398 | 731429 | 731 |
| 731353 | 731368 | 731398 | 731429 | 731460 | 731 |

Columns 7 through 12

| 730668 | 730699 | 730730 |
| :--- | :--- | :--- |
| 731490 | 731521 | 731551 |
| 731521 | 731551 | 731582 |

## 730760 <br> 731582

731613
730791
731613
731641
730

731521
731551
731582
731
7316

Columns 13 through 18

| 730852 | 730883 |
| :--- | :--- |
| 731672 | 731702 |

730911
730942
730972
7310
731672731702
731733
731763
731794
731825
7318
731702731733
731763
731794
7318
Columns 19 through 24

| 731033 | 731064 | 731095 | 731125 | 731156 | 731 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 731855 | 731886 | 731916 | 731947 | 731978 | 732 |
| 731886 | 731916 | 731947 | 731978 | 732007 | 732 |

11-1513

Columns 25 through 30

| 731217 | 731248 | 731276 | 731307 | 731337 | 731368 |
| :--- | ---: | :--- | :--- | :--- | :--- |
| 732038 | 732068 | 732099 | 732129 | 732160 | 732191 |
| 732068 | 732099 | 732129 | 732160 | 732191 | 732221 |
|  |  |  |  |  |  |
| amns 31 | through 36 |  |  |  |  |
|  |  |  |  | 731490 | 731521 |

Columns 37 through 42

| 731582 | 731613 |
| :--- | ---: |
| 732403 | 732433 |
| 732433 | 732464 |
|  |  |
| umns 43 | through 48 |


| 731763 | 731794 | 731825 | 731855 | 731886 | 731916 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 732586 | 732617 | 732647 | 732678 | 732709 | 732737 |
| 732617 | 732647 | 732678 | 732709 | 732737 | 732768 |

Columns 49 through 54

| 731947 | 731978 | 732007 | 732038 | 732068 | 732099 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 732768 | 732798 | 732829 | 732859 | 732890 | 732921 |
| 732798 | 732829 | 732859 | 732890 | 732921 | 732951 |

Columns 55 through 60

| 732129 | 732160 | 732191 | 732221 | 732252 | 732282 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 732951 | 732982 | 733012 | 733043 | 733074 | 733102 |
| 732982 | 733012 | 733043 | 733074 | 733102 | 733133 |

Columns 61 through 66

| 732313 | 732344 |
| :--- | ---: |
| 733133 | 733163 |
| 733163 | 733194 |
|  |  |
| umns 67 | through 72 |


| 732494 | 732525 | 732556 |
| :--- | :--- | :--- |
| 733316 | 733347 | 733377 |

732586
733408
732617
732
733316733347
733377
733439
733439
733468
733
$733347 \quad 733377$
733408
732403
733224
733255
732433

733286

Columns 67 through 72

Columns 73 through 78

| 732678 | 732709 | 732737 | 732768 |
| :--- | :--- | :--- | :--- |
| 733499 | 733529 | 733560 | 733590 |
| 733529 | 733560 | 733590 | 733621 |

732798
733621
$733529 \quad 733560$
733590
733621
733652

Columns 79 through 84
$732859 \quad 732890$
733682733713
732921
732951
733774
732982
733

733743
733743
733805
733805
7338
733713
733774
733833
7338

Columns 85 through 90

| 733043 | 733074 | 733102 | 733133 | 733163 | 733 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 733864 | 733894 | 733925 | 733955 | 733986 | $734($ |
| 733894 | 733925 | 733955 | 733986 | 734017 | $734($ |

Columns 91 through 96

| 733224 | 733255 | 733286 | 733316 | 733347 | 7333 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 734047 | 734078 | 734108 | 734139 | 734170 | 734 |
| 734078 | 734108 | 734139 | 734170 | 734198 | 7342 |

Columns 97 through 102

| 733408 | 733439 |
| :--- | ---: |
| 734229 | 734259 |
| 734259 | 734290 |
|  |  |
| umns 103 through 108 |  |


| 733590 | 733621 | 733652 | 733682 | 733713 | 733743 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 734412 | 734443 | 734473 | 734504 | 734535 | 734563 |
| 734443 | 734473 | 734504 | 734535 | 734563 | 734594 |

Columns 109 through 114

| 733774 | 733805 | 733833 |
| :--- | :--- | :--- |
| 734594 | 734624 | 734655 |
| 734624 | 734655 | 734685 |

733864
734685
733894
734716

733925
734716
734747 734747 734777
$733955 \quad 733986$
$734777 \quad 734808734838$

734047
734869
734078
734108
$734777 \quad 734808$
734838
734900
734900
734929
734808
734838
734869
734929
734960

Columns 121 through 126

| 734139 | 734170 | 734198 | 734229 | 734259 | 734290 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 734960 | 734990 | 735021 | 735051 | 735082 | 735113 |
| 734990 | 735021 | 735051 | 735082 | 735113 | 735143 |

Columns 127 through 132

| 734320 | 734351 | 734382 | 734412 | 734443 | 734473 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 735143 | 735174 | 735204 | 735235 | 735266 | 735294 |
| 735174 | 735204 | 735235 | 735266 | 735294 | 735325 |


| 734504 | 734535 |
| :--- | :--- |
| 735325 | 735355 |
| 735355 | 735386 |

734563
735386
735416

734594
735416
735447

734624
735447
735478

734 735 735

Columns 139 through 144

734685
$735508 \quad 735539$
735539735569

Columns 145 through 150

735690735720735751
$735720 \quad 735751 \quad 735781$

Columns 151 through 156

735051
735873735904
735904735934
35113
735934
735965
735965
735996

Columns 157 through 162

735235735266
736055736085
736085
736116
736116
736146
736146
736177

Columns 163 through 168

735416
$736238 \quad 736269$
736269
736299

Columns 169 through 174
$735600 \quad 735631$
735659
735690
735720

## mbscfamounts

$736421 \quad 736451 \quad 736482$

736482
736512

736512 736543

736543 736574

736574 736604

Columns 175 through 180

73660473663573666
$736635 \quad 736665$
736696
736727
735904
736727
735934 736755 736786

Columns 181 through 186

| 735965 | 735996 | 736024 |
| :--- | :--- | :--- |
| 736786 | 736816 | 736847 |
| 736816 | 736847 | 736877 |

736055 736877 736908

736085
736116
736908 736939 736816

736847
736877
736939 736969

Columns 187 through 192

| 736146 | 736177 | 736208 |
| :--- | :--- | :--- |
| 736969 | 737000 | 737030 |

736238
737061
737092
736269
736299
737030
737000
737030
737061
737092
737120
737120

Columns 193 through 198

| 736330 | 736361 | 736390 |
| :--- | :--- | :--- |
| 737151 | 737181 | 737212 |
| 737181 | 737212 | 737242 |

736421
737242
737273
736451
737273
736482 737304

737304
737334

Columns 199 through 204

| 736512 | 736543 | 736574 |
| :--- | :--- | :--- |
| 737334 | 737365 | 737395 |
| 737365 | 737395 | 737426 |

736604
737426
737457
736635
736665
737334 737365
737426
737457
737485
737485
737516

Columns 205 through 210

| 736696 | 736727 | 736755 | 736786 | 736816 | 736847 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 737516 | 737546 | 737577 | 737607 | 737638 | 737669 |

737546
737577

Columns 211 through 216

| 736877 | 736908 |
| :--- | ---: |
| 737699 | 737730 |
| 737730 | 737760 |
|  |  |
| umns 217 | through 222 |


| 737061 | 737092 | 737120 |
| :--- | :--- | :--- |
| 737882 | 737912 | 737943 |
| 737912 | 737943 | 737973 |

Columns 223 through 228

| 737242 | 737273 |
| :--- | ---: |
| 738065 | 738096 |
| 738096 | 738126 |
|  |  |
| umns 229 | through 234 |


| 737426 | 737457 | 737485 |
| :--- | :--- | :--- |
| 738247 | 738277 | 738308 |
| 738277 | 738308 | 738338 |

Columns 235 through 240

| 737607 | 737638 |
| :--- | ---: |
| 738430 | 738461 |
| 738461 | 738491 |
|  |  |
| umns 241 | through 246 |


| 737791 | 737822 | 737851 | 737882 | 737912 | 7379 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 738612 | 738642 | 738673 | 738703 | 738734 | 738 |
| 738642 | 738673 | 738703 | 738734 | 738765 | 738 |

11-1519

## mbscfamounts

Columns 247 through 252

| 737973 | 738004 | 738035 | 738065 | 738096 | 738126 |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 738795 | 738826 | 738856 | 738887 | 738918 | 738946 |
| 738826 | 738856 | 738887 | 738918 | 738946 | 738977 |
|  |  |  |  |  |  |
| amns 253 through 258 |  |  |  |  |  |
|  |  |  |  |  |  |
| 738157 | 738188 | 738216 | 738247 | 738277 | 738308 |
| 738977 | 739007 | 739038 | 739068 | 739099 | 739130 |
| 739007 | 739038 | 739068 | 739099 | 739130 | 739160 |

Columns 259 through 264
$739160739191 \quad 739221$
739191
738430
739252
738461
738491

739283
739283
73931
739312
739343

Columns 265 through 270

| 738522 | 738553 |
| :--- | ---: |
| 739343 | 739373 |
| 739373 | 739404 |
|  |  |
| mans 271 | through 276 |


| 738703 | 738734 | 738765 |
| :--- | :--- | :--- |
| 739526 | 739557 | 739587 |

738795
738826
738856
739526739557
739618
739649
739649
739677
739557
739587
739618
738612
739434
739465
738642
738673
739465
739496
739496 739526

Columns 277 through 282

| 738887 | 738918 | 738946 | 738977 | 739007 | 739038 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 739708 | 739738 | 739769 | 739799 | 739830 | 739861 |
| 739738 | 739769 | 739799 | 739830 | 739861 | 739891 |

Columns 283 through 288

| 739068 | 739099 | 739130 | 739160 | 739191 | 7392 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 739891 | 739922 | 739952 | 739983 | 740014 | 7400 |
| 739922 | 739952 | 739983 | 740014 | 740042 | 7400 |

Columns 289 through 294

| 739252 | 739283 | 739312 |
| :--- | :--- | :--- |
| 740073 | 740103 | 740134 |

739343
740164
739373
740073740103

740195
740195
740
740

Columns 295 through 300
$740256 \quad 740287 \quad 740317$
740348
$740287 \quad 740317$
740348
740379
739557
740379
740407

739

Columns 301 through 306

| 739618 | 739649 |
| :--- | :--- |
| 740438 | 740468 |
| 740468 | 740499 |

739677
739708
739738
739
740499
740529
740529
740560
740560
740
740591
740

Columns 307 through 312

| 739799 | 739830 | 739861 | 739891 | 739922 | 7399 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 740621 | 740652 | 740682 | 740713 | 740744 | 740 |
| 740652 | 740682 | 740713 | 740744 | 740773 | 7408 |

Columns 313 through 318

| 739983 | 740014 |
| :--- | ---: |
| 740804 | 740834 |
| 740834 | 740865 |
|  |  |
| umns 319 | through 324 |

Columns 319 through 324

11-1521

| 740164 | 740195 | 740226 |
| :--- | :--- | :--- |
| 740987 | 741018 | 741048 |
| 741018 | 741048 | 741079 |

740256
741079
741110

740287
741110
741138
740317
741138
741169

Columns 325 through 330

| 740348 | 740379 | 740407 | 740438 |
| :--- | :--- | :--- | :--- |
| 741169 | 741199 | 741230 | 741260 |

740468
740499
741291
741322

Columns 331 through 336

| 740529 | 740560 | 740591 | 740621 |
| :--- | :--- | :--- | :--- |
| 741352 | 741383 | 741413 | 741444 |
| 741383 | 741413 | 741444 |  |

740652
NaN
NaN
740682
NaN
NaN

Columns 337 through 342

| 740713 | 740744 |
| ---: | ---: |
| NaN | NaN |
| NaN | NaN |

740773
NaN
NaN
740804
NaN
NaN

740834
NaN
NaN

Columns 343 through 348

| 740895 | 740926 |
| ---: | ---: |
| NaN | NaN |
| NaN | NaN |

Columns 349 through 354

| 741079 | 741110 |
| ---: | ---: |
| NaN | NaN |
| NaN | NaN |

741138
NaN
NaN

> 741169
> NaN

NaN

```
741199
NaN
NaN
```

Columns 355 through 360

| 741260 | 741291 | 741322 | 741352 | 741383 |
| ---: | ---: | ---: | ---: | ---: |
| NaN | NaN | NaN | NaN | NaN |
| NaN | NaN | NaN | NaN | NaN |

Column 361
741444
NaN
NaN
TFactors $=$
Columns 1 through 7

| 0 | 1.0667 | 2.0667 | 3.0667 | 4.0667 | 5.0667 | 6.066 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0.9333 | 1.9333 | 2.9333 | 3.9333 | 4.9333 | 5.933 |
| 0 | 0.9333 | 1.9333 | 2.9333 | 3.9333 | 4.9333 | 5.933 |

Columns 8 through 14

| 7.0667 | 8.0667 | 9.0667 | 10.0667 | 11.0667 | 12.0667 | 13.066 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 6.9333 | 7.9333 | 8.9333 | 9.9333 | 10.9333 | 11.9333 | 12.933 |
| 6.9333 | 7.9333 | 8.9333 | 9.9333 | 10.9333 | 11.9333 | 12.933 |

Columns 15 through 21

| 14.0667 | 15.0667 | 16.0667 | 17.0667 | 18.0667 | 19.0667 | 20.066 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 13.9333 | 14.9333 | 15.9333 | 16.9333 | 17.9333 | 18.9333 | 19.933 |
| 13.9333 | 14.9333 | 15.9333 | 16.9333 | 17.9333 | 18.9333 | 19.933 |

Columns 22 through 28

| 21.0667 | 22.0667 | 23.0667 | 24.0667 | 25.0667 | 26.0667 | 27.066 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 20.9333 | 21.9333 | 22.9333 | 23.9333 | 24.9333 | 25.9333 | 26.933 |
| 20.9333 | 21.9333 | 22.9333 | 23.9333 | 24.9333 | 25.9333 | 26.933 |

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Columns 29 through 35

| 28.0667 | 29.0667 | 30.0667 | 31.0667 | 32.0667 | 33.0667 | 34.0667 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 27.9333 | 28.9333 | 29.9333 | 30.9333 | 31.9333 | 32.9333 | 33.9333 |
| 27.9333 | 28.9333 | 29.9333 | 30.9333 | 31.9333 | 32.9333 | 33.9333 |

Columns 36 through 42

| 35.0667 | 36.0667 | 37.0667 | 38.0667 | 39.0667 | 40.0667 | 41.0667 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 34.9333 | 35.9333 | 36.9333 | 37.9333 | 38.9333 | 39.9333 | 40.9333 |
| 34.9333 | 35.9333 | 36.9333 | 37.9333 | 38.9333 | 39.9333 | 40.9333 |

Columns 43 through 49

| 42.0667 | 43.0667 | 44.0667 | 45.0667 | 46.0667 | 47.0667 | 48.0667 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 41.9333 | 42.9333 | 43.9333 | 44.9333 | 45.9333 | 46.9333 | 47.9333 |
| 41.9333 | 42.9333 | 43.9333 | 44.9333 | 45.9333 | 46.9333 | 47.9333 |

Columns 50 through 56

| 49.0667 | 50.0667 | 51.0667 | 52.0667 | 53.0667 | 54.0667 | 55.0667 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 48.9333 | 49.9333 | 50.9333 | 51.9333 | 52.9333 | 53.9333 | 54.9333 |
| 48.9333 | 49.9333 | 50.9333 | 51.9333 | 52.9333 | 53.9333 | 54.9333 |

Columns 57 through 63

| 56.0667 | 57.0667 | 58.0667 | 59.0667 | 60.0667 | 61.0667 | 62.0667 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 55.9333 | 56.9333 | 57.9333 | 58.9333 | 59.9333 | 60.9333 | 61.9333 |
| 55.9333 | 56.9333 | 57.9333 | 58.9333 | 59.9333 | 60.9333 | 61.9333 |

Columns 64 through 70

| 63.0667 | 64.0667 | 65.0667 | 66.0667 | 67.0667 | 68.0667 | 69.0667 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 62.9333 | 63.9333 | 64.9333 | 65.9333 | 66.9333 | 67.9333 | 68.9333 |
| 62.9333 | 63.9333 | 64.9333 | 65.9333 | 66.9333 | 67.9333 | 68.9333 |
|  |  |  |  |  |  |  |
| Columns 71 through 77 |  |  |  |  |  |  |


| 70.0667 | 71.0667 | 72.0667 | 73.0667 | 74.0667 | 75.0667 | 76.066 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 69.9333 | 70.9333 | 71.9333 | 72.9333 | 73.9333 | 74.9333 | 75.933 |
| 69.9333 | 70.9333 | 71.9333 | 72.9333 | 73.9333 | 74.9333 | 75.933 |

Columns 78 through 84

| 77.0667 | 78.0667 | 79.0667 | 80.0667 | 81.0667 | 82.0667 | 83.066 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 76.9333 | 77.9333 | 78.9333 | 79.9333 | 80.9333 | 81.9333 | 82.933 |
| 76.9333 | 77.9333 | 78.9333 | 79.9333 | 80.9333 | 81.9333 | 82.933 |

Columns 85 through 91

| 84.0667 | 85.0667 | 86.0667 | 87.0667 | 88.0667 | 89.0667 | 90.066 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 83.9333 | 84.9333 | 85.9333 | 86.9333 | 87.9333 | 88.9333 | 89.933 |
| 83.9333 | 84.9333 | 85.9333 | 86.9333 | 87.9333 | 88.9333 | 89.9333 |
|  |  |  |  |  |  |  |
| Columns 92 through 98 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| 91.0667 | 92.0667 | 93.0667 | 94.0667 | 95.0667 | 96.0667 | 97.066 |
| 90.9333 | 91.9333 | 92.9333 | 93.9333 | 94.9333 | 95.9333 | 96.933 |
| 90.9333 | 91.9333 | 92.9333 | 93.9333 | 94.9333 | 95.9333 | 96.9333 |

Columns 99 through 105

| 98.0667 | 99.0667 | 100.0667 | 101.0667 | 102.0667 | 103.0667 | 104.066 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| 97.9333 | 98.9333 | 99.9333 | 100.9333 | 101.9333 | 102.9333 | 103.933 |
| 97.9333 | 98.9333 | 99.9333 | 100.9333 | 101.9333 | 102.9333 | 103.933 |

Columns 106 through 112

| 105.0667 | 106.0667 | 107.0667 | 108.0667 | 109.0667 | 110.0667 | 111.066 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 104.9333 | 105.9333 | 106.9333 | 107.9333 | 108.9333 | 109.9333 | 110.9333 |
| 104.9333 | 105.9333 | 106.9333 | 107.9333 | 108.9333 | 109.9333 | 110.9333 |
|  |  |  |  |  |  |  |

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| 112.0667 | 113.0667 | 114.0667 | 115.0667 | 116.0667 | 117.0667 | 118.0667 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 111.9333 | 112.9333 | 113.9333 | 114.9333 | 115.9333 | 116.9333 | 117.9333 |
| 111.9333 | 112.9333 | 113.9333 | 114.9333 | 115.9333 | 116.9333 | 117.9333 |

Columns 120 through 126

| 119.0667 | 120.0667 | 121.0667 | 122.0667 | 123.0667 | 124.0667 | 125.0667 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 118.9333 | 119.9333 | 120.9333 | 121.9333 | 122.9333 | 123.9333 | 124.9333 |
| 118.9333 | 119.9333 | 120.9333 | 121.9333 | 122.9333 | 123.9333 | 124.9333 |

Columns 127 through 133

| 126.0667 | 127.0667 | 128.0667 | 129.0667 | 130.0667 | 131.0667 | 132.0667 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 125.9333 | 126.9333 | 127.9333 | 128.9333 | 129.9333 | 130.9333 | 131.9333 |
| 125.9333 | 126.9333 | 127.9333 | 128.9333 | 129.9333 | 130.9333 | 131.9333 |

Columns 134 through 140

| 133.0667 | 134.0667 | 135.0667 | 136.0667 | 137.0667 | 138.0667 | 139.0667 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 132.9333 | 133.9333 | 134.9333 | 135.9333 | 136.9333 | 137.9333 | 138.9333 |
| 132.9333 | 133.9333 | 134.9333 | 135.9333 | 136.9333 | 137.9333 | 138.9333 |

Columns 141 through 147

| 140.0667 | 141.0667 | 142.0667 | 143.0667 | 144.0667 | 145.0667 | 146.0667 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 139.9333 | 140.9333 | 141.9333 | 142.9333 | 143.9333 | 144.9333 | 145.9333 |
| 139.9333 | 140.9333 | 141.9333 | 142.9333 | 143.9333 | 144.9333 | 145.9333 |
|  |  |  |  |  |  |  |
| Columns 148 through 154 |  |  |  |  |  |  |


| 147.0667 | 148.0667 | 149.0667 | 150.0667 | 151.0667 | 152.0667 | 153.0667 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 146.9333 | 147.9333 | 148.9333 | 149.9333 | 150.9333 | 151.9333 | 152.9333 |
| 146.9333 | 147.9333 | 148.9333 | 149.9333 | 150.9333 | 151.9333 | 152.9333 |

Columns 155 through 161
$154.0667 \quad 155.0667 \quad 156.0667 \quad 157.0667 \quad 158.0667159 .0667160 .0667$

| 153.9333 | 154.9333 | 155.9333 | 156.9333 | 157.9333 | 158.9333 | 159.933 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 153.9333 | 154.9333 | 155.9333 | 156.9333 | 157.9333 | 158.9333 | 159.933 |
| Columns 162 through 168 |  |  |  |  |  |  |
| 161.0667 | 162.0667 | 163.0667 | 164.0667 | 165.0667 | 166.0667 | 167.066 |
| 160.9333 | 161.9333 | 162.9333 | 163.9333 | 164.9333 | 165.9333 | 166.933 |
| 160.9333 | 161.9333 | 162.9333 | 163.9333 | 164.9333 | 165.9333 | 166.933 |
| Columns 169 through 175 |  |  |  |  |  |  |
| 168.0667 | 169.0667 | 170.0667 | 171.0667 | 172.0667 | 173.0667 | 174.066 |
| 167.9333 | 168.9333 | 169.9333 | 170.9333 | 171.9333 | 172.9333 | 173.933 |
| 167.9333 | 168.9333 | 169.9333 | 170.9333 | 171.9333 | 172.9333 | 173.933 |

Columns 176 through 182

| 175.0667 | 176.0667 | 177.0667 | 178.0667 | 179.0667 | 180.0667 | 181.066 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 174.9333 | 175.9333 | 176.9333 | 177.9333 | 178.9333 | 179.9333 | 180.933 |
| 174.9333 | 175.9333 | 176.9333 | 177.9333 | 178.9333 | 179.9333 | 180.9333 |

Columns 183 through 189

| 182.0667 | 183.0667 | 184.0667 | 185.0667 | 186.0667 | 187.0667 | 188.066 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 181.9333 | 182.9333 | 183.9333 | 184.9333 | 185.9333 | 186.9333 | 187.9333 |
| 181.9333 | 182.9333 | 183.9333 | 184.9333 | 185.9333 | 186.9333 | 187.933 |
|  |  |  |  |  |  |  |
| Columns 190 through 196 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| 189.0667 | 190.0667 | 191.0667 | 192.0667 | 193.0667 | 194.0667 | 195.066 |
| 188.9333 | 189.9333 | 190.9333 | 191.9333 | 192.9333 | 193.9333 | 194.9333 |
| 188.9333 | 189.9333 | 190.9333 | 191.9333 | 192.9333 | 193.9333 | 194.933 |

Columns 197 through 203

| 196.0667 | 197.0667 | 198.0667 | 199.0667 | 200.0667 | 201.0667 | 202.066 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 195.9333 | 196.9333 | 197.9333 | 198.9333 | 199.9333 | 200.9333 | 201.933 |

Columns 204 through 210

| 203.0667 | 204.0667 | 205 |
| :--- | :--- | :--- |
| 202.9333 | 203.9333 | 204 |
| 202.9333 | 203.9333 | 204 |
|  |  |  |
| Columns 211 | through 217 |  |


| 210.0667 | 211.0667 | 212.0667 | 213.0667 | 214.0667 | 215.0667 | 216.0667 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 209.9333 | 210.9333 | 211.9333 | 212.9333 | 213.9333 | 214.9333 | 215.9333 |
| 209.9333 | 210.9333 | 211.9333 | 212.9333 | 213.9333 | 214.9333 | 215.9333 |

Columns 218 through 224

| 217.0667 | 218.0667 | 219 |
| :--- | :--- | :--- |
| 216.9333 | 217.9333 | 218 |
| 216.9333 | 217.9333 | 218. |
| Columns 225 | through | 231 |


| 224.0667 | 225.0667 | 226. |
| :--- | :--- | :--- |
| 223.9333 | 224.9333 | 225. |
| 223.9333 | 224.9333 | 225. |
|  |  |  |
| Columns 232 through 238 |  |  |


| 231.0667 | 232.0667 | 233.0667 | 234.0667 | 235.0667 | 236.0667 | 237.0667 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 230.9333 | 231.9333 | 232.9333 | 233.9333 | 234.9333 | 235.9333 | 236.9333 |
| 230.9333 | 231.9333 | 232.9333 | 233.9333 | 234.9333 | 235.9333 | 236.9333 |

Columns 239 through 245

| 238.0667 | 239.0667 | 240.0667 | 241.0667 | 242.0667 | 243.0667 | 244.0667 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 237.9333 | 238.9333 | 239.9333 | 240.9333 | 241.9333 | 242.9333 | 243.9333 |
| 237.9333 | 238.9333 | 239.9333 | 240.9333 | 241.9333 | 242.9333 | 243.9333 |

Columns 246 through 252

| 245.0667 | 246.0667 | 247.0667 | 248.0667 | 249.0667 | 250.0667 | 251.066 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 244.9333 | 245.9333 | 246.9333 | 247.9333 | 248.9333 | 249.9333 | 250.933 |
| 244.9333 | 245.9333 | 246.9333 | 247.9333 | 248.9333 | 249.9333 | 250.933 |
|  |  |  |  |  |  |  |
| Columns 253 through 259 |  |  |  |  |  |  |


| 252.0667 | 253.0667 | 254 |
| :--- | :--- | :--- |
| 251.9333 | 252.9333 | 253 |
| 251.9333 | 252.9333 | 253 |
|  |  |  |
| Columns 260 through | 266 |  |


| 259.0667 | 260.0667 | 261.0667 | 262.0667 | 263.0667 | 264.0667 | 265.066 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 258.9333 | 259.9333 | 260.9333 | 261.9333 | 262.9333 | 263.9333 | 264.933 |
| 258.9333 | 259.9333 | 260.9333 | 261.9333 | 262.9333 | 263.9333 | 264.933 |
|  |  |  |  |  |  |  |
| Columns 267 through 273 |  |  |  |  |  |  |


| 266.0667 | 267.0667 | 268.0667 | 269.0667 | 270.0667 | 271.0667 | 272.066 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 265.9333 | 266.9333 | 267.9333 | 268.9333 | 269.9333 | 270.9333 | 271.933 |
| 265.9333 | 266.9333 | 267.9333 | 268.9333 | 269.9333 | 270.9333 | 271.933 |


| 273.0667 | 274.0667 | 275 |
| :--- | ---: | ---: |
| 272.9333 | 273.9333 | 274 |
| 272.9333 | 273.9333 | 274 |
| Columns 281 | through | 287 |


| 280.0667 | 281.0667 | 282.0667 | 283.0667 | 284.0667 | 285.0667 | 286.066 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 279.9333 | 280.9333 | 281.9333 | 282.9333 | 283.9333 | 284.9333 | 285.9333 |
| 279.9333 | 280.9333 | 281.9333 | 282.9333 | 283.9333 | 284.9333 | 285.9333 |

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Columns 288 through 294

| 287.0667 | 288.0667 | 289.0667 | 290.0667 | 291.0667 | 292.0667 | 293.0667 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 286.9333 | 287.9333 | 288.9333 | 289.9333 | 290.9333 | 291.9333 | 292.9333 |
| 286.9333 | 287.9333 | 288.9333 | 289.9333 | 290.9333 | 291.9333 | 292.9333 |

Columns 295 through 301

| 294.0667 | 295.0667 | 296.0667 | 297.0667 | 298.0667 | 299.0667 | 300.0667 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 293.9333 | 294.9333 | 295.9333 | 296.9333 | 297.9333 | 298.9333 | 299.9333 |
| 293.9333 | 294.9333 | 295.9333 | 296.9333 | 297.9333 | 298.9333 | 299.9333 |

Columns 302 through 308

| 301.0667 | 302.0667 | 303. |
| :--- | :--- | :--- |
| 300.9333 | 301.9333 | 302. |
| 300.9333 | 301.9333 | 302. |
|  |  |  |
| Columns | 309 through | 315 |


| 308.0667 | 309.0667 | 310.0667 | 311.0667 | 312.0667 | 313.0667 | 314.0667 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 307.9333 | 308.9333 | 309.9333 | 310.9333 | 311.9333 | 312.9333 | 313.9333 |
| 307.9333 | 308.9333 | 309.9333 | 310.9333 | 311.9333 | 312.9333 | 313.9333 |

Columns 316 through 322

| 315.0667 | 316.0667 | 317.0667 | 318.0667 | 319.0667 | 320.0667 | 321.0667 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 314.9333 | 315.9333 | 316.9333 | 317.9333 | 318.9333 | 319.9333 | 320.9333 |
| 314.9333 | 315.9333 | 316.9333 | 317.9333 | 318.9333 | 319.9333 | 320.9333 |

Columns 323 through 329

| 322.0667 | 323.0667 | 324.0667 | 325.0667 | 326.0667 | 327.0667 | 328.0667 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 321.9333 | 322.9333 | 323.9333 | 324.9333 | 325.9333 | 326.9333 | 327.9333 |
| 321.9333 | 322.9333 | 323.9333 | 324.9333 | 325.9333 | 326.9333 | 327.9333 |

Columns 330 through 336

| 329.0667 | 330.0667 | 331.0667 | 332.0667 | 333.0667 | 334.0667 | 335.066 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 328.9333 | 329.9333 | 330.9333 | 331.9333 | 332.9333 | NaN | Na |
| 328.9333 | 329.9333 | 330.9333 | 331.9333 | NaN | NaN | Na |
| Columns 337 through 343 |  |  |  |  |  |  |
| 336.0667 | 337.0667 | 338.0667 | 339.0667 | 340.0667 | 341.0667 | 342.066 |
| NaN | NaN | NaN | NaN | NaN | NaN | Na |
| NaN | NaN | NaN | NaN | NaN | NaN | Na |
| Columns 344 through 350 |  |  |  |  |  |  |
| 343.0667 | 344.0667 | 345.0667 | 346.0667 | 347.0667 | 348.0667 | 349.066 |
| NaN | NaN | NaN | NaN | NaN | NaN | Na |
| NaN | NaN | NaN | NaN | NaN | NaN | Na |
| Columns 351 through 357 |  |  |  |  |  |  |
| 350.0667 | 351.0667 | 352.0667 | 353.0667 | 354.0667 | 355.0667 | 356.066 |
| NaN | NaN | NaN | NaN | NaN | NaN | Na |
| NaN | NaN | NaN | NaN | NaN | NaN | Na |
| Columns 358 through 361 |  |  |  |  |  |  |
| 357.0667 | 358.0667 | 359.0667 | 360.0667 |  |  |  |
| NaN | NaN | NaN | NaN |  |  |  |
| NaN | NaN | NaN | NaN |  |  |  |

Factors =

Columns 1 through 7

| 1.0000 | 0.9992 | 0.9982 | 0.9970 | 0.9957 | 0.9942 | 0.992 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1.0000 | 0.9944 | 0.9887 | 0.9828 | 0.9769 | 0.9711 | 0.965 |
| 1.0000 | 0.9942 | 0.9883 | 0.9824 | 0.9766 | 0.9707 | 0.964 |

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Columns 8 through 14

| 0.9907 | 0.9887 | 0.9865 | 0.9841 | 0.9816 | 0.9789 | 0.9761 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.9595 | 0.9538 | 0.9481 | 0.9424 | 0.9368 | 0.9311 | 0.9255 |
| 0.9592 | 0.9534 | 0.9477 | 0.9420 | 0.9364 | 0.9307 | 0.9251 |

Columns 15 through 21

| 0.9731 | 0.9699 | 0.9666 | 0.9631 | 0.9594 | 0.9556 | 0.9517 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.9199 | 0.9144 | 0.9089 | 0.9034 | 0.8979 | 0.8925 | 0.8871 |
| 0.9195 | 0.9140 | 0.9085 | 0.9030 | 0.8975 | 0.8921 | 0.8866 |

Columns 22 through 28

| 0.9475 | 0.9433 | 0.9389 | 0.9343 | 0.9296 | 0.9247 | 0.9197 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.8817 | 0.8763 | 0.8710 | 0.8657 | 0.8604 | 0.8552 | 0.8499 |
| 0.8813 | 0.8759 | 0.8706 | 0.8653 | 0.8600 | 0.8547 | 0.8495 |

Columns 29 through 35

| 0.9146 | 0.9093 | 0.9039 | 0.8985 | 0.8932 | 0.8878 | 0.8825 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.8447 | 0.8396 | 0.8344 | 0.8293 | 0.8242 | 0.8191 | 0.8140 |
| 0.8443 | 0.8391 | 0.8339 | 0.8288 | 0.8237 | 0.8186 | 0.8136 |

Columns 36 through 42

| 0.8772 | 0.8720 | 0.8668 | 0.8616 | 0.8564 | 0.8512 | 0.8461 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.8090 | 0.8040 | 0.7990 | 0.7941 | 0.7892 | 0.7842 | 0.7794 |
| 0.8085 | 0.8035 | 0.7985 | 0.7936 | 0.7887 | 0.7837 | 0.7789 |

Columns 43 through 49

| 0.8410 | 0.8359 | 0.8309 | 0.8258 | 0.8208 | 0.8159 | 0.8109 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.7745 | 0.7697 | 0.7649 | 0.7601 | 0.7553 | 0.7506 | 0.7458 |
| 0.7740 | 0.7692 | 0.7643 | 0.7595 | 0.7548 | 0.7500 | 0.7453 |

Columns 50 through 56

| 0.8060 | 0.8011 | 0.7962 | 0.7913 | 0.7865 | 0.7817 | 0.776 § |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.7411 | 0.7365 | 0.7318 | 0.7272 | 0.7226 | 0.7180 | 0.713 |
| 0.7406 | 0.7359 | 0.7313 | 0.7267 | 0.7220 | 0.7175 | 0.712 |

Columns 57 through 63

| 0.7722 | 0.7674 | 0.7627 | 0.7580 | 0.7533 | 0.7487 | 0.744 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.7089 | 0.7044 | 0.6999 | 0.6954 | 0.6910 | 0.6865 | 0.682 |
| 0.7084 | 0.7038 | 0.6993 | 0.6949 | 0.6904 | 0.6860 | 0.681 |

Columns 64 through 70
0.7395
0.7349
0.6777
0.6734
0.6772
0.6728
Columns 71 through 77
0.7303
0.7258
0.7213
0.7168
0.712
0.6690
0.6647
0.6604
0.6561
0.651
0.6684
0.6641
0.6598
0.6555
$0.651 ?$

| 0.7079 | 0.7035 | 0.6991 | 0.6947 | 0.6903 | 0.6860 | 0.681 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.6476 | 0.6434 | 0.6392 | 0.6350 | 0.6309 | 0.6267 | 0.622 |
| 0.6470 | 0.6428 | 0.6386 | 0.6344 | 0.6302 | 0.6261 | 0.622 |

Columns 78 through 84

| 0.6774 | 0.6731 | 0.6688 | 0.6646 | 0.6604 | 0.6562 | $0.652($ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.6185 | 0.6144 | 0.6104 | 0.6063 | 0.6023 | 0.5983 | 0.594 |
| 0.6179 | 0.6138 | 0.6097 | 0.6057 | 0.6016 | 0.5976 | 0.593 |

Columns 85 through 91

| 0.6478 | 0.6437 | 0.6396 | 0.6355 | 0.6314 | 0.6274 | 0.623 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.5903 | 0.5864 | 0.5825 | 0.5785 | 0.5747 | 0.5708 | 0.566 |
| 0.5897 | 0.5857 | 0.5818 | 0.5779 | 0.5740 | 0.5701 | 0.566 |

[^10]| 0.6193 | 0.6153 | 0.6113 | 0.6074 | 0.6035 | 0.5995 | 0.5956 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.5631 | 0.5593 | 0.5555 | 0.5517 | 0.5479 | 0.5442 | 0.5405 |
| 0.5624 | 0.5586 | 0.5548 | 0.5510 | 0.5473 | 0.5435 | 0.5398 |

Columns 99 through 105

| 0.5917 | 0.5879 | 0.5840 | 0.5802 | 0.5764 | 0.5726 | 0.5688 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.5368 | 0.5331 | 0.5294 | 0.5257 | 0.5221 | 0.5185 | 0.5149 |
| 0.5361 | 0.5324 | 0.5287 | 0.5250 | 0.5214 | 0.5178 | 0.5142 |

Columns 106 through 112

| 0.5651 | 0.5614 | 0.5576 | 0.5539 | 0.5503 | 0.5466 | 0.5429 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.5113 | 0.5077 | 0.5042 | 0.5006 | 0.4971 | 0.4936 | 0.4901 |
| 0.5106 | 0.5070 | 0.5034 | 0.4999 | 0.4964 | 0.4929 | 0.4894 |

Columns 113 through 119

| 0.5393 | 0.5357 | 0 |
| :--- | :--- | :--- |
| 0.4866 | 0.4832 | 0 |
| 0.4859 | 0.4824 | 0 |
|  |  |  |
| Columns 120 | through | 126 |


| 0.5144 | 0.5109 | 0.5074 | 0.5040 | 0.5005 | 0.4971 | 0.4937 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.4628 | 0.4594 | 0.4561 | 0.4528 | 0.4495 | 0.4462 | 0.4430 |
| 0.4620 | 0.4587 | 0.4554 | 0.4520 | 0.4487 | 0.4455 | 0.4422 |

Columns 127 through 133

| 0.4903 | 0.4869 | 0.4835 | 0.4802 | 0.4769 | 0.4735 | 0.4702 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.4397 | 0.4365 | 0.4333 | 0.4301 | 0.4269 | 0.4237 | 0.4205 |
| 0.4389 | 0.4357 | 0.4325 | 0.4293 | 0.4261 | 0.4229 | 0.4197 |

11-1534

| 0.4670 | 0.4637 | 0.4604 | 0.4572 | 0.4540 | 0.4508 | 0.4476 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.4174 | 0.4143 | 0.4111 | 0.4080 | 0.4049 | 0.4019 | 0.3988 |
| 0.4166 | 0.4134 | 0.4103 | 0.4072 | 0.4041 | 0.4011 | 0.398 |

Columns 141 through 147

| 0.4444 | 0.4412 | 0.4381 | 0.4349 | 0.4318 | 0.4287 | 0.425 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.3958 | 0.3927 | 0.3897 | 0.3867 | 0.3837 | 0.3808 | 0.377 |
| 0.3949 | 0.3919 | 0.3889 | 0.3859 | 0.3829 | 0.3799 | 0.376 |

Columns 148 through 154

| 0.4226 | 0.4195 | 0.4165 | 0.4134 | 0.4104 | 0.4074 | 0.4044 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.3748 | 0.3719 | 0.3690 | 0.3661 | 0.3632 | 0.3603 | 0.3574 |
| 0.3740 | 0.3711 | 0.3681 | 0.3652 | 0.3623 | 0.3594 | 0.3566 |

Columns 155 through 161

| 0.4014 | 0.3985 | 0.3955 | 0.3926 | 0.3897 | 0.3868 | 0.383 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.3546 | 0.3517 | 0.3489 | 0.3461 | 0.3433 | 0.3405 | 0.337 |
| 0.3537 | 0.3509 | 0.3480 | 0.3452 | 0.3424 | 0.3396 | 0.336 |

Columns 162 through 168

| 0.3810 | 0.3781 | 0.3753 | 0.3724 | 0.3696 | 0.3668 | 0.364 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.3350 | 0.3322 | 0.3295 | 0.3267 | 0.3240 | 0.3213 | 0.318 |
| 0.3341 | 0.3313 | 0.3286 | 0.3258 | 0.3231 | 0.3204 | 0.317 |

Columns 169 through 175

| 0.3612 | 0.3584 | 0.3557 | 0.3529 | 0.3502 | 0.3475 | 0.3448 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.3160 | 0.3133 | 0.3106 | 0.3080 | 0.3054 | 0.3027 | 0.300 |
| 0.3151 | 0.3124 | 0.3097 | 0.3071 | 0.3045 | 0.3018 | 0.299 |

Columns 176 through 182

| 0.3421 | 0.3394 | 0.3367 | 0.3340 | 0.3314 | 0.3287 | 0.326 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

11-1535

| 0.2975 | 0.2950 | 0.2924 | 0.2898 | 0.2873 | 0.2847 | 0.2822 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.2966 | 0.2940 | 0.2915 | 0.2889 | 0.2863 | 0.2838 | 0.2813 |

Columns 183 through 189

| 0.3235 | 0.3209 | 0.3183 | 0.3157 | 0.3132 | 0.3106 | 0.3081 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.2797 | 0.2772 | 0.2747 | 0.2722 | 0.2698 | 0.2673 | 0.2649 |
| 0.2788 | 0.2763 | 0.2738 | 0.2713 | 0.2688 | 0.2663 | 0.2639 |

Columns 190 through 196
0.3055
0.3030
0.3005
0.2980
0.2955
0.2931
0.2906
0.2624
0.2600
0.2576
0.2552
0.2542
0.2518
0.2504
0.2480
0.2615
0.2590
0.2566
0.2542
0.2494
0.2470

Columns 197 through 203

| 0.2881 | 0.2857 | 0.2833 | 0.2809 | 0.2784 | 0.2760 | 0.2737 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.2457 | 0.2433 | 0.2410 | 0.2386 | 0.2363 | 0.2340 | 0.2317 |
| 0.2447 | 0.2423 | 0.2400 | 0.2376 | 0.2353 | 0.2330 | 0.2307 |

Columns 204 through 210

| 0.2713 | 0.2689 | 0.2666 | 0.2642 | 0.2619 | 0.2596 | 0.2573 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.2294 | 0.2271 | 0.2249 | 0.2226 | 0.2204 | 0.2181 | 0.2159 |
| 0.2284 | 0.2261 | 0.2239 | 0.2216 | 0.2193 | 0.2171 | 0.2149 |

Columns 211 through 217

| 0.2550 | 0.2527 | 0.2504 | 0.2481 | 0.2458 | 0.2436 | 0.2414 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.2137 | 0.2115 | 0.2093 | 0.2071 | 0.2049 | 0.2027 | 0.2005 |
| 0.2126 | 0.2104 | 0.2082 | 0.2060 | 0.2038 | 0.2017 | 0.1995 |

Columns 218 through 224
0.2391
0.2369
0.2347
0.2325
0.2303
0.2281
0.2259
0.1984
0.1962
0.1941
0.1920
0.1899
0.1877
0.1856
0.1973
0.1952
0.1931
0.1909
0.1888
0.1867
0.184

Columns 225 through 231

| 0.2238 | 0.2216 | 0.2195 | 0.2173 | 0.2152 | 0.2131 | 0.2110 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.1836 | 0.1815 | 0.1794 | 0.1773 | 0.1753 | 0.1732 | 0.1712 |
| 0.1825 | 0.1804 | 0.1783 | 0.1763 | 0.1742 | 0.1722 | 0.170 |

Columns 232 through 238
0.2089
0.2068
0.2047
0.2027
0.2006
0.1986
0.196
0.1692
0.1671
0.1651
0.1631
0.1611
0.1591
0.157
0.1681
0.1661
0.1640
0.1620
0.1600
0.1581
0.156

Columns 239 through 245

| 0.1945 | 0.1925 | 0.1904 | 0.1884 | 0.1864 | 0.1844 | 0.182 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.1552 | 0.1532 | 0.1513 | 0.1493 | 0.1474 | 0.1455 | 0.143 |
| 0.1541 | 0.1521 | 0.1502 | 0.1482 | 0.1463 | 0.1444 | 0.142 |

Columns 246 through 252
0.1805
0.1785
0.1416
0.1397
0.1405
0.1386
Columns 253 through 259
0.1766
0.1746
0.1727
0.1707
0.168
0.1378
0.1359
0.1341
0.1322
0.130
0.1367
0.1348
0.1329
0.1311
0.129

| 0.1669 | 0.1650 | 0.1631 | 0.1612 | 0.1593 | 0.1575 | 0.155 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.1285 | 0.1266 | 0.1248 | 0.1229 | 0.1211 | 0.1193 | 0.117 |
| 0.1273 | 0.1255 | 0.1236 | 0.1218 | 0.1200 | 0.1181 | 0.116 |

Columns 260 through 266

| 0.1537 | 0.1519 | 0.1500 | 0.1482 | 0.1464 | 0.1446 | 0.142 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.1157 | 0.1139 | 0.1121 | 0.1103 | 0.1085 | 0.1068 | 0.105 |
| 0.1145 | 0.1127 | 0.1109 | 0.1091 | 0.1074 | 0.1056 | 0.103 |

Columns 267 through 273

| 0.1409 | 0.1391 | 0.1374 | 0.1356 | 0.1338 | 0.1320 | 0.1303 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.1032 | 0.1015 | 0.0998 | 0.0980 | 0.0963 | 0.0946 | 0.0929 |
| 0.1021 | 0.1003 | 0.0986 | 0.0968 | 0.0951 | 0.0934 | 0.0917 |

Columns 274 through 280

| 0.1285 | 0.1268 | 0 |
| :---: | :---: | :---: |
| 0.0912 | 0.0895 | 0 |
| 0.0900 | 0.0883 | 0 |
| Columns 281 | through | 287 |


| 0.1165 | 0.1148 | 0.1131 | 0.1114 | 0.1097 | 0.1080 | 0.1064 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0794 | 0.0778 | 0.0761 | 0.0745 | 0.0728 | 0.0712 | 0.0696 |
| 0.0782 | 0.0765 | 0.0749 | 0.0732 | 0.0716 | 0.0700 | 0.0684 |

Columns 288 through 294

| 0.1047 | 0.1031 | 0.1014 | 0.0998 | 0.0982 | 0.0966 | 0.0950 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0680 | 0.0664 | 0.0648 | 0.0632 | 0.0616 | 0.0600 | 0.0584 |
| 0.0667 | 0.0651 | 0.0635 | 0.0619 | 0.0603 | 0.0587 | 0.0572 |

Columns 295 through 301

| 0.0933 | 0.0917 | 0.0902 | 0.0886 | 0.0870 | 0.0854 | 0.0838 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0568 | 0.0553 | 0.0537 | 0.0522 | 0.0506 | 0.0491 | 0.0476 |
| 0.0556 | 0.0540 | 0.0525 | 0.0509 | 0.0494 | 0.0478 | 0.0463 |

Columns 302 through 308

| 0.0823 | 0.0807 | 0.0792 | 0.0776 | 0.0761 | 0.0746 | 0.0730 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0460 | 0.0445 | 0.0430 | 0.0415 | 0.0400 | 0.0385 | 0.0370 |
| 0.0448 | 0.0432 | 0.0417 | 0.0402 | 0.0387 | 0.0372 | 0.0357 |

11-1538

Columns 309 through 315

| 0.0715 | 0.0700 | 0.0685 | 0.0670 | 0.0655 | 0.0640 | 0.062 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0355 | 0.0340 | 0.0325 | 0.0311 | 0.0296 | 0.0281 | 0.026 |
| 0.0342 | 0.0327 | 0.0312 | 0.0298 | 0.0283 | 0.0268 | 0.025 |

Columns 316 through 322

| 0.0610 | 0.0596 | 0.0581 | 0.0566 | 0.0552 | 0.0537 | 0.052 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0252 | 0.0238 | 0.0223 | 0.0209 | 0.0195 | 0.0180 | 0.016 |
| 0.0239 | 0.0225 | 0.0210 | 0.0196 | 0.0181 | 0.0167 | 0.015 |

Columns 323 through 329

| 0.0508 | 0.0494 | 0 |
| :--- | :--- | :--- |
| 0.0152 | 0.0138 | 0 |
| 0.0139 | 0.0125 | 0 |
|  |  |  |
| olumns 330 | through | 336 |


| 0.0409 | 0.0395 | 0 |
| :--- | :--- | ---: |
| 0.0055 | 0.0041 | 0 |
| 0.0041 | 0.0027 | 0 |
|  |  |  |
| olumns 337 | through | 343 |

$$
\begin{array}{rrr}
0.0313 & 0.0299 & 0.0286 \\
\mathrm{NaN} & \mathrm{NaN} & \mathrm{NaN} \\
\mathrm{NaN} & \mathrm{NaN} & \mathrm{NaN}
\end{array}
$$

0.0272
NaN NaN

Columns 344 through 350
0.0219
0.0205

NaN
NaN
0.0192

NaN
NaN
0.0179
NaN
0.016

NaN
NaN
0.0153
NaN
NaN
0.032

NaI NaI

[^11]
## mbscfamounts

| 0.0127 | 0.0114 | 0.0101 | 0.0088 | 0.0076 | 0.0063 | 0.0050 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| NaN | NaN | NaN | NaN | NaN | NaN | NaN |
| NaN | NaN | NaN | NaN | NaN | NaN | NaN |

Columns 358 through 361

| 0.0038 | 0.0025 | 0.0012 | 0 |
| ---: | ---: | ---: | ---: |
| NaN | NaN | NaN | NaN |
| NaN | NaN | NaN | NaN |

Each output is a 3-by-361 element matrix padded with | $\mathrm{NaN} \mid \mathrm{s}$ wherever elements are missing.

## Calculate Payment, Principal, Interest, and Prepayment for a Single Mortgage

Given a mortgage with the following characteristics, compute payments, principal, interest, and prepayment.

Define the mortgage characteristics.

```
Settle = datenum('17-April-2002');
Maturity = datenum('1-Jan-2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
CouponRate = 0.075;
Delay = 14;
PrepaySpeed = 100;
```

Use mbscfamonts to evaluate the mortgage.
[Payment, Principal, Interest, Prepayment] = ... mbscfamounts(Settle, Maturity, IssueDate, GrossRate, ... CouponRate, Delay, PrepaySpeed)
Payment =
Columns 1 through 7
$-0.0033$ 0.0118 0.0120 0.0121 0.0120 0.0119 ..... 0.011
Columns 8 through ..... 14
0.0118 0.0117 ..... 0.0117
0.0116 0.0115 0.0115 0.011
Columns 15 through ..... 21
0.0114 0.0113 0.0112 0.0112 ..... 0.0111
0.0110 0.011
Columns 22 through ..... 28
0.0109 0.0109 0.0108 0.0107 0.0107 0.0106 0.010
Columns 29 through ..... 35
0.0105 0.0105 0.0104 0.0103 0.0103 0.0102 0.010
Columns 36 through ..... 42
0.0101 0.0101 0.0100 0.0099 0.0099 0.0098 0.009
Columns 43 through ..... 49
0.0097 0.0097 0.0096 0.0096 0.0095 0.0095 0.009
Columns 50 through ..... 56
0.0094 0.0093 0.0093 0.0092 0.0092 0.0091 0.009
Columns 57 through ..... 63
0.0090 0.0089 0.0089 0.0088 0.0088 0.0087 ..... 0.008

Columns 64 through 70
$0.0087 \quad 0.0086$
0.0086
0.0085
0.0085
0.0084
0.0084

Columns 71 through 77
$\begin{array}{lllllll}0.0083 & 0.0083 & 0.0082 & 0.0082 & 0.0081 & 0.0081 & 0.0080\end{array}$

Columns 78 through 84
0.0080
0.0079
0.0079
0.0079
0.0078
0.0078
0.0077

Columns 85 through 91
$\begin{array}{lllllll}0.0077 & 0.0076 & 0.0076 & 0.0075 & 0.0075 & 0.0075 & 0.0074\end{array}$

Columns 92 through 98
0.0074
0.0073
0.0073
0.0073
0.0072
0.0072
0.0071

Columns 99 through 105
0.0071
0.0070
0.0070
0.0070
0.0069
0.0069
0.0068

Columns 106 through 112
0.0068
0.0068
0.0067
0.0067
0.0066
0.0066
0.0066

Columns 113 through 119

$$
\begin{array}{lllllll}
0.0065 & 0.0065 & 0.0065 & 0.0064 & 0.0064 & 0.0063 & 0.0063
\end{array}
$$

Columns 120 through 126
0.0063
0.0062
0.0062
0.0062
0.0061
0.0061
0.0061
Columns 127 through ..... 133
0.0060 0.0060 0.0059 0.0059 0.0059 0.0058 0.0058
Columns 134 through ..... 140
0.0058 0.0057 0.0057 0.0057 0.0056 0.0056 0.005
Columns 141 through ..... 147
0.0055 0.0055 0.0055 0.0054 0.0054 0.0054 0.005
Columns 148 through ..... 154
0.0053 0.0053 0.0052 0.0052 0.0052 0.0052 0.005
Columns 155 through ..... 161
0.0051 0.0051 0.0050 0.0050 0.0050 0.0049 0.004
Columns 162 through ..... 168
0.0049 0.0048 0.0048 0.0048 0.0048 0.0047 0.004
Columns 169 through ..... 175
0.0047 0.0046 0.0046 0.0046 0.0046 0.0045 0.004
Columns 176 through ..... 182
0.0045 0.0044 0.0044 0.0044 0.0044 0.0043 0.004
Columns 183 through ..... 189
0.0043 0.0043 0.0042 0.0042 0.0042 0.0041 0.004
Columns 190 through ..... 196
0.0041
0.0041
0.0040
0.0040
0.0040
0.0040
0.0039

Columns 197 through 203
0.0039
0.0039
0.0039
0.0038
0.0038
0.0038
0.0038

Columns 204 through 210

$$
\begin{array}{lllllll}
0.0037 & 0.0037 & 0.0037 & 0.0037 & 0.0036 & 0.0036 & 0.0036
\end{array}
$$

Columns 211 through 217
$0.0036 \quad 0.0035 \quad 0.003$
0.0035
0.0035
0.0035
0.0034

Columns 218 through 224
$\begin{array}{lllllll}0.0034 & 0.0034 & 0.0034 & 0.0033 & 0.0033 & 0.0033 & 0.0033\end{array}$
Columns 225 through 231

$$
\begin{array}{lllllll}
0.0033 & 0.0032 & 0.0032 & 0.0032 & 0.0032 & 0.0031 & 0.0031
\end{array}
$$

Columns 232 through 238
$\begin{array}{lllllll}0.0031 & 0.0031 & 0.0031 & 0.0030 & 0.0030 & 0.0030 & 0.0030\end{array}$
Columns 239 through 245

$$
\begin{array}{lllllll}
0.0030 & 0.0029 & 0.0029 & 0.0029 & 0.0029 & 0.0029 & 0.0028
\end{array}
$$

Columns 246 through 252
$\begin{array}{lllllll}0.0028 & 0.0028 & 0.0028 & 0.0028 & 0.0027 & 0.0027 & 0.0027\end{array}$

Columns 253 through 259
0.0027
0.0027
0.0026
0.0026
0.0026
0.0026
0.002

Columns 260 through 266
0.0025
0.0025
0.0025
0.0025
0.0025
0.0024
0.002

Columns 267 through 273
$\begin{array}{lllllll}0.0024 & 0.0024 & 0.0024 & 0.0024 & 0.0023 & 0.0023 & 0.002\end{array}$
Columns 274 through 280
$\begin{array}{lllllll}0.0023 & 0.0023 & 0.0023 & 0.0022 & 0.0022 & 0.0022 & 0.002\end{array}$
Columns 281 through 287
$\begin{array}{lllllll}0.0022 & 0.0021 & 0.0021 & 0.0021 & 0.0021 & 0.0021 & 0.002\end{array}$
Columns 288 through 294
0.0020
0.0020
0.0020
0.0020
0.0020
0.0020
0.002

Columns 295 through 301
0.0019
0.0019
0.0019
0.0019
0.0019
0.0019
0.0018

Columns 302 through 308

| 0.0018 | 0.0018 | 0.0018 | 0.0018 | 0.0018 | 0.0017 | 0.0017 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Columns 309 through 315
0.0017
0.0017
0.0017
0.0017
0.0017
0.0016
0.001

Columns 316 through 322
0.0016
0.0016
0.0016
0.0016
0.0016
0.0015
0.001

11-1545
Columns 323 through ..... 329
0.0015 0.0015 0.0015 0.0015 0.0015 0.0014 0.0014
Columns 330 through ..... 334
0.0014 0.0014 0.0014 0.0014 0.0014
Principal =Columns 1 through 6

| 731323 | 731337 | 731368 | 731398 | 731429 |
| :--- | :--- | :--- | :--- | :--- |

Columns 7 through ..... 12
731490731521731551731582731613731641
Columns 13 through ..... 18731672731702731733731763731794731825
Columns 19 through 24
731855 731886731916731947731978732007Columns 25 through 30732038732068732099732129732160732191
Columns 31 through 36
732221 732252732282732313732344732372
Columns 37 through ..... 42

732403
732433

Columns 43 through 48
$732586 \quad 732617$

Columns 49 through 54
$732768 \quad 732798$

Columns 55 through 60

732951 - 732982

Columns 61 through 66

733133
733163

Columns 67 through 72

733316733347

Columns 73 through 78

733499733529
733560
733590
733621

Columns 79 through 84

733682733713

Columns 85 through 90

733864733894

Columns 91 through 96

732464

732647
732678
732709

732829

733012
733043

733194
733224
733255

733408
733377
733439
733

733743
733774
733805

7338

| 734047 | 734078 | 734108 | 734139 | 734170 | 734198 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Columns 97 through 102 |  |  |  |  |  |
| 734229 | 734259 | 734290 | 734320 | 734351 | 734382 |
| Columns 103 through 108 |  |  |  |  |  |
| 734412 | 734443 | 734473 | 734504 | 734535 | 734563 |
| Columns 109 through 114 |  |  |  |  |  |
| 734594 | 734624 | 734655 | 734685 | 734716 | 734747 |
| Columns 115 through 120 |  |  |  |  |  |
| 734777 | 734808 | 734838 | 734869 | 734900 | 734929 |
| Columns 121 through 126 |  |  |  |  |  |
| 734960 | 734990 | 735021 | 735051 | 735082 | 735113 |
| Columns 127 through 132 |  |  |  |  |  |
| 735143 | 735174 | 735204 | 735235 | 735266 | 735294 |
| Columns 133 through 138 |  |  |  |  |  |
| 735325 | 735355 | 735386 | 735416 | 735447 | 735478 |
| Columns 139 through 144 |  |  |  |  |  |
| 735508 | 735539 | 735569 | 735600 | 735631 | 735659 |
| Columns 145 through 150 |  |  |  |  |  |
| 735690 | 735720 | 735751 | 735781 | 735812 | 735843 |


Columns 205 through 210

| 737516 | 737546 | 737577 | 737607 | 737638 | 737669 |
| :--- | :--- | :--- | :--- | :--- | :--- |

Columns 211 through 216
737699 737730737760737791737822737851
Columns 217 through 222
737882 737912737943737973738004738035
Columns 223 through ..... 228738065738096738126738157738188738216
Columns 229 through ..... 234738247738277738308738338738369738400Columns 235 through 240$738430 \quad 738461$738491738522738553738581
Columns 241 through 246
738612 738642738673738703738734738765Columns 247 through 252738795738826738856738887738918738946
Columns 253 through 258
738977 739007739038739068739099739130
Columns 259 through ..... 264


740804
740834
740865

Columns 319 through 324
$740987-741018$

Columns 325 through 330

741169

Columns 331 through 334
$741352 \quad 741383 \quad 741413 \quad 74144$

Interest =

Columns 1 through 7

$$
\begin{array}{lllllll}
0 & 0.9333 & 1.9333 & 2.9333 & 3.9333 & 4.9333 & 5.9333
\end{array}
$$

Columns 8 through 14
8.9333
9.9333
10.9333
11.9333
12.9333

Columns 15 through 21
13.9333
14.9333
15.9333
16.9333
17.9333
18.9333
19.9333

Columns 22 through 28
20.9333
21.9333
22.9333
23.9333
24.9333
25.9333
26.9333

Columns 29 through 35
27.9333
28.9333
29.9333
30.9333
31.9333
32.9333
33.9333


```
    97.9333 98.9333 99.9333 100.9333 101.9333 102.9333 103.9333
Columns 106 through 112
104.9333 105.9333 106.9333 107.9333 108.9333 109.9333 110.9333
Columns 113 through 119
111.9333 112.9333 113.9333 114.9333 115.9333 116.9333 117.9333
Columns }120\mathrm{ through 126
118.9333 119.9333 120.9333 121.9333 122.9333 123.9333 124.9333
Columns 127 through 133
125.9333 126.9333 127.9333 128.9333 129.9333 130.9333 131.9333
Columns 134 through 140
132.9333 133.9333 134.9333 135.9333 136.9333 137.9333 138.9333
Columns 141 through 147
139.9333 140.9333 141.9333 142.9333 143.9333 144.9333 145.9333
Columns 148 through 154
146.9333 147.9333 148.9333 149.9333 150.9333 151.9333 152.9333
Columns 155 through 161
153.9333 154.9333 155.9333 156.9333 157.9333 158.9333 159.9333
Columns 162 through 168
```

```
160.9333 161.9333 162.9333 163.9333 164.9333 165.9333 166.933?
Columns 169 through 175
167.9333 168.9333 169.9333 170.9333 171.9333 172.9333 173.933'
Columns 176 through 182
174.9333 175.9333 176.9333 177.9333 178.9333 179.9333 180.933
Columns 183 through 189
181.9333 182.9333 183.9333 184.9333 185.9333 186.9333 187.933)
Columns }190\mathrm{ through 196
188.9333 189.9333 190.9333 191.9333 192.9333 193.9333 194.933'
Columns 197 through 203
195.9333 196.9333 197.9333 198.9333 199.9333 200.9333 201.933)
Columns 204 through 210
202.9333 203.9333 204.9333 205.9333 206.9333 207.9333 208.933!
Columns 211 through 217
209.9333 210.9333 211.9333 212.9333 213.9333 214.9333 215.9333
Columns 218 through 224
216.9333 217.9333 218.9333 219.9333 220.9333 221.9333 222.933
Columns 225 through 231
223.9333 224.9333 225.9333 226.9333 227.9333 228.9333 229.9333
```

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## mbscfamounts

```
Columns 232 through 238
230.9333 231.9333 232.9333 233.9333 234.9333 235.9333 236.9333
Columns 239 through 245
237.9333 238.9333 239.9333 240.9333 241.9333 242.9333 243.9333
Columns 246 through 252
244.9333 245.9333 246.9333 247.9333 248.9333 249.9333 250.9333
Columns 253 through 259
251.9333 252.9333 253.9333 254.9333 255.9333 256.9333 257.9333
Columns 260 through 266
258.9333 259.9333 260.9333 261.9333 262.9333 263.9333 264.9333
Columns 267 through 273
265.9333 266.9333 267.9333 268.9333 269.9333 270.9333 271.9333
Columns 274 through 280
272.9333 273.9333 274.9333 275.9333 276.9333 277.9333 278.9333
Columns 281 through 287
279.9333 280.9333 281.9333 282.9333 283.9333 284.9333 285.9333
Columns 288 through 294
286.9333 287.9333 288.9333 289.9333 290.9333 291.9333 292.9333
```

```
    Columns 295 through 301
    293.9333 294.9333 295.9333 296.9333 297.9333 298.9333 299.933)
    Columns 302 through 308
    300.9333 301.9333 302.9333 303.9333 304.9333 305.9333 306.933'
    Columns 309 through 315
    307.9333 308.9333 309.9333 310.9333 311.9333 312.9333 313.9333
    Columns 316 through 322
    314.9333 315.9333 316.9333 317.9333 318.9333 319.9333 320.9333
    Columns 323 through 329
    321.9333 322.9333 323.9333 324.9333 325.9333 326.9333 327.9333
    Columns 330 through 334
    328.9333 329.9333 330.9333 331.9333 332.9333
Prepayment =
    Columns 1 through 7
    1.0000 0.9944 0.9887 0.9828 0.9769 0.9711 0.965?
Columns 8 through 14
    0.9595 0.9538 0.9481 0.9424 0.9368 0.9311 0.925!
Columns 15 through 21
```

0.9199
0.9144
0.9089
0.9034
0.8979
0.8925
0.8871

Columns 22 through 28
0.8817
0.8763
0.8710
0.8657
0.8604
0.8552
0.8499

Columns 29 through 35
0.84470 .8396
0.8344
0.8293
0.8242
0.8191
0.8140

Columns 36 through 42
$0.8090 \quad 0.8040$
0.7990
0.7941
0.7892
0.7842
0.7794

Columns 43 through 49
0.7745
0.7697
0.7649
0.7601
0.7553
0.7506
0.7458

Columns 50 through 56
0.7411
0.7365
0.7318
0.7272
0.7226
0.7180
0.7134

Columns 57 through 63
$0.7089 \quad 0.7044$
0.6999
0.6954
0.6910
0.6865
0.6821

Columns 64 through 70
0.6777
0.6734
0.6690
0.6647
0.6604
0.6561
0.6519

Columns 71 through 77
0.6476
0.6434
0.6392
0.6350
0.6309
0.6267
0.6226

Columns 78 through 84
0.6185
0.6144
0.6104
0.6063
0.6023
0.5983
0.5943

Columns 85 through 91
0.5903
0.5864
0.5825
0.5785
0.5747
0.5708
0.566

Columns 92 through 98
0.5631
0.5593
0.5555
0.5517
0.5479
0.5442
0.540

Columns 99 through 105
0.5368
0.5331
0.5294
0.5257
0.5221
0.5185
0.514

Columns 106 through 112
0.5113
0.5077
0.5042
0.5006
0.4971
0.4936
0.490

Columns 113 through 119
0.4866
0.4832
0.4797
0.4763
0.4729
0.4695
0.466

Columns 120 through 126
$\begin{array}{lllllll}0.4628 & 0.4594 & 0.4561 & 0.4528 & 0.4495 & 0.4462 & 0.443 \text { ( }\end{array}$
Columns 127 through 133
0.4397
0.4365
0.4333
0.4301
0.4269
0.4237
0.420

Columns 134 through 140
$\begin{array}{lllllll}0.4174 & 0.4143 & 0.4111 & 0.4080 & 0.4049 & 0.4019 & 0.3988\end{array}$
Columns 141 through 147
$\begin{array}{lllllll}0.3958 & 0.3927 & 0.3897 & 0.3867 & 0.3837 & 0.3808 & 0.377\end{array}$

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Columns 148 through 154
0.3748
0.3719
0.3690
0.3661
0.3632
0.3603
0.3574

Columns 155 through 161
0.3546
0.3517
0.3489
0.3461
0.3433
0.3405
0.3377

Columns 162 through 168
0.3350
0.3322
0.3295
0.3267
0.3240
0.3213
0.3186

Columns 169 through 175
$\begin{array}{lllllll}0.3160 & 0.3133 & 0.3106 & 0.3080 & 0.3054 & 0.3027 & 0.3001\end{array}$

Columns 176 through 182
0.2975
0.2950
0.2924
0.2898
0.2873
0.2847
0.2822

Columns 183 through 189
0.2797
0.2772
0.2747
0.2722
0.2698
0.2673
0.2649

Columns 190 through 196
$\begin{array}{lllllll}0.2624 & 0.2600 & 0.2576 & 0.2552 & 0.2528 & 0.2504 & 0.2480\end{array}$

Columns 197 through 203
0.2457
0.2433
0.2410
0.2386
0.2363
0.2340
0.2317

Columns 204 through 210
$\begin{array}{lllllll}0.2294 & 0.2271 & 0.2249 & 0.2226 & 0.2204 & 0.2181 & 0.2159\end{array}$

Columns 211 through 217
0.2137
0.2115
0.2093
0.2071
0.2049
0.2027
0.200

Columns 218 through 224
0.1984
0.1962
0.1941
0.1920
0.1899
0.1877
0.185

Columns 225 through 231
0.1836
0.1815
0.1794
0.1773
0.1753
0.1732
0.171

Columns 232 through 238
0.1692
0.1671
0.1651
0.1631
0.1611
0.1591
0.157

Columns 239 through 245
$\begin{array}{lllllll}0.1552 & 0.1532 & 0.1513 & 0.1493 & 0.1474 & 0.1455 & 0.143\end{array}$
Columns 246 through 252
0.1416
0.1397
0.1378
0.1359
0.1341
0.1322
0.130

Columns 253 through 259
0.1285
0.1266
0.1248
0.1229
0.1211
0.1193
0.117

Columns 260 through 266
0.1157
0.1139
0.1121
0.1103
0.1085
0.1068
0.105

Columns 267 through 273
$\begin{array}{lllllll}0.1032 & 0.1015 & 0.0998 & 0.0980 & 0.0963 & 0.0946 & 0.092\end{array}$

Columns 274 through 280
0.0912
0.0895
0.0878
0.0861
0.0844
0.0827
0.0811

Columns 281 through 287
0.0794
0.0778
0.0761
0.0745
0.0728
0.0712
0.0696

Columns 288 through 294
$0.0680 \quad 0.0664 \quad 0.064$
0.0632
0.0616
0.0600
0.0584

Columns 295 through 301

## $0.0568 \quad 0.0553$ <br> Columns 302 through 308

0.0537
0.0522
0.0506
0.0491
0.0476
0.0460
0.0445
0.0430
0.0415
0.0400
0.0385
0.0370

Columns 309 through 315

$$
\begin{array}{lllllll}
0.0355 & 0.0340 & 0.0325 & 0.0311 & 0.0296 & 0.0281 & 0.0267
\end{array}
$$

Columns 316 through 322
$\begin{array}{lllllll}0.0252 & 0.0238 & 0.0223 & 0.0209 & 0.0195 & 0.0180 & 0.0166\end{array}$
Columns 323 through 329

$$
\begin{array}{lllllll}
0.0152 & 0.0138 & 0.0124 & 0.0110 & 0.0096 & 0.0082 & 0.0068
\end{array}
$$

Columns 330 through 334
0.0055
0.0041
0.0027
0.0014

0

References [1] PSA Uniform Practices, SF-4

See Also mbspassthrough | mbsnoprepay | cmoseqcf | cmoschedcf | cmosched

## Purpose Convexity of mortgage pool given price

```
Syntax Convexity = mbsconvp(Price, Settle, Maturity, IssueDate, GrossRate,
```

CouponRate, Delay, PrepaySpeed, PrepayMatrix)

## Arguments

| Price | Clean price for every $\$ 100$ face value. |
| :--- | :--- |
| Settle | Settlement date. A serial date number or date <br> string. Settle must be earlier than Maturity. |
| Maturity | Maturity date. A serial date number or date <br> string. |
| IssueDate | Issue date. A serial date number or date string. |
| GrossRate | Gross coupon rate (including fees), in decimal. |
| CouponRate | Net coupon rate, in decimal. Default = <br> GrossRate. |
| Delay | Delay in days. <br> (Optional) Relation of the conditional payment |
| PrepaySpeed | rate (CPR) to the benchmark model. Default = 0. <br> Set PrepaySpeed to [] if you input a customized <br> prepayment matrix. |
| PrepayMatrix | (Optional) Used only when PrepaySpeed is <br> unspecified. Customized prepayment vector. A |
|  | NaN-padded matrix of size <br> max(TermRemaining) -by-NMBS. Each column <br> corresponds to each mortgage-backed security, |
| and each row corresponds to each month after |  |
| settlement. |  |

All inputs (except PrepayMatrix) are number of mortgage-backed securities (NMBS) by 1 vectors.

[^12]computes mortgage-backed security convexity, given time information, price at settlement, and optionally, a prepayment model.

Note If you specify the PSA or FHA model, it will be seasoned with how long the debt has been outstanding (the loan's age).

## Examples Compute a Mortgage-Backed Security Convexity

This example shows how to compute a mortgage-backed security convexity, given a mortgage-backed security with the following characteristics.

```
Price = 101;
Settle = '15-Apr-2002';
Maturity = '1 Jan 2030';
IssueDate = '1-Jan-2000';
GrossRate = 0.08125;
CouponRate = 0.075;
Delay = 14;
Speed = 100;
Convexity = mbsconvp(Price, Settle, Maturity, IssueDate,...
GrossRate, CouponRate, Delay, Speed)
```

Convexity =
71.6299

## References

[1] PSA Uniform Practices, SF-49
See Also mbsconvy | mbsdurp | mbspassthrough | mbsnoprepay | mbsdury

Purpose Convexity of mortgage pool given yield
Syntax Convexity = mbsconvy(Yield, Settle, Maturity, IssueDate, GrossRate, CouponRate, Delay, PrepaySpeed, PrepayMatrix)

## Arguments

## Description

Convexity = mbsconvy(Yield, Settle, Maturity, IssueDate, GrossRate, CouponRate, Delay, PrepaySpeed, PrepayMatrix) computes mortgage-backed security convexity, given time information, semiannual mortgage yield, and optionally, a prepayment model.

Note If you specify the PSA or FHA model, it will be seasoned with how long the debt has been outstanding (the loan's age).

## Examples

## Compute the Convexity of a Mortgage Pool Given Yield

This example shows how to compute the convexity of mortgage pool given yield for a mortgage-backed security with the following characteristics.

```
Yield = 0.07125;
Settle = '15-Apr-2002';
Maturity = '1 Jan 2030';
IssueDate = '1-Jan-2000';
GrossRate = 0.08125;
Speed = 100;
CouponRate = 0.075;
Delay = 14;
```

Convexity = mbsconvy(Yield, Settle, Maturity, IssueDate, ...
GrossRate, CouponRate, Delay, Speed)
Convexity =
72.8263

## References

See Also mbsconvp | mbsdurp | mbsdury

Purpose
Syntax

Arguments

Duration of mortgage pool given price
[YearDuration, ModDuration] = mbsdurp(Price, Settle, Maturity, IssueDate, GrossRate, CouponRate, Delay, PrepaySpeed, PrepayMatrix)

Price Clean price for every $\$ 100$ face value.
Settle Settlement date. A serial date number or date string. Settle must be earlier than or equal to Maturity.
Maturity Maturity date. A serial date number or date string.
IssueDate Issue date. A serial date number or date string.
GrossRate Gross coupon rate (including fees), in decimal.
CouponRate (Optional) Net coupon rate, in decimal. Default = GrossRate.

Delay (Optional) Delay in days.
PrepaySpeed (Optional) Relation of the conditional payment rate $(\mathrm{CPR})$ to the benchmark model. Default $=0$. Set PrepaySpeed to [ ] if you input a customized prepayment matrix.
PrepayMatrix (Optional) Used only when PrepaySpeed is unspecified. Customized prepayment vector. A NaN-padded matrix of size max (TermRemaining)-by-NMBS. Each column corresponds to each mortgage-backed security, and each row corresponds to each month after settlement.

All inputs (except PrepayMatrix) are number of mortgage-backed securities (NMBS) by 1 vectors.

## Description

[YearDuration, ModDuration] = mbsdurp(Price, Settle, Maturity, IssueDate, GrossRate, CouponRate, Delay, PrepaySpeed, PrepayMatrix) computes the mortgage-backed security Macaulay (YearDuration) in years and modified (ModDuration) durations in years, given time information, price at settlement, and optionally, a prepayment model.

Note If you specify the PSA or FHA model, it will be seasoned with how long the debt has been outstanding (the loan's age).

## Examples Find the Duration of a Mortgage Pool

This example shows how to find the duration of mortgage pool given a mortgage-backed security with the following characteristics.

```
Price = 101;
Settle = datenum('15-Apr-2002');
Maturity = datenum('1 Jan 2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
CouponRate = 0.075;;
Delay = 14;
Speed = 100;
```

[YearDuration, ModDuration] = mbsdurp(Price, Settle, Maturity,...
IssueDate, GrossRate, CouponRate, Delay, Speed)

YearDuration $=$
6.4380

ModDuration =

## mbsdurp

### 6.2080

## References [1] PSA Uniform Practices, SF-49

See Also mbsconvp | mbsconvy | mbsdury

## Purpose

Duration of mortgage pool given yield

## Arguments

## Description

\(\left.$$
\begin{array}{ll}\text { Yield } & \text { Mortgage yield, compounded monthly, in decimal. } \\
\text { Settle } & \begin{array}{l}\text { Settlement date. A serial date number or date } \\
\text { string. Settle must be earlier than Maturity. }\end{array} \\
\text { Maturity } & \text { Maturity date. A serial date number or date string. } \\
\text { IssueDate } & \text { Issue date. A serial date number or date string. } \\
\text { GrossRate } & \text { Gross coupon rate (including fees), in decimal. } \\
\text { CouponRate } & \begin{array}{l}\text { Net coupon rate, in decimal. Default = GrossRate. }\end{array} \\
\text { Delay } & \begin{array}{l}\text { Delay in days. } \\
\text { (Optional) Relation of the conditional payment }\end{array} \\
\text { PrepaySpeed } & \begin{array}{l}\text { rate (CPR) to the benchmark model. Default = 0. } \\
\text { Set PrepaySpeed to [] if you input a customized } \\
\text { prepayment matrix. }\end{array}
$$ <br>

(Optional) Used only when PrepaySpeed is\end{array}\right\}\)| (Onspecified. Customized prepayment vector. A |
| :--- |
| NaN-padded matrix of size |
| max(TermRemaining)-by-NMBS. Each column |
| corresponds to each mortgage-backed security, |
| and each row corresponds to each month after |

All inputs (except PrepayMatrix) are number of mortgage-backed securities (NMBS) by 1 vectors.
[YearDuration, ModDuration] = mbsdury(Yield, Settle, Maturity, IssueDate, GrossRate, CouponRate, Delay, PrepaySpeed, PrepayMatrix) computes the mortgage-backed security

Macaulay (YearDuration) and Modified (ModDuration) durations, given time information, yield to maturity, and optionally, a prepayment model.

Note If you specify the PSA or FHA model, it will be seasoned with how long the debt has been outstanding (the loan's age).

## Examples

## Find the Duration of a Mortgage Pool Given the Yield

This example shows how to find the duration of mortgage pool given a mortgage-backed security with the following characteristics.

```
Yield = 0.07298413;
Settle = '15-Apr-2002';
Maturity = '1 Jan 2030';
IssueDate = '1-Jan-2000';
GrossRate = 0.08125;
Speed = 100;
CouponRate = 0.075;
Delay = 14;
```

[YearDuration, ModDuration] = mbsdury(Yield, Settle, Maturity,...
IssueDate, GrossRate, CouponRate, Delay, Speed)
YearDuration =
6.4380
ModDuration =
6.2080

## References [1] PSA Uniform Practices, SF-49

See Also mbsconvp | mbsconvy | mbsdurp

## mbsnoprepay

Purpose

## Syntax

## Arguments

## Description

## Examples

End-of-month mortgage cash flows and balances without prepayment
[Balance, Interest, Payment, Principal] = mbsnoprepay(OriginalBalance, GrossRate, Term)

OriginalBalance Original face value in dollars. GrossRate Gross coupon rate (including fees), in decimal.

Term Term of the mortgage in months.
All inputs are number of mortgage-backed securities (NMBS)-by-1 vectors.
[Balance, Interest, Payment, Principal] = mbsnoprepay(OriginalBalance, GrossRate, Term) computes end-of-month mortgage balance, interest payments, principal payments, and cash flow payments with zero prepayment rate.

The function returns amortizing cash flows and balances over a specified term with no prepayment. When the lengths of pass-throughs are not the same, MATLAB software pads the shorter ones with NaN .

Balance lists the end-of-month balances over the life of the pass-through.

Interest lists all end-of-month interest payments over the life of the pass-through.

Payment lists all end-of-month payments over the life of the pass-through.

Principal lists all scheduled end-of-month principal payments over the life of the pass-through.

All outputs are Term-by-1 vectors.

Given mortgage pools with the following characteristics, compute an amortization schedule.

```
OriginalBalance = 400000000;
CouponRate = 0.08125;
Term = [357; 355]; % Three- and five-month old mortgage pools.
[Balance, Interest, Payment, Principal] = ...
mbsnoprepay(OriginalBalance, CouponRate, Term);
```


## mbsoas2price

Purpose Price given option-adjusted spread

```
Syntax Price = mbsoas2price(ZeroCurve, OAS, Settle, Maturity, IssueDate,
GrossRate, CouponRate, Delay, Interpolation, PrepaySpeed,
PrepayMatrix)
```

ZeroCurve A matrix of three columns:

- Column 1: Serial date numbers.
- Column 2: Spot rates with maturities corresponding to the dates in Column 1, in decimal (for example, 0.075).
- Column 3: Compounding of the rates in Column 2. (This is the agency spot rate on the settlement date.)

| OAS | Option-adjusted spreads in basis points. |
| :---: | :---: |
| Settle | Settlement date (scalar only). A serial date number or date string. Date when option-adjusted spread is calculated. Settle must be earlier than Maturity. |
| Maturity | Maturity date. Scalar or vector in serial date number or date string format. |
| IssueDate | Issue date. A serial date number or date string. |
| GrossRate | Gross coupon rate (including fees), in decimal. |
| CouponRate | (Optional) Net coupon rate, in decimal. <br> Default $=$ GrossRate . |
| Delay | (Optional) Delay (in days) between payment from homeowner and receipt by bondholder. Default $=0$ (no delay between payment and receipt). |

## mbsoas2price

Interpolation Interpolation method. Computes the corresponding spot rates for the bond's cash flow. Available methods are (0) nearest, (1) linear, and (2) cubic spline. Default =1. See interp1 for more information.

PrepaySpeed (Optional) Relation of the conditional payment rate (CPR) to the benchmark model. Default = end of month's CPR. Set PrepaySpeed to [] if you input a customized prepayment matrix.

PrepayMatrix (Optional) Customized prepayment matrix. A matrix of size max (TermRemaining)-by-NMBS. Missing values are padded with NaNs. Each column corresponds to a mortgage-backed security, and each row corresponds to each month after settlement.

All inputs (except PrepayMatrix) are number of mortgage-backed securities (NMBS) by 1 vectors.

## Description

Price = mbsoas2price(ZeroCurve, OAS, Settle, Maturity, IssueDate, GrossRate, CouponRate, Delay, Interpolation, PrepaySpeed, PrepayMatrix) computes the clean price of a pass-through security for each $\$ 100$ face value of outstanding principal.

## Examples Compute the Theoretical Price of a Mortgage Pool

Given an option-adjusted spread, a spot curve, and a prepayment assumption, compute theoretical price of a mortgage pool. First, create the bonds matrix:

```
Bonds = [datenum('11/21/2002') 0 100 0 2 1;
    datenum('02/20/2003') 0 100 0 2 1;
    datenum('07/31/2004') 0.03 100 2 3 1;
    datenum('08/15/2007') 0.035 100 2 3 1;
    datenum('08/15/2012') 0.04875 100 2 3 1;
    datenum('02/15/2031') 0.05375 100 2 3 1];
```


## mbsoas2price

Choose a settlement date.
Settle = datenum('20-Aug-2002');
Assume the following clean prices for the bonds:

```
Prices = [ 98.97467;
    98.58044;
    100.10534;
    98.18054;
    101.38136;
    99.25411];
```

Use the following formula to compute spot compounding for the bonds:
SpotCompounding = 2*ones(size(Prices));
Compute the zero curve.
[ZeroRatesP, CurveDatesP] = zbtprice(Bonds, Prices, Settle); ZeroCurve = [CurveDatesP, ZeroRatesP, SpotCompounding]

ZeroCurve =

| $1.0 \mathrm{e}+05$ * |  |  |
| :--- | :--- | :--- |
|  |  |  |
| 7.3154 | 0.0000 | 0.0000 |
| 7.3163 | 0.0000 | 0.0000 |
| 7.3216 | 0.0000 | 0.0000 |
| 7.3327 | 0.0000 | 0.0000 |
| 7.3510 | 0.0000 | 0.0000 |
| 7.4185 | 0.0000 | 0.0000 |

Assign the following parameters:

```
OAS = [26.0502; 28.6348; 31.2222];
Maturity = datenum('02-Jan-2030');
IssueDate = datenum('02-Jan-2000');
GrossRate = 0.08125;
CouponRate = 0.075;
Delay = 14;
Interpolation = 1;
PrepaySpeed = [0 50 100];
```

Calculate the theoretical price from the option-adjusted spread.
Price = mbsoas2price(ZeroCurve, OAS, Settle, Maturity, ... IssueDate, GrossRate, CouponRate, Delay, Interpolation, ... PrepaySpeed)

Price =
95.0006
95.0006
95.0006

See Also mbsprice2oas | mbsyield2oas | mbsoas2yield

## mbsoas2yield

Purpose Yield given option-adjusted spread

```
Syntax
[MYield, BEMBSYield] = mbsoas2yield(ZeroCurve, OAS, Settle,
Maturity, IssueDate, GrossRate, CouponRate, Delay, Interpolation,
PrepaySpeed, PrepayMatrix)
```

Arguments
ZeroCurve

OAS
Settle

Maturity

IssueDate
GrossRate
CouponRate

Delay

A matrix of three columns:

- Column 1: Serial date numbers.
- Column 2: Spot rates with maturities corresponding to the dates in Column 1, in decimal (for example, 0.075).
- Column 3: Compounding of the rates in Column 2. (This is the agency spot rate on the settlement date.)

Option-adjusted spreads in basis points.
Settlement date (scalar only). A serial date number or date string. Date when option-adjusted spread is calculated. Settle must be earlier than Maturity.

Maturity date. Scalar or vector in serial date number or date string format.

Issue date. A serial date number or date string.
Gross coupon rate (including fees), in decimal.
(Optional) Net coupon rate, in decimal. Default $=$ GrossRate .
(Optional) Delay (in days) between payment from homeowner and receipt by bondholder. Default = 0 (no delay between payment and receipt).

| Interpolation | Interpolation method. Computes the <br> corresponding spot rates for the bond's cash flow. <br> Available methods are (0) nearest, (1) linear, and <br> (2) cubic spline. Default = 1. See interp1 for <br> more information. |
| :--- | :--- |
| PrepaySpeed | (Optional) Relation of the conditional payment <br> rate (CPR) to the benchmark model. Default = <br> end of month's CPR. Set PrepaySpeed to [ ] if <br> you input a customized prepayment matrix. |
| PrepayMatrix | (Optional) Customized prepayment matrix. A |
|  | matrix of size max (TermRemaining) -by-NMBS. <br> Missing values are padded with NaNs. Each |
| column corresponds to a mortgage-backed |  |
| security, and each row corresponds to each |  |
| month after settlement. |  |

All inputs (except PrepayMatrix) are number of mortgage-backed securities (NMBS) by 1 vectors.

## Description

[MYield, BEMBSYield] = mbsoas2yield(ZeroCurve, OAS, Settle, Maturity, IssueDate, GrossRate, CouponRate, Delay, Interpolation, PrepaySpeed, PrepayMatrix) computes the mortgage and bond-equivalent yields of a pass-through security.
MYield is the yield to maturity of the mortgage-backed security (the mortgage yield). This yield is compounded monthly ( 12 times per year). For example:
0.075 (7.5\%)

BEMBSYield is the corresponding bond equivalent yield of the mortgage-backed security. This yield is compounded semiannually (two times per year). For example:
0.0761 (7.61\%)

## mbsoas2yield

## Examples Compute the Theoretical Yield to Maturity of a Mortgage Pool

Given an option-adjusted spread, a spot curve, and a prepayment assumption, compute the theoretical yield to maturity of a mortgage pool. First, create the bonds matrix:

| Bonds $=$ | $[$ datenum('11/21/2002')  <br>  datenum('02/20/2003') | 0 | 100 | 0 | 2 | $1 ;$ |
| ---: | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 0 | 100 | 0 | 2 | $1 ;$ |  |
|  | datenum('07/31/2004') | 0.03 | 100 | 2 | 3 | $1 ;$ |
|  | datenum('08/15/2007') | 0.035 | 100 | 2 | 3 | $1 ;$ |
|  | datenum('08/15/2012') | 0.04875 | 100 | 2 | 3 | $1 ;$ |
|  | datenum('02/15/2031') | 0.05375 | 100 | 2 | 3 | $1] ;$ |

Choose a settlement date.
Settle = datenum('20-Aug-2002');
Assume the following clean prices for the bonds:

```
Prices = [ 98.97467;
    98.58044;
    100.10534;
    98.18054;
    101.38136;
    99.25411];
```

Use the following formula to compute spot compounding for the bonds:
SpotCompounding = 2*ones(size(Prices));
Compute the zero curve.
[ZeroRatesP, CurveDatesP] = zbtprice(Bonds, Prices, Settle);
ZeroCurve = [CurveDatesP, ZeroRatesP, SpotCompounding]

ZeroCurve =
$1.0 \mathrm{e}+05$ *

| 7.3154 | 0.0000 | 0.0000 |
| :--- | :--- | :--- |
| 7.3163 | 0.0000 | 0.0000 |
| 7.3216 | 0.0000 | 0.0000 |
| 7.3327 | 0.0000 | 0.0000 |
| 7.3510 | 0.0000 | 0.0000 |
| 7.4185 | 0.0000 | 0.0000 |

Assign the following parameters:

```
OAS = [26.0502; 28.6348; 31.2222];
Maturity = datenum('02-Jan-2030');
IssueDate = datenum('02-Jan-2000');
GrossRate = 0.08125;
CouponRate = 0.075;
Delay = 14;
Interpolation = 1;
PrepaySpeed = [0 50 100];
```

Compute the mortgage yield and bond equivalent mortgage yield.
[MYield BEMBSYield] = mbsoas2yield(ZeroCurve, OAS, Settle, ... Maturity, IssueDate, GrossRate, CouponRate, Delay, ... Interpolation, PrepaySpeed)

MYield =
0.0802
0.0814
0.0828

BEMBSYield =

$$
\begin{aligned}
& 0.0816 \\
& 0.0828 \\
& 0.0842
\end{aligned}
$$

See Also
mbsprice2oas | mbsyield2oas | mbsoas2price

## mbspassthrough

## Purpose

Mortgage pool cash flows and balances with prepayment

## Syntax

[Balance, Payment, Principal, Interest, Prepayment] = mbspassthrough(OriginalBalance, GrossRate, OriginalTerm, TermRemaining, PrepaySpeed, PrepayMatrix)

## Arguments

## Description

[Balance, Payment, Principal, Prepayment, Interest] = mbspassthrough(OriginalBalance, GrossRate, OriginalTerm, TermRemaining, PrepaySpeed, PrepayMatrix)

All outputs are TermRemaining-by-1 vectors of end-of-month values.
Balance is a matrix for the principal balance at end of month.

## mbspassthrough

Payment is a matrix for the total monthly payment.
Principal is a matrix for the principal portion of the payment.
Interest is a matrix for the interest portion of the payment.
Prepayment is a matrix that indicates any unscheduled principal payment.

By default, the securities are seasoned. The applicable CPR depends upon TermRemaining based on a 30 -year prepayment model (PSA or FHA). You may supply a different CPR vector of size TermRemaining-by-1.

## Examples Compute the Cash Flow of Principal, Interest, and Prepayment of a Pass-Through Security

This example shows how to compute the cash flows and balances of a 3 -month old mortgage pool with original term of 360 months, assuming a prepayment speed of 100 .

```
OriginalBalance = 100000;
GrossRate = 0.08125;
OriginalTerm = 360;
TermRemaining = 357;
PrepaySpeed = 100;
[Balance, Payment, Principal, Interest, Prepayment] =...
mbspassthrough(OriginalBalance, GrossRate, OriginalTerm,...
TermRemaining, PrepaySpeed)
```

Balance =
$1.0 \mathrm{e}+04$ *
9.9866
9.9715
9.9548

$$
\begin{aligned}
& 9.9363 \\
& 9.9161 \\
& 9.8942 \\
& 9.8707 \\
& 9.8454 \\
& 9.8185 \\
& 9.7900 \\
& 9.7598 \\
& 9.7280 \\
& 9.6946 \\
& 9.6596 \\
& 9.6230 \\
& 9.5848 \\
& 9.5450 \\
& 9.5037 \\
& 9.4609 \\
& 9.4166 \\
& 9.3708 \\
& 9.3235 \\
& 9.2748 \\
& 9.2247 \\
& 9.1731 \\
& 9.1202 \\
& 9.0659 \\
& 9.0119 \\
& 8.9582 \\
& 8.9047 \\
& 8.8515 \\
& 8.7985 \\
& 8.7458 \\
& 8.6934 \\
& 8.6412 \\
& 8.5893 \\
& 8.5376 \\
& 8.4862 \\
& 8.4351 \\
& 8.3842
\end{aligned}
$$

$$
\begin{aligned}
& 8.3335 \\
& 8.2831 \\
& 8.2329 \\
& 8.1830 \\
& 8.1333 \\
& 8.0839 \\
& 8.0347 \\
& 7.9858 \\
& 7.9371 \\
& 7.8886 \\
& 7.8404 \\
& 7.7924 \\
& 7.7446 \\
& 7.6971 \\
& 7.6498 \\
& 7.6027 \\
& 7.5559 \\
& 7.5093 \\
& 7.4629 \\
& 7.4168 \\
& 7.3708 \\
& 7.3251 \\
& 7.2797 \\
& 7.2344 \\
& 7.1894 \\
& 7.1445 \\
& 7.0999 \\
& 7.0556 \\
& 7.0114 \\
& 6.9674 \\
& 6.9237 \\
& 6.8802 \\
& 6.8368 \\
& 6.7937 \\
& 6.7508 \\
& 6.7082 \\
& 6.6657
\end{aligned}
$$

$$
\begin{aligned}
& 6.6234 \\
& 6.5813 \\
& 6.5395 \\
& 6.4978 \\
& 6.4563 \\
& 6.4151 \\
& 6.3740 \\
& 6.3331 \\
& 6.2925 \\
& 6.2520 \\
& 6.2117 \\
& 6.1716 \\
& 6.1317 \\
& 6.0920 \\
& 6.0525 \\
& 6.0132 \\
& 5.9741 \\
& 5.9351 \\
& 5.8964 \\
& 5.8578 \\
& 5.8194 \\
& 5.7812 \\
& 5.7432 \\
& 5.7054 \\
& 5.6678 \\
& 5.6303 \\
& 5.5930 \\
& 5.5559 \\
& 5.5189 \\
& 5.4822 \\
& 5.4456 \\
& 5.4092 \\
& 5.3730 \\
& 5.3369 \\
& 5.3010 \\
& 5.2653 \\
& 5.2298
\end{aligned}
$$

$$
\begin{aligned}
& 5.1944 \\
& 5.1592 \\
& 5.1241 \\
& 5.0893 \\
& 5.0545 \\
& 5.0200 \\
& 4.9856 \\
& 4.9514 \\
& 4.9174 \\
& 4.8835 \\
& 4.8497 \\
& 4.8162 \\
& 4.7827 \\
& 4.7495 \\
& 4.7164 \\
& 4.6834 \\
& 4.6507 \\
& 4.6180 \\
& 4.5855 \\
& 4.5532 \\
& 4.5211 \\
& 4.4890 \\
& 4.4572 \\
& 4.4254 \\
& 4.3939 \\
& 4.3624 \\
& 4.3312 \\
& 4.3000 \\
& 4.2691 \\
& 4.2382 \\
& 4.2075 \\
& 4.1770 \\
& 4.1466 \\
& 4.1163 \\
& 4.0862 \\
& 4.0562 \\
& 4.0264
\end{aligned}
$$

$$
\begin{aligned}
& 3.9967 \\
& 3.9671 \\
& 3.9377 \\
& 3.9084 \\
& 3.8792 \\
& 3.8502 \\
& 3.8213 \\
& 3.7926 \\
& 3.7640 \\
& 3.7355 \\
& 3.7071 \\
& 3.6789 \\
& 3.6508 \\
& 3.6229 \\
& 3.5950 \\
& 3.5673 \\
& 3.5398 \\
& 3.5123 \\
& 3.4850 \\
& 3.4578 \\
& 3.4307 \\
& 3.4038 \\
& 3.3770 \\
& 3.3503 \\
& 3.3237 \\
& 3.2972 \\
& 3.2709 \\
& 3.2447 \\
& 3.2186 \\
& 3.1926 \\
& 3.1668 \\
& 3.1410 \\
& 3.1154 \\
& 3.0899 \\
& 3.0645 \\
& 3.0393 \\
& 3.0141
\end{aligned}
$$

## mbspassthrough

$$
\begin{aligned}
& 2.9891 \\
& 2.9641 \\
& 2.9393 \\
& 2.9146 \\
& 2.8900 \\
& 2.8655 \\
& 2.8412 \\
& 2.8169 \\
& 2.7928 \\
& 2.7687 \\
& 2.7448 \\
& 2.7210 \\
& 2.6972 \\
& 2.6736 \\
& 2.6501 \\
& 2.6267 \\
& 2.6034 \\
& 2.5802 \\
& 2.5571 \\
& 2.5342 \\
& 2.5113 \\
& 2.4885 \\
& 2.4658 \\
& 2.4432 \\
& 2.4208 \\
& 2.3984 \\
& 2.3761 \\
& 2.3539 \\
& 2.3318 \\
& 2.3099 \\
& 2.2880 \\
& 2.2662 \\
& 2.2445 \\
& 2.2229 \\
& 2.2014 \\
& 2.1800 \\
& 2.1587
\end{aligned}
$$

2.1374
2.1163
2.0953
2.0743
2.0535
2.0327
2.0120
1.9915
1.9710
1.9506
1.9303
1.9100
1.8899
1.8698
1.8499
1.8300
1.8102
1.7905
1.7709
1.7513
1.7319
1.7125
1.6932
1.6740
1.6549
1.6359
1.6169
1.5980
1.5792
1.5605
1.5419
1.5233
1.5048
1.4864
1.4681
1.4498
1.4317

$$
\begin{aligned}
& 1.4136 \\
& 1.3956 \\
& 1.3776 \\
& 1.3597 \\
& 1.3419 \\
& 1.3242 \\
& 1.3066 \\
& 1.2890 \\
& 1.2715 \\
& 1.2541 \\
& 1.2367 \\
& 1.2194 \\
& 1.2022 \\
& 1.1851 \\
& 1.1680 \\
& 1.1510 \\
& 1.1341 \\
& 1.1172 \\
& 1.1004 \\
& 1.0837 \\
& 1.0671 \\
& 1.0505 \\
& 1.0340 \\
& 1.0175 \\
& 1.0011 \\
& 0.9848 \\
& 0.9685 \\
& 0.9524 \\
& 0.9362 \\
& 0.9202 \\
& 0.9042 \\
& 0.8883 \\
& 0.8724 \\
& 0.8566 \\
& 0.8409 \\
& 0.8252 \\
& 0.8096
\end{aligned}
$$

$$
\begin{aligned}
& 0.7940 \\
& 0.7786 \\
& 0.7631 \\
& 0.7478 \\
& 0.7325 \\
& 0.7172 \\
& 0.7020 \\
& 0.6869 \\
& 0.6719 \\
& 0.6569 \\
& 0.6419 \\
& 0.6270 \\
& 0.6122 \\
& 0.5974 \\
& 0.5827 \\
& 0.5680 \\
& 0.5535 \\
& 0.5389 \\
& 0.5244 \\
& 0.5100 \\
& 0.4956 \\
& 0.4813 \\
& 0.4670 \\
& 0.4528 \\
& 0.4387 \\
& 0.4246 \\
& 0.4105 \\
& 0.3965 \\
& 0.3826 \\
& 0.3687 \\
& 0.3548 \\
& 0.3411 \\
& 0.3273 \\
& 0.3136 \\
& 0.3000 \\
& 0.2864 \\
& 0.2729
\end{aligned}
$$

$$
\begin{array}{r}
0.2594 \\
0.2460 \\
0.2326 \\
0.2193 \\
0.2060 \\
0.1928 \\
0.1796 \\
0.1665 \\
0.1534 \\
0.1403 \\
0.1273 \\
0.1144 \\
0.1015 \\
0.0887 \\
0.0759 \\
0.0631 \\
0.0504 \\
0.0377 \\
0.0251 \\
0.0125 \\
0 \\
\hline
\end{array}
$$

729.8136
727.8383
725.7432
723.5292
721.1972
718.7479
716.1826
713.5021
710.7075
707.8002
704.7812
701.6518
698.4134
695.0674
691.6152
688.0582
684.5195
680.9990
677.4966
674.0122
670.5458
667.0971
663.6663
660.2530
656.8573
653.4791
650.1182
646.7747
643.4483
640.1390
636.8468
633.5715
630.3130
627.0713
623.8463
620.6378
617.4459

```
614.2704
611.1112
607.9682
604.8414
601.7307
598.6360
595.5572
592.4942
589.4470
586.4155
583.3996
580.3991
577.4141
574.4445
571.4901
568.5509
565.6269
562.7178
559.8238
556.9446
554.0802
551.2306
548.3956
545.5752
542.7693
539.9778
537.2007
534.4379
531.6893
528.9548
526.2343
523.5279
520.8354
518.1567
515.4919
512.8407
510.2031
```

```
507.5791
504.9687
502.3716
499.7879
497.2175
494.6603
492.1162
489.5853
487.0673
484.5623
482.0702
479.5909
477.1244
474.6705
472.2293
469.8006
467.3844
464.9807
462.5893
460.2102
457.8433
455.4886
453.1460
450.8155
448.4969
446.1903
443.8955
441.6126
439.3414
437.0818
434.8339
432.5976
430.3727
428.1593
425.9573
423.7666
421.5871
```

```
419.4189
417.2618
415.1158
412.9809
410.8569
408.7439
406.6417
404.5503
402.4697
400.3998
398.3406
396.2919
394.2538
392.2261
390.2089
388.2020
386.2055
384.2192
382.2432
380.2773
378.3215
376.3758
374.4401
372.5144
370.5985
368.6925
366.7963
364.9099
363.0332
361.1661
359.3086
357.4607
355.6223
353.7933
351.9737
350.1635
348.3626
```

```
346.5710
344.7886
343.0153
341.2512
339.4961
337.7501
336.0130
334.2849
332.5657
330.8553
329.1537
327.4609
325.7767
324.1013
322.4344
320.7761
319.1264
317.4851
315.8523
314.2278
312.6117
311.0040
309.4045
307.8132
306.2301
304.6552
303.0883
301.5295
299.9788
298.4360
296.9011
295.3742
293.8550
292.3437
290.8402
289.3444
287.8563
```

286.3759
284.9030
283.4378
281.9800
280.5298
279.0871
277.6517
276.2237
274.8031
273.3898
271.9838
270.5849
269.1933
267.8088
266.4315
265.0612
263.6980
262.3418
260.9926
259.6503
258.3149
256.9864
255.6647
254.3498
253.0417
251.7403
250.4456
249.1576
247.8761
246.6013
245.3330
244.0713
242.8160
241.5672
240.3248
239.0888
237.8592

```
236.6359
235.4189
234.2081
233.0036
231.8052
230.6131
229.4270
228.2471
227.0732
225.9054
224.7435
223.5877
222.4377
221.2937
220.1556
219.0234
217.8969
216.7763
215.6614
214.5522
213.4488
212.3510
211.2589
210.1724
209.0915
208.0161
206.9463
205.8820
204.8231
203.7697
202.7217
201.6791
200.6419
199.6100
198.5834
197.5621
196.5460
```

195.5352
194.5295
193.5290
192.5337
191.5435
190.5584
189.5784
188.6034
187.6334
186.6684
185.7083
184.7532
183.8030
182.8577
181.9173
180.9817
180.0509
179.1249
178.2037
177.2872
176.3754
175.4683
174.5658
173.6680
172.7749
171.8863
171.0023
170.1228
169.2479
168.3774
167.5114
166.6499
165.7928
164.9402
164.0919
163.2480
162.4084
161.5731
160.7421
159.9154
159.0930
158.2748
157.4608
156.6509
155.8453
155.0438
154.2464
153.4531
152.6639
151.8787
151.0976
150.3205
149.5474
148.7783
148.0131
147.2519
146.4946
145.7411
144.9916
144.2459
143.5040
142.7660
142.0317
141.3013
140.5746
139.8516
139.1323
138.4168
137.7049
136.9967
136.2921
135.5911
134.8938
134.2000

## mbspassthrough

$$
\begin{aligned}
& 133.5098 \\
& 132.8232 \\
& 132.1401 \\
& 131.4605 \\
& 130.7844 \\
& 130.1118 \\
& 129.4426 \\
& 128.7769 \\
& 128.1146 \\
& 127.4557 \\
& 126.8002 \\
& 126.1480
\end{aligned}
$$

```
Principal =
```

    66.8837
    67.2915
    67.6904
    68.0802
    68.4607
    68.8317
    69.1929
    69.5442
    69.8854
    70.2163
    70.5367
    70.8464
    71.1453
    71.4331
    71.7098
    71.9751
    72.2289
    72.4710
    72.7012
    72.9195
    73.1257
    73.3197
73.5013
73.6704
73.8269
73.9706
74.1016
74.2196
74.3379
74.4563
74.5749
74.6937
74.8127
74.9319
75.0512
75.1708
75.2905
75.4105
75.5306
75.6509
75.7714
75.8921
76.0130
76.1341
76.2554
76.3769
76.4985
76.6204
76.7425
76.8647
76.9871
77.1098
77.2326
77.3557
77.4789
77.6023
77.7259
77.8497

> 77.9738
> 78.0980
> 78.2224
> 78.3470
> 78.4718
> 78.5968
> 78.7220
> 78.8474
> 78.9730
> 79.0988
> 79.2248
> 79.3510
> 79.4774
> 79.6040
> 79.7308
> 79.8579
> 79.9851
> 80.1125
> 80.2401
> 80.3679
> 80.4960
> 80.6242
> 80.7526
> 80.8813
> 81.0101
> 81.1392
> 81.2684
> 81.3979
> 81.5275
> 81.6574
> 81.7875
> 81.9178
> 82.0483
> 82.1790
> 82.3099
> 82.4410
> 82.5723

```
82.7039
82.8356
82.9676
83.0997
83.2321
83.3647
83.4975
83.6305
83.7637
83.8972
84.0308
84.1647
84.2988
84.4330
84.5675
84.7023
84.8372
84.9723
85.1077
85.2433
85.3791
85.5151
85.6513
85.7877
85.9244
86.0613
86.1984
86.3357
86.4732
86.6110
86.7489
86.8871
87.0256
87.1642
87.3030
87.4421
87.5814
```

> 87.7209
> 87.8607
> 88.0006
> 88.1408
> 88.2812
> 88.4218
> 88.5627
> 88.7038
> 88.8451
> 88.9866
> 89.1284
> 89.2704
> 89.4126
> 89.5550
> 89.6977
> 89.8405
> 89.9837
> 90.1270
> 90.2706
> 90.4144
> 90.5584
> 90.7027
> 90.8472
> 90.9919
> 91.1368
> 91.2820
> 91.4274
> 91.5731
> 91.7189
> 91.8650
> 92.0114
> 92.1580
> 92.3048
> 92.4518
> 92.5991
> 92.7466
> 92.8943

```
93.0423
93.1905
93.3390
93.4877
93.6366
93.7858
93.9352
94.0848
94.2347
94.3848
94.5352
94.6857
94.8366
94.9877
95.1390
95.2905
95.4423
95.5944
95.7466
95.8992
96.0519
96.2049
96.3582
96.5117
96.6654
96.8194
96.9737
97.1281
97.2829
97.4378
97.5931
97.7485
97.9042
98.0602
98.2164
98.3729
98.5296
```

98.6865
98.8437
99.0012
99.1589
99.3169
99.4751
99.6335
99.7923
99.9512
100.1104
100.2699
100.4297
100.5896
100.7499
100.9104
101.0711
101.2321
101.3934
101.5549
101.7167
101.8787
102.0410
102.2036
102.3664
102.5294
102.6928
102.8564
103.0202
103.1843
103.3487
103.5133
103.6782
103.8434
104.0088
104.1745
104.3404
104.5067
104.6731
104.8399
105.0069
105.1742
105.3417
105.5095
105.6776
105.8459
106.0145
106.1834
106.3526
106.5220
106.6917
106.8616
107.0319
107.2024
107.3732
107.5442
107.7155
107.8871
108.0590
108.2311
108.4035
108.5762
108.7492
108.9224
109.0959
109.2697
109.4438
109.6181
109.7927
109.9676
110.1428
110.3183
110.4940
110.6700
110.8463

```
111.0229
111.1998
111.3769
111.5543
111.7320
111.9100
112.0883
112.2669
112.4457
112.6248
112.8042
112.9839
113.1639
113.3442
113.5247
113.7056
113.8867
114.0681
114.2498
114.4318
114.6141
114.7967
114.9796
115.1627
115.3462
115.5299
115.7140
115.8983
116.0829
116.2679
116.4531
116.6386
116.8244
117.0105
117.1969
117.3836
117.5706
```

117.7579
117.9455
118.1333
118.3215
118.5100
118.6988
118.8879
119.0773
119.2670
119.4570
119.6473
119.8379
120.0288
120.2200
120.4115
120.6033
120.7954
120.9878
121.1806
121.3736
121.5670
121.7606
121.9546
122.1488
122.3434
122.5383
122.7335
122.9290
123.1249
123.3210
123.5175
123.7142
123.9113
124.1087
124.3064
124.5044
124.7027

```
602.9217
599.3185
595.7331
592.1653
588.6150
585.0822
581.5668
578.0686
574.5877
571.1238
567.6769
564.2469
560.8338
557.4374
554.0576
550.6945
547.3477
544.0174
540.7034
537.4056
534.1240
530.8584
527.6088
524.3750
521.1571
517.9549
514.7683
511.5973
508.4417
505.3016
502.1768
499.0671
495.9727
492.8933
489.8289
486.7795
483.7448
```

```
480.7250
477.7198
474.7292
471.7531
468.7915
465.8443
462.9114
459.9927
457.0882
454.1978
451.3213
448.4588
445.6102
442.7753
439.9541
437.1466
434.3527
431.5723
428.8053
426.0516
423.3113
420.5841
417.8701
415.1692
412.4813
409.8064
407.1443
404.4950
401.8585
399.2346
396.6234
394.0247
391.4384
388.8646
386.3031
383.7539
381.2169
```

```
323.8314
```

321.5797
319.3387
317.1082
314.8883
312.6789
310.4799
308.2913
306.1129
303.9449
301.7870
299.6393
297.5017
295.3742

```
293.2566
291.1489
289.0512
286.9633
284.8851
282.8167
280.7580
278.7089
276.6693
274.6393
272.6188
270.6077
268.6059
266.6135
264.6304
262.6565
260.6917
258.7361
256.7896
254.8520
252.9235
251.0039
249.0932
247.1914
245.2983
243.4140
241.5383
239.6713
237.8130
235.9632
234.1219
232.2890
230.4646
228.6486
226.8409
225.0416
223.2504
```

```
221.4675
219.6927
217.9260
216.1674
214.4168
212.6742
210.9396
209.2129
207.4940
205.7829
204.0796
202.3840
200.6962
199.0160
197.3433
195.6783
194.0208
192.3708
190.7282
189.0930
187.4652
185.8447
184.2315
182.6256
181.0269
179.4353
177.8509
176.2735
174.7033
173.1400
171.5837
170.0344
168.4920
166.9564
165.4277
163.9058
162.3906
```

```
160.8822
159.3804
157.8853
156.3968
154.9149
153.4395
151.9706
150.5082
149.0522
147.6027
146.1595
144.7226
143.2920
141.8677
140.4497
139.0378
137.6321
136.2325
134.8390
133.4516
132.0702
130.6948
129.3254
127.9619
126.6043
125.2525
123.9066
122.5665
121.2322
119.9036
118.5808
117.2636
115.9520
114.6461
113.3458
112.0510
110.7617
```

```
109.4780
108.1997
106.9268
105.6593
104.3973
103.1405
101.8891
100.6430
    99.4021
    98.1664
    96.9360
    95.7107
    94.4906
    93.2756
    92.0657
    90.8608
    89.6610
    88.4661
    87.2763
    86.0914
    84.9114
    83.7363
    82.5661
    81.4007
    80.2401
    79.0844
    77.9333
    76.7870
    75.6455
    74.5086
    73.3763
    72.2487
    71.1257
    70.0073
    68.8934
    67.7841
    66.6792
```

$$
\begin{aligned}
& 65.5788 \\
& 64.4829 \\
& 63.3914 \\
& 62.3044 \\
& 61.2217 \\
& 60.1433 \\
& 59.0693 \\
& 57.9996 \\
& 56.9341 \\
& 55.8730 \\
& 54.8160 \\
& 53.7633 \\
& 52.7147 \\
& 51.6703 \\
& 50.6301 \\
& 49.5940 \\
& 48.5619 \\
& 47.5340 \\
& 46.5100 \\
& 45.4902 \\
& 44.4743 \\
& 43.4623 \\
& 42.4544 \\
& 41.4504 \\
& 40.4503 \\
& 39.4540 \\
& 38.4617 \\
& 37.4732 \\
& 36.4885 \\
& 35.5076 \\
& 34.5305 \\
& 33.5572 \\
& 32.5876 \\
& 31.6217 \\
& 30.6595 \\
& 29.7010 \\
& 28.7461
\end{aligned}
$$

```
    27.7949
    26.8473
    25.9033
    24.9628
    24.0259
    23.0926
    22.1627
    21.2363
    20.3134
    19.3940
    18.4780
    17.5654
    16.6562
    15.7503
    14.8478
    13.9487
    13.0528
    12.1603
    11.2710
    10.3850
    9.5022
    8.6226
    7.7463
    6.8731
    6.0031
    5.1362
    4.2725
    3.4118
    2.5543
    1.6998
    0.8484
```

Prepayment =
66.8676
83.5494

```
100.2000
116.8108
133.3731
149.8785
166.3183
182.6840
198.9672
215.1593
231.2520
247.2369
263.1058
278.8505
294.4628
309.9347
325.2583
340.4258
355.4294
370.2615
384.9145
399.3813
413.6545
427.7270
441.5919
455.2424
468.6720
465.8799
463.1015
460.3369
457.5858
454.8483
452.1242
449.4136
446.7162
444.0322
441.3613
438.7035
436.0588
```

```
433.4271
430.8083
428.2023
425.6091
423.0286
420.4608
417.9056
415.3628
412.8326
410.3147
407.8091
405.3158
402.8347
400.3657
397.9088
395.4638
393.0308
390.6097
388.2004
385.8029
383.4171
381.0428
378.6802
376.3291
373.9894
371.6611
369.3441
367.0384
364.7439
362.4606
360.1883
357.9271
355.6769
353.4376
351.2091
348.9915
346.7846
```

```
344.5884
342.4029
340.2279
338.0634
335.9095
333.7659
331.6327
329.5098
327.3972
325.2947
323.2024
321.1202
319.0481
316.9859
314.9337
312.8913
310.8588
308.8361
306.8230
304.8197
302.8260
300.8418
298.8672
296.9021
294.9464
293.0000
291.0630
289.1353
287.2168
285.3074
283.4072
281.5161
279.6340
277.7609
275.8968
274.0415
272.1951
```

```
270.3575
268.5286
266.7085
264.8970
263.0941
261.2998
259.5140
257.7367
255.9679
254.2074
252.4553
250.7115
248.9759
247.2485
245.5294
243.8183
242.1154
240.4204
238.7335
237.0546
235.3835
233.7204
232.0650
230.4175
228.7777
227.1456
225.5212
223.9044
222.2952
220.6935
219.0994
217.5127
215.9334
214.3616
212.7971
211.2399
209.6899
```

208.1472
206.6117
205.0834
203.5621
202.0480
200.5409
199.0408
197.5476
196.0614
194.5821
193.1097
191.6441
190.1852
188.7331
187.2877
185.8490
184.4170
182.9915
181.5726
180.1603
178.7544
177.3551
175.9621
174.5756
173.1954
171.8215
170.4540
169.0927
167.7376
166.3887
165.0460
163.7094
162.3789
161.0545
159.7361
158.4238
157.1173

```
155.8169
154.5223
153.2336
151.9507
150.6736
149.4024
148.1368
146.8770
145.6229
144.3744
143.1316
141.8943
140.6626
139.4365
138.2158
137.0007
135.7910
134.5866
133.3877
132.1942
131.0059
129.8230
128.6453
127.4729
126.3057
125.1437
123.9869
122.8352
121.6886
120.5470
119.4105
118.2791
117.1526
116.0311
114.9145
113.8029
112.6961
```


## mbspassthrough

$$
\begin{array}{r}
111.5942 \\
110.4972 \\
109.4049 \\
108.3175 \\
107.2347 \\
106.1568 \\
105.0835 \\
104.0149 \\
102.9509 \\
101.8916 \\
100.8369 \\
99.7868 \\
98.7412 \\
97.7002 \\
96.6636 \\
95.6316 \\
94.6040 \\
93.5808 \\
92.5620 \\
91.5476 \\
90.5376 \\
89.5319 \\
88.5306 \\
87.5335 \\
86.5407 \\
85.5521 \\
84.5677 \\
83.5875 \\
82.6115 \\
81.6397 \\
80.6720 \\
79.7084 \\
78.7488 \\
77.7934 \\
76.8419 \\
75.8945 \\
74.9511
\end{array}
$$

```
74.0116
73.0761
72.1445
71.2169
70.2931
69.3731
68.4571
67.5448
66.6363
65.7316
64.8307
63.9335
63.0400
62.1503
61.2642
60.3817
59.5029
58.6277
57.7561
56.8881
56.0236
55.1626
54.3052
53.4513
52.6008
51.7538
50.9103
50.0701
49.2334
48.4000
47.5700
46.7434
45.9200
45.1000
44.2833
43.4698
42.6596
```

$$
\begin{aligned}
& 41.8526 \\
& 41.0488 \\
& 40.2482 \\
& 39.4508 \\
& 38.6566 \\
& 37.8655 \\
& 37.0775 \\
& 36.2927 \\
& 35.5109 \\
& 34.7322 \\
& 33.9566 \\
& 33.1839 \\
& 32.4144 \\
& 31.6478 \\
& 30.8842 \\
& 30.1236 \\
& 29.3659 \\
& 28.6112 \\
& 27.8593 \\
& 27.1104 \\
& 26.3644 \\
& 25.6213 \\
& 24.8810 \\
& 24.1435 \\
& 23.4089 \\
& 22.6770 \\
& 21.9480 \\
& 21.2217 \\
& 20.4982 \\
& 19.7774 \\
& 19.0594 \\
& 18.3440 \\
& 17.6314 \\
& 16.9215 \\
& 16.2142 \\
& 15.5095 \\
& 14.8075
\end{aligned}
$$

14.1081
13.4113
12.7171
12.0255
11.3365
10.6499
9.9660
9.2845
8.6055
7.9290
7.2550
6.5835
5.9144
5.2477
4.5834
3.9215
3.2621
2.6050
1.9502
1.2978
0.6477
0.0000

## mbsprice

## Purpose <br> Mortgage-backed security price given yield

Syntax<br>[Price, AccrInt] = mbsprice(Yield, Settle, Maturity, IssueDate, GrossRate, CouponRate, Delay, PrepaySpeed, PrepayMatrix)

## Arguments

| Yield | Mortgage yield, compounded monthly (in decimal). |
| :--- | :--- |
| Settle | Settlement date. A serial date number or date <br> string. Settle must be earlier than Maturity. |
| Maturity | Maturity date. A serial date number or date string. |
| IssueDate | Issue date. A serial date number or date string. |
| GrossRate | Gross coupon rate (including fees), in decimal. <br> CouponRate <br> Delay <br>  <br> (Optional) Net coupon rate, in decimal. <br> Default = GrossRate. <br> (Optional) Delay (in days) between payment from |
| PrepaySpeed | homeowner and receipt by bondholder. Default = 0 <br> (no delay between payment and receipt. <br> (Optional) Relation of the conditional payment rate |
|  | (CPR) to the benchmark model. Default =0 (no <br> prepayment). Set PrepaySpeed to [] if you input a <br> customized prepayment matrix. |
| PrepayMatrix | (Optional) Customized prepayment matrix. A <br> matrix of size max (TermRemaining) -by-NMBS. |
|  | Missing values are padded with NaNs. Each column <br> corresponds to a mortgage-backed security, and each <br> row corresponds to each month after settlement. |

All inputs (except PrepayMatrix) are number of mortgage-backed securities (NMBS) by 1 vectors.

## Description

[Price, AccrInt] = mbsprice(Yield, Settle, Maturity, IssueDate, GrossRate, CouponRate, Delay, PrepaySpeed,

PrepayMatrix) computes a mortgage-backed security price, given time information, mortgage yield at settlement, and optionally, a prepayment model.

All outputs are scalar values.
Price is the clean price for every $\$ 100$ face value of the securities.
AccrInt is the accrued interest of the mortgage-backed securities.

## Examples Determine the Mortgage-Backed Security Price Given the Yield

This example shows how to determine the mortgage-backed security price given a mortgage-backed security with the following characteristics.

```
Yield = 0.0725;
Settle = datenum('15-Apr-2002');
Maturity = datenum('1 Jan 2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
CouponRate = 0.075;
Delay = 14;
Speed = 100;
[Price AccrInt] = mbsprice(Yield, Settle, Maturity, IssueDate,...
GrossRate, CouponRate, Delay, Speed)
```

Price =
101.3147

## AccrInt =

0.2917

## mbsprice

References [1] PSA Uniform Practices, SF-49
See Also mbsyield

## Purpose

Option-adjusted spread given price

Syntax<br>OAS = mbsprice2oas(ZeroCurve, Price, Settle, Maturity, IssueDate, GrossRate, CouponRate, Delay, Interpolation PrepaySpeed, PrepayMatrix)

## Arguments

| ZeroCurve | A matrix of three columns: <br> - Column 1: Serial date numbers. <br> - Column 2: Spot rates with maturities corresponding to the dates in Column 1, in decimal (for example, 0.075). <br> - Column 3: Compounding of the rates in Column 2. Values are 1 (annual), 2 (semiannual, 3 (three times per year), 4 (quarterly), 6 (bimonthly), 12 (monthly), and -1 (continuous). |
| :---: | :---: |
| Price | Clean price for every $\$ 100$ face value of bond issue. |
| Settle | Settlement date (scalar only). A serial date number or date string. Date when option-adjusted spread is calculated. Settle must be earlier than Maturity. |
| Maturity | Maturity date. Scalar or vector in serial date number or date string format. |
| IssueDate | Issue date. A serial date number or date string. |
| GrossRate | Gross coupon rate (including fees), in decimal. |
| CouponRate | (Optional) Net coupon rate, in decimal. Default $=$ GrossRate . |

## mbsprice2oas

| Delay | (Optional) Delay (in days) between payment <br> from homeowner and receipt by bondholder. <br> Default = 0 (no delay between payment and <br> receipt. |
| :--- | :--- |
| Interpolation | Interpolation method. Computes the <br> corresponding spot rates for the bond's cash <br> flow. Available methods are (0) nearest, (1) <br> linear, and (2) cubic spline. Default = 1. See <br> interp1 for more information. |
| PrepaySpeed | (Optional) Relation of the conditional payment <br> rate (CPR) to the benchmark model. Default = <br> end of month's CPR. Set PrepaySpeed to [] if |
| PrepayMatrix | you input a customized prepayment matrix. <br> (Optional) Customized prepayment matrix. A |
|  | matrix of size max (TermRemaining) -by-NMBS. |
|  | Missing values are padded with NaNs. Each <br> column corresponds to a mortgage-backed <br> security, and each row corresponds to each <br> month after settlement. |

All inputs (except PrepayMatrix) are number of mortgage-backed securities (NMBS) by 1 vectors.

## Description

## Examples Calculate the Option-Adjusted Spread of a 30-Year Fixed-Rate Mortgage

Calculate the option-adjusted spread of a 30-year fixed-rate mortgage with about a 28 -year weighted average maturity remaining, given assumptions of 0,50 , and 100 PSA prepayments. First, create the bonds matrix:

| Bonds = [datenum('11/21/2002') | 0 | 100 | 0 | 2 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| datenum('02/20/2003') | 0 | 100 | 0 | 2 | 1; |
| datenum('07/31/2004') | 0.03 | 100 | 2 | 3 | 1; |
| datenum('08/15/2007') | 0.035 | 100 | 2 | 3 | ; |
| datenum('08/15/2012') | 0.04875 | 100 | 2 | 3 | ; |
| datenum('02/15/2031') | 0.05375 | 100 | 2 | 3 | $1]$ |

Choose a settlement date.
Settle = datenum('20-Aug-2002');
Assume the following clean prices for the bonds:

```
Prices = [ 98.97467;
    98.58044;
    100.10534;
    98.18054;
    101.38136;
    99.25411];
```

Use the following formula to compute spot compounding for the bonds:

```
SpotCompounding = 2*ones(size(Prices));
```

Compute the zero curve.

```
[ZeroRatesP, CurveDatesP] = zbtprice(Bonds, Prices, Settle);
```

ZeroCurve = [CurveDatesP, ZeroRatesP, SpotCompounding]

## ZeroCurve =

$1.0 \mathrm{e}+05$ *

| 7.3154 | 0.0000 | 0.0000 |
| :--- | :--- | :--- |
| 7.3163 | 0.0000 | 0.0000 |
| 7.3216 | 0.0000 | 0.0000 |
| 7.3327 | 0.0000 | 0.0000 |

## mbsprice2oas

| 7.3510 | 0.0000 | 0.0000 |
| :--- | :--- | :--- |
| 7.4185 | 0.0000 | 0.0000 |

Assign the following parameters:

```
Price = 95;
Maturity = datenum('02-Jan-2030');
IssueDate = datenum('02-Jan-2000');
GrossRate = 0.08125;
CouponRate = 0.075;
Delay = 14;
Interpolation = 1;
PrepaySpeed = [0; 50; 100];
Interpolation = 1;
```

Compute the option-adjusted spread.
OAS = mbsprice2oas(ZeroCurve, Price, Settle, Maturity, ... IssueDate, GrossRate, CouponRate, Delay, Interpolation, ... PrepaySpeed)

OAS =
26.0508
28.6355
31.2232

See Also
mbsoas2price | mbsoas2yield | mbsyield2oas

## Purpose

Implied PSA prepayment speeds given price

## Syntax

[ImpSpdOnPrc, ImpSpdOnDur, ImpSpdOnCnv] = mbsprice2speed(Price, Settle, Maturity, IssueDate, GrossRate, PrepayMatrix, CouponRate, Delay)

## Arguments

## Description

Price Clean price for every $\$ 100$ face value.
Settle Settlement date. A serial date number or date string. Settle must be earlier than Maturity.
Maturity Maturity date. A serial date number or date string.
IssueDate Issue date. A serial date number or date string.
GrossRate Gross coupon rate (including fees), in decimal.
PrepayMatrix Customized prepayment matrix. A matrix of size max (TermRemaining)-by-NMBS. Missing values are padded with NaNs. Each column corresponds to a mortgage-backed security, and each row corresponds to each month after settlement.

CouponRate (Optional) Net coupon rate, in decimal. Default $=$ GrossRate .

Delay (Optional) Delay (in days) between payment from homeowner and receipt by bondholder. Default $=0$ (no delay between payment and receipt.

All inputs (except PrepayMatrix) are number of mortgage-backed securities (NMBS)-by-1 vectors.
[ImpSpdOnPrc, ImpSpdOnDur, ImpSpdOnCnv] = mbsprice2speed(Price, Settle, Maturity, IssueDate, GrossRate, PrepayMatrix, CouponRate, Delay) computes PSA prepayment speeds implied by pool prices and projected (user-defined) prepayment vectors. The calculated PSA speed produces the same

## mbsprice2speed

price, modified duration, or modified convexity, depending upon the output requested.

ImpSpdOnPrc calculates the equivalent PSA benchmark prepayment speed for the pass-through to carry the same price.

ImpSpdOnDur calculates the equivalent PSA benchmark prepayment speed for the pass-through to carry the same modified duration.

ImpSpdOnCnv calculates the equivalent PSA benchmark prepayment speed for the pass-through to carry the same modified convexity.

All outputs are NMBS-by- 1 vectors.

## Examples Compute PSA Prepayment Speeds

This example shows how to compute the equivalent PSA benchmark prepayment speeds for a mortgage pool with the following characteristics and prepayment matrix.

```
Price = 101;
Settle = datenum('1-Jan-2000');
Maturity = datenum('1-Jan-2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
PrepayMatrix = 0.005*ones(360,1);
CouponRate = 0.075;
Delay = 14;
[ImpSpdOnPrc, ImpSpdOnDur, ImpSpdOnCnv] = ...
mbsprice2speed(Price,Settle, Maturity, IssueDate, ...
GrossRate, PrepayMatrix, CouponRate, Delay)
```

ImpSpdOnPrc =
118.5980
ImpSpdOnDur =118.3946
ImpSpdOnCnv =109.5115
References [1] PSA Uniform Practices, SF-49
See Also mbsprice | mbsyield2speed

| Purpose | Weighted average life of mortgage pool |
| :---: | :---: |
| Compatibility | PSA |
| Syntax | WAL = mbswal(Settle, Maturity, IssueDate, GrossRate, Coupon <br> Delay, PrepaySpeed, PrepayMatrix) |
| Arguments | Settle $\quad$Settlement date. A serial date number or date string. <br> Settle must be earlier than Maturity. |
|  | Maturity Maturity date. A serial date number or date string. |
|  | IssueDate Issue date. A serial date number or date string. |
|  | GrossRate Gross coupon rate (including fees), in decimal. |
|  | CouponRate (Optional) Net coupon rate, in decimal. <br> Default = GrossRate. |
|  | Delay <br> (Optional) Delay (in days) between payment from homeowner and receipt by bondholder. Default $=0$ (no delay between payment and receipt). |
|  | PrepaySpeed (Optional) Relation of the conditional payment rate (CPR) to the benchmark model. Default = end of month's CPR. Set PrepaySpeed to [ ] if you input a customized prepayment matrix. |
|  | PrepayMatrix (Optional) Customized prepayment matrix. A matrix of size max (TermRemaining)-by-NMBS. Missing values are padded with NaNs. Each column corresponds to a mortgage-backed security, and each row corresponds to each month after settlement. |

All inputs (except PrepayMatrix) are number of mortgage-backed securities (NMBS) by 1 vectors.

## Description

WAL = mbswal(Settle, Maturity, IssueDate, GrossRate, CouponRate, Delay, PrepaySpeed, PrepayMatrix) computes the
weighted average life, in number of years, of a mortgage pool, as measured from the settlement date.

## Examples

WAL =
10.5477

References [1] PSA Uniform Practices, SF-49
See Also mbspassthrough

## mbsyield

Purpose

## Syntax

## Arguments

## Description

Mortgage-backed security yield given price
[MYield, BEMBSYield] = mbsyield(Price, Settle, Maturity, IssueDate, GrossRate, CouponRate, Delay, PrepaySpeed, PrepayMatrix)

Price Clean price for every $\$ 100$ face value.
Settle Settlement date. A serial date number or date string. Settle must be earlier than Maturity.

Maturity Maturity date. A serial date number or date string.
IssueDate Issue date. A serial date number or date string.
GrossRate Gross coupon rate (including fees), in decimal.
CouponRate (Optional) Net coupon rate, in decimal. Default = GrossRate.

Delay (Optional) Delay (in days) between payment from homeowner and receipt by bondholder. Default $=0$
 (CPR) to the benchmark model. Default = 0 (no prepayment). Set PrepaySpeed to [ ] if you input a customized prepayment matrix.

PrepayMatrix (Optional) Customized prepayment matrix. A matrix of size max(TermRemaining)-by-NMBS. Missing values are padded with NaNs. Each column corresponds to a mortgage-backed security, and each row corresponds to each month after settlement.

All inputs (except PrepayMatrix) are number of mortgage-backed securities (NMBS) by 1 vectors.
[MYield, BEMBSYield] = mbsyield(Price, Settle, Maturity, IssueDate, GrossRate, CouponRate, Delay, PrepaySpeed, PrepayMatrix) computes a mortgage-backed security yield to maturity
and the bond equivalent yield, given time information, price at settlement, and optionally, a prepayment model.

MYield is the yield to maturity of the mortgage-backed security (the mortgage yield). This yield is compounded monthly (12 times a year).
BEMBSYield is the corresponding bond equivalent yield of the mortgage-backed security. This yield is compounded semiannually (two times a year).

## Examples

## Determine a Mortgage-Backed Security Yield Given the Price

This example shows how to determine the mortgage-backed security yield, given a mortgage-backed security with the following characteristics.

```
Price = 102;
Settle = '15-Apr-2002';
Maturity = '1 Jan 2030';
IssueDate = '1-Jan-2000';
GrossRate = 0.08125;
CouponRate = 0.075;
Delay = 14;
Speed = 100;
```

[MYield, BEMBSYield] = mbsyield(Price, Settle, Maturity, ...
IssueDate, GrossRate, CouponRate, Delay, Speed)
MYield =
0.0715
BEMBSYield =
0.0725

## mbsyield

## Determine Multiple Mortgage-Backed Securities Yields Given the Price

This example shows how to determine multiple mortgage-backed securities yields, given a portfolio of mortgage-backed securities with the following characteristics.

```
Price = 102;
Settle = datenum(['13-Feb-2000';'17-Apr-2002';'17-May-2002';...
'13-Jan-2000']);
Maturity = datenum('1-Jan-2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
CouponRate = [0.075; 0.07875; 0.0775; 0.08125];
Delay = 14;
Speed = 100;
[MYield, BEMBSYield] = mbsyield(Price, Settle, Maturity,...
IssueDate, GrossRate, CouponRate, Delay, Speed)
MYield =
    0.0717
    0.0751
    0.0739
    0.0779
BEMBSYield =
    0.0728
    0.0763
    0.0750
    0.0791
```

References [1] PSA Uniform Practices, SF-49
See Also mbsprice

## mbsyield2oas

Purpose
Syntax

Arguments

Option-adjusted spread given yield
OAS = mbsyield2oas(ZeroCurve, Yield, Settle, Maturity, IssueDate, GrossRate, CouponRate, Delay, Interpolation PrepaySpeed, PrepayMatrix)

ZeroCurve A matrix of three columns:

- Column 1: Serial date numbers.
- Column 2: Spot rates with maturities corresponding to the dates in Column 1, in decimal (for example, 0.075).
- Column 3: Compounding of the rates in Column 2. Values are 1 (annual), 2 (semiannual, 3 (three times per year), 4 (quarterly), 6 (bimonthly), 12 (monthly), and -1 (continuous).
$\left.\begin{array}{ll}\text { Yield } & \begin{array}{l}\text { Mortgage yield, compounded monthly (in decimal). } \\ \text { Settle }\end{array} \\ \text { Settlement date (scalar only). A serial date number } \\ \text { or date string. Date when option-adjusted spread is } \\ \text { calculated. Settle must be earlier than Maturity. }\end{array}\right\}$

Interpolation Interpolation method. Computes the corresponding spot rates for the bond's cash flow. Available methods are (0) nearest, (1) linear, and (2) cubic spline. Default $=1$. See interp1 for more information.

PrepaySpeed (Optional) Relation of the conditional payment rate (CPR) to the benchmark model. Default = end of month's CPR. Set PrepaySpeed to [ ] if you input a customized prepayment matrix.
PrepayMatrix (Optional) Customized prepayment matrix. A matrix of size max (TermRemaining)-by-NMBS. Missing values are padded with NaNs. Each column corresponds to a mortgage-backed security, and each row corresponds to each month after settlement.

All inputs (except PrepayMatrix) are number of mortgage-backed securities (NMBS) by 1 vectors.

| Description $\quad$ | OAS = mbsyield2oas(ZeroCurve, Yield, Settle, Maturity, <br>  <br>  <br>  <br>  <br>  <br>  <br>  <br>  <br> PrepaySpeed, GrossRate, CouponRate, Delay, Interpolation, <br> in basis points. |
| :--- | :--- |

Examples Calculate the Option-Adjusted Spread of a 30-Year Fixed-Rate Mortgage Pool

Calculate the option-adjusted spread of a 30-year, fixed-rate mortgage pool with about 28 -year weighted average maturity left, given assumptions of 0,50 , and 100 PSA prepayments. First, create the bonds matrix:


## mbsyield2oas

Choose a settlement date.
Settle = datenum('20-Aug-2002');
Assume the following clean prices for the bonds:

```
Prices = [ 98.97467;
    98.58044;
    100.10534;
    98.18054;
    101.38136;
    99.25411];
```

Use the following formula to compute spot compounding for the bonds:
SpotCompounding = 2*ones(size(Prices));
Compute the zero curve.
[ZeroRatesP, CurveDatesP] = zbtprice(Bonds, Prices, Settle); ZeroCurve = [CurveDatesP, ZeroRatesP, SpotCompounding]

ZeroCurve =
$1.0 \mathrm{e}+05$ *

| 7.3154 | 0.0000 | 0.0000 |
| :--- | :--- | :--- |
| 7.3163 | 0.0000 | 0.0000 |
| 7.3216 | 0.0000 | 0.0000 |
| 7.3327 | 0.0000 | 0.0000 |
| 7.3510 | 0.0000 | 0.0000 |
| 7.4185 | 0.0000 | 0.0000 |

Assign the following parameters:

```
Price = 95;
Maturity = datenum('02-Jan-2030');
IssueDate = datenum('02-Jan-2000');
GrossRate = 0.08125;
CouponRate = 0.075;
Delay = 14;
Interpolation = 1;
PrepaySpeed = [0 50 100];
```

Compute the yield, and from the yield, compute the option-adjusted spread.

```
[mbsyld, beyld] = mbsyield(Price, Settle, ...
Maturity, IssueDate, GrossRate, CouponRate, Delay, PrepaySpeed);
OAS = mbsyield2oas(ZeroCurve, mbsyld, Settle, ...
Maturity, IssueDate, GrossRate, CouponRate, Delay, ...
Interpolation, PrepaySpeed)
```

OAS =
26.0508
28.6355
31.2232

See Also
mbsoas2price | mbsoas2yield | mbsprice2oas

## mbsyield2speed

Purpose
Syntax

Arguments

## Description

Implied PSA prepayment speeds given yield
[ImpSpdOnYld, ImpSpdOnDur, ImpSpdOnCnv] = mbsyield2speed(Yield, Settle, Maturity, IssueDate, GrossRate, PrepayMatrix, CouponRate, Delay)

| Yield | Mortgage yield, compounded monthly, in decimal. |
| :--- | :--- |
| Settle | Settlement date. A serial date number or date <br> string. Settle must be earlier than Maturity. |
| Maturity | Maturity date. A serial date number or date string. |
| IssueDate | Issue date. A serial date number or date string. |
| GrossRate | Gross coupon rate (including fees), in decimal. |
| PrepayMatrix | Customized prepayment matrix. A matrix of size <br> max(TermRemaining)-by-NMBS. Missing values <br> are padded with NaNs. Each column corresponds <br> to a mortgage-backed security, and each row <br> corresponds to each month after settlement. |
| CouponRate | (Optional) Net coupon rate, in decimal. |
| DelayDefault = GrossRate. <br> (Optional) Delay (in days) between payment from |  |
|  | homeowner and receipt by bondholder. Default = 0 <br> (no delay between payment and receipt). |

All inputs (except PrepayMatrix) are number of mortgage-backed securities (NMBS) by 1 vectors.
[ImpSpdOnYld, ImpSpdOnDur, ImpSpdOnCnv] = mbsyield2speed(Yield, Settle, Maturity, IssueDate, GrossRate, PrepayMatrix, CouponRate, Delay) computes PSA prepayment speeds implied by pool yields and projected (user-defined) prepayment vectors. The calculated PSA speed produces the same
yield, modified duration, or modified convexity, depending upon the output requested.

ImpSpdOnPrc calculates the equivalent PSA benchmark prepayment speed for the pass-through to carry the same price.

ImpSpdOnDur calculates the equivalent PSA benchmark prepayment speed for the pass-through to carry the same modified duration.

ImpSpdOnCnv calculates the equivalent PSA benchmark prepayment speed for the pass-through to carry the same modified convexity.

All outputs are NMBS-by- 1 vectors.

## Examples

Calculate the equivalent PSA benchmark prepayment speeds for a security with these characteristics and prepayment matrix.

```
Yield = 0.065;
Settle = datenum('1-Jan-2000');
Maturity = datenum('1-Jan-2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
PrepayMatrix = 0.005*Ones(360,1);
CouponRate = 0.075;
Delay = 14;
```

[ImpSpdOnYld, ImpSpdOnDur, ImpSpdOnCnv] = ...
mbsyield2speed(Yield, Settle, Maturity, IssueDate, GrossRate, ...
PrepayMatrix, CouponRate, Delay)
ImpSpdOnYld =
117.7644
ImpSpdOnDur =
116.7436
ImpSpdOnCnv =

## mbsyield2speed

108.3309

References [1] PSA Uniform Practices, SF-49
See Also mbsyield | mbsprice2speed

| Purpose | Benchmark default |
| :---: | :---: |
| Syntax | [ADRPSA, MDRPSA] = psaspeed2default(DefaultSpeed) |
| Arguments | DefaultSpeed Annual speed relative to the benchm <br> benchmark $=100$ |
| Description | [ADRPSA, MDRPSA] = psaspeed2default(DefaultSpeed) the benchmark default on the performing balance of mortga securities per PSA benchmark speed. <br> ADRPSA is the PSA default rate, in decimal (360-by-1). <br> MDRPSA is the PSA monthly default rate, in decimal (360-by- |
| Examples | Compute the Benchmark Default Rates on the Perf Balance of Mortgage-Backed Securities Per PSA Be Speed |
|  | This example shows how to compute the benchmark default performing balance of mortgage-backed securities per PSA speed, given a mortgage-backed security with annual speed PSA default benchmark. |
|  | DefaultSpeed = 100; |
|  | [ADRPSA, MDRPSA] = psaspeed2default(DefaultSpeed) |
|  | ADRPSA $=$ |
|  | 0.0002 |
|  | 0.0004 |
|  | 0.0006 |
|  | 0.0008 |
|  | 0.0010 |

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## See Also

psaspeed2rate

## psaspeed2rate

Purpose Single monthly mortality rate given PSA speed
Syntax [CPRPSA, SMMPSA]= psaspeed2rate(PSASpeed)

Arguments PSASpeed Any value $>0$ representing the annual speed relative to the benchmark. PSA benchmark $=100$.

Description [CPRPSA, SMMPSA] = psaspeed2rate(PSASpeed) calculates vectors of PSA prepayments, each containing 360 prepayment elements, to represent the 360 months in a 30 -year mortgage pool.

CPRPSA is the PSA conditional prepayment rate, in decimal [360-by-1].
SMMPSA is the PSA single monthly mortality rate, in decimal [360-by-1].

## Examples Compute the Prepayment and Mortality Rates

This example shows how to compute the prepayment and mortality rates, given a mortgage-backed security with annual speed set at the PSA default benchmark.

```
PSASpeed = [100 200];
[CPRPSA, SMMPSA]= psaspeed2rate(PSASpeed);
% view the plot of the output
psaspeed2rate(PSASpeed)
```



See Also
psaspeed2default

## stepcpncfamounts

Purpose Cash flow amounts and times for bonds and stepped coupons

Syntax<br>[CFlows, CDates, CTimes] = stepcpncfamounts(Settle, Maturity, ConvDates, CouponRates, Period, Basis, EndMonthRule, Face)

| Arguments | Settle | Settlement date. A scalar or vector of serial date numbers. Settle must be earlier than Maturity. |
| :---: | :---: | :---: |
|  | Maturity | Maturity date. A scalar or vector of serial date numbers. |
|  | ConvDates | Matrix of serial date numbers representing conversion dates after Settle. Size = number of instruments by maximum number of conversions. Fill unspecified entries with NaN. |
|  | CouponRates | Matrix indicating the coupon rates for each bond in decimal form. Size = number of instruments by maximum number of conversions +1 . First column of this matrix contains rates applicable between Settle and the first conversion date (date in the first column of ConvDates). Fill unspecified entries with NaN. See Note below. |
|  | Period | (Optional) Coupons per year of the bond. A vector of integers. Allowed values are $0,1,2,3,4,6$, and 12 . Default $=2$. |


| Basis | (Optional) Day-count basis of the instrument. A |
| :---: | :---: |
|  | - $0=$ actual/actual (default) |
|  | - $1=30 / 360$ (SIA) |
|  | - $2=\mathrm{actual} / 360$ |
|  | - 3 = actual/365 |
|  | - $4=30 / 360$ (BMA) |
|  | - $5=30 / 360$ (ISDA) |
|  | - $6=30 / 360$ (European) |
|  | - $7=$ actual/365 (Japanese) |
|  | - $8=$ actual/actual (ICMA) |
|  | - 9 = actual/360 (ICMA) |
|  | - $10=\mathrm{actual} / 365$ (ICMA) |
|  | - $11=30 / 360 \mathrm{E}$ (ICMA) |
|  | - $12=$ actual/actual (ISDA) |
|  | - 13 = BUS/252 |
|  | For more information, see basis on page Glossary-1. |
| EndMonthRule | (Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. $1=$ set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |
| Face | (Optional) Face value of each bond in the portfolio. Default $=100$. |

## stepcpncfamounts

All arguments must be scalars or number of bonds (NUMBONDS)-by-1 vectors, except for ConvDates and CouponRates.

Note ConvDates has the same number of rows as CouponRates to reflect the same number of bonds. However, ConvDates has one less column than CouponRates. This situation is illustrated by


## Description

[CFlows, CDates, CTimes] = stepcpncfamounts(Settle, Maturity, ConvDates, CouponRates, Period, Basis, EndMonthRule, Face) returns matrices of cash flow amounts, cash flow dates, and time factors for a portfolio of NUMBONDS stepped-coupon bonds.

CFlows is a matrix of cash flow amounts. The first entry in each row vector is a negative number indicating the accrued interest due at settlement. If no accrued interest is due, the first column is 0 .

CDates is a matrix of cash flow dates in serial date number form. At least two columns are always present, one for settlement and one for maturity.

CTimes is a matrix of time factors for the SIA semiannual price/yield conversion.

DiscountFactor = (1 + Yield/2).^(-TFactor)
Time factors are in units of semiannual coupon periods. In computing time factors, use SIA actual/actual conventions for all time factor calculations.

> Note For bonds with fixed coupons, use cfamounts. If you use a fixed-coupon bond with stepcpncfamounts, MATLAB software generates an error.

## Examples

This example generates stepped cash flows for three different bonds, all paying interest semiannually. Their life span is about 18 to 19 years each:

- Bond A has two conversions, but the first one occurs on the settlement date and immediately expires.
- Bond B has three conversions, with conversion dates exactly on the coupon dates.
- Bond C has three conversions, with some conversion dates not on the coupon dates. It has the longest maturity. This case illustrates that only cash flows for full periods after conversion dates are affected, as illustrated below.


The following table illustrates the interest rate characteristics of this bond portfolio.

| Bond A Dates | Bond A Rates | Bond B Dates | Bond B Rates | Bond C Dates | Bond C Rates |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Settle (02-Aug-92) | 7.5\% | Settle (02-Aug-92) | 7.5\% | Settle (02-Aug-92) | 2.5\% |
| First <br> Conversion (02-Aug-92) | 8.875\% | First Conversion (15-Jun-97)) | 8.875\% | First Conversion (14-Jun-97)) | 5.0\% |


| Bond A <br> Dates | Bond A <br> Rates | Bond B <br> Dates | Bond B <br> Rates | Bond C <br> Dates | Bond C <br> Rates |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Second <br> Conversion <br> (15-Jun-03) | $9.25 \%$ | Second <br> Conversion <br> (15-Jun-01) | $9.25 \%$ | Second <br> Conversion <br> (14-Jun-01) | $7.5 \%$ |
| Maturity <br> (15-Jun-10) | NaN | Third <br> Conversion <br> (15-Jun-05) | $10.0 \%$ | Third <br> Conversion <br> (14-Jun-05) | $10.0 \%$ |
| Maturity | NaN | Maturity <br> (15-Jun-11) | NaN |  |  |

```
Settle = datenum('02-Aug-1992');
ConvDates = [datenum('02-Aug-1992'), datenum('15-Jun-2003'),...
    nan;
    datenum('15-Jun-1997'), datenum('15-Jun-2001'),...
    datenum('15-Jun-2005');
    datenum('14-Jun-1997'), datenum('14-Jun-2001'),\ldots.
    datenum('14-Jun-2005')];
Maturity = [datenum('15-Jun-2010');
    datenum('15-Jun-2010');
    datenum('15-Jun-2011')];
CouponRates = [0.075 0.08875 0.0925 nan;
    0.075 0.08875 0.0925 0.1;
    0.025 0.05 0.0750 0.1];
Basis = 1;
Period = 2;
EndMonthRule = 1;
Face = 100;
```

Call stepcpncfamounts to compute cash flows and timings.
[CFlows, CDates, CTimes] = stepcpncfamounts(Settle, Maturity, ...

```
ConvDates, CouponRates);
```

Visualize the third bond cash flows (2.5-5-7.5-10) using the cfplot function.

```
cfplot(CDates(3,:),CFlows(3,:));
xlabel('Dates in Serial Number Format')
ylabel('Relative Amounts of Cash Flows')
title('CashFlow of 2.5 - 5 - 7.5 - 10 Stepped Coupon Bond')
```



See Also stepcpnprice | stepcpnyield | cfplot<br>Related<br>Examples<br>- "Cash Flows from Stepped-Coupon Bonds" on page 6-12<br>- "Price and Yield of Stepped-Coupon Bonds" on page 6-14

## Purpose

Price bond with stepped coupons
Syntax
[Price, AccruedInterest] = stepcpnprice(Yield, Settle, Maturity, ConvDates, CouponRates, Period, Basis, EndMonthRule, Face)

## Arguments

$\left.\begin{array}{ll}\text { Yield } & \begin{array}{l}\text { Scalar or vector containing yield to maturity of } \\ \text { instruments. }\end{array} \\ \text { Settle } & \begin{array}{l}\text { Settlement date. A scalar or vector of serial date } \\ \text { numbers. Settle must be earlier than Maturity. }\end{array} \\ \text { Maturity } & \begin{array}{l}\text { Maturity date. A scalar or vector of serial date } \\ \text { numbers. }\end{array} \\ \text { ConvDates } & \begin{array}{l}\text { Matrix of serial date numbers representing } \\ \text { conversion dates after Settle. Size = number of } \\ \text { instruments by maximum number of conversions. }\end{array} \\ \text { CouponRates } & \begin{array}{l}\text { Fill unspecified entries with NaN. }\end{array} \\ & \begin{array}{l}\text { Matrix indicating the coupon rates for each bond } \\ \text { in decimal form. Size = number of instruments } \\ \text { by maximum number of conversions + 1. First } \\ \text { column of this matrix contains rates applicable } \\ \text { between Settle and the first conversion date } \\ \text { (date in the first column of ConvDates). Fill }\end{array} \\ \text { unspecified entries with NaN. See Note below. }\end{array}\right\}$

| Basis | (Optional) Day-count basis of the instrument. A vector of integers. |
| :---: | :---: |
|  | - $0=$ actual/actual (default) |
|  | - $1=30 / 360$ (SIA) |
|  | - 2 = actual/360 |
|  | - 3 = actual/365 |
|  | - $4=30 / 360$ (BMA) |
|  | - $5=30 / 360$ (ISDA) |
|  | - $6=30 / 360$ (European) |
|  | - $7=$ actual/365 (Japanese) |
|  | - $8=$ actual/actual (ICMA) |
|  | - $9=$ actual/360 (ICMA) |
|  | - $10=\mathrm{actual} / 365$ (ICMA) |
|  | - $11=30 / 360 \mathrm{E}$ (ICMA) |
|  | - $12=\mathrm{actual} / \mathrm{actual}$ (ISDA) |
|  | - 13 = BUS/252 |
|  | For more information, see basis on page Glossary-1. |
| EndMonthRule | (Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. $1=$ set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |
| Face | (Optional) Face value of each bond in the portfolio. Default $=100$. |

All arguments must be scalars or number of bonds (NUMBONDS)-by-1 vectors, except for ConvDates and CouponRates.

Note ConvDates has the same number of rows as CouponRate to reflect the same number of bonds. However, ConvDates has one less column than CouponRate. This situation is illustrated by


## Description

[Price, AccruedInterest] = stepcpnprice(Yield, Settle, Maturity, ConvDates, CouponRates, Period, Basis, EndMonthRule, Face) computes the price of bonds with stepped coupons given the yield to maturity. The function supports any number of conversion dates.

Price is a NUMBONDS-by- 1 vector of clean prices.
AccruedInterest is a NUMBONDS-by-1 vector of accrued interest payable at settlement dates.

Note For bonds with fixed coupons, use bndprice. If you use a fixed-coupon bond with stepcpnprice, you will receive the error: incorrect number of inputs.

Examples Compute the price and accrued interest due on a portfolio of stepped-coupon bonds having a yield of $7.221 \%$, given three conversion scenarios:

- Bond A has two conversions, the first one falling on the settle date and immediately expiring.


## stepcpnprice

- Bond B has three conversions, with conversion dates exactly on the coupon dates.
- Bond C has three conversions, with one or more conversion dates not on coupon dates. This case illustrates that only cash flows for full periods after conversion dates are affected, as illustrated below.


The following table illustrates the interest rate characteristics of this bond portfolio.

| Bond A Dates | Bond A Rates | Bond B Dates | Bond B Rates | Bond C <br> Dates | Bond C Rates |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Settle (02-Aug-92) | 7.5\% | Settle (02-Aug-92) | 7.5\% | Settle (02-Aug-92) | 7.5\% |
| First Conversion (02-Aug-92) | 8.875\% | First <br> Conversion <br> (15-Jun-97) | 8.875\% | First Conversion (14-Jun-97) | 8.875\% |
| Second Conversion (15-Jun-03) | 9.25\% | Second Conversion (15-Jun-01) | 9.25\% | Second Conversion (14-Jun-01) | 9.25\% |
| Maturity <br> (15-Jun-10) | NaN | Third Conversion (15-Jun-05) | 10.0\% | Third Conversion (14-Jun-05) | 10.0\% |
|  |  | Maturity <br> (15-Jun-10) | NaN | Maturity <br> (15-Jun-10) | NaN |

```
Yield = 0.07221;
Settle = datenum('02-Aug-1992');
ConvDates = [datenum('02-Aug-1992'), datenum('15-Jun-2003'),...
    nan;
    datenum('15-Jun-1997'), datenum('15-Jun-2001'),...
```

```
    datenum('15-Jun-2005');
    datenum('14-Jun-1997'), datenum('14-Jun-2001'),...
    datenum('14-Jun-2005')];
Maturity = datenum('15-Jun-2010');
CouponRates = [0.075 0.08875 0.0925 nan;
    0.075 0.08875 0.0925 0.1;
    0.075 0.08875 0.0925 0.1];
Basis = 1;
Period = 2;
EndMonthRule = 1;
Face = 100;
[Price, AccruedInterest] = ...
stepcpnprice(Yield, Settle, Maturity, ConvDates, CouponRates, ...
Period, Basis, EndMonthRule, Face)
Price =
    117.3824
    113.4339
    113.4339
```

AccruedInterest =
1.1587
0.9792
0.9792
References This function adheres to SIA Fixed Income Securities Formulas for Price, Yield, and Accrued Interest, Volume 1, 3rd edition, pp. 120-123, on zero-coupon instruments pricing.
stepcpnprice | stepcpnyield | tbillprice | stepcpncfamounts | cdprice | bndprice

## stepcpnprice

Related

- "Cash Flows from Stepped-Coupon Bonds" on page 6-12
Examples
- "Price and Yield of Stepped-Coupon Bonds" on page 6-14

| Purpose | Yield to maturity of bond with stepped coupons |
| :---: | :---: |
| Syntax | Yield = stepcpnyield(Price, Settle, Maturity, ConvDates, CouponRate, Period, Basis, EndMonthRule, Face) |
| Arguments | Price $\quad$ Vector containing price of the bonds. |
|  | Settle Settlement date. A vector of serial date numbers. Settle must be earlier than Maturity. |
|  | Maturity Maturity date. A vector of serial date numbers. |
|  | ConvDates <br> Matrix of serial date numbers representing conversion dates after Settle. Size = number of instruments by maximum number of conversions. Fill unspecified entries with NaN. |
|  | CouponRates <br> Matrix indicating the coupon rates for each bond in decimal form. Size = number of instruments by maximum number of conversions +1 . First column of this matrix contains rates applicable between Settle and the first conversion date (date in the first column of ConvDates). Fill unspecified entries with NaN. See Note below. |
|  | Period <br> (Optional) Coupons per year of the bond. A vector of integers. Allowed values are $0,1,2,3,4,6$, and 12. Default $=2$. |

Basis (Optional) Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.

EndMonthRule (Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. $1=$ set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
(Optional) Face value of each bond in the portfolio. Default $=100$.

All arguments must be number of bonds (NUMBONDS)-by-1 vectors, except for ConvDates and CouponRate.

Note ConvDates has the same number of rows as CouponRate to reflect the same number of bonds. However, ConvDates has one less column than CouponRate. This situation is illustrated by


```
Rate1 Rate2 Rate3
```


## Description

Yield = stepcpnyield(Price, Settle, Maturity, ConvDates, CouponRate, Period, Basis, EndMonthRule, Face) computes the yield to maturity of bonds with stepped coupons given the price. The function supports any number of conversion dates.

Yield is a NUMBONDS-by-1 vector of yields to maturity in decimal form.

Note For bonds with fixed coupons, use bndyield. You will receive the error incorrect number of inputs if you use a fixed-coupon bond with stepcpnyield.

## Examples

Find the yield to maturity of three stepped-coupon bonds of known price, given three conversion scenarios:

- Bond A has two conversions, the first one falling on the settle date and immediately expiring.
- Bond B has three conversions, with conversion dates exactly on the coupon dates.
- Bond C has three conversions, with one or more conversion dates not on coupon dates. This case illustrates that only cash flows for full periods after conversion dates are affected, as illustrated below.


## stepcpnyield



The following table illustrates the interest rate characteristics of this bond portfolio.

| Bond A Dates | Bond A Rates | Bond B Dates | Bond B Rates | Bond C Dates | Bond C <br> Rates |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Settle (02-Aug-92) | 7.5\% | Settle (02-Aug-92) | 7.5\% | Settle (02-Aug-92) | 7.5\% |
| First Conversion (02-Aug-92) | 8.875\% | First <br> Conversion <br> (15-Jun-97) | 8.875\% | First <br> Conversion <br> (14-Jun-97) | 8.875\% |
| Second <br> Conversion <br> (15-Jun-03) | 9.25\% | Second Conversion (15-Jun-01) | 9.25\% | Second Conversion (14-Jun-01) | 9.25\% |
| Maturity(15-Jun-10) | NaN | Third <br> Conversion <br> (15-Jun-05) | 10.0\% | Third Conversion (14-Jun-05) | 10.0\% |
|  |  | Maturity (15-Jun-10) | NaN | Maturity (15-Jun-10) | NaN |

```
format long
Price = [117.3824; 113.4339; 113.4339];
Settle = datenum('02-Aug-1992');
ConvDates = [datenum('02-Aug-1992'), datenum('15-Jun-2003'), nan;
datenum('15-Jun-1997'), datenum('15-Jun-2001'), datenum('15-Jun-2005');
datenum('14-Jun-1997'), datenum('14-Jun-2001'), datenum('14-Jun-2005')];
Maturity = datenum('15-Jun-2010');
CouponRates = [0.075 0.08875 0.0925 nan;
```

```
        0.075 0.08875 0.0925 0.1;
    0.075 0.08875 0.0925 0.1];
Basis = 1;
Period = 2;
EndMonthRule = 1;
Face = 100;
Yield = stepcpnyield(Price, Settle, Maturity, ConvDates, ...
CouponRates, Period, Basis, EndMonthRule, Face)
Yield =
0.07221440204915
0.07221426780036
0.07221426780036
```


## References This function adheres to SIA Fixed Income Securities Formulas for Price, Yield, and Accrued Interest, Volume 1, 3rd edition, pp. 120-123, on zero-coupon instruments pricing.

See Also stepcpnprice | bndprice | cdprice | stepcpncfamounts | stepcpnprice | tbillprice | zeroprice
Related- "Cash Flows from Stepped-Coupon Bonds" on page 6-12- "Price and Yield of Stepped-Coupon Bonds" on page 6-14

Purpose

## Syntax

Arguments

## Description

Future prices of Treasury bonds given spot price
[QtdFutPrice, AccrInt] = tfutbyprice(SpotCurve, Price, SettleFut, MatFut, ConvFactor, CouponRate, Maturity, Interpolation)

SpotCurve Treasury spot curve; a number of futures (NFUT) by 3 matrix in the form of [SpotDates SpotRates Compounding].
Allowed compounding values are $-1,1,2$ (default), 3 , 4 , and 12 , where -1 is continuous compounding.
Price Scalar or vector containing prices of Treasury bonds or notes per $\$ 100$ notional. Use bndprice for theoretical value of bond.
SettleFut Scalar or vector of identical elements containing settlement date of futures contract.
MatFut Scalar or vector containing maturity dates (or anticipated delivery dates) of futures contract.
ConvFactor Conversion factor. See convfactor.
CouponRate Scalar or vector containing underlying bond annual coupon in decimal.
Maturity $\quad$ Scalar or vector containing underlying bond maturity.

Interpolation (Optional) Interpolation method. Available methods are (0) nearest, (1) linear, and (2) cubic. Default $=1$. See interp1 for more information.

Inputs (except SpotCurve) must either be a scalar or a vector of size equal to the number of Treasury futures (NFUT) by 1 or 1-by-NFUT.
[QtdFutPrice, AccrInt] = tfutbyprice(SpotCurve, Price, SettleFut, MatFut, ConvFactor, CouponRate, Maturity,

Interpolation) computes future prices of Treasury notes and bonds given the spot price. The output arguments are:

- QtdFutPrice - Quoted futures price, per $\$ 100$ notional.
- AccrInt - Accrued interest due at delivery date, per $\$ 100$ notional.

In addition, you can use the Financial Instruments Toolbox method getZeroRates for an IRDataCurve object with a Dates property to create a vector of dates and data acceptable for tfutbyprice. For more information, see "Converting an IRDataCurve or IRFunctionCurve Object" on page 9-40.

## Examples Determine the Future Prices of Treasury Bonds Given the Spot Price

This example shows how to determine the future price of two Treasury bonds based upon a spot rate curve constructed from data for November 14, 2002.

```
% construct spot curve from Nov 14, data
Bonds = [datenum('02/13/2003'), 0;
        datenum('05/15/2003'), 0;
        datenum('10/31/2004'), 0.02125;
        datenum('11/15/2007'), 0.03;
        datenum('11/15/2012'), 0.04;
        datenum('02/15/2031'), 0.05375];
Yields = [1.20; 1.25; 1.86; 2.99; 4.02; 4.93]/100;
Settle = datenum('11/15/2002');
[ZeroRates, CurveDates] = ...
zbtyield(Bonds, Yields, Settle);
SpotCurve = [CurveDates, ZeroRates];
% calculate a particular bond's future quoted price
RefDate = [datenum('1-Dec-2002'); datenum('1-Mar-2003')];
```

```
MatFut = [datenum('15-Dec-2002'); datenum('15-Mar-2003')];
Maturity = [datenum('15-Aug-2009');datenum('15-Aug-2010')];
CouponRate = [0.06;0.0575];
ConvFactor = convfactor(RefDate, Maturity, CouponRate);
Price = [114.416; 113.171];
Interpolation = 1;
[QtdFutPrice, AccrInt] = tfutbyprice(SpotCurve, Price, Settle, ...
MatFut, ConvFactor, CouponRate, Maturity, Interpolation)
```

QtdFutPrice =
114.0409
113.4029

## Accrint =

1.9891
0.4448

| See Also | convfactor I tfutbyyield |
| :--- | :--- |
| Related | - "Computing Treasury Bill Price and Yield" |
| Examples |  |
| Concepts | - "Treasury Bills Defined" |

Purpose Future prices of Treasury bonds given current yield
Syntax [QtdFutPrice, AccrInt] = tfutbyyield(SpotCurve, Yield, SettleFut, Mat ConvFactor, CouponRate, Maturity, Interpolation)
Arguments
Description[QtdFutPrice, AccrInt] = tfutbyyield(SpotCurve, Yield,SettleFut, MatFut, ConvFactor, CouponRate, Maturity,Interpolation) computes future prices of Treasury notes and bonds
given current yields of Treasury bonds/notes. The output arguments are:

- QtdFutPrice - Quoted futures price, per $\$ 100$ notional.
- AccrInt - Accrued Interest due at delivery date, per $\$ 100$ notional.

In addition, you can use the Financial Instruments Toolbox method getZeroRates for an IRDataCurve object with a Dates property to create a vector of dates and data acceptable for tfutbyyield. For more information, see "Converting an IRDataCurve or IRFunctionCurve Object" on page 9-40.

## Examples

## Determine Future Prices of Treasury Bonds Given the Current Yield

This example shows how to determine the future price of two Treasury bonds based upon a spot rate curve constructed from data for November 14, 2002.

```
% construct spot curve from Nov 14, data
Bonds = [datenum('02/13/2003'), 0;
        datenum('05/15/2003'), 0;
        datenum('10/31/2004'), 0.02125;
        datenum('11/15/2007'), 0.03;
        datenum('11/15/2012'), 0.04;
        datenum('02/15/2031'), 0.05375];
Yields = [1.20; 1.25; 1.86; 2.99; 4.02; 4.93]/100;
Settle = datenum('11/15/2002');
[ZeroRates, CurveDates] = ...
zbtyield(Bonds, Yields, Settle);
SpotCurve = [CurveDates, ZeroRates];
% calculate a particular bond's future quoted price
RefDate = [datenum('1-Dec-2002'); datenum('1-Mar-2003')];
```

```
MatFut = [datenum('15-Dec-2002'); datenum('15-Mar-2003')];
Maturity = [datenum('15-Aug-2009');datenum('15-Aug-2010')];
CouponRate = [0.06;0.0575];
ConvFactor = convfactor(RefDate, Maturity, CouponRate);
Yield = [0.03576; 0.03773];
Interpolation = 1;
[QtdFutPrice, AccrInt] = tfutbyyield(SpotCurve, Yield, Settle, ...
MatFut, ConvFactor, CouponRate, Maturity, Interpolation)
```


## QtdFutPrice =

114.0416
113.4034

Accrint =
1.9891
0.4448

| See Also | convfactor I tfutbyprice |
| :--- | :--- |
| Related | - "Computing Treasury Bill Price and Yield" |
| Examples |  |
| Concepts | - "Treasury Bills Defined" |

Purpose Implied repo rates for Treasury bond future given price

```
Syntax ImpliedRepo = tfutimprepo(ReinvestData, Price, QtdFutPrice, Settle,
MatFut, ConvFactor, CouponRate, Maturity)
```


## Arguments

| ReinvestData | Number of futures (NFUT) by 2 matrix of rates <br> and bases for the reinvestment of intervening <br> coupons in the form of <br> [ReinvestRate ReinvestBasis]. <br> ReinvestRate is the simple reinvestment rate, <br> in decimal. Specify ReinvestBasis as $0=$ not <br> reinvested, $2=$ actual/360, or $3=$ actual/ $/ 365$. |
| :--- | :--- |
| Price | Current bond price per $\$ 100$ notional. |
| QtdFutPrice | Quoted bond futures price per $\$ 100$ notional. |
| Settle | Settlement/valuation date of futures contract. <br> MatFut |
| ConvFactor |  |
| futures contract. |  |$\quad$| Conversion factor. See convfactor. |
| :--- |
| CouponRate |$\quad$| Underlying bond annual coupon, in decimal. |
| :--- |

Inputs (except ReinvestData) must either be a scalar or a vector of size equal to the number of Treasury futures (NFUT) by 1 or 1-by-NFUT.

## Description

ImpliedRepo = tfutimprepo(ReinvestData, Price, QtdFutPrice, Settle, MatFut, ConvFactor, CouponRate, Maturity) computes the implied repo rate that prevents arbitrage of Treasury bond futures, given the clean price at the settlement and delivery dates.

ImpliedRepo is the implied annual repo rate, in decimal, with an actual/360 basis.

## Examples Compute the Implied Repo Rates for Treasury Bond Futures Given the Price

This example shows how to compute the implied repo rate given the following set of data.

```
ReinvestData = [0.018 3];
Price = [114.4160; 113.1710];
QtdFutPrice = [114.1201; 113.7090];
Settle = datenum('11/15/2002');
MatFut = [datenum('15-Dec-2002'); datenum('15-Mar-2003')];
ConvFactor = [1; 0.9854];
CouponRate = [0.06; 0.0575];
Maturity = [datenum('15-Aug-2009'); datenum('15-Aug-2010')];
ImpliedRepo = tfutimprepo(ReinvestData, Price, QtdFutPrice, ...
Settle, MatFut, ConvFactor, CouponRate, Maturity)
```


## ImpliedRepo =

0.0200
0.0200

## See Also tfutpricebyrepo \| tfutyieldbyrepo

Related

- "Computing Treasury Bill Price and Yield"
Examples

Concepts •"Treasury Bills Defined"

| Purpose | Calculates Treasury bond futures price given the implied repo rates |
| :---: | :---: |
| Syntax | ```[QtdFutPrice AccrInt] = tfutpricebyrepo(RepoData, ReinvestData, Price, Settle, MatFut, ConvFactor, CouponRate, Maturity)``` |
| Arguments | RepoData Number of futures (NFUT) by 2 matrix of simple term repo/funding rates in decimal and their bases in the form of [RepoRate RepoBasis]. <br> Specify RepoBasis as $2=$ actual/360 or 3 = actual/365. |
|  | ReinvestData Number of futures (NFUT) by 2 matrix of rates and bases for the reinvestment of intervening coupons in the form of [ReinvestRate ReinvestBasis]. <br> ReinvestRate is the simple reinvestment rate, in decimal. Specify ReinvestBasis as $0=$ not reinvested, $2=$ actual $/ 360$, or $3=\mathrm{actual} / 365$. |
|  | Price Quoted clean prices of Treasury bonds per $\$ 100$ notional at Settle. |
|  | Settle Settlement/valuation date of futures contract. |
|  | MatFut $\quad$Maturity date (or anticipated delivery dates) of <br> futures contract. |
|  | ConvFactor Conversion factor. See convfactor. |
|  | CouponRate Underlying bond annual coupon, in decimal. |
|  | Maturity Underlying bond maturity date. |
|  | Inputs (except RepoData and ReinvestData) must either be a scalar or a vector of size equal to the number of Treasury futures (NFUT) by 1 or 1-by-NFUT. |
| Description | [QtdFutPrice AccrInt] = tfutpricebyrepo(RepoData, ReinvestData, Price, Settle, MatFut, ConvFactor, |

## tfutpricebyrepo

CouponRate, Maturity) computes the theoretical futures bond price given the settlement price, the repo/funding rates, and the reinvestment rate.

QtdFutPrice is the quoted futures price, per $\$ 100$ notional.
AccrInt is the accrued interest due at the delivery date, per $\$ 100$ notional.

## Examples Compute Treasury Bond Futures Price Given the Implied Repo Rates

This example shows how to compute the quoted futures price and accrued interest due on the target delivery date, given the following data.

```
RepoData = [0.020 2];
ReinvestData = [0.018 3];
Price = [114.416; 113.171];
Settle = datenum('11/15/2002');
MatFut = [datenum('15-Dec-2002'); datenum('15-Mar-2003')];
ConvFactor = [1 ; 0.9854];
CouponRate = [0.06;0.0575];
Maturity = [datenum('15-Aug-2009'); datenum('15-Aug-2010')];
```

[QtdFutPrice AccrInt] = tfutpricebyrepo(RepoData, ...
ReinvestData, Price, Settle, MatFut, ConvFactor, CouponRate, ...
Maturity)

QtdFutPrice =
114.1201
113.7090

Accrint =

## tfutpricebyrepo

1.9891
0.4448

## See Also tfutimprepo \| tfutyieldbyrepo

Related Examples

Concepts • "Treasury Bills Defined"

## tfutyieldbyrepo

Purpose Calculates Treasury bond futures yield given the implied repo rates
Syntax FwdYield = tfutyieldbyrepo(RepoData, ReinvestData, Yield, Settle, MatFut, ConvFactor, CouponRate, Maturity)
Arguments
Description

| RepoData | Number of futures (NFUT) by 2 matrix of simple term <br> repo/funding rates in decimal and their bases in the <br> form of [RepoRate RepoBasis]. <br> Specify RepoBasis as $2=$ actual/360 or <br> $3=$ actual/365. |
| :--- | :--- |
| ReinvestData | Number of futures (NFUT) by 2 matrix of rates and <br> bases for the reinvestment of intervening coupons <br> in the form of <br> [ReinvestRate ReinvestBasis]. <br> ReinvestRate is the simple reinvestment rate, <br> in decimal. Specify ReinvestBasis as 0 not <br> reinvested, $2=$ actual/360, or $3=$ actual/365. |
| Yield | Yield to maturity of Treasury bonds per $\$ 100$ <br> notional at Settle. |
| Settle | Settlement/valuation date of futures contract. |
| MatFut | Maturity date (or anticipated delivery dates) of <br> futures contract. |
| ConvFactor | Conversion factor. See convfactor. |
| CouponRate | Underlying bond annual coupon, in decimal. |
| Maturity | Underlying bond maturity date. |

Inputs (except RepoData and ReinvestData) must either be a scalar or a vector of size equal to the number of Treasury futures (NFUT) by 1 or 1-by-NFUT.
the theoretical futures bond yield given the settlement yield, the repo/funding rate, and the reinvestment rate.
FwdYield is the forward yield to maturity, in decimal, compounded semiannually.

## Examples Compute the Treasury Bond Futures Yield Given the Implied Repo Rates

This example shows how to compute the quoted futures bond yield, given the following data.

```
RepoData = [0.020 2];
ReinvestData = [0.018 3];
Yield = [0.0215; 0.0257];
Settle = datenum('11/15/2002');
MatFut = [datenum('15-Dec-2002'); datenum('15-Mar-2003')];
ConvFactor = [1; 0.9854];
CouponRate = [0.06; 0.0575];
Maturity = [datenum('15-Aug-2009'); datenum('15-Aug-2010')];
FwdYield = tfutyieldbyrepo(RepoData, ReinvestData, Yield,...
Settle, MatFut, ConvFactor, CouponRate, Maturity)
```

FwdYield =
0.0221
0.0282
See Also tfutimprepo \| tfutpricebyrepo
Related
Examples "Computing Treasury Bill Price and Yield"

## Concepts

- "Treasury Bills Defined"

| Purpose | Convert IRDataCurve object to RateSpec |
| :---: | :---: |
| Class | @IRDataCurve |
| Syntax | $F=$ toratespec(CurveObj, InpDates) |
| Arguments | $\begin{array}{ll}\text { CurveObj } & \text { Interest-rate curve object that is constructed using } \\ & \text { IRDataCurve. }\end{array}$ |
|  | $\begin{array}{ll}\text { InpDates } \quad \text { Vector of input dates using MATLAB date format. } \\ & \text { The input dates must be after the settle date. }\end{array}$ |
| Description | F = toratespec(CurveObj, InpDates) returns a RateSpec object that is identical to the RateSpec structure created by the Financial Instruments Toolbox function intenvset. |
| Examples | Convert an IRDataCurve Object to a RateSpec |
|  | This example shows how to convert an IRDataCurve object to a RateSpec. First, an IRDataCurve object is created using the function IRDataCurve constructor with Dates and Data, then this object is converted to a RateSpec structure using the toRateSpec method. ```Data = [2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100; Dates = daysadd(today,[360 2*360 3*360 5*360 7*360 10*360 20*360 30*3 irdc = IRDataCurve('Forward',today,Dates,Data); irdc.toRateSpec(today+30:30:today+365)``` |
|  | ans = |
|  | ```FinObj: 'RateSpec' Compounding: 2 Disc: [12x1 double] Rates: [12\times1 double] EndTimes: [12x1 double]``` |

## toRateSpec

```
    StartTimes: [12x1 double]
    EndDates: [12\times1 double]
    StartDates: 735617
ValuationDate: 735617
        Basis: 0
EndMonthRule: 1
```

How To
-"@IRDataCurve" on page A-7


## toRateSpec

StartDates: 735617<br>ValuationDate: 735617<br>Basis: 0<br>EndMonthRule: 1

How To<br>- "@IRFunctionCurve" on page A-12

| Purpose | Price zero-coupon instruments given yield |
| :---: | :---: |
| Syntax | Price = zeroprice(Yield, Settle, Maturity, Period, Basis, EndMonthRule) |
| Arguments | $\begin{array}{ll} \text { Yield } & \begin{array}{l} \text { Scalar or vector containing yield to maturity of } \\ \text { instruments. } \end{array} \end{array}$ |
|  | Settle Settlement date. A vector of serial date numbers <br> or date strings. Settle must be earlier than <br> Maturity. |
|  | $\begin{array}{ll} \text { Maturity } & \begin{array}{l} \text { Maturity date. A vector of serial date numbers or } \\ \text { date strings. } \end{array} \end{array}$ |
|  | Period (Optional) Scalar or vector specifying number of quasi-coupons per year. Default $=2$. |
|  | Basis (Optional) Day-count basis of the bond. A vector of integers. |
|  | - $0=$ actual/actual (default) |
|  | - $1=30 / 360$ (SIA) |
|  | - 2 = actual/360 |
|  | - $3=$ actual/365 |
|  | - $4=30 / 360$ (BMA) |
|  | - $5=30 / 360$ (ISDA) |
|  | - $6=30 / 360$ (European) |
|  | - $7=\mathrm{actual} / 365$ (Japanese) |
|  | - $8=$ actual/actual (ICMA) |
|  | - 9 = actual/360 (ICMA) |
|  | - $10=$ actual/365 (ICMA) |
|  | - $11=30 / 360 \mathrm{E}$ (ICMA) |

- $12=$ actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.

EndMonthRule (Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. $0=$ ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.

Description

## Algorithms

Price = zeroprice(Yield, Settle, Maturity, Period, Basis, EndMonthRule) calculates the prices for a portfolio of general short and long term zero-coupon instruments given the yield of reference bonds. In other words, if the zero-coupon computed with this yield is used to discount the reference bond, the value of that reference bond will be equal its price.

Price is a column vector containing a price for each zero-coupon instrument per $\$ 100$ notional.

When there is less than one quasi-coupon, the function uses a simple yield based upon (Period) x (Number of Days in quasi coupon period) day-year. The default period is 2 and the default number of days is 180 , which makes the user-supplied yield a simple yield on a 360-day year.
For longer term computations (more than one quasi-coupon), use the bond equivalent yield based upon present value (or compounding).

To compute the price when there is 1 or 0 quasi-coupon periods to redemption, zeroprice uses the formula

$$
\text { Price }=\frac{R V}{1+\left(\frac{D S R}{E} \times \frac{Y}{M}\right)}
$$

Quasi-coupon periods are the coupon periods that would exist if the bond were paying interest at a rate other than zero.

When there is more than one quasi-coupon period to the redemption date, zeroprice uses the formula

$$
\text { Price }=\frac{R V}{\left(1+\frac{Y}{M}\right)^{N_{q}-1+\frac{D S C}{E}}}
$$

The elements of the equations are defined as follows.

| Variable | Definition |
| :--- | :--- |
| $D S C$ | Number of days from settlement date to next quasi-coupon <br> date as if the security paid periodic interest. |
| $D S R$ | Number of days from settlement date to the redemption <br> date (call date, put date, and so on). |
| $E$ | Number of days in quasi-coupon period. |
| $M$ | Number of quasi-coupon periods per year (standard for <br> the particular security involved). |
| $N q$ | Number of quasi-coupon periods between settlement date <br> and redemption date. If this number contains a fractional <br> part, raise it to the next whole number. |
| $P r i c e$ | Dollar price per $\$ 100$ par value. |
| $R V$ | Redemption value. |
| $Y$ | Annual yield (decimal) when held to redemption. |

## zeroprice

## Examples Compute the Price of a Short-Term Zero-Coupon Instrument

This example shows how to compute the price of a short-term zero-coupon instrument.

```
Settle = '24-Jun-1993';
Maturity = '1-Nov-1993';
Period = 2;
Basis = 0;
Yield = 0.04;
Price = zeroprice(Yield, Settle, Maturity, Period, Basis)
```

Price =
98.6066

## Compute the Prices of a Portfolio of Two Zero-Coupon Instruments

This example shows how to compute the prices of a portfolio of two zero-coupon instruments, one short-term, and the other long-term.

```
Settle = '24-Jun-1993';
Maturity = ['01-Nov-1993'; '15-Jan-2024'];
Basis = [0; 1];
Yield = [0.04; 0.1];
Price = zeroprice(Yield, Settle, Maturity, [], Basis)
```

Price =
98.6066
5.0697
ReferencesSee Alsobndprice | cdprice | tbillprice | zeroyield
Related - "Computing Treasury Bill Price and Yield"
Examples- "Pricing Treasury Notes" on page 6-8- "Pricing Corporate Bonds" on page 6-10
Concepts - "Measuring Zero-Coupon Bond Function Quality" on page 6-7

Purpose Yield of zero-coupon instruments given price

```
Syntax Yield = zeroyield(Price, Settle, Maturity, Period, Basis,
EndMonthRule)
```


## Arguments

| Price | Scalar or vector containing prices of instruments. |
| :--- | :--- |
| Settle | Settlement date. A vector of serial date numbers or <br> date strings. Settle must be earlier than Maturity. |
| Maturity | Maturity date. A vector of serial date numbers or <br> date strings. |

Period (Optional) Scalar or vector specifying number of quasi-coupons per year. Default $=2$.

Basis (Optional) Day-count basis of the bond. A vector of integers.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- $2=$ actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
EndMonthRule (Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. $0=$ ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. $1=$ set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.

## Description

Algorithms
To compute the yield when there is zero or one quasi-coupon periods to redemption, zeroyield uses the formula

$$
\text { Yield }=\left(\frac{R V-P}{P}\right) \times\left(\frac{M \times E}{D S R}\right)
$$

Quasi-coupon periods are the coupon periods which would exist if the bond was paying interest at a rate other than zero. The first term calculates the yield on invested dollars. The second term converts this yield to a per annum basis.

When there is more than one quasi-coupon period to the redemption date, zeroyield uses the formula

$$
\text { Yield }=\left(\left(\frac{R V}{P}\right)^{\frac{1}{N q-1+\frac{D S C}{E}}}-1\right) \times M
$$

The elements of the equations are defined as follows.

| Variable | Definition |
| :--- | :--- |
| $D S C$ | Number of days from the settlement date to next <br> quasi-coupon date as if the security paid periodic interest. |
| $D S R$ | Number of days from the settlement date to redemption <br> date (call date, put date, and so on). |
| $E$ | Number of days in quasi-coupon period. |
| $M$ | Number of quasi-coupon periods per year (standard for the <br> particular security involved). |
| $N q$ | Number of quasi-coupon periods between the settlement <br> date and redemption date. If this number contains a <br> fractional part, raise it to the next whole number. |
| $P$ | Dollar price per $\$ 100$ par value. |

## Variable Definition

$R V \quad$ Redemption value.
Yield Annual yield (decimal) when held to redemption.

## Examples Compute the Yield of a Short-Term Zero-Coupon Instrument

This example shows how to compute the yield of a short-term zero-coupon instrument.

```
Settle = '24-Jun-1993';
Maturity = '1-Nov-1993';
Basis = 0;
Price = 95;
Yield = zeroyield(Price, Settle, Maturity, [], Basis)
```

Yield =
0.1490

## Compute the Yield of a Short-Term Zero-Coupon Instrument Using a Day-Count Basis of 30/360 (SIA)

This example shows how to compute the yield of a short-term zero-coupon instrument using a day-count basis of 30/360 (SIA).

```
Settle = '24-Jun-1993';
Maturity = '1-Nov-1993';
Basis = 1;
Price = 95;
Yield = zeroyield(Price, Settle, Maturity, [], Basis)
```


## zeroyield

Yield =
0.1492

## Compute the Yield of a Long-Term Zero-Coupon Instrument

This example shows how to ompute the yield of a long-term zero-coupon instrument.

```
Settle = '24-Jun-1993';
Maturity = '15-Jan-2024';
Basis = 0;
Price = 9;
Yield = zeroyield(Price, Settle, Maturity, [], Basis)
```

Yield =
0.0804

| References | [1] Mayle, Jan. Standard Securities Calculation Methods. New York: <br> Securities Industry Association, Inc. Vol. 1, 3rd ed., 1993, ISBN <br> 1-882936-01-9. Vol. 2, 1994, ISBN 1-882936-02-7. |
| :--- | :--- |
| See Also | bndyield \| cdyield | tbillyield | zeroprice |
| Related | - "Computing Treasury Bill Price and Yield" <br> Examples <br> - "Pricing Treasury Notes" on page 6-8 <br> - "Pricing Corporate Bonds" on page 6-10 |
| Concepts | - "Measuring Zero-Coupon Bond Function Quality" on page 6-7 |

# Derivatives Pricing Options 

- "Pricing Options Structure" on page B-2
- "Customizing the Structure" on page B-5


## Pricing Options Structure

In this section...<br>"Introduction" on page B-2<br>"Default Structure" on page B-2

## Introduction

The MATLAB Options structure provides additional input to most pricing functions. The Options structure

- Tells pricing functions how to use the interest-rate tree to calculate instrument prices.
- Determines what additional information the Command Window displays along with instrument prices.
- Tells pricing functions which method to use in pricing barrier options.

The pricing options structure is primarily used in the pricing of interest-rate-based financial derivatives. However, the BarrierMethod field in the structure allows you to use it in pricing equity barrier options as well.

You provide pricing options in an optional Options argument passed to a pricing function. (See, for example, bondbyhjm, bdtprice, barrierbycrr, barrierbyeqp, or barrierbyitt.)

## Default Structure

If you do not specify the Options argument in the call to a pricing function, the function uses a default structure. To observe the default structure, use derivset without any arguments.

```
Options = derivset
Options =
```

    Diagnostics: 'off'
        Warnings: 'on'
    ConstRate: 'on'
    
## BarrierMethod: 'unenhanced'

The Options structure has four fields: Diagnostics, Warnings, ConstRate, and BarrierMethod.

## Diagnostics Field

Diagnostics indicates whether additional information is displayed if the tree is modified. The default value for this option is 'off'. If Diagnostics is set to 'on' and ConstRate is set to 'off', the pricing functions display information such as the number of nodes in the last level of the tree generated for pricing purposes.

## Warnings Field

Warnings indicates whether to display warning messages when the input tree is not adequate for accurately pricing the instruments. The default value for this option is 'on'. If both ConstRate and Warnings are 'on', a warning is displayed if any of the instruments in the input portfolio has a cash flow date between tree dates. If ConstRate is 'off', and Warnings is 'on', a warning is displayed if the tree is modified to match the cash flow dates on the instruments in the portfolio.

## ConstRate Field

ConstRate indicates whether the interest rates should be assumed constant between tree dates. By default this option is 'on', which is not an arbitrage-free assumption. Consequently the pricing functions return an approximate price for instruments featuring cash flows between tree dates. Instruments featuring cash flows only on tree nodes are not affected by this option and return exact (arbitrage-free) prices. When ConstRate is 'off ', the pricing function finds the cash flow dates for all instruments in the portfolio. If these cash flows do not align exactly with the tree dates, a new tree is generated and used for pricing. This new tree features the same volatility and initial rate specifications of the input tree but contains tree nodes for each date in which at least one instrument in the portfolio has a cash flow. Keep in mind that the number of nodes in a tree grows exponentially with the number of tree dates. Consequently, setting ConstRate 'off' dramatically increases the memory and processor demands on the computer.

## BarrierMethod Field

When using binomial trees to price barrier options, you may require a large number of tree steps to achieve an accurate result when tree nodes do not align with the barrier level. With the BarrierMethod field, the toolbox provides an enhancement method that improves the accuracy of the results without having to use large trees.

The BarrierMethod field can be set to 'unenhanced' (default) or 'interp'. If you specify 'unenhanced ', no correction calculation is used. Otherwise, if you specify 'interp', the toolbox provides an enhanced valuation by interpolating between nodes on barrier boundaries.

You specify the barrier method in the last input argument, Options, of the functions barrierbycrr, barrierbyeqp, crrprice, or eqpprice. Options is a structure that you create with the function derivset. Using derivset, you specify whether to use the enhanced or the unenhanced method.

For more information about this algorithm, see Derman, E., I. Kani, D. Ergener and I. Bardhan, "Enhanced Numerical Methods for Options with Barriers," Financial Analysts Journal, (Nov. - Dec. 1995), pp. 65-74.

## Customizing the Structure

Customize the Options structure by passing property name/property value pairs to the derivset function.

As an example, consider an Options structure with ConstRate 'off' and Diagnostics 'on'.

```
Options = derivset('ConstRate', 'off', 'Diagnostics', 'on')
```

Options =
Diagnostics: 'on'
Warnings: 'on'
ConstRate: 'off'
BarrierMethod: 'unenhanced'

To obtain the value of a specific property from the Options structure, use derivget.

CR = derivget(Options, 'ConstRate')
CR $=$
Off

Note Use derivset and derivget to construct the Options structure. These functions are guaranteed to remain unchanged, while the implementation of the structure itself may be modified in the future.

Now observe the effects of setting ConstRate 'off'. Obtain the tree dates from the HJM tree.

TreeDates = [HJMTree.TimeSpec.ValuationDate;... HJMTree.TimeSpec.Maturity]

TreeDates =

730486

730852
731217
731582
731947
datedisp(TreeDates)
01-Jan-2000
01-Jan-2001
01-Jan-2002
01-Jan-2003
01-Jan-2004
All instruments in HJMInstSet settle on January 1, 2000, and all have cash flows once a year, with the exception of the second bond, which features a period of 2. This bond has cash flows twice a year, with every other cash flow consequently falling between tree dates. You can extract this bond from the portfolio to compare how its price differs by setting ConstRate to 'on' and 'off'.

```
BondPort = instselect(HJMInstSet, 'Index', 2);
```

instdisp(BondPort)
Index Type CouponRate Settle Maturity Period Basis...
1 Bond 0.04 01-Jan-2000 01-Jan-2004 2 NaN...

First price the bond with ConstRate 'on' (default).

```
format long
[BondPrice, BondPriceTree] = hjmprice(HJMTree, BondPort)
Warning: Not all cash flows are aligned with the tree. Result will
be approximated.
BondPrice =
```

    97.52801411736377
    BondPriceTree =
FinObj: 'HJMPriceTree'

```
PBush: {1x5 cell}
AIBush: {[0] [1x1x2 double] ... [1x4x2 double] [1x8 double]}
    tObs: [0 1 2 3 4]
```

Now recalculate the price of the bond setting ConstRate 'off '.

```
OptionsNoCR = derivset('ConstR', 'off')
OptionsNoCR =
Diagnostics: 'off'
    Warnings: 'on'
    ConstRate: 'off'
[BondPriceNoCR, BondPriceTreeNoCR] = hjmprice(HJMTree,...
BondPort, OptionsNoCR)
Warning: Not all cash flows are aligned with the tree. Rebuilding
tree.
BondPriceNoCR =
    97.53342361674437
BondPriceTreeNoCR =
FinObj: 'HJMPriceTree'
    PBush: {1x9 cell}
AIBush: {1x9 cell}
    tObs: [0 0.5000 1 1.5000 2 2.5000 3 3.5000 4]
```

As indicated in the last warning, because the cash flows of the bond did not align with the tree dates, a new tree was generated for pricing the bond. This pricing method returns more accurate results since it guarantees that the process is arbitrage-free. It also takes longer to calculate and requires more memory. The tobs field of the price tree structure indicates the increased memory usage. BondPriceTree.tObs has only five elements, while BondPriceTreeNoCR.tObs has nine. While this may not seem like a large difference, it has a dramatic effect on the number of states in the last node.

```
size(BondPriceTree.PBush{end})
```

```
ans =
18
size(BondPriceTreeNoCR.PBush{end})
ans =
    128
```

The differences become more obvious by examining the price trees with treeviewer.
treeviewer(BondPriceTree, BondPort)

treeviewer(BondPriceTreeNoCR, BondPort)


All = [Delta ./ Price, Gamma ./ Price, Vega ./ Price, Price]

All =

| -2.76 | 10.43 | 0.00 | 98.72 |
| ---: | ---: | ---: | ---: |
| -3.56 | 16.64 | -0.00 | 97.53 |
| -166.18 | 13235.59 | 700.96 | 0.05 |
| -2.76 | 10.43 | 0.00 | 98.72 |
| -0.01 | 0.03 | 0 | 100.55 |
| 46.95 | 1090.63 | 14.91 | 6.28 |
| -969.85 | 173969.77 | 1926.72 | 0.05 |
| -76.39 | 287.00 | 0.00 | 3.690 |

## Bibliography

- "Black-Derman-Toy (BDT) Modeling" on page C-2
- "Heath-Jarrow-Morton (HJM) Modeling" on page C-3
- "Hull-White (HW) and Black-Karasinski (BK) Modeling" on page C-4
- "Cox-Ross-Rubinstein (CRR) Modeling" on page C-5
- "Implied Trinomial Tree (ITT) Modeling" on page C-6
- "Leisen-Reimer Tree (LR) Modeling" on page C-7
- "Equal Probabilities Tree (EQP) Modeling" on page C-8
- "Closed-Form Solutions Modeling" on page C-9
- "Financial Derivatives" on page C-10
- "Fitting Interest-Rate Curve Functions" on page C-11
- "Interest-Rate Modeling Using Monte Carlo Simulation" on page C-12
- "Bootstrapping a Swap Curve" on page C-13
- "Bond Futures" on page C-14
- "Credit Derivatives" on page C-15


## Black-Derman-Toy (BDT) Modeling

A description of the Black-Derman-Toy interest-rate model can be found in:
Black, Fischer, Emanuel Derman, and William Toy, "A One Factor Model of Interest Rates and its Application to Treasury Bond Options," Financial Analysts Journal, January - February 1990.

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An introduction to Heath-Jarrow-Morton modeling, used extensively in Financial Instruments Toolbox software, can be found in:

Jarrow, Robert A., Modelling Fixed Income Securities and Interest Rate Options, McGraw-Hill, 1996, ISBN 0-07-912253-1.

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A description of the Hull-White model and its Black-Karasinski modification can be found in:

Hull, John C., Options, Futures, and Other Derivatives, Prentice-Hall, 1997, ISBN 0-13-186479-3

You can find additional information about the Hull-White single-factor model used in this toolbox in these papers:

Hull, J., and A. White, "Numerical Procedures for Implementing Term Structure Models I: Single-Factor Models," Journal of Derivatives, 1994.

Hull, J., and A. White, "Using Hull-White Interest Rate Trees," Journal of Derivatives, 1996.

## Cox-Ross-Rubinstein (CRR) Modeling

To learn about the Cox-Ross-Rubinstein model, see:
Cox, J. C., S. A. Ross, and M. Rubinstein, "Option Pricing: A Simplified
Approach," Journal of Financial Economics, Number 7, 1979, pp. 229-263.

## Implied Trinomial Tree (ITT) Modeling

To learn about the Implied Trinomial Tree model, see:
Chriss, Neil A., E. Derman, and I. Kani, "Implied trinomial trees of the volatility smile," Journal of Derivatives, 1996.

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To learn about the Leisen-Reimer model, see:
Leisen D.P., M. Reimer, "Binomial Models for Option Valuation - Examining and Improving Convergence," Applied Mathematical Finance, Number 3, 1996, pp. 319-346.

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To learn about the Equal Probabilities model, see:
Chriss, Neil A.,Black Scholes and Beyond: Option Pricing Models, McGraw-Hill, 1996, ISBN 0-7863-1025-1.

## Closed-Form Solutions Modeling

To learn about the Bjerksund-Stensland 2002 model, see:
Bjerksund, P. and G. Stensland, Closed-Form Approximation of American Options, Scandinavian Journal of Management, 1993, Vol. 9, Suppl., pp. S88-S99.

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(http://brage.bibsys.no/nhh/bitstream/URN:NBN:no-bibsys_brage_22301/1/bjerksund\ r

## Financial Derivatives

You can find information on the creation of financial derivatives and their role in the marketplace in numerous sources. Among those consulted in the development of Financial Instruments Toolbox software are:

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## Fitting Interest-Rate Curve Functions

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"Zero-coupon yield curves: technical documentation," BIS Papers, Bank for International Settlements, Number 25, October, 2005.

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O’Kane, D., Modelling Single-name and Multi-name Credit Derivatives, Wiley Finance, 2008, pp. 156-169.

C Bibliography

## American option

An option that can be exercised any time until its expiration date. Contrast with European option on page Glossary-6.

## arbitrary cash flow instrument

A set of generic cash flow amounts for which a price needs to be established.

## Asian option

An option whose payoff depends upon the average price of the underlying asset over a certain period of time.

## asset-or-nothing option

A digital option that pays the value of the underlying security if the option expires in the money.

## barrier option

An option that is activated or deactivated only if the price of the underlying asset crosses a barrier. See also knock-in on page Glossary-8 and knock-out on page Glossary-8. If the option fails to execute, the seller may pay to the purchaser a predetermined rebate on page Glossary-10.

## barrier option

An option that is activated or deactivated only if the price of the underlying asset crosses a barrier. See also knock-in on page Glossary-8 and knock-out on page Glossary-8. If the option fails to execute, the seller may pay to the purchaser a predetermined rebate on page Glossary-10.

## basis

Day count basis determines how interest accrues over time for various instruments and the amount transferred on interest payment dates. The calculation of accrued interest for dates between payments also uses day count basis. Day count basis is a fraction of Number of interest accrual days/Days in the relevant coupon period. Supported day count conventions and basis values are:

| Basis <br> Value | Day Count Convention |
| :--- | :--- |
| 0 | actual/actual (default) - Number of days in both a period <br> and a year is the actual number of days. |
| 1 | $30 / 360$ SIA - Year fraction is calculated based on a 360 <br> day year with 30-day months, after applying the following |
| 2 | actual/360 - Number of days in a period is equal to the <br> actual number of days, however the number of days in a year <br> is 360. |
| $\mathbf{3}$ | actual/365 - Number of days in a period is equal to the <br> actual number of days, however the number of days in a year <br> is 365 (even in a leap year). |
| 4 | 30/360 PSA - Number of days in every month is set to 30 <br> (including February). If the start date of the period is either <br> the 31st of a month or the last day of February, the start <br> date is set to the 30th, while if the start date is the 30th of a <br> month and the end date is the 31st, the end date is set to the <br> 30th. The number of days in a year is 360. |
| 5 | 30/360 ISDA - Number of days in every month is set to 30, <br> except for February where it is the actual number of days. If <br> the start date of the period is the 31st of a month, the start <br> date is set to the 30th while if the start date is the 30th of a <br> month and the end date is the 31st, the end date is set to the <br> 30th. The number of days in a year is 360. |
| 6 | 30E /360 - Number of days in every month is set to 30 <br> except for February where it is equal to the actual number <br> of days. If the start date or the end date of the period is the <br> 31st of a month, that date is set to the 30th. The number of <br> days in a year is 360. |
| 7 | actual/365 Japanese - Number of days in a period is equal <br> to the actual number of days, except for leap days (29th <br> February) which are ignored. The number of days in a year <br> is 365 (even in a leap year). |

Glossary-2

| Basis <br> Value | Day Count Convention |
| :--- | :--- |
| 8 | actual/actual ICMA - Number of days in both a period and <br> a year is the actual number of days and the compounding <br> frequency is annual. |
| 9 | actual/360 ICMA - Number of days in a period is equal to <br> the actual number of days, however the number of days in a <br> year is 360 and the compounding frequency is annual. |
| 10 | actual/365 ICMA - Number of days in a period is equal <br> to the actual number of days, however the number of days <br> in a year is 365 (even in a leap year) and the compounding <br> frequency is annual. |
| 11 | $30 / 360$ ICMA - Number of days in every month is set to 30, <br> except for February where it is equal to the actual number of <br> days. If the start date or the end date of the period is the 31st <br> of a month, that date is set to the 30th. The number of days <br> in a year is 360 and the compounding frequency is annual. |
| 12 | actual/365 ISDA - The day count fraction is calculated using <br> the following formula: (Actual number of days in period <br> that fall in a leap year / 366) + (Actual number of <br> days in period that fall in a normal year / 365 ). |
| 13 | bus/252 - The number of days in a period is equal to the <br> actual number of business days. The number of business <br> days in a year is 252. |

## basket option

An option that provides a payoff dependent on the value of a portfolio of assets.

## beta

The price volatility of a financial instrument relative to the price volatility of a market or index as a whole. Beta is most commonly used with respect to equities. A high-beta instrument is riskier than a low-beta instrument.

## binomial model

A method in which the probability over time of each possible price or rate follows a binomial distribution. The basic assumption is that prices or rates can move to only two values (one higher and one lower) over any short time period. See also trinomial model on page Glossary-12.

## Black-Derman-Toy (BDT) model

A model for pricing interest rate derivatives where all security prices and rates depend upon the short rate (annualized one-period interest rate).

## bond

A long-term debt security with fixed interest payments and fixed maturity date.

## bond option

The right to sell a bond back to the issuer (put) or to redeem a bond from its current owner (call) at a specific price and on a specific date.

## bushy tree

A tree of prices or interest rates in which the number of branches increases exponentially relative to observation times; branches never recombine. Opposite of a recombining tree on page Glossary-10.
call

1. An option to buy a certain quantity of a stock or commodity for a specified price within a specified time. See also put on page Glossary-9.
2. A demand to submit bonds to the issuer for redemption before the maturity date.
call swaption
Allows the option buyer to enter into an interest rate swap in which the buyer of the option pays the fixed rate and receives the floating rate.

## callable bond

A bond that allows the issuer to buy back the bond at a predetermined price at specified future dates. The bond contains an embedded call option; that is, the holder has sold a call option to the issuer. See also puttable bond on page Glossary-9.

## cap

Interest-rate option that guarantees that the rate on a floating-rate loan will not exceed a certain level.

## caplet

An interim cap component in a multiperiod interest-rate cap agreement.

## cash-or-nothing option

A digital option that pays some fixed amount of cash if the option expires in the money.

## compound option

An option on an option, such as a call on a call, a put on a put, a call on a put, or a put on a call.

## delta

The rate of change of the price of a derivative security relative to the price of the underlying asset; that is, the first derivative of the curve that relates the price of the derivative to the price of the underlying security.

## derivative

A financial instrument that is based on some underlying asset. For example, an option is a derivative instrument based on the right to buy or sell an underlying instrument.

## deterministic model

An interest rate model in which the values of the rates in the next time step are determined solely by the values of the rates in the current time step.

## digital option

An option whose payout is fixed after the underlying stock exceeds the predetermined threshold or strike price.

## discount factor

Coefficient used to compute the present value of future cash flows.

## dollar sensitivity

Sensitivity reported as a dollar price change instead of a percentage price change.

## down-and-in

A type of barrier option on page Glossary-1 that becomes active if the barrier is reached from above. See also knock-in on page Glossary-8.

## down-and-out

A type of barrier option on page Glossary-1 that becomes deactivated if the barrier is reached from above. See also knock-out on page Glossary-8.

## European option

An option that can be exercised only on its expiration date. Contrast with American option on page Glossary-1.

## ex-dividend date

Date when a declared dividend belongs to the seller rather than the buyer.

## exercise price

The price set for buying an asset (call) or selling an asset (put). The strike price.

## exotic option

Any nonstandard option. Opposite of vanilla option on page Glossary-12.

## fixed lookback option

Strike price is fixed at purchase. The underlying is priced at its highest or lowest level, depending whether it is a call or put, during the life of the option rather than expiring at market.

## fixed-rate note

A long-term debt security with preset interest rate and maturity, by which the interest must be paid. The principal may or may not be paid at maturity.

## floating lookback option

Strike price is fixed at maturity. For a call, the price is fixed at the lowest price during the life of the option; for a put it is fixed at the highest price.

## floating-rate note

A security similar to a bond, but in which the note's interest rate is reset periodically, relative to a reference index rate, to reflect fluctuations in market interest rates.

## floor

Interest-rate option that guarantees that the rate on a floating-rate loan will not fall below a certain level.

## floorlet

One of the interim period floors in a multiple period floor agreement.

## forward curve

The curve of forward interest rates vs. maturity dates for bonds.

## forward rate

The future interest rate of a bond inferred from the term structure, especially from the yield curve of zero-coupon bonds, calculated from the growth factor of an investment in a zero held until maturity.

## gamma

The rate of change of delta for a derivative security relative to the price of the underlying asset; that is, the second derivative of the option price relative to the security price.

## gap option

A digital option in which one strike decides if the option is in or out of money and another strike decides the size the size of the payoff.

## Heath-Jarrow-Morton (HJM) model

A model of the interest rate term structure that works with a type of interest rate tree called a bushy tree on page Glossary-4.

## hedge

A securities transaction that reduces or offsets the risk on an existing investment position.

## instrument set

A collection of financial assets. A portfolio.

## inverse discount

A factor by which the present value of an asset is multiplied to find its future value. The reciprocal of the discount factor.

## irregular coupon

A bond interest payment for more or less than six-months' interest. The first coupon on many bonds is irregular because payment is other than six months from the dated date.

## knock-in

A barrier option on page Glossary-1 that is activated when the price of the underlying asset achieves a designated target. There are two types: up-and-in on page Glossary-12 and down-and-in on page Glossary-6.

## knock-out

A barrier option on page Glossary-1 that is deactivated when the price of the underlying asset achieves a designated target. There are two types: up-and-out on page Glossary-12 and down-and-out on page Glossary-6.

## Lambda

The percentage change in an option price divided by the percentage change in an underlying price.

## least-squares method

A mathematical method of determining the best fit of a curve to a series of observations by choosing the curve that minimizes the sum of the squares of all deviations from the curve.

## long rate

The yield on a zero-coupon Treasury bond.

## lookback option

An option that reduces uncertainties associated with the timing of market entry. Lookback options can be either fixed lookback option on page Glossary-6 and floating lookback option on page Glossary-6.

## mean reversion

The tendency of a variable to return to its mean value after reaching a point of excessive positive or negative valuation relative to the mean.

## option

A right to buy or sell specific securities or commodities at a stated price (exercise or strike price) within a specified time. An option is a type of derivative.

## per-dollar sensitivity

The dollar sensitivity on page Glossary-10 divided by the corresponding instrument price.

## portfolio

A collection of financial assets. Also called an instrument set.

## price tree structure

A MATLAB structure that holds all pricing information.

## price vector

A vector of instrument prices.

## pricing options structure

A MATLAB structure that defines how the price tree is used to find the price of instruments in the portfolio, and how much additional information is displayed in the command window when the pricing function is called.

## put

An option to sell a stipulated amount of stock or securities within a specified time and at a fixed exercise price. See also call on page Glossary-4.

## put swaption

Allows the option buyer to enter into an interest rate swap in which the buyer of the option receives the fixed rate and pays the floating rate.

## puttable bond

A bond that allows the holder to redeem the bond at a predetermined price at specified future dates. The bond contains an embedded put option; that is, the holder has bought a put option. See also callable bond on page Glossary-4.

## rainbow option

A single option linked to two or more underlying assets. In order for the option to pay off, all the underlying assets must move in the intended direction.

## rate specification

A MATLAB structure that holds all information needed to identify completely the evolution of interest rates.

## rebate

A predetermined amount of money paid to the purchaser of a barrier option on page Glossary-1 if the option fails to execute.

## recombining tree

A tree of prices or interest rates whose branches recombine over time. Opposite of a bushy tree on page Glossary-4.

## self-financing hedge

A trading strategy whereby the value of a portfolio after rebalancing is equal to its value at any previous time.

## sensitivity

The "what if" relationship between variables; the degree to which changes in one variable cause changes in another variable. A specific synonym is volatility. See also dollar sensitivity on page Glossary-5.

## short rate

The annualized one-period interest rate.

## sinking fund bond

A sinking fund bond is a coupon bond with a sinking fund provision. This provision obligates the issuer to amortize portions of the principal prior to maturity, affecting bond prices since the time of the principal repayment changes.

## spot curve, spot yield curve

See zero curve, zero-coupon yield curve on page Glossary-13.

## spot rate

The current interest rate appropriate for discounting a cash flow of some given maturity.

## spread

For options, a combination of call or put options on the same stock with differing exercise prices or maturity dates.

## stepped coupon bond

A step-up and step-down bond is a debt security with a predetermined coupon structure over time.

## stochastic model

Involving or containing a random variable or variables; involving chance or probability.

## strike

Exercise a put or call option.

## strike price

See exercise price on page Glossary-6.

## supershare option

A digital option that pays out a proportion of the assets underlying a portfolio if the asset lies between a lower and an upper bound at the expiry of the option.

## swap

A contract between two parties to exchange cash flows in the future according to some formula.

## swaption

An option on an interest rate swap. It grants the option buyer the right to enter into an interest rate swap at a future date.

## time specification

A MATLAB structure that represents the mapping between times and dates for interest rate quoting.

## trinomial model

A method in which the basic assumption is that prices or rates can move to one of three possible values over any short time period. At any time step the price or rate direction can be upward, neutral, or downward. See also binomial model on page Glossary-4.

## under-determined system

A set of simultaneous equations in which the number of independent variables exceeds the number of equations in the set, leading to an infinite number of solutions.

## up-and-in

A type of barrier option on page Glossary- 1 that becomes active if the barrier is reached from below. See also knock-in on page Glossary-8.

## up-and-out

A type of barrier option on page Glossary- 1 that becomes deactivated if the barrier is reached from below. See also knock-out on page Glossary-8.

## vanilla option

A common option, such as a put or call. Opposite of exotic option on page Glossary-6.

## vanilla swap

A swap on page Glossary-11 agreement to exchange a fixed rate for a floating rate.

## vega

The rate of change in the price of a derivative security relative to the volatility of the underlying security. When vega is large, the security is sensitive to small changes in volatility.

## volatility specification

A MATLAB structure that specifies the forward rate volatility process.

## yields

The zero coupon rate.

## yield curve

The zero curve.

## yield volatility

The zero coupon volatilities.

## zero curve, zero-coupon yield curve

A yield curve for zero-coupon bonds; zero rates versus maturity dates. Since the maturity and duration (Macaulay duration) are identical for zeros, the zero curve is a pure depiction of supply/demand conditions for loanable funds across a continuum of durations and maturities. Also known as spot curve or spot yield curve.

## zero-coupon bond, or zero

A bond that, instead of carrying a coupon, is sold at a discount from its face value, pays no interest during its life, and pays the principal only at maturity.


[^0]:    Settle $=$ '17-Jul-2009'; \% valuation date for the CDS

[^1]:    \% Compute all mark-to-market values for this scenario. We use an

[^2]:    IRFitOptions

[^3]:    Concepts

    - "Asian Option" on page 3-35

[^4]:    See Also
    | bondbyzero | cfbyzero | fixedbyzero | swapbyzero | intenvset

[^5]:    Note The Settle date for every floating-rate note is set to the ValuationDate of the HW Tree. The floating-rate note argument Settle is ignored.

[^6]:    Note The Settle date for every floating-rate note is set to the ValuationDate of the HW Tree. The floating-rate note argument Settle is ignored.

[^7]:    Note The Settle date for every floating-rate note is set to the ValuationDate of the HW tree. The floating-rate note argument Settle is ignored.

[^8]:    Note The Settle date for every floating-rate note is set to the ValuationDate of the HW tree. The floating-rate note argument settle is ignored.

[^9]:    Concepts

    - basis on page Glossary-1

[^10]:    Columns 92 through 98

[^11]:    Columns 351 through 357

[^12]:    Convexity = mbsconvp(Price, Settle, Maturity, IssueDate, GrossRate, CouponRate, Delay, PrepaySpeed, PrepayMatrix)

